HySIM: A Hybrid Spectrum and Information Market for TV White Space Networks

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Abstract—We propose a hybrid spectrum and information market for database-assisted TV white space networks, where a geo-location white space database serves as the platform for both the spectrum market and the information market. We study the interactions among the database operator, the spectrum licensee, and unlicensed users systematically, using a three-layer hierarchical model. In Layer I, the licensee negotiates with the database regarding the commission fee of using the spectrum market platform. In Layer II, the database and the licensee compete for selling information or channels to unlicensed users. In Layer III, unlicensed users determine whether to buy the exclusive usage right of licensed channels from the licensee, or to buy the information regarding unlicensed channels from the database. Analyzing such a three-layer model is challenging, due to the coexistence of both positive and negative network externalities in the information market. We characterize the market equilibrium systematically, and analyze how the network externalities affect the equilibrium behaviours of all parties involved. Our numerical results show that the proposed hybrid market can improve the network profit more than 80%, compared with a pure information market. Meanwhile, the achieved network profit is very close to the coordinated benchmark (e.g., the gap is less than 4%).

I. INTRODUCTION

A. Background

With the explosive growth of mobile smartphones and bandwidth-hungry wireless applications, radio spectrum has become increasingly scarce and congested [1]. The UHF/VHF frequency band originally assigned for broadcast television services (hereafter called TV channels) has been viewed as a high potential spectrum band for supporting new wireless broadband services. First, in some places especially rural areas, TV channels may not be fully allocated (licensed) to TV licensees. That is, some TV channels may be unlicensed to any TV licensee. These vacant (unlicensed) channels are often referred to as TV white spaces [2], [3], and can be potentially used for supporting non-TV wireless services. Second, even the licensed TV channels (i.e., those licensed to some TV licensees) may be under a low utilization at some time [4], and hence can be opportunistically reused by unlicensed non-TV wireless services with the permissions of licensees.

To effectively exploit the (unlicensed) TV white spaces while not harming the interests of licensed devices (of TV licensees), some spectrum regulators (such as FCC in the USA [5] and Ofcom in the UK [6]) have proposed a database-assisted TV white space network architecture, and the industry has started to adopt such an architecture [7]–[9]. Specifically, in this database-assisted architecture, unlicensed devices obtain the list of available TV white spaces via querying a certified white space database called geo-location. To achieve this, the geo-location database needs to house and periodically updated a repository of TV licensees (e.g., their network infrastructures and channel occupancies). Before using any TV white spaces, unlicensed devices first report their location information to the database, and then database computes and returns the available TV white space list. According to the current policies of FCC and Ofcom, these unlicensed TV white spaces must be used in a licensed-exempt (shared) manner [5], [6]. Hence, interference management is an important issue when multiple unlicensed users sharing the same TV white space.

Moreover, spectrum regulators in many counties (including FCC in the USA) also allows the spectrum licensees to temporarily lease their licensed channels to unlicensed devices [10]. Note that an unlicensed device can use the leased channels exclusively, depending on the agreement with the associated spectrum licensee. Such a secondary spectrum leasing (trading) requires a market platform connecting the buyers (unlicensed users) and sellers (spectrum licensees). The geo-location database can potentially serve as such a platform (see, e.g., SpecEx [11]) due to its proximity to both spectrum licensees and unlicensed devices. Based on the above, the geo-location database can facilitate the unlicensed access to both unlicensed and licensed TV channels.

B. Motivation

In this work, we study the business modelling for a database-assisted TV white space network, with both unlicensed TV white spaces and licensed TV channels. Recently, researchers have proposed several business and marketing models related to the database-assisted TV channel sharing, which can be categorized into two classes: Spectrum Market [12]–[14] and Information Market [15], [16].

The spectrum market model mainly deals with the trading of licensed TV channels between spectrum licensees and unlicensed users [12]–[14]. The key idea is to let spectrum licensees temporarily lease their under-utilized channels to unlicensed users for additional revenue. The database serves as a market platform facilitating this trading process. A commercial example of such a database-provided spectrum market platform is SpecEx [11], operated by Spectrum Bridge.

The information market model has been recently proposed and analyzed in [15], [16], for the unlicensed TV channels (i.e., TV white spaces) sharing. In their models, the geo-location database sells the advanced information regarding the

1The (secondary) spectrum market has been extensively used in dynamic spectrum access networks (see, e.g., [18]–[22]), where auction, contract, and pricing are commonly used in theoretic models. The auction and contract models usually focus on the information asymmetry. In this work, we mainly focus on the interplay between the spectrum market and information market. Hence, we will consider the basic pricing model for the spectrum market.

2For example, it may act as a spectrum broker or agent, purchasing spectrum from licensees and then reselling the purchased spectrum to unlicensed users.
quality of TV white spaces to the unlicensed users for profit. The key motivation of such an information market model is that the database knows more information regarding TV white spaces than unlicensed users,\(^3\) which can be potentially used by unlicensed users for improving their performances. A practical example of information market is White Space Plus [17], again operated by Spectrum Bridge.

In the above studies, researches studied the spectrum market (for licensed TV channels) and information market (for unlicensed TV white spaces) separately. In practice, however, the licensed TV channels and unlicensed TV white spaces often co-exist at the same location. Some users may prefer to lease the licensed TV channels for the exclusive usage, while other users may prefer to share the free unlicensed TV white spaces with others. Hence, a joint study of both spectrum market and information market is highly desirable. This motivates our study of a hybrid spectrum and information market for database-assisted TV white space networks.

