# A Contract-Based Incentive Mechanism for Crowdsourced Wireless Community Networks

Qian Ma, Lin Gao, Ya-Feng Liu, and Jianwei Huang

***Abstract*—Crowdsourced wireless community networks enable individual users to share their private Wi-Fi access points (APs) with each other, hence can achieve a large Wi-Fi coverage with a low deployment cost. This paper presents the first Wi-Fi sharing mechanism design for the community network operator under incomplete information, where the quality of each user-provided Wi-Fi access is his private information. Specifically, we propose a *contract-based* incentive mechanism, where the operator offers a set of contract items to users, each consisting of a Wi-Fi access price (that a user can charge others who access his AP) and a subscription fee (that a user needs to pay the operator). Different from prior contract mechanisms for wireless networks, here each user's best contract choice depends not only on his private information, but also on other users' choices. This greatly complicates the contract design, as the operator needs to analyze the equilibrium choices of all users, rather than the best choice of each single user. We derive the feasible contract that guarantees the user participation and truthful information disclosure *under the equilibrium*. Our analysis shows that a higher type user (who provides a higher quality access) is more likely to choose a higher price and subscription fee. Simulation results further show that when increasing the ratio of higher type users in the system, the operator can gain more profit, while counter-intuitively, offering lower prices and subscription fees for all users.**

## I. Introduction

### A. Background and Motivation

Wi-Fi technology is playing an increasingly important role in mobile and wireless communication today. According to the Cisco report [1], more than 50% of global mobile data traffic will be carried via Wi-Fi by 2016. However, the ubiquitous deployment of Wi-Fi network is usually restricted by the limited coverage (generally tens of meters [2]) of a single Wi-Fi access point (AP). Hence, to provide a city-wide or nation-wide Wi-Fi network, the network operator needs to deploy a large number of APs, which is usually expensive.[1]

On the other hand, home Wi-Fi access is becoming more and more popular during the past several years. According to [4], 66% of global households had deployed Wi-Fi APs by the end of 2014, and this percentage is expected to grow to nearly 90% by 2019. This motivates the network operator to study a new type of Wi-Fi network that relies on aggregating (crowdsourcing) the large number of existing home Wi-Fi APs deployed by individual users, instead of deploying new Wi-Fi APs all by herself. Such a Wi-Fi network is usually referred to as *Crowdsourced Wireless Community Network* [5].

Specifically, crowdsourced wireless community network enables a set of individual users, who own private home Wi-Fi APs, to form a community and share their home Wi-Fi APs with each other [5]. These individual AP owners (APOs) in the community network are called *community members*.[2] By leveraging the large number of existing home Wi-Fi APs, the wireless community network operator can achieve a large Wi-Fi coverage (e.g., city-wide or nation-wide coverage) with a low deployment cost. A successful example of such a community network is FON [6], the world largest Wi-Fi community network with more than 17 million member Wi-Fi APs globally. Obviously, in such a community network, the network operator plays a critical role in providing a high level of security for community members through proper hardware (e.g., specialized router) and software solutions [6]. Another equally important issue faced by the operator is the *incentive mechanism* design, as these individual APOs may not know each other personally, hence may lack the proper incentives to share Wi-Fi with each other [7]–[12].

In this work, we focus on studying the incentive issues in a crowdsourced wireless community network managed by a network operator. Several recent studies have been devoted to such an issue, including user behavior analysis [7], [8], pricing scheme design [9], [10], and competition analysis [11], [12]. All of these works focused on the *complete* information scenario, where the network operator is assumed to know detailed attributes of all APOs, e.g., user's daily mobility pattern and quality of the provided Wi-Fi access. In practice, however, it is often difficult for the operator to obtain such information.[3] To this end, we consider a more general *incomplete* information scenario in this work, where each APO has certain *private* information not known by the operator and other APOs. In particular, we will study the incentive issues under incomplete information, and aim to design an *incentive mechanism* that (i) encourages individual APOs to join the community network and share their home Wi-Fi APs properly, and (ii) offers a considerable revenue for the operator to manage such a community network.

### B. Model and Problem Formulation

We consider a crowdsourced wireless community network, consisting of a set of individual APOs (community members)

This work is supported by the General Research Funds (Project Number CUHK 412713 and 14202814) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

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[1]For example, KT, LG, and SK Telecom in South Korea have invested 44 million dollars to provide Wi-Fi access at 10,000 hotspots in Seoul [3].

[2]In this work, we will use "individual user", "individual APO", and "community member" interchangeably.

[3]Take the Wi-Fi access quality on an AP as example. It depends on many factors, such as the quality of a user's backhaul, Wi-Fi protocol, transmission power, wireless bandwidth, backhaul bandwidth, and network congestion.

978-1-5090-1311-1/16/$31.00 

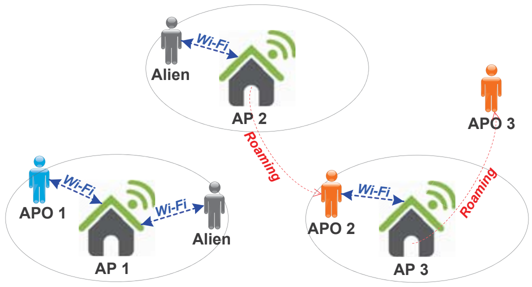


**Figure 1:** An Example of Wireless Community Network

who share their private APs with each other. The community network is not only open to community members, but also open to those users (called *Aliens*) who do not own Wi-Fi APs. Figure 1 illustrates a community network with 3 APOs, where APO 1 stays at home and accesses his own AP, APO 2 roams to APO 3's home location and accesses AP 3, and APO 3 roams to a blank location without any member Wi-Fi coverage. Moreover, there are 2 Aliens in the network, accessing APs 1 and 2, respectively.

