Coopetition between LTE Unlicensed and Wi-Fi: A Reverse Auction with Allocative Externalities

Haoran Yu*, George Iosifidis†, Jianwei Huang*, and Leandros Tassiulas†
*Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong
†Department of Electrical Engineering, and the Yale Institute for Network Science, Yale University

Abstract—**Motivated by the recent efforts in extending LTE to the unlicensed spectrum, we propose a novel spectrum sharing framework for the *coopetition* (*i.e.*, cooperation and competition) between LTE and Wi-Fi in the unlicensed band. Basically, the LTE network chooses to work in one of the two modes: in the *competition mode*, it randomly accesses an unlicensed channel, and interferes with a Wi-Fi access point; in the *cooperation mode*, it onloads a Wi-Fi access point's traffic in exchange for the full access of the corresponding channel. Because the LTE network works in an interference-free manner in the *cooperation mode*, it can achieve a much larger total data rate (comparing to the *competition mode*) to serve both its own users and the Wi-Fi users under proper channel conditions. To achieve the maximum potential of this novel coopetition framework, we design a reverse auction mechanism, where the LTE provider is the auctioneer (buyer), and the Wi-Fi access point owners (APOs) are the bidders who compete to sell their channels to the LTE provider. An APO's bid indicates the data rate that it would like the LTE provider to offer in the *cooperation mode*. We show that the auction involves the *allocative externalities*, *i.e.*, the cooperation between the LTE provider and an APO benefits other APOs who are not directly involved in this cooperation. As a result, a particular APO's bidding strategy is affected by its belief about other APOs' bidding strategies. This makes our analysis much more challenging than that of the standard second-price auction, where bidding truthfully is a weakly dominant strategy. We characterize the APOs' unique equilibrium bidding strategies, and analyze the LTE provider's optimal reserve rate that maximizes its payoff for a general APO type distribution. Our analysis shows that only when the LTE throughput exceeds a threshold, the LTE provider will choose a reasonably large reserve rate to cooperate with the APOs; otherwise, it will restrict the reserve rate to a small value and work in the *competition mode*.**

I. Introduction

A. Motivations

The proliferation of mobile devices has led to an explosion of the global mobile traffic, which is estimated to grow to 24.2 exabytes per month by 2019 [1]. To accommodate the rapidly growing mobile traffic, 3GPP has been working on the standards for LTE to operate in unlicensed 5GHz band [2]. By extending LTE to the unlicensed spectrum, the LTE provider can significantly expand its network capacity, and tightly integrate its control over the licensed and unlicensed bands [3]. Furthermore, since the LTE technology has an efficient framework of traffic management (*e.g.,* congestion control), it is capable of achieving a much higher spectral efficiency than Wi-Fi networks in the unlicensed spectrum if there is no competition between these two technologies [4]. Key market players, such as AT&T, Verizon, T-Mobile, Qualcomm, and Ericsson, have already demonstrated the great potential of LTE in the unlicensed band through experiments [4], and have formed several coalitions (*e.g.*, LTE-U Forum [5] and EVOLVE [6]) to promote this promising technology.

A key technical challenge for LTE working in the unlicensed spectrum is that it can significantly degrade the Wi-Fi network performance if there is no effective co-channel interference avoidance mechanism. To address this issue, industries have proposed two major mechanisms for LTE/Wi-Fi coexistence: (a) Qualcomm's carrier-sensing adaptive transmission (CSAT) scheme [7], where the LTE transmission follows a periodic on/off pattern, creating the interference-free zones for Wi-Fi during certain periods, and (b) Ericsson's "Listen-Before-Talk" (LBT) scheme [8], where LTE transmits only when it senses the channel being idle for at least certain duration. However, the practical testing results based on the above mechanisms are not up to prior expectations. In particular, a series of experiments by Google revealed that both mechanisms impact severely the performance of Wi-Fi [9]: for the CSAT mechanism, since Wi-Fi is not designed in anticipation of LTE's activity, it cannot respond well to LTE's on-off cycling, and its transmission is severely affected; for the LBT mechanism, it is hard to choose the proper backoff time and transmission length for LTE to fairly coexist with Wi-Fi. Therefore, a harmonious coexistence between LTE and Wi-Fi is still open for the discussion, which motivates our study in this work.