C. Contributions

In this paper, we model and study a Hybrid Spectrum and Information Market (HySIM) for a database-assisted TV white space network, in which the geo-location database serves as (i) a spectrum market platform, for trading (under-utilized) licensed TV channels between spectrum licensees and unlicensed users, and (ii) an information market platform, for trading advanced information (regarding the unlicensed TV white spaces) between the database itself and unlicensed users. Unlicensed users can choose to lease the licensed TV channels from licensees (via the database) for the exclusive usage, or to share the free TV white spaces with others. In the latter case, users can further decide whether to purchase the advanced information regarding these TV white spaces from the database to enhance the performance.

Figure 1 illustrates such a database-provided HySIM framework, where user 1 and user 2 lease the licensed TV channels from the spectrum licensee (via the database-provided platform), and user 3 and user 4 share the free unlicensed TV white spaces with others. User 4 further purchases the advanced information to improve its performance.

In order to thoroughly understand the user behavior in such a hybrid market as well as the market evolution and equilibrium, we formulate the interactions among the geo-location database (operator), the spectrum licensee, and unlicensed users as a three-layer hierarchical model:

1) Layer I: Commission Negotiation (in Section V): In the first layer, the database and the spectrum licensee negotiate on the commission fee that the spectrum licensee needs to pay for using the spectrum market platform. Specifically, the database, as the market platform, helps the spectrum licensee to display, advertise, and sell the under-utilized TV channels to unlicensed users. Accordingly, it takes some commission fee from each successful transaction between the spectrum licensee and unlicensed users. In this work, we consider the revenue sharing commission scheme, where the spectrum licensee shares a fixed percentage of revenue with the database.\(^4\) We study the commission negotiation (for the revenue sharing percentage) using the Nash bargaining theory [23].

2) Layer II: Price Competition Game (in Section IV): In the second layer, the database and the spectrum licensee compete with each other for selling information or channels to unlicensed users. The spectrum licensee decides the price of the licensed TV channels, and the database decides the price of the advanced information (regarding the unlicensed TV channels). Hence, the interplay of both prices forms a price competition game with heterogeneous items. We analyze the equilibrium of such a price competition game using the supermodular game theory [24].

3) Layer III: User Behavior and Market Dynamics (in Section III): In the third layer, unlicensed users decide their best purchasing decisions, given the database’s information price and the spectrum licensee’s channel price. Note that a user’s best purchasing behaviour changes with other users’ purchasing behaviours, due to the negative and positive network externalities of the information market (see Section II-C for details). We will show how the market dynamically evolves along the user purchasing behaviour dynamics, and what is the market equilibrium point it evolves to.

In summary, we list the main contributions as follows.

- **Novelty and Practical Significance**: To the best of our knowledge, this is the first paper that studies a hybrid spectrum and information market, for promoting the unlicensed spectrum access to both licensed and unlicensed TV channels.

- **Modeling and Solution Techniques**: We formulate the interactions as a three-layer hierarchical model, and analyze the model by backward induction, using market equilibrium theory, supermodular game theory, and Nash bargaining theory.

- **Performance Evaluations**: Numerical results show that the proposed hybrid market can improve network profit more than 80%, compared with a pure information market. The gap between our achieved network profit and the coordinated benchmark is less than 4%.

\(^3\)For example, based on the knowledge about the network infrastructures of TV licensees and their channel occupancies, the database can predict the average interference (from licensed devices) on each channel at each location.

\(^4\)Another widely-used commission scheme is the wholesale pricing scheme, where the database charges the spectrum licensee a fixed price for each successful transaction, regardless of the spectrum licensee’s actual revenue. We will study this commission scheme in our future work.
II. SYSTEM MODEL

We consider a database-assisted TV white space network with a geo-location database and a set of unlicensed users (devices) in a particular region (e.g., a city). Unlicensed users can use the unlicensed TV channels (i.e., TV white spaces) in a shared manner (e.g., using CDMA or CSMA). Meanwhile, there is a spectrum licensee, who owns the licensed channels and wants to lease the under-utilized channels to unlicensed users for additional revenue.\(^5\) Different from the unlicensed TV white spaces, the licensed TV channels can be used by unlicensed users in an exclusive manner (with the permission of the licensee). Therefore, users can enjoy a better performance (e.g., a higher data rate or a lower interference) on the licensed channels. For convenience, let \(\pi_l \geq 0\) denote the (licensed) channel price set by the spectrum licensee.

A. Geo-location Database

1) Basic Service: According to the regulation policy (e.g., [5]), it is mandatory for a geo-location white space database to provide the following information for any unlicensed user: (i) the list of TV white spaces (i.e., unlicensed TV channels), and (ii) the transmission constraint (e.g., maximum transmission power) on each channel in the list. The database needs to provide this basic (information) service free of charge for any unlicensed user.

2) Advanced Service: Beyond the basic information, the database can also provide certain advanced information regarding the quality of TV channels (as Spectrum Bridge did in [17]), which we call the advanced (information) service. Such an advanced information can be rather general, and a typical example is “the interference level on each channel” used in [15], [16]. With the advanced information, the user is able to choose a channel with the highest quality (e.g., with the lowest interference level). Hence, the database can sell this advanced information to users for a profit. This leads to an information market. For convenience, let \(\pi_a \geq 0\) denote the (advanced) information price of the database.