Inspired by the success of FON [6] and the existing studies [7], [8], we consider two different sharing schemes for community members (APOs), corresponding to two membership types: *Linus* and *Bill*. As a Linus, an APO can get free Wi-Fi access at other APs in the community, and meanwhile he has to share his own AP without any monetary compensation. As a Bill, an APO needs to pay for accessing other APs in the community, and meanwhile he can earn some money from sharing his own AP with others.[4] Such a dual membership scheme has achieved a great success in practice (e.g., FON [6]), as it captures two major motivations for APOs to join the community [7]: *getting free Wi-Fi access* and *earning money*. Moreover, Aliens always need to pay for accessing any AP in the community, as they do not contribute to the community. Therefore, as far as the network operator is concerned, she will get all the money collected at a Linus's AP (paid by Bills and Aliens), while only part at a Bill's AP.

In this work, we are interested in the following key problems in such a community network with dual membership:

- *Membership Selection*: which type of membership will each APO select (i.e., Bill or Linus)?
- *Wi-Fi Pricing*: how should the network operator set the Wi-Fi access price on each AP?
- *Revenue Division*: how should the network operator share the revenue collected at each Bill's AP with the Bill?[5]

We formulate the interactions between the network operator and APOs as a *two-stage* decision process. In Stage I, the network operator decides the pricing and revenue division strategy. In Stage II, each APO chooses the membership, given the operator's pricing and revenue division strategy.

We aim to design a pricing and revenue division strategy that guarantees the user participation and truthful information disclosure. Such a design is challenging due to several reasons. First, each APO may be associated with certain *private information* (e.g., his mobility pattern and the quality of his provided Wi-Fi access), which cannot be observed by the network operator. Hence, the network operator may not be able to predict each APO's best response precisely. To this end, some incentive compatible mechanisms are necessary for eliciting the private information of APOs. Second, APOs' decisions are coupled with each other, as one APO's membership choice may affect other APOs' payoffs, hence affect their membership choices. Thus, the comprehensive analysis of APOs' equilibrium membership choices is challenging, even under the fixed pricing and revenue division strategy [8].

### *C. Solution and Contributions*

Contract theory [15] is a promising theoretic tool for dealing with problems under incomplete information, and has been recently used in analyzing various wireless resource allocation problems, such as spectrum trading [16], cooperative spectrum sharing [17], [18], and data offloading [19]. In this work, we adopt contract theory to solve the operator's pricing and revenue division problem under incomplete information.

*1) Model Scenario:* We consider the following structured model scenario: (i) all APOs are classified into different *types* based on their private information; and (ii) the network operator knows the distribution of APO type in the system,[6] but not the detailed type of each APO. We consider a simplified yet representative scenario, where the private information of each APO is *the quality of his provided Wi-Fi access*.

*2) Contract-based Operation:* We then propose a contract-based operation scheme: (i) the network operator offers a *contract* to APOs, which contains a set of *contract items*, each consisting of a Wi-Fi access price and a subscription fee *for Bills*;[7] and (ii) each APO chooses the best membership, and the best contract item if choosing to be Bill.

*3) Contract-Theoretic Analysis:* We perform a systematic contract-theoretic analysis. We analyze the incentive compatibility (IC) and individual rationality (IR) constraints for the contract, and derive the *feasible* contract which incentivizes each APO to choose the proper membership and contract item intended for his type.

The key difference between the contract in this work and the existing contracts [16]–[19] for wireless networks is that in our contract, each APO's best choice depends not only on his private information (as in standard contracts), but also on other APOs' choices. This greatly complicates the contract design, as the operator needs to analyze the *equilibrium* choices of all APOs, rather than the best choice of each single APO. We summarize the key contributions as follows.

- *Novel Model:* To our best knowledge, this is the first work that studies the incentive mechanism design for Wi-Fi sharing in crowdsourced wireless community networks under incomplete information.
- *Novel Solution:* We propose a novel contract-based mechanism, which differs form the existing contract mechanisms for wireless networks, in that users' choices are coupled with each other. We analyze the feasibility of the

[4] In the example of FON [6], a Bill obtains a portion of the money paid by other users (Bills and Aliens) who access his AP.

[5] The operator will always get all revenue collected at a Linus's AP.

[6] The network operator can obtain the distribution of APO type through, for example, historical information or long-term learning.

[7] That is, the revenue division between the network operator and each Bill is via the subscription fee that the Bill pays the operator.

proposed contract systematically, based on the analysis of user equilibrium choice.

- *Practical Insight:* Our analysis helps understand how different users choose their Wi-Fi sharing schemes, which facilitates the network operator to better optimize her profit in different scenarios. Specifically, we find that (i) a higher type user (who provides a higher quality Wi-Fi access) is more willing to be Bill and choose larger price and subscription fee; and (ii) when increasing the higher type users, the operator can gain more profit, but setting lower prices and subscription fees to all users.

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we provide the contract formulation. In Section IV, we derive the sufficient and necessary conditions for a feasible contract. We provide simulation results in Section V, and conclude in Section VI.

## II. System Model

We consider a crowdsourced wireless community network, consisting of a set $\mathcal{N} = \{1, ..., N\}$ of $N$ APOs (community members) organized by a network operator. Each APO can access not only his own AP, but also other members' APs. The community network is also open to a set of $N_A = aN$ *Aliens* (users) who do not own Wi-Fi APs, where $a > 0$ is the ratio between Aliens and APOs. Aliens have to pay for accessing the community network through, for example, purchasing Wi-Fi passes from the operator.

To ensure the security, privacy, and quality-of-service (QoS), each APO splits his Wi-Fi channel into a *private channel* for himself and a *public channel* for other users, as was done in FON [13]. Hence, the access of other users to an AP (on the public channel) will not affect the APO's own QoS (on the private channel), but will affect roaming users' (hence reducing the Wi-Fi access quality on the public channel).

*1) APO Classification:* The Wi-Fi access provided by different APOs for other users (on the public channel) may have different *Qualities*, depending on both the wireless characteristics (such as Wi-Fi standard, (public channel) bandwidth, channel fading, and network congestion) and the location popularity. For example, the Wi-Fi access in a popular location may be more valuable, hence adopts a carrier grade Wi-Fi standard that provides a better quality than a regular home Wi-Fi [14]. The service quality will affect other users' connection time on this Wi-Fi, hence affect the APO's revenue (see Section III-A2 for details).