B. Contributions

We propose a novel framework for LTE's operation in the unlicensed spectrum: it works in either the *competition mode* or the *cooperation mode*. For the *competition mode*, the LTE network simply coexists with the Wi-Fi networks, and shares the channel. For the *cooperation mode*, the LTE network fully occupies a Wi-Fi access point's channel and the corresponding Wi-Fi access point does not transmit, which avoids the co-channel interference and hence generates a high throughput. As a compensation, the LTE network allocates some throughput to the access point's users based on the access point's request. Since LTE usually has a much higher spectral

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efficiency than Wi-Fi, such a cooperation can lead to a *win-win* situation for both networks under proper network conditions.

In order to maximize the performance of both networks and achieve the most efficient utilization of the spectrum, we need to determine *the conditions for the cooperation and also the amount of Wi-Fi traffic that the LTE network should serve*. These questions lie in the heart of this solution and become very challenging when there are more than one Wi-Fi network in range. The problem is further complicated as there is no centralized decision maker in such a system and every network (LTE or Wi-Fi) wants to maximize its own throughput.

To address these issues, in the first part of our work, we introduce a reserve auction where the LTE provider is the auctioneer (buyer) and wants to fully obtain the channel from one of the Wi-Fi access point owners (APOs).[1] The LTE provider announces the maximum data rate (*i.e.,* reserve rate) that it is willing to allocate to serve the users of the winning APO. Then the APOs report whether they are willing to cooperate and what are the data rates that they request from the LTE provider. If no APO wants to cooperate, the LTE network works in the *competition mode*, and randomly accesses an access point's channel; otherwise, it works in the *cooperation mode*. This is a particular challenging auction since it induces the *positive allocative externalities* [10]: the cooperation between the LTE provider and an APO benefits other APOs that are not involved in this collaboration, because the other APOs can avoid the potential interference generated by the LTE network under the *competition mode*.

In the second part of our work, we start to analyze the reverse auction by characterizing the APOs' bidding strategies under different possible values of the reserve rate, and prove that such bidding strategies are unique. Our study shows that for some APOs, the data rates they request from the LTE provider are lower than the rates they can obtain by themselves without the LTE's interference. Intuitively, such a low request motivates the LTE network to work in the *cooperation mode* rather than the *competition mode*. In the latter case, the APOs may receive even lower data rates due to the co-channel interference from the LTE network.

In the third part of our work, we further analyze the reverse auction by computing the optimal reserve rate that maximizes the LTE network's payoff. The LTE network's payoff has different function forms, depending on the region the reserve rate belongs to. We analyze the optimal reserve rate by jointly considering all the reserve rate regions. We show that when the LTE network's throughout exceeds a threshold, it will choose a reasonably large reserve rate and cooperate with some APOs; otherwise, it will restrict the reserve rate to a small value, and eventually work in the *competition mode*.

The main contributions of this work are as follows:

- *Proposal of the LTE/Wi-Fi coopetition framework:* We propose a solution for the efficient co-existence and cooperation of these two competitive technologies. Specifically, for the cooperation between LTE and Wi-Fi, LTE onloads Wi-Fi's traffic and generates a high total throughput by fully occupying the channel. Unlike current technical solutions, our proposal can leash the full potential of the novel LTE unlicensed technology, and foster its quick and widespread adoption.
- *Equilibrium analysis of the reverse auction with the allocative externalities:* We characterize the APOs' bidding strategies under a fixed reserve rate, and show that they are the unique bidding strategies at the equilibrium.
- *Characterization of the optimal reserve rate:* We analyze the reserve rate that maximizes the LTE network's payoff, and investigate its relation with the LTE throughput. Through simulation, we show that the optimal reserve rate is non-decreasing in the LTE throughput, non-increasing in the LTE's data rate discounting factor, and increasing in the APOs' data rate discounting factor.