3) Leasing Service: As mentioned previously, the geo-location database can also serve as a spectrum market platform for trading licensed channels between the spectrum licensee and unlicensed users, which we call the leasing service. By doing this, the spectrum licensee shares a fixed percentage \(\delta \in [0,1]\) of revenue with the database, which we call the revenue share commission scheme (RSS).

B. Unlicensed User

Unlicensed users can choose either to purchase the licensed channel from the licensee for the exclusive usage, or to share the free unlicensed TV white spaces with others (with or without advanced information). We assume that all licensed and unlicensed TV channels have the same bandwidth (e.g., 6MHz in the USA), and each user only needs one channel (either licensed or unlicensed) at a particular time. Formally, we denote \(s \in \{b,a,l\}\) as the strategy of a user, where

(i) \(s = b\): Choose the basic service (i.e., share TV white spaces with others, without the advanced information);

(ii) \(s = a\): Choose the advanced service (i.e., share TV white spaces with others, with the advanced information);

(iii) \(s = l\): Choose the leasing service (i.e., lease the licensed channel from the licensee for an exclusive usage).

We further denote \(B, A,\) and \(L\) as the expected utility that a user can achieve from choosing the basic service \((s = b)\), the advanced service \((s = a)\), and the leasing service \((s = l)\), respectively. The payoff of a user is defined as the difference between the achieved utility and the service cost (i.e., the information price when choosing the advanced service, or the leasing price if choosing the leasing service). Let \(\theta\) denote the user’s evaluation for the achieved utility. Then, the payoff of a user with an evaluation parameter \(\theta\) can be written as

\[
\Pi^{\theta}_{s} = \begin{cases} 
\theta \cdot B, & \text{if } s = b, \\
\theta \cdot A - \pi_a, & \text{if } s = a, \\
\theta \cdot L - \pi_l, & \text{if } s = l.
\end{cases}
\]

Each user is rational and will choose a strategy \(s \in \{b,a,l\}\) that maximizes its payoff. Note that different users may have different values of \(\theta\) (e.g., depending on application types), hence have different choices. That is, users are heterogeneous in term of \(\theta\). For convenience, we assume that \(\theta\) is uniformly distributed in \([0,1]\) for all users.\(^6\)

Let \(\eta_b, \eta_a,\) and \(\eta_l\) denote the fraction of users choosing the basic service, the advanced service, and the leasing service, respectively. For convenience, we refer to \(\eta_b, \eta_a,\) and \(\eta_l\) as the market shares of the basic service, the advanced service, and the leasing service, respectively. Obviously, \(\eta_b + \eta_a + \eta_l = 1\). Then, the normalized payoffs (profits) of the spectrum licensee and the database are, respectively,

\[
\begin{align*}
\Pi^{\text{DB}}_L & \triangleq \Pi^{\text{DB}}_{(l)} = \pi_l \cdot \eta_l \cdot (1 - \delta), \\
\Pi^{\text{DL}}_b & \triangleq \Pi^{\text{DL}}_{(b)} = \pi_a \cdot \eta_b + \pi_l \cdot \eta_l \cdot \delta.
\end{align*}
\]

C. Externalities of the Information Market

There are two types of network externalities coexisting in the information market: (i) negative externality, which characterizes the increase of congestion (or the decrease of user performance) when more users sharing the same TV white space, and (ii) positive externality, which characterizes the increase of advanced information accuracy (or the increase of user performance) when more users purchasing the information. Next we quantify such negative and positive network externalities analytically.

We first have the following intuitive observations for a user’s expected utilities of three strategy choices:

- \(L\) is a constant and independent of \(\eta_a, \eta_b,\) and \(\eta_l\). This is because a user uses the licensed channels in an exclusive manner, hence its performance (on licensed channels) does not depend on the activities of others.

- \(B\) is non-increasing in \(\eta_a + \eta_b\) (the total fraction of users using TV white space) due to the congestion effect. This is because more users using TV white spaces (in a shared manner) will increase the level of congestion on these channel, hence reduce the performance of each user.

\(^5\)In case there are multiple spectrum licensees, we assume that they are coordinated by the single entity. We will leave the case with multiple competitive spectrum licensees to a future work.

\(^6\)This assumption is commonly used in the existing literature. Relaxing to more general distributions often does not change the main insights [25], [26].
A is non-increasing in $\eta_A + \eta_B$, due to the congestion effect (similar as that of $B$). This leads to the negative network externality of the information market.

A is non-decreasing in $\eta_A$, given a fixed value of $\eta_A + \eta_B$. This is because more users purchasing the advanced information will increase the quality of the information [15], [16]. This leads to the positive network externality.

Based on the above observations, we write $B$ as a non-increasing function $f(\cdot)$ of $\eta_A + \eta_B$ (or equivalently, $1 - \eta_A$),

$$B \triangleq f(\eta_A + \eta_B),$$

and write $A$ as the combination of a non-increasing function $f(\cdot)$ of $\eta_A + \eta_B$ and a non-decreasing function $g(\cdot)$ of $\eta_A$,

$$A \triangleq f(\eta_A + \eta_B) + g(\eta_A).$$

Note that $f(\cdot)$ reflects the congestion effect in the information market, and is identical in $B$ and $A$ (as users experience the same congestion effect in both basic and advanced services), and $g(\cdot)$ reflects the performance gain induced by the advanced information, i.e., the value of advanced information.

Without loss of generality, we introduce the following assumption.