We classify all APOs into $K$ *types*, according to the quality of Wi-Fi access (on the public channel) that they provide for other users.[8] Let $\theta_k$ denote the Wi-Fi access quality provided by a type-$k$ APO, and let $N_k$ denote the number of type-$k$ APOs. Without loss of generality, we assume that $\theta_1 < \theta_2 < \cdots < \theta_K$. It is important to note that the exact type (quality) of each APO is *private information*, and cannot be observed by the network operator or other APOs. Similar as in many existing literatures [16]–[19], we assume that the distribution information regarding APO type (i.e., $\theta_k$ and $N_k$, $\forall k \in \mathcal{K}$, where $\mathcal{K} = \{1, ..., K\}$) is public information and known by the network operator and all APOs.

[8] Our discrete model can approximate a continuous distribution of Wi-Fi access quality when $K$ is large.

**Table I:** A Summary of Three User Types

| **Type** | **Pay for using other APs** | **Paid by sharing his AP** | **Subscription fee** |
|---|---|---|---|
| Linus (member) | No | No | No |
| Bill (member) | Yes | Yes | Yes |
| Alien (non-member) | Yes | N.A. | N.A. |

*2) Membership:* Similar as in [6]–[8], an APO can share his AP in two different ways, corresponding to two different membership types: *Linus* and *Bill*. Specifically,

- As a Linus, an APO can get free Wi-Fi access at other APs in the community, and meanwhile he has to share his own AP without any monetary compensation.
- As a Bill, an APO needs to pay for accessing other APs in the community, and meanwhile he can earn some money from sharing his own AP with others.

Specifically, the network operator will design a *Wi-Fi access price* $p_n$ for each APO $n \in \mathcal{N}$.[9] This price $p_n$ is designed for charging other users (Bills and Aliens) who access AP $n$. When an APO $n$ chooses to be a Bill, he needs to pay for accessing every other AP $m \neq n$ at a unit price $p_m$ (per unit Wi-Fi connection time), and meanwhile he can charge other users (Bills and Aliens) for accessing his AP at a unit price $p_n$. When an APO $n$ chooses to be a Linus, he neither pays for accessing other APs, nor charges other users for accessing his AP. In this case, the network operator will instead charge other users (Bills and Aliens) for accessing AP $n$.

Moreover, the network operator will charge a fixed *subscription fee* $\delta_n$ for each Bill (APO) $n \in \mathcal{N}$.[10] Note that a Linus does not need to pay the subscription fee. Besides, Aliens always pay for accessing any AP in the community, without any subscription fee. For clarity, we summarize the properties of three user types in Table I.

*3) Mobility:* As we focus on studying the impact of Wi-Fi access quality in this work, we consider a homogeneous mobility pattern for APOs: *(i) each APO stays at home with probability $\eta$, and roams outside home with probability $1 - \eta$, and (ii) when roaming outside, each APO roams to every other AP with the same probability $\frac{1-\eta}{N}$.*[11] Moreover, we assume that each Alien roams to each AP with the same probability $\frac{1}{N}$. With more complicated notations, our analytical framework can be extended to the heterogeneous mobility pattern, although we may not be able to obtain the closed-form result. We will study the impact of heterogeneous mobility patterns on the mechanism in our future work.

## III. Contract Formulation

Due to the private information, it is difficult for the operator to predict the optimal membership choice of each APO, and

[9] A special case of our model is that the price (and the subscription fee defined later) does not depend on APO index $n$. In this case, the contract design problem is significantly simplified, as the operator only needs to optimize one contract choice for all APOs.

[10] Note that $\delta_n$ can be negative, which is essentially a bonus from the operator to incentivize the APO to be a Bill.

[11] Here we use the approximation $\frac{1-\eta}{N-1} \approx \frac{1-\eta}{N}$ as $N$ is usually large.

**Table II:** Key Notations

| | |
|---|---|
| $\mathcal{K}$ | The set of APO types, $\mathcal{K} = \{1, 2, \ldots, K\}$ |
| $\theta_k$ | The Wi-Fi quality provided by the type-$k$ APOs |
| $N_k$ | The number of the type-$k$ APOs |
| $\mathcal{B}$ | The set of APO types choosing Bills |
| $\mathcal{L}$ | The set of APO types choosing Linus |
| $N_B$ | The number of Bills, $N_B = \sum_{k\in\mathcal{B}} N_k$ |
| $N_L$ | The number of Linus, $N_L = \sum_{k\in\mathcal{L}} N_k$ |
| $N_A$ | The number of Aliens, $N_A = aN$ |
| $a$ | The ratio between Aliens and APOs, $a \geq 0$ |
| $\eta$ | The probability of an APO staying at home |

design the proper price and subscription fee to optimize her revenue. This motivates us to consider the approach of *contract design*, where the operator will provide a range of choices (*contract items*) for each APO to choose [15].

More specifically, each contract item is a combination of price and subscription fee, denoted by $\phi \triangleq (p, \delta)$. A special combination $\phi_0 = (0, 0)$ indicates the membership choice of Linus. Based on the revelation principle [20], the operator needs to design one contract item for each type of APOs, to induce them to reveal their types truthfully. Hence a *contract* is such a list of contract items, denoted by

$$\Phi = \{\phi_k \triangleq (p_k, \delta_k),\ k \in \mathcal{K}\}, \tag{1}$$

where $\phi_k$ denotes the contract item designed for the type-$k$ APOs. Note that the same type APOs will choose the same contract item, as they have the same preference, which will be justified later in Section III-B.

A contract is *feasible* if any APO is willing to choose the contract item designed for his type (i.e., achieving the maximum payoff under the item designed for his type). Mathematically, a contract is feasible if it satisfies Incentive Compatibility (IC) constraint and Individual Rationality (IR) constraint [15]. We will analyze the contract feasibility in Section IV systematically.