[1]We consider one LTE network and multiple Wi-Fi access points, since the LTE network has a larger coverage than the Wi-Fi access points, and the Wi-Fi access points are already very popular and exist in many areas.

C. Related Work

Several recent works have studied the coexistence of LTE and Wi-Fi in the unlicensed spectrum. Zhang *et al.* in [11] discussed the typical deployment scenarios and the coexistence with Wi-Fi for LTE in the unlicensed spectrum. Li *et al.* in [12] applied stochastic geometry to characterize the main performance metrics (*e.g.,* SINR coverage probability) for the neighboring LTE and Wi-Fi networks in the unlicensed spectrum. Zhang *et al.* in [13] proposed a new MAC protocol that allows LTE to friendly coexist with Wi-Fi. However, these results did not consider the cooperation between LTE and Wi-Fi, which is the main focus of our paper.

In terms of the auction with the *allocative externalities*, the most relevant works are [10] and [14]. Jehiel and Moldovanu in [10] provided a systematic study of the second-price forward auction with the allocative externalities. They characterized the bidders' bidding strategies in the equilibrium for general payoff functions. However, they did not prove the uniqueness of the equilibrium strategies. Bagwell *et al.* in [14] studied a special example in the WTO system, where the retaliation rights were allocated through a first-price forward auction among different countries. The auction involves the positive allocative externalities, and the authors showed the uniqueness of the countries' bidding strategies. In contrast, our work considers a second-price reverse auction. Furthermore, in our problem, the bidders' equilibrium strategies have different expressions under different reserve rates announced by the auctioneer, which makes our analysis of the optimal reserve rate quite different from [10] and [14].

II. System Model

A. Basic Settings

We study the problem in a certain period (*e.g.*, one hour), and consider a scenario where there is a LTE small cell network and two Wi-Fi access points. The LTE small cell network is owned by a LTE provider, and the two Wi-Fi access points have different owners, *i.e.,* APO 1 and APO 2. We assume that the two Wi-Fi APOs occupy different unlicensed channels, *i.e.*, channel 1 and channel 2, so that they do not interfere with each other. The LTE small cell network has a larger coverage area than the Wi-Fi access points. Furthermore, it can work in either channel 1 or channel 2, and cause interference to the corresponding access point in the channel. Notice that we focus on the interaction between one LTE small cell network and two Wi-Fi APOs, and assume that the APOs occupy different channels. Such a simplified model helps us gain key insights into the proposed reverse auction framework.[2]

APOs' Rates: We use r_i $(i = 1, 2)$ to denote the total data rate that APO i can achieve to serve its users when it *fully* occupies channel i. Due to the uncertainty of the network conditions and users' demands, we assume that r_i is a continuous random variable drawn from interval $[r_{\min}, r_{\max}]$ $(r_{\min}, r_{\max} \geq 0)$, and follows a probability distribution function (PDF) $f(\cdot)$ and a cumulative distribution function (CDF) $F(\cdot)$.[3] We assume that $f(\cdot) > 0$ for all $r \in [r_{\min}, r_{\max}]$. Moreover, we assume that r_i is the private information of APO i,[4] while functions $f(\cdot)$ and $F(\cdot)$ are the common knowledge.

LTE's Dual Modes: For the LTE provider, we assume that when it *fully* occupies one channel (either channel 1 or channel 2), it has the same total data rate $R_{\text{LTE}} > 0$. The LTE provider can operate its network in one of the following modes:

(i) In the *competition mode*, the LTE provider randomly chooses channel i $(i = 1, 2)$ with an equal probability and coexists with APO i. The co-channel interference decreases both the data rates of the LTE provider and the corresponding APO. We use $\delta \in (0, 1)$ and $\eta \in (0, 1)$ to denote LTE's and the APO's data rate discounting factors, respectively;

(ii) In the *cooperation mode*, the LTE provider reaches an agreement with APO i $(i = 1, 2)$, where APO i stops transmission and the LTE provider fully occupies channel i. In this case, there is no co-channel interference, and the LTE provider's data rate is simply R_{LTE}. As a compensation, the LTE provider will serve APO i's users with a guaranteed data rate $r_{\text{pay}} \in [0, R_{\text{LTE}}]$. Which APO to cooperate and what the value r_{pay} should be will be determined through a reverse auction design in the next subsection.