**Assumption 1.** $L \geq A \geq B$.

The reason behind $L \geq A$ is that there is no congestion on the licensed channels, and the reason behind $A \geq B$ is that users can achieve additional gain $g(\eta_A)$ from the advanced information. Note that if $A < B$, then users will never choose the advanced service even with a zero information price $\pi_A$. In this case, our model degenerates to a monopoly spectrum market (where the spectrum licensee is the monopolist). If $L < B$, then users will never choose the leasing service even with a zero channel price $\pi_A$. In this case, our model degenerates to the pure information market as in [15]. In this sense, our hybrid market model generalizes both the pure spectrum market and pure information market.

To facilitate the later analysis, we further introduce the following assumptions on functions $f(\cdot)$ and $g(\cdot)$.

**Assumption 2.** $f(\cdot)$ is non-negative, non-increasing, convex, and continuously differentiable.

**Assumption 3.** $g(\cdot)$ is non-negative, non-decreasing, concave, and continuously differentiable.

The non-increasing and convexity assumption of $f(\cdot)$ reflects the increasing of marginal performance degradation under congestion, and is widely used in wireless networks with congestion effect (see, e.g., [26], [27] and references therein). The non-decreasing and concavity assumption of $g(\cdot)$ reflects the diminishing of marginal performance improvement induced by the advanced information. In this work, we use the generic functions $f(\cdot)$ and $g(\cdot)$, which can generalize many practical scenarios with the explicit advanced information definition (e.g., those defined in [15], [16], where the advanced information is the interference level on each channel). We provide more detailed discussions about these generic functions and practical scenarios in our online technical report [29].
Intuitively, $\theta_{LB}$ denotes the smallest $\theta$ such that a type-$\theta$ user prefers the leasing service than the basic service; $\theta_{AB}$ denotes the smallest $\theta$ such that a type-$\theta$ user prefers the advanced service than the basic service; and $\theta_{LA}$ denotes the smallest $\theta$ such that a type-$\theta$ user prefers the leasing service than the advanced service. Notice that $A$ and $B$ are functions of initial market shares $\{\eta^0_A, \eta^0_B, \eta^0_L\}$. Hence, $\theta_{LB}$, $\theta_{AB}$, and $\theta_{LA}$ are also functions of $\{\eta^0_L, \eta^0_A, \eta^0_B\}$.

Figure 3 illustrates the relationship of $\theta_{LB}$, $\theta_{AB}$, and $\theta_{LA}$. Intuitively, the users with a high utility evaluation factor $\theta$ are more willing to choose the leasing service in order to achieve a large utility; the users with a low $\theta$ are more willing to choose the basic service so that they will pay zero service cost; the users with a middle $\theta$ are more willing to choose the advanced service, in order to achieve a relatively large utility with a relatively low service cost. Notice that when the information price $\pi_\alpha$ is high or the information value (i.e., $A - B$) is low, we could have $\theta_{LB} < \theta_{AB}$, in which no users will choose the advanced service (as illustrated in the lower subfigure).

Next we characterize the new market shares (called the "derived market shares") resulting from the users' best choices mentioned above. Such derived market shares are important for analyzing the user behavior dynamics and market evolutions in the next subsection. Recall that $\eta$ is uniformly distributed in $[0, 1]$. Then, given any initial market shares $\{\eta^0_A, \eta^0_B, \eta^0_L\}$, the newly derived market shares $\{\eta_{LB}, \eta_{LA}\}$ are

- If $\theta_{LB} > \theta_{AB}$, then $\eta_{LB} = 1 - \theta_{LA}$ and $\eta_{LA} = \theta_{LA} - \theta_{AB}$;
- If $\theta_{LB} \leq \theta_{AB}$, then $\eta_{LB} = 1 - \theta_{LB}$ and $\eta_{LA} = 0$.

Formally, we have the following derived market shares.$^9$

**Lemma 1.** Given any initial market shares $\eta^0_A$ and $\eta^0_B$, the derived market shares $\eta_{LB}$ and $\eta_{LA}$ are given by

$$
\begin{cases}
\eta_{LB} = \max \{1 - \max \{\theta_{LB}, \theta_{AB}\}, 0\}, \\
\eta_{LA} = \max \{\min \{\theta_{LA}, 1\} - \theta_{LB}, 0\}.
\end{cases}
$$

The results in Lemma 1 assume that all users update the best strategies once and simultaneously. Since $\theta_{LB}$, $\theta_{AB}$, and $\theta_{LA}$ are functions of initial market shares $\{\eta^0_A, \eta^0_B, \eta^0_L\}$, the derived market shares $\{\eta_{LB}, \eta_{LA}\}$ are also functions of $\{\eta^0_L, \eta^0_A, \eta^0_B\}$, and hence can be written as $\eta_{LB}(\eta^0_L, \eta^0_A)$ and $\eta_{LA}(\eta^0_L, \eta^0_A)$.

**B. Market Dynamics and Market Equilibrium**

When the market shares change, the users’ payoffs (on the advanced service and basic service) change accordingly, as $A$ and $B$ change. As a result, users will update their best strategies continuously, hence the market shares will evolve dynamically, until reaching a stable point (called market equilibrium). In this subsection, we will study such a market dynamics and equilibrium, given the prices $\{\pi_L, \pi_A\}$.