### A. Operator Payoff

We now characterize the network operator's profit under a given feasible contract $\Phi = \{\phi_k, k \in \mathcal{K}\}$, where each APO chooses the contract item designed for his type. Let $\mathcal{L} \triangleq \{k \in \mathcal{K} \mid \phi_k = \phi_0\}$ denote the set of APO types choosing Linus, and $N_L = \sum_{k\in\mathcal{L}} N_k$ denote the number of Linus. Let $\mathcal{B} \triangleq \{k \in \mathcal{K} \mid \phi_k \neq \phi_0\}$ denote the set of APO types choosing Bills, and $N_B = \sum_{k\in\mathcal{B}} N_k$ denote the number of Bills. For more clarity, we list the key notations in Table II.

*1) Profit Achieved from a Bill's AP:* If a type-$k$ APO is a Bill who chooses $\phi_k = (p_k, \delta_k)$, the operator's profit achieved from this AP is simply the subscription fee $\delta_k$.

*2) Profit Achieved from a Linus's AP:* If a type-$k$ APO is a Linus who chooses $\phi_k = \phi_0$, the network operator will charge other users (Bills and Aliens) accessing this AP directly, at a price $p_0$ per unit Wi-Fi connection time. Hence, the network operator's profit achieved from this AP is the payment from Bills and Aliens accessing this AP.

Notice that the expected number of Bills accessing this AP is $\sum_{i\in\mathcal{B}} \frac{1-\eta}{N} N_i = \frac{1-\eta}{N} N_B$, and the expected number of Aliens accessing this AP is $\frac{N_A}{N}$. We further denote the average Wi-Fi connection time of a Bill or Alien on this (type-$k$) AP under the price $p$ as $d_k(p)$. Intuitively, $d_k(p)$ reflects a Bill's or Alien's demand for using a type-$k$ AP, which is non-negative, monotonically decreasing with $p$, and monotonically increasing with $\theta_k$. For example, one widely-used demand function in literatures (e.g., [21], [22]) is:[12]

$$d_k(p) = \frac{1}{1+p/\theta_k}. \tag{2}$$

Thus, the total profit from this type-$k$ Linus's AP is:

$$\left(\frac{1-\eta}{N} N_B + \frac{N_A}{N}\right) p_0 d_k(p_0), \tag{3}$$

where $p_0$ is the price charged by the operator on Linus's AP.

*3) Total Profit:* To summarize, the network operator's profit achieved from all APs can be computed as follows:

$$\sum_{k\in\mathcal{L}} \left[N_k \left(\frac{1-\eta}{N} N_B + \frac{N_A}{N}\right) p_0 d_k(p_0)\right] + \sum_{k\in\mathcal{B}} N_k \delta_k. \tag{4}$$

### B. APO Payoff

We now define the payoff of each APO under a feasible contract $\Phi = \{\phi_k, k \in \mathcal{K}\}$.

*1) Payoff of a Linus:* If a type-$k$ APO chooses to be a Linus (i.e., $\phi_k = \phi_0$), he neither pays for using other APs, nor gains from sharing his own AP. Hence, his payment is zero, and his payoff is simply the difference between utility and cost,

$$u_k(\phi_0) = u_k(0, 0) = \eta U_{\text{H}} + (1-\eta) U_{\text{R}} - C_{\text{S}}, \tag{5}$$

where $U_{\text{H}}$ is the utility when staying at home and accessing his own AP, $U_{\text{R}}$ is the utility when roaming outside and accessing other APs, and $C_{\text{S}}$ is the cost of serving other users.

*2) Payoff of a Bill:* If a type-$k$ APO chooses to be a Bill (i.e., $\phi_k \neq \phi_0$), his payment consists of (i) the revenue earned at his own AP, (ii) the payment for using other APs, and (iii) the subscription fee to the network operator.

First, the revenue earned at his own AP equals the payment of other Bills and Aliens accessing his AP. The average Wi-Fi connection time of a Bill or Alien on this (type-$k$) AP under the price $p_k$ is $d_k(p_k)$. The average payment of a Bill or Alien for accessing this AP is

$$g_k(p_k) \triangleq p_k d_k(p_k), \tag{6}$$

which monotonically increases with $\theta_k$, as the demand $d_k(p_k)$ monotonically increases with $\theta_k$. The expected number of paying users (including the other $N_k - 1$ Bills with type-$k$, all Bills with other types, and all Aliens) accessing this AP can be computed as follows:

$$\begin{aligned} \omega &\triangleq \left[\frac{1-\eta}{N}(N_k - 1) + \sum_{i\in\mathcal{B}, i\neq k} \frac{1-\eta}{N} N_i + \frac{1}{N} N_A\right] \\ &= \frac{1-\eta}{N}(N_B - 1) + \frac{N_A}{N}. \end{aligned} \tag{7}$$

Note that $w$ is same for all Bills, and does not depend on $k$. Thus, the revenue earned at his own AP is $\omega g_k(p_k)$.

Second, the expected payment of this (type-$k$) APO for accessing other APs includes (i) the payment for accessing the other $N_k - 1$ Bills' APs with type-$k$, (ii) the payment for accessing all Bills' APs with other types, and (iii) the payment for accessing Linus' APs, which can be calculated as

$$\begin{aligned} \beta_k \triangleq \frac{1-\eta}{N} \Big[ & (N_k - 1) g_k(p_k) + \sum_{i\in\mathcal{B}, i\neq k} N_i g_i(p_i) \\ & + \sum_{j\in\mathcal{L}} N_j g_j(p_0) \Big]. \end{aligned} \tag{8}$$

[12] Note that our analysis is general and can be applied to other demands.

Third, the subscription fee to the network operator is $\delta_k$. Hence, a type-$k$ Bill APO's total revenue (when choosing $\phi_k$) is $\omega g_k(p_k) - \delta_k - \beta_k$. Accordingly, his payoff is[13]

$$u_k(\phi_k) = u_k(p_k, \delta_k) = \omega g_k(p_k) - \delta_k - \beta_k + \eta U_{\text{H}} + (1-\eta)U_{\text{R}} - C_{\text{S}}. \quad (9)$$

By (9), it is easy to see that *the payoff of an APO depends not only on his contract item choice, but also on other APOs' choices.* For example, the first term in $\omega$ depends on how many APOs choosing to be Bills, the second term in $\beta_k$ depends on how other Bills choosing prices, and the last term in $\beta_k$ depends on how many APOs choosing to be Linus. Such a strategy coupling makes our contract design problem very different from and much more challenging than traditional contract models in literatures [16]–[19].