For convenience, we introduce a virtual discrete-time system with slots $t = 1, 2, \ldots$, where users change their decisions at the beginning of every slot, based on the derived market shares in the previous slot. Let $\{\eta^t_A, \eta^t_B\}$ denote the market shares derived at the end of slot $t$ (which serve as the initial market shares in the next slot $t + 1$). We further denote $\Delta \eta_L$ and $\Delta \eta_A$ as the changes (dynamics) of market shares between two successive time slots, e.g., $t$ and $t + 1$, that is,

$$
\begin{align*}
\Delta \eta_L(\eta^t_A, \eta^t_B) &= \eta^{t+1}_L - \eta^t_L, \\
\Delta \eta_A(\eta^t_A, \eta^t_B) &= \eta^{t+1}_A - \eta^t_A,
\end{align*}
$$

where $(\eta^{t+1}_L, \eta^{t+1}_A)$ are the derived market share in slot $t + 1$, which can be computed by Lemma 1. Obviously, if both $\Delta \eta_L$ and $\Delta \eta_A$ are zero in a slot $t + 1$, i.e., $\eta^{t+1}_L = \eta^t_L$ and $\eta^{t+1}_A = \eta^t_A$, then users will no longer change their strategies in the future. This implies that the market achieves a stable state, which we call the market equilibrium. Formally,

**Definition 1 (Market Equilibrium).** A pair of market shares $\eta^* = (\eta^*_A, \eta^*_B)$ is a market equilibrium of Layer III, if

$$
\begin{align*}
\Delta \eta_L(\eta^*_A, \eta^*_B) &= 0, \quad \text{and} \quad \Delta \eta_A(\eta^*_A, \eta^*_B) = 0.
\end{align*}
$$

Next, we study the existence and uniqueness of the market equilibrium, and further characterize the market equilibrium analytically. These results are important for analyzing the price competition game in Layer II (Section IV).

**Lemma 2 (Existence).** Given any feasible price pair $\{\pi_L, \pi_A\}$, there exists at least one market equilibrium.

**Lemma 3 (Uniqueness).** Given any feasible price pair $\{\pi_L, \pi_A\}$, there exists a unique market equilibrium $\eta^*_A, \eta^*_B$ if there exists a tuple $(\eta_A, \eta_B)$ with $\eta_A + \eta_B \leq 1$ such that

$$
\frac{g'(\eta_A)}{g(\eta_A)} \cdot \frac{L - B}{A} \leq 1. \quad (7)
$$

As an example, if the information value $g(\eta_A)$ (reflecting the positive network externality) does not increase very fast with $\eta_A$, then (7) is satisfied and there exists a unique equilibrium. Note that condition (7) is sufficient but not necessary for the uniqueness. In particular, we observe through numerical simulations that in some cases, the market converges to a unique equilibrium for a wide range of prices under which the condition (7) is not satisfied. Nevertheless, the sufficient condition (7) leads to the insight that if the change of positive network externality is dramatic in $\eta_A$, there may exist multiple equilibrium points. Note that even if there exist multiple equilibrium points, the market always converges to a unique one of them given the initial market shares, as here we consider deterministic market dynamics.

For a better understanding, we illustrate the dynamics of market shares in Figure 4. The $x$-axis denotes the leasing service’s market share $\eta_A$, and the $y$-axis denotes the advanced service’s market share $\eta_B$. Notice that a feasible pair of market shares $\{\eta_A, \eta_B\}$ satisfies $\eta_A + \eta_B \leq 1$. Each grey arrow denotes the dynamics of market shares under a particular initial market share (at the starting point of the arrow). For example, from the initial market shares $\eta^0 = \{\eta_A, \eta_B\} = \{0.6, 0\}$, the market will evolve to $\eta^1 = \{0.32, 0.16\}$, and then

$^9$Here, $g'(\eta_A)$ is the first-order derivative of $g(\cdot)$ with respect to $\eta_A$. Note that $A$ is a function of $\eta_A$ and $\eta_B$, and $B$ is a function of $\eta_A$. 

\[ \eta^2 = \{0.35, 0.2\}, \] and eventually converge to the equilibrium point \( \eta^* = (0.33, 0.24) \). The red curve denotes the isoline of \( \Delta \eta_L(\eta_L, \eta_A) = 0 \), and the blue curve denotes the isoline of \( \Delta \eta_A(\eta_L, \eta_A) = 0 \). By Definition 1, the intersection between blue and red curves is the market equilibrium point. In this example, there is a unique market equilibrium point.

Suppose the uniqueness condition (7) is satisfied. We characterize the unique equilibrium by the following theorem.

**Theorem 1 (Market Equilibrium).** Suppose the uniqueness condition (7) holds. For any feasible pair \((\pi_L, \pi_A)\), the unique market equilibrium is given by

(a) If \( \theta_{LB}(\eta_L, \eta_A)|_{\eta_L=0} \leq \theta_{AB}(\eta_L, \eta_A)|_{\eta_L=0} \), then there is a unique market equilibrium \( \eta^* = \{\eta^*_L, \eta^*_A\} \) satisfying

\[ \eta^*_L = 1 - \theta_{LB}(\eta^*_L, \eta^*_A), \text{ and } \eta^*_A = 0; \]  

(b) If \( \theta_{LB}(\eta_L, \eta_A)|_{\eta_L=0} > \theta_{AB}(\eta_L, \eta_A)|_{\eta_L=0} \), then there is a unique market equilibrium \( \eta^* = \{\eta^*_L, \eta^*_A\} \) satisfying

\[ \begin{cases} \eta^*_L = 1 - \theta_{LA}(\eta^*_L, \eta^*_A), \\ \eta^*_A = \theta_{LA}(\eta^*_L, \eta^*_A) - \theta_{AB}(\eta^*_L, \eta^*_A). \end{cases} \]  

Intuitively, we can obtain the derived market shares by substituting (8) or (9) into (4). Then, we can check the above derived market shares satisfy the equilibrium condition (6). Please refer to [29] for the detailed proof.

IV. LAYER II – PRICE COMPETITION GAME EQUILIBRIUM

In this section, we study the price competition between the database and the spectrum licensee in Layer II, given the commission negotiation solution in Layer I and based on the market equilibrium prediction in Layer III.

A. Price Competition Game

We first define the price competition game (PCG) and the associated Nash equilibrium explicitly.

**Definition 2 (Price Competition Game – PCG).**

- **Players:** The database and the spectrum licensee;
- **Strategies:** The database’s strategy is the nonnegative information price \( \pi_L \), and the spectrum licensee’s strategy is the nonnegative channel price \( \pi_A \);
- **Payoffs:** The players’ payoffs are defined in (2).

For clarity, we write the (unique) market equilibrium \( \eta^* = \{\eta^*_L, \eta^*_A\} \) in Layer III as functions of prices \((\pi_L, \pi_A)\), i.e., \( \eta^*_L(\pi_L, \pi_A) \) and \( \eta^*_A(\pi_L, \pi_A) \). Intuitively, we can interpret \( \eta^*_L(\cdot) \) and \( \eta^*_A(\cdot) \) as the demand functions of the spectrum licensee and the database, respectively. Based on (2), the payoffs of the spectrum licensee and the database can be written as:

\[ \begin{align*} 
\Pi_{DB}^{\pi_L}(\pi_L, \pi_A) &= \pi_L \cdot \eta^*_L(\pi_L, \pi_A) \cdot (1 - \delta), \\
\Pi_{SL}^{\pi_A}(\pi_L, \pi_A) &= \pi_A \cdot \eta^*_A(\pi_L, \pi_A) + \pi_L \cdot \eta^*_L(\pi_L, \pi_A) \cdot \delta. 
\end{align*} \]  

We refer to the Nash equilibrium of the PCG as Pricing Equilibrium. Formally.

**Definition 3 (Pricing Equilibrium).** A pair of prices \((\pi^*_L, \pi^*_A)\) is a pricing equilibrium of Layer II if

\[ \begin{align*} 
\pi^*_L &= \underset{\pi_L \geq 0}{\text{arg max}} \ \Pi_{DB}^{\pi_L}(\pi_L, \pi^*_A), \\
\pi^*_A &= \underset{\pi_A \geq 0}{\text{arg max}} \ \Pi_{SL}^{\pi_A}(\pi^*_L, \pi_A). 
\end{align*} \]  

Note that directly solving the pricing equilibrium of PCG is challenging, due to the difficulty in obtaining the closed-form characterization of the market equilibrium \( \{\eta^*_L(\pi_L, \pi_A), \eta^*_A(\pi_L, \pi_A)\} \) under a particular price pair \((\pi_L, \pi_A)\). To this end, we transform the original price competition game into an equivalent demand competition game. The key idea is to view the achieved market shares (i.e., demands) as strategies of the database and the spectrum licensee, and view the prices as functions of market shares.

B. Demand Competition Game

We notice that under the uniqueness condition (7), there is a one-to-one correspondence between the market equilibrium \( \{\eta^*_L, \eta^*_A\} \) and the prices \( \{\pi_L, \pi_A\} \) in Layer III. In this sense, once the spectrum licensee and the database choose the prices \( \{\pi_L, \pi_A\} \), they have equivalently chosen the respective market shares \( \{\eta^*_L, \eta^*_A\} \).

Hence, we can define an equivalent demand competition game, where the strategy of each player is its demand or market share \( \eta_L \) for the spectrum licensee and \( \eta_A \) for the database, and the prices \( \{\pi_L, \pi_A\} \) are functions of market shares \( \{\eta_L, \eta_A\} \). Substitute \( \theta_{LA} = \frac{\pi^*_L}{\pi^*_L + \pi^*_A} \) and \( \theta_{AB} = \frac{\pi^*_A}{\pi^*_L + \pi^*_A} \) into (9), we can derive the inverse functions of (9), i.e., the prices as functions of market shares,\(^{10}\)

\[ \begin{align*} 
\pi_L(\eta_L, \eta_A) &= (1 - \eta_L) \cdot (L - f(1 - \eta_L) - g(\eta_A)) \\
&\quad + (1 - \eta_L - \eta_A) \cdot g(\eta_A), \\
\pi_A(\eta_L, \eta_A) &= (1 - \eta_A - \eta_L) \cdot g(\eta_A). 
\end{align*} \]  

Accordingly, the payoffs of two players can be written as:

\[ \begin{align*} 
\Pi_{DB}^{\eta_L}(\eta_L, \eta_A) &= \pi_L(\eta_L, \eta_A) \cdot \eta_L \cdot (1 - \delta), \\
\Pi_{SL}^{\eta_A}(\eta_L, \eta_A) &= \pi_A(\eta_L, \eta_A) \cdot \eta_A + \pi_L(\eta_L, \eta_A) \cdot \eta_L \cdot \delta. 
\end{align*} \]  

\(^{10}\)We omit the trivial case in (8), where the database has a zero market share, as this will never the case at the pricing equilibrium of Layer II.
Next, we formally define the demand competition game (DCG) and the associated Nash equilibrium.

**Definition 4 (Demand Competition Game – DCG).**

- **Players:** The database and the spectrum licensee.
- **Strategies:** The database’s strategy is its market share \( \eta_\lambda \), and the spectrum licensee’s strategy is its market share \( \eta_I \).
- **Payoffs:** The players’ payoffs are defined in (13).