To make the model more practical and to facilitate the later analysis, we make the following assumptions:

(a) $0 \le p_k \le p_{\max}$, $\forall k \in \mathcal{K}$;
(b) $g_k(p)$ increases with $p \in [0, p_{\max}]$, $\forall k \in \mathcal{K}$;
(c) $g_i(p') - g_i(p) > g_j(p') - g_j(p)$, for any types $i > j$ and prices $p' > p$ (where $p, p' \in [0, p_{\max}]$).

Assumption (a) states that all prices are upper-bounded by $p_{\max}$, considering the potential competition from cellular.[14] Assumption (b) implies that the Wi-Fi connection demand of APOs (on other APs) is inelastic with price $p$ [23]. Assumption (c) states that when increasing the price by a same amount, the increased payment achieved by a higher type APO (i.e., with a higher quality) is larger than that by a lower-type APO. This is usually referred to as "*increasing differences*" in economics [25], and is satisfied for many widely-used demand functions, e.g., (2) and the one in [21] [22].

## IV. Feasibility of the Contract

The network operator needs to design a feasible contract to APOs, such that each APO chooses the contract item designed for his type. In this section, we discuss the sufficient and necessary conditions for a contract to be feasible.

Formally, a contract is feasible, if and only if (iff) it satisfies the following Incentive Compatibility (IC) and Individual Rationality (IR) constraints defined below.

**Definition 1** (Incentive Compatibility – IC)**.** *A contract is incentive compatible, if each type-$k$ APO maximizes his payoff when choosing the contract item $\phi_k$ intended for his type $k$ than any other contract item $\phi_i, i \neq k$, i.e.,*

$$u_k(\phi_k) \ge u_k(\phi_i), \quad \forall k, i \in \mathcal{K}. \quad (10)$$

**Definition 2** (Individual Rationality – IR)**.** *A contract is individual rational, if each type-$k$ APO achieves a non-negative payoff when choosing the item $\phi_k$ intended for his type k, i.e.,*

$$u_k(\phi_k) \ge 0, \forall k \in \mathcal{K}. \quad (11)$$

Obviously, if a contract satisfies the IC and IR constraints, each APO will choose the contract item designed for his type. Hence, we can equivalently say that he truthfully reveals his type (private information) to the operator.

[13] In the rest of the paper, we set $\eta U_{\text{H}} + (1-\eta)U_{\text{R}} - C_{\text{S}} = 0$.

[14] The Wi-Fi access service is usually a complement to the cellular access service [23], and hence a higher-priced Wi-Fi service is less competitive and may drive all APOs to the cellular network when roaming [24].

### A. Necessary Conditions

We now characterize the necessary conditions for a feasible contract. For notational convenience, we rewrite a contract as $\Phi = \{(0,0), k \in \mathcal{L}\} \cup \{(p_k, \delta_k), k \in \mathcal{B}\}$, where $\mathcal{L}$ and $\mathcal{B}$ are the type sets of Linus and Bills, respectively.

**Lemma 1.** *If a contract $\Phi = \{(0,0), k \in \mathcal{L}\} \cup \{(p_k, \delta_k), k \in \mathcal{B}\}$ is feasible, then*

$$p_i > p_j \Longleftrightarrow \delta_i > \delta_j, \quad \forall i, j \in \mathcal{B}.$$

*Proof:* We prove this lemma using the IC constraint.

$\Rightarrow$: We first prove that if $p_i > p_j$, then $\delta_i > \delta_j$.

For any type-$j$ ($\forall j \in \mathcal{B}$) APO, the following IC constraint must be satisfied:

$$u_j(p_j, \delta_j) \ge u_j(p_i, \delta_i), \forall i \in \mathcal{B}$$

which is

$$\omega g_j(p_j) - \delta_j - \beta_j \ge \omega g_j(p_i) - \delta_i - \beta_j.$$

This implies that

$$\delta_i - \delta_j \ge \omega(g_j(p_i) - g_j(p_j)) > 0.$$

$\Leftarrow$: We now prove that if $\delta_i > \delta_j$, then $p_i > p_j$.

For any type-$i$ ($\forall i \in \mathcal{B}$) APO, the following IC constraint must be satisfied:

$$u_i(p_i, \delta_i) \ge u_i(p_j, \delta_j), \forall j \in \mathcal{B}$$

which is

$$\omega g_i(p_i) - \delta_i - \beta_i \ge \omega g_i(p_j) - \delta_j - \beta_i.$$

This implies that

$$g_i(p_i) - g_i(p_j) \ge \frac{\delta_i - \delta_j}{\omega} > 0.$$

Since $g_i(p)$ monotonically increases with $p$, we have

$$p_i > p_j.$$

■

Lemma 1 shows that in a feasible contract, a larger Wi-Fi access price corresponds to a larger subscription fee (for Bill).

**Lemma 2.** *If a contract $\Phi = \{(0,0), k \in \mathcal{L}\} \cup \{(p_k, \delta_k), k \in \mathcal{B}\}$ is feasible, then*

$$\theta_i > \theta_j \Longrightarrow p_i > p_j, \quad \forall i, j \in \mathcal{B}.$$

*Proof:* We prove this lemma by contradiction. Assume to the contrary that there exists $\theta_i > \theta_j$ and $p_i < p_j$.

Considering the IC constraints for the type-$i$ ($\forall i \in \mathcal{B}$) APOs and the type-$j$ ($\forall j \in \mathcal{B}$) APOs, we have

$$u_i(\phi_i) \ge u_i(\phi_j), \; u_j(\phi_j) \ge u_j(\phi_i),$$

which are equivalent to

$$\omega g_i(p_i) - \delta_i - \beta_i \ge \omega g_i(p_j) - \delta_j - \beta_i,$$
$$\omega g_j(p_j) - \delta_j - \beta_j \ge \omega g_j(p_i) - \delta_i - \beta_j.$$

Combining the above two inequations, we have

$$g_i(p_i) - g_i(p_j) \ge g_j(p_i) - g_j(p_j).$$

This contradicts with $p_i < p_j$, since $g_i(p') - g_i(p) > g_j(p') - g_j(p)$, for any prices $p' > p$ . ■

Lemma 2 shows that in a feasible contract, a higher type APO will be designed with a higher Wi-Fi access price.

**Lemma 3.** *If a contract $\Phi = \{(0,0), k \in \mathcal{L}\} \cup \{(p_k, \delta_k), k \in \mathcal{B}\}$ is feasible, then there exists a critical APO type $m \in \{1, 2, \ldots, K+1\}$, such that $k \in \mathcal{L}$ for all $k < m$ and $k \in \mathcal{B}$ for all $k \geq m$, i.e.,*

$$\mathcal{L} = \{1, 2, \ldots, m-1\}, \mathcal{B} = \{m, m+1, \ldots, K\}. \quad (12)$$

*Proof:* We prove this lemma by contradiction. Assume to the contrary that the subscripts $k$ in the Bill set $\mathcal{B}$ is not consecutive.

Suppose the type-$j$ APOs choose to be Linus, but the type-$i$ ($i < j$) APOs choose to be Bills.

Considering the IC constraint for the type-$j$ (Linus) APO, we have

$$u_j(p_i, \delta_i) \leq u_j(0, 0),$$

equivalently,

$$\mu g_j(p_i) - \delta_i - \nu \leq 0.$$

Then for the type-$i$ ($i < j$) (Bill) APO, we have

$$u_i(p_i, \delta_i) = \mu g_i(p_i) - \delta_i - \nu < \mu g_j(p_i) - \delta_i - \nu = u_j(p_i, \delta_i) \leq 0$$

which contradicts with the IC constraint for the type-$i$ (Bill) APO.

Hence, for a feasible contract $\Phi = \{\phi_k, \forall k \in \mathcal{K}\}$, there exists a critical AP type $m$ such that $k \in \mathcal{L}$ for all $k < m$, and $k \in \mathcal{B}$ for all $k \geq m$. ■

Lemma 3 shows that there exists a critical APO type: *All APOs with types lower than the critical type will choose to be Linus, and all APOs with types higher than or equal to the critical type will choose to be Bills.* In particular, all APOs are Bills when $m = 1$, and all APOs are Linus when $m = K+1$.

### B. Sufficient and Necessary Conditions

We can show that the above necessary conditions together are also sufficient for a contract to be feasible. For notational convenience, we denote $\mu \triangleq \frac{1-\eta}{N} N_B + \frac{N_A}{N}$ as the expected number of Bills and Aliens connecting to an AP, and $\nu \triangleq \frac{1-\eta}{N} \cdot \left(\sum_{i \in \mathcal{B}} N_i g_i(p_i) + \sum_{i \in \mathcal{L}} N_i g_i(p_0)\right)$ as the expected payment of an APO when choosing to be Bill.

**Theorem 1** (Feasible Contract)**.** *A contract $\Phi = \{\phi_k, k \in \mathcal{K}\}$ is feasible, if and only if the following conditions hold:*[15]

$$\mathcal{L} = \{1, \ldots, m-1\}, \mathcal{B} = \{m, \ldots, K\}, \ m \in \{1, ..., K+1\}, \quad (13)$$

$$0 \leq p_0 \leq p_{\max}, 0 \leq p_i \leq p_k \leq p_{\max}, \ \forall k, i \in \mathcal{B}, i < k, \quad (14)$$

$$\delta_k \geq \mu g_j(p_k) - \nu, \ \forall k \in \mathcal{B}, \forall j \in \mathcal{L}, \quad (15)$$

$$\delta_k \leq \omega g_k(p_k) - \beta_k, \ \forall k \in \mathcal{B}, \quad (16)$$

$$\omega\left(g_i(p_k) - g_i(p_i)\right) \leq \delta_k - \delta_i \leq \omega\left(g_k(p_k) - g_k(p_i)\right), \quad \forall k, i \in \mathcal{B}, i < k. \quad (17)$$

*Proof:* The first condition has been proved by Lemma 3. We prove the remaining conditions by mathematical induction.

[15] Here, (15) can be derived from the IC constraints of Linus APOs, i.e., a type-$j$ (Linus) APO will achieve a non-positive payoff, if he chooses any contract item designed for Bills, i.e., $(p_k, \delta_k), k \in \mathcal{B}$. Similarly, (16) and (17) can be derived from the IR and IC constraints of Bill APOs.

We denote $\Phi(m-1+n)$ as a subset of $\Phi$ which contains the first $m-1+n$ contract items in $\Phi$, i.e., $\Phi(m-1+n) = \{(0,0), k \in \mathcal{L}\} \cup \{(p_k, \delta_k), k \in \mathcal{B}\}$ with $|L| = m-1$ and $|B| = n$. Let $\Phi(m-1+n)$ be a contract for the network which contains the first $m-1+n$ types of AP owners of the original network.

Obviously, $m = K+1$ indicates that all AP owners choose to be Linus. In the following proof, we consider the case where $m \leq K$.

We first verify that $\Phi(m) = \{(0,0), \forall k = 1, 2, \cdots, m-1\} \cup \{(p_m, \delta_m)\}$ is feasible. The conditions for a contract to be feasible are the IR and IC constraints for all APOs. Obviously,

$$\omega g_m(p_m) - \delta_m - \delta_m \geq 0,$$
$$\mu g_j(p_m) - \delta_m - \nu \leq 0, \ \forall j \in \mathcal{L}$$

Thus $\Phi(m)$ is a feasible contract.