Similarly, we refer to the Nash equilibrium of the DCG as the Demanding Equilibrium. Formally,

**Definition 5 (Demanding Equilibrium).** A pair of market shares \((\eta^*_\lambda, \eta^*_I)\) is a demanding equilibrium, if

\[
\begin{align*}
\eta^*_\lambda &= \arg \max \Pi^*_\lambda(\eta_\lambda, \eta^*_I), \\
\eta^*_I &= \arg \max \Pi^*_I(\eta^*_I, \eta_I).
\end{align*}
\]  

(14)

The following lemma shows the equivalence between the original PCG and the above DCG.

**Lemma 4 (Equivalence).** If \((\eta^*_\lambda, \eta^*_I)\) is a demanding equilibrium of DCG, then \((\pi^*_\lambda, \pi^*_I)\) given by (12) is a pricing equilibrium of the original PCG.

**C. Game Equilibrium**

Due to equivalence of PCG and DCG, we focus on finding the demanding equilibrium of DCG in the following. Specifically, we show that the DCG is a supermodular game (with minor strategy transformation), and hence the demanding equilibrium of DCG can be easily obtained by using the supermodular game theory [24].

**Lemma 5 (Existence of Demanding Equilibrium).** The DCG is a supermodular game with respect to \( \eta_\lambda \) and \( -\eta_I \). Hence, there exists at least one demanding equilibrium.

The following lemma further gives the uniqueness condition for the demanding equilibrium of DCG.

**Lemma 6 (Uniqueness of Demanding Equilibrium).** The DCG has a unique demanding equilibrium \((\eta^*_\lambda, \eta^*_I)\), if

\[
\frac{\partial^2 \Pi^*_\lambda(\eta_\lambda, \eta_I)}{\partial (-\eta_\lambda)^2} \geq \frac{\partial^2 \Pi^*_\lambda(\eta_\lambda, \eta_I)}{\partial (-\eta_I) \partial \eta_\lambda} \text{ and } \frac{\partial^2 \Pi^*_I(\eta^*_I, \eta_I)}{\partial (-\eta_I)^2} \geq \frac{\partial^2 \Pi^*_I(\eta^*_I, \eta_I)}{\partial \eta_I \partial (-\eta_I)}.
\]

The above uniqueness conditions are quite general, and follow the standard supermodular game theory. Next, we provide a special example to illustrate these conditions more intuitively. Consider the following example: \( f(\eta_\lambda + \eta_I) = \alpha_1 - \beta_1 \cdot (\eta_\lambda + \eta_I) \) and \( g(\eta_\lambda) = \beta_2 \cdot \eta_\lambda \). That is, both positive and negative network effects change linearly with the respective market shares. By Lemma 6, we can easily obtain the following uniqueness condition: \( L - \alpha_1 - \beta_1 > \beta_2 \). Namely, if \( L \) is large enough or \( \alpha_1 \) (or \( \beta_1, \beta_2 \)) is small enough, there is a unique demanding equilibrium.

Once we obtain the demanding equilibrium \((\eta^*_\lambda, \eta^*_I)\) of DCG, we can immediately obtain the pricing equilibrium \((\pi^*_\lambda, \pi^*_I)\) of the original PCG by (12). It is notable that we may not be able to derive the closed-form expression of demanding equilibrium of DCG, as we do not assume any specific function forms of \( f(\cdot) \) and \( g(\cdot) \). Nevertheless, thanks to the nice property of supermodular game, we can easily compute the demanding equilibrium of DCG through, for example, the simple best response iteration (see [29] for details).

**V. Layer I – Commission Bargaining Solution**

In this section, we study the commission negotiation among the database and the spectrum licensee in Layer I, based on their predictions of the pricing equilibrium in Layer II and the market equilibrium in Layer III. The purpose of this negotiation is to find a feasible revenue sharing percentage \( \delta \in [0, 1] \) that is satisfactory for both the database and the spectrum licensee. We formulate the commission negotiation problem as a bargaining problem, and study the bargaining solution by using the Nash bargaining theory [23].

Following the Nash bargaining framework [23], we first derive the database’s and the spectrum licensee’s payoffs when reaching an agreement and when not reaching any agreement (hence reaching the disagreement). Specifically, when reaching an agreement \( \delta \), their payoffs are \( \Pi^*_I(\delta) \) and \( \Pi^*_I(\delta) \) derived in Section IV, respectively. When not reaching any agreement (i.e., reaching the disagreement), the spectrum licensee’s profit is \( \Pi^*_I(0) = 0 \), and the database’s profit is \( \Pi^*_I(\delta) = \pi^*_\lambda \cdot \eta^*_I(\pi^*_I) \), where \( \pi^*_I \) and \( \eta^*_I(\pi^*_I) \) are the database’s optimal price and the corresponding market share in the pure information market. Then, the Nash bargaining solution \( \delta^* \) is given by

\[
\max_{\delta \in [0,1]} \left( \Pi^*_I(\delta) - \Pi^*_I(0) \right) \cdot \left( \Pi^*_I(\delta) - \Pi^*_I(\delta) \right) \cdot \left( \Pi^*_I(\delta) - \Pi^*_I(\delta) \right)
\]

s. t. \( \Pi^*_I(\delta) \geq \Pi^*_I(\delta) \geq \Pi^*_I(\delta) \).