We then show that if $\Phi(m-1+k) = \{(0,0), \forall i = 1, \cdots, m-1\} \cup \{(p_i, \delta_i), \forall i = m, \cdots, m-1+k\}$ ($k \geq 1$) is a feasible contract, $\Phi(m+k)$ is also feasible. To achieve this, we need to prove that (I) for the new type $\theta_{m+k}$, the IC and IR constraints are satisfied, i.e.,

$$\begin{cases} \omega g_{m+k}(p_{m+k}) - \delta_{m+k} - \beta_{m+k} \\ \quad \geq \omega g_{m+k}(p_i) - \delta_i - \beta_{m+k}, \ \forall i = m, \ldots, m-1+k, \\ \omega g_{m+k}(p_{m+k}) - \delta_{m+k} - \beta_{m+k} \geq 0. \end{cases}$$

and (II) for the existing types $\theta_1, \ldots, \theta_{m-1}, \theta_m, \cdots, \theta_{m-1+k}$, the IC constraints are still satisfied in the presence of type $\theta_{m+k}$, i.e.,

$$\begin{cases} \mu g_j(p_{m+k}) - \delta_{m+k} - \nu \leq 0, \ \forall j = 1, \ldots, m-1, \\ \omega g_i(p_i) - \delta_i - \beta_i \geq \omega g_i(p_{m+k}) - \delta_{m+k} - \beta_i, \\ \quad \forall i = m, \ldots, m-1+k. \end{cases}$$

(Proof of I) Now we prove the IC and IR constraints for the new type $\theta_{m+k}$.

Since $p_{m+k} \geq p_i$ and $\delta_{m+k} - \delta_i \leq \omega\left(g_{m+k}(p_{m+k}) - g_{m+k}(p_i)\right), \ \forall i = m, \ldots, m-1+k$, we have

$$\omega g_{m+k}(p_{m+k}) - \delta_{m+k} - \beta_{m+k}$$
$$\geq \omega g_{m+k}(p_i) - \delta_i - \beta_{m+k}, \ \forall i = m, \ldots, m-1+k,$$

which proves that the IC constraint is satisfied.

Since

$$\delta_{m+k} \leq \omega g_{m+k}(p_{m+k}) - \delta_{m+k}$$

we have

$$\omega g_{m+k}(p_{m+k}) - \delta_{m+k} - \delta_{m+k} \geq 0$$

which proves that the IR constraint is satisfied.

(Proof of II) Here we prove the IC constraints for the existing types $\theta_1, \ldots, \theta_{m-1}, \theta_m, \cdots, \theta_{m-1+k}$, in the presence of $\theta_{m+k}$.

Since

$$\delta_{m+k} \geq \mu g_j(p_{m+k}) - \nu, \forall j = 1, \cdots, m-1,$$

we have

$$\mu g_j(p_{m+k}) - \delta_{m+k} - \nu \leq 0, \ \forall j = 1, \ldots, m-1,$$

which proves that the IC constraints for the Linus AP owners in the set $\mathcal{L} = \{1, \cdots, m-1\}$ are satisfied.

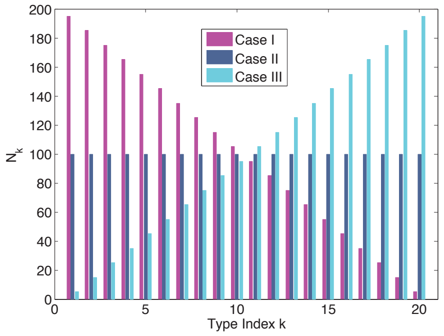


**Figure 2:** Distribution of APOs (Case I: low type APO dominant; Case II: uniform distribution; Case III: high type APO dominant)

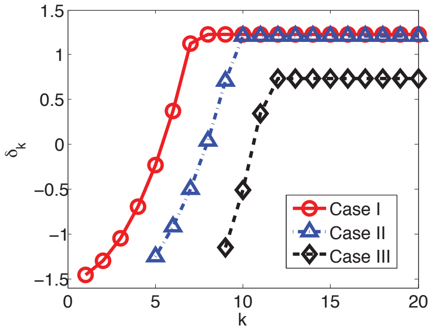


**Figure 3:** Subscription Fees for Bills (Case I: low type APO dominant; Case II: uniform distribution; Case III: high type APO dominant)

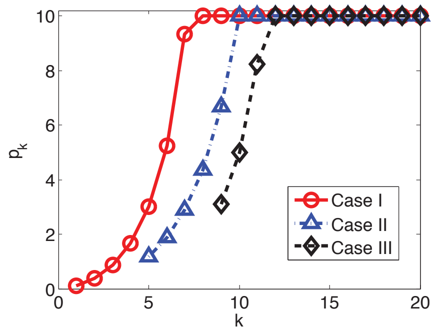


**Figure 4:** Wi-Fi Access Prices for Bills (Case I: low type APO dominant; Case II: uniform distribution; Case III: high type APO dominant)

Since $p_i \leq p_{m+k}$ and $\delta_{m+k} - \delta_i \geq \omega\left(g_i(p_{m+k}) - g_i(p_i)\right)$, $\forall i = m, \ldots, m-1+k$, we have

$$\omega g_i(p_i) - \delta_i - \beta_i \geq \omega g_i(p_{m+k}) - \delta_{m+k} - \beta_i,$$
$$\forall i = m, \ldots, m-1+k,$$

which proves the IC constraints for the Bill AP owners in the set $\mathcal{B} = \{m, \cdots, m-1+k\}$.

Up to present, we have proved that (i) $\Phi(m)$ is feasible, and (ii) if $\Phi(m-1+k)$ (for $k \geq 1$) is feasible, then $\Phi(m+k)$ is feasible. It follows that $\Phi = \Phi(K)$ is feasible. ∎

## V. Simulation Results

In this section, we numerically study the contract-based Wi-Fi sharing mechanism. We simulate a network with $N = 2000$ APOs, which are classified into $K = 20$ types, associated with Wi-Fi qualities $\boldsymbol{\theta} = \{1, 2, \ldots, 20\}$. Each APO stays at home with probability $\eta = 0.5$, and the maximal unit price is $p_{\max} = 10$. We study the contracts under three different APO type distributions, as shown in Figure 2, where Case I is *low type APO dominant*, Case II is *uniform distribution*, and Case III is *high type APO dominant*. The demand to a type-$k$ AP (with quality $\theta_k$) is $d_k(p) = \frac{1}{1+p/\theta_k}$ defined in (2).

### A. Contract

In this subsection, we assume that the ratio between Aliens and APOs is $a = 0.25$. We study the feasible contracts under the three cases. The contract item $\phi_0 = (0, 0)$ for Linus is implicitly included in the contract. We numerically find the contract which yields the maximum profit for the operator, i.e., the optimal contract. Figure 3 and Figure 4 show the optimal contract items for Bills, i.e., the best subscription fees and the optimal prices, under the three cases. Those APO types that do not have representations in the Figures are those choosing to be Linus. Specifically, in Case I, no APO chooses Linus; in Case II, the first 4 types of APOs choose Linus; and in Case III, the first 8 types of APOs choose Linus.

**Observation 1.** *More types of APOs choose to be Linus in Case III where the population is high type APO dominant.*

The reason is that higher type APOs charge higher prices (and many of them charge the highest price $p_{\max}$). In the high type APO dominant case (Case III), on average it is more expensive to use other APs as a Bill. As a result, a lower type APO's payoff as a Bill is smaller due to the higher payment on other APs. Hence, more APOs will choose to be Linus.

**Observation 2.** *The subscription fee can be negative (Fig. 3).*

As a lower type APO charges a smaller price at his own AP but pays higher prices to higher type APOs, his payoff can be negative without proper compensation from the operator. Hence, the network operator will give bonus to the lower type APO for joining the community and sharing Wi-Fi.

**Observation 3.** *The subscription fee is the highest when the population is low type APO dominant (Figure 3).*

In the low type APO dominant case (Case I), each type APO's payoff as a Bill is higher comparing with other cases, since lower type APOs charge smaller prices and on average it is less expensive to use other APs. Hence, the network operator will set higher subscription fees to extract more profit.

**Observation 4.** *The price is the highest when the population is low type APO dominant (Figure 4).*

The highest price (Figure 4) corresponds to the highest subscription fee (Figure 3) in feasible contracts. An APO will choose a higher price since he needs to pay a higher subscription fee to the operator.

Figure 5 plots each APO's payoff in the three cases.

**Observation 5.** *The higher type Bill APO gains more payoff than the lower type Bill APO.*

The reason is that the higher type APO receives more revenue on his own AP and pays less to other APs, hence achieves a payoff higher than that of a lower type APO.

**Observation 6.** *Each Bill APO's payoff is the largest when the population is low type APO dominant.*

In the low type APO dominant case (Case I), on average each Bill APO pays less to use other APs. This turns out to the dominant factor in determining the payoff, despite of the fact that every Bill APO pays more to the operator (Observation 3) and charges other users more (Observation 4).

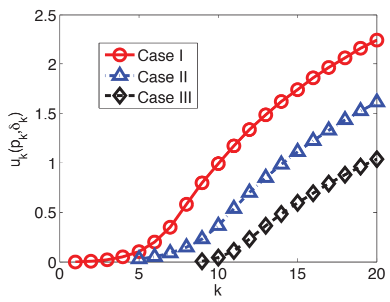


**Figure 5:** Payoffs of Bill APOs

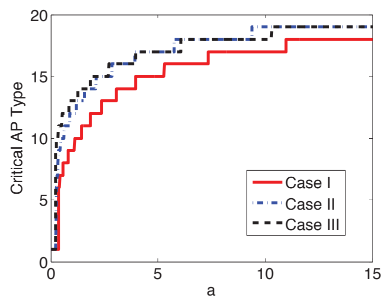


**Figure 6:** Critical AP Type

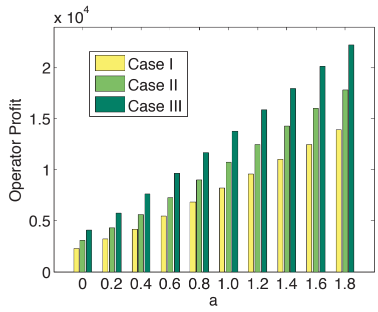


**Figure 7:** Profit of the Network Operator

### B. Impact of Alien Number

We study how the ratio $a = \frac{N_A}{N}$ affects the contract. Figure 6 shows the relationship between $a$ and the critical AP type $m$ in the three cases.

**Observation 7.** *The number of APO types choosing to be Linus increases with the number of Aliens.*

Recall the operator's total profit defined in (4), i.e.,

$$\sum_{k\in\mathcal{L}} \left[N_k \left((1-\eta)\frac{N_B}{N} + a\right) g_k(p_0)\right] + \sum_{k\in\mathcal{B}} N_k\delta_k.$$

The operator sets the same price on all Linus APs as $p_0 = p_{max}$. As $a$ increases, the term $ag_k(p_{max})$ becomes increasingly important for the operator's total revenue. Hence, in the optimal contract, the operator increases $m$ to gain more revenue from Aliens' Wi-Fi access on Linus' APs.

Figure 7 shows the relationship between $a$ and the network operator's profit in the three cases under different ratio $a$, or equivalently, the number of Aliens $N_A = aN$ since $N$ is fixed.

**Observation 8.** *The network operator's profit increases with the number Aliens, and the profit is largest when the population is high type APO dominant.*

The operator's profit increases with $a$, as more Aliens can bring more revenue. The operator's profit is the largest in the high type APO dominant case (Case III), because as the quality of communication increases, users generate more demand in the network, and hence the operator gains more revenue.

## VI. Conclusion

In this paper, we proposed a novel contract mechanism for crowdsourced wireless community networks under incomplete information. Different from existing contract mechanisms designed for wireless networks, the proposed contract considers the unique coupling among users' contract item choices, hence is much more complicated to design. We analyzed the feasibility of the proposed contract systematically, and provided simulation results to illustrate the contract and the profit of the operator. In our working paper, we studied the optimal contract which yields the maximum profit for the operator. In the future, it is important to study a more general case with heterogenous mobility pattern and asymmetric user demands. It is interesting to study the impact of network congestion when considering the limited network capacity of each Wi-Fi AP.

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