(15)

Note that analytically solving (15) may be difficult, as it is hard to characterize the analytical forms of \( \Pi^*_I(\delta) \) and \( \Pi^*_I(\delta) \). Nevertheless, we notice that the bargaining variable \( \delta \) lies in a closed and bounded range \([0, 1]\), and the objective function of (15) is bounded. Hence, there must be an optimal solution for (15), which can be easily found by using many one-dimensional search methods provided in [28].

**Theorem 2 (Bargaining Solution).** There exists an optimal solution for problem (15). In addition, if the objective function of (15) is monotonic, the optimal solution is unique.

**VI. Numerical Results**

In this section, we perform numerical studies to illustrate the market solution (e.g., the bargaining solution in Layer I and the pricing equilibrium in Layer II) and evaluate the system performance (e.g., the network profit, the database’s profit, and the licensee’s profit) in our proposed framework.

1) Simulation Configurations: For an illustrative purpose, we characterize the negative and positive network externalities (of the information market) by the following functions:

\[
\begin{align*}
f(\eta_\lambda + \eta_I) &= \alpha_1 - \beta_1 \cdot (\eta_\lambda + \eta_I)^{\gamma_1}, \\
g(\eta_\lambda) &= \beta_2 \cdot \eta_\lambda^{\gamma_2}.
\end{align*}
\]

We can easily check that these two functions satisfy Assumption 2 and Assumption 3. Moreover, these functions are good approximations of the actual negative and positive network externalities, when the advanced information is defined as the interference level on each channel as in [15], [16].
3. With the above setting, if all users choose the leasing service (purchasing the licensed TV channels), i.e., $\eta_h = 1$ and $\eta_a = \eta_b = 0$, then the utilities of choosing basic or advanced service is $B = A = \alpha_1$; if all users choose the advanced service (sharing the unlicensed TV white spaces with the advanced information), i.e., $\eta_a = 1$ and $\eta_h = \eta_b = 0$, then the utilities of choosing basic and advanced services are $B = \alpha_1 - \beta_1$ and $A = \alpha_1 - \beta_1 + \beta_2$, respectively.

To further characterize which externality dominates the information market, we derive the first-order derivative of $A = f(\eta_a + \eta_b) + g(\eta_a)$ with respect to $\eta_a$:

$$\frac{\partial A}{\partial \eta_a} = -\gamma_1 \cdot \beta_1 \cdot (\eta_a + \eta_b)^{\gamma_1 - 1} + \gamma_2 \cdot \beta_2 \cdot (\eta_a)^{\gamma_2 - 1}.$$ 

If $\frac{\partial A}{\partial \eta_a} > 0$, the utility of users purchasing the advance information increases with the percentage $\eta_a$ of users purchasing the advanced information (i.e., the size of the information market). In this case, we will say that the positive network externality is dominant. On the other hand, if $\frac{\partial A}{\partial \eta_a} < 0$, the utility of users purchasing the advance information increases with $\eta_a$, and we will say that the negative network externality is dominant.

For easy comparison, we set $\gamma_1 = \gamma_2$ and change the value of $\beta_1$ and $\beta_2$ to achieve different degrees of network externality. Specifically, we introduce $\lambda = \beta_1 / \beta_2$ to represent the degree of network externality: (i) If $\lambda < (\eta_a/\eta_b+\eta_a)^{\gamma_1 - 1}$, then $\frac{\partial A}{\partial \eta_a} > 0$, hence the information market is positive externality dominant; and (ii) If $\lambda > (\eta_a/\eta_b+\eta_a)^{\gamma_1 - 1}$, then $\frac{\partial A}{\partial \eta_a} < 0$, hence the information market is negative externality dominant.

3) Illustration of Pricing Equilibrium: We now illustrate the pricing equilibrium of the PCG in Layer II.

Figure 6 shows the pricing equilibrium vs $L$, under both strongly positive (red curve) and negative externalities (blue curve). We can see that the equilibrium information price of the database (denoted by DB) slightly increases with $L$, while the equilibrium channel price of the licensee (denote by LH) significantly increases with $L$, under both externality cases. This is because a larger $L$ can attract more users to purchase the licensed channels, and thus allows the licensee to charge a higher channel price. Accordingly, the database can charge a slightly higher information price.

From Figure 6, we can also see that the equilibrium channel price of the licensee under the strongly negative externality is higher than that under the strongly positive externality, while the equilibrium information price of the database is just the opposite. This is because under the strongly negative externality, a small increase of users in the information market will dramatically decreases the quality of unlicensed TV channels. Hence, the spectrum licensee can potentially charge a higher channel price as more users are willing to purchase the licensed channels, while the database has to charge a lower information price to retain users.

4) System Performance: We now evaluate the network profit (i.e., the aggregate profit of the database and the licensee) in our proposed framework. For an illustrative purpose, we

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12Due to space limit, we leave the evaluations of the database’s profit and the spectrum licensee’s profit in the online technical report [29].
In this paper, we proposed a database-provided hybrid spectrum and information market, and analyzed the interactions among the database, the licensee, and the unlicensed users systematically. We also analyzed how the network externalities (of the information market) affect these interactions. Our work not only captures the performance gain introduced by the hybrid market, but also characterizes the impact of positive and negative network externalities on the market equilibrium behaviors of all parties involved. There are several possible directions to extend this work. One is to consider an oligopoly scenario with multiple databases (hence multiple platforms). In this scenario, databases compete with each other for unlicensed users as well as for spectrum licensees.

**REFERENCES**