B. Second-Price Reverse Auction Design

We design a second-price reverse auction, where the LTE provider is the auctioneer (buyer) and the APOs are the bidders (sellers). The private type of APO i is r_i (*i.e.*, the data rate when it fully occupies channel i), and APO i's item for sale is the right of fully occupying channel i. Since we assume that the LTE provider cannot occupy two channels at the same time, the LTE provider is only interested in obtaining one item from one of the APOs. Different from the standard reverse auction where the auctioneer pays the winner money to obtain the item, here the LTE provider serves the winning APO's users with the rate r_{pay} as the payment.

[2]For ease of exposition, we use "LTE provider" and "LTE network" interchangeably. Similarly, we use "APO" and "access point" interchangeably.

[3]We assume that r_1 and r_2 follow the same distribution, and hence both functions $f(\cdot)$ and $F(\cdot)$ are independent of index i. We will study problem with the non-identical variables r_1 and r_2 in our future work.

[4]The value of r_i can be estimated based on the historical information of users' activities.

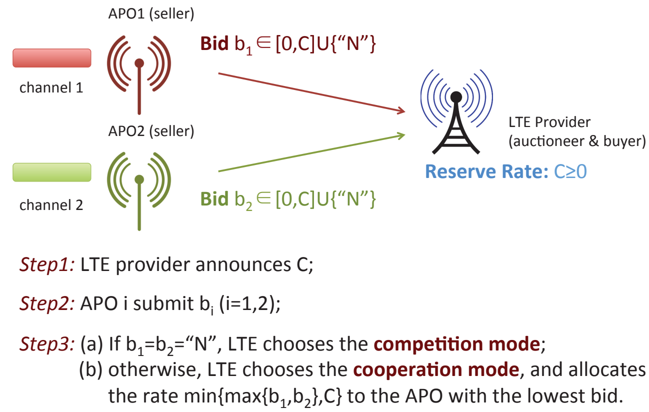

Fig. 1: Illustration of the Reverse Auction.

Reserve Rate and Bids: At the beginning of the auction, the LTE provider announces its reserve rate $C \in [0, \infty)$, which shows the maximum data rate it will accept to serve the winning APO's users. After observing the reserve rate C, APO i submits a bid $b_i \in [0, C] \cup \{\text{"N"}\}$: (a) $b_i \in [0, C]$ indicates the data rate that APO i requests the LTE provider to serve APO i's users; (b) $b_i = \text{"N"}$ means that APO i does not want to sell its item (*i.e.*, the right of fully occupying channel i) to the LTE provider.[5] We define the vector of APOs' bids as

$$\boldsymbol{b} \triangleq (b_1, b_2). \tag{1}$$

The auction design is illustrated in Fig. 1.

Auction Outcomes: Next we discuss the possible auction outcomes based on the different values of $\boldsymbol{b}$ and C.

We first define relations $\prec$ and $\preceq$ for the comparison between b_i and b_j $(i \neq j, i, j = 1, 2)$. We say $b_i \prec b_j$ if and only if (a) $b_i, b_j \in [0, C]$ and $b_i < b_j$, or (b) $b_i \in [0, C]$ and $b_j = \text{"N"}$. Furthermore, we say $b_i \preceq b_j$ if and only if $b_i \prec b_j$ or $b_i = b_j$. We also calculate $\min\{C, \text{"N"}\} = C$.

The auction has the following possible outcomes:

(a) When $b_1 \prec b_2$, then APO 1 is the winner, and leaves channel 1 to the LTE provider. The LTE provider works in the *cooperation mode* and fully occupies channel 1. Furthermore, the LTE provider serves APO 1's users with a rate $r_{\text{pay}} = \min\{b_2, C\}$, which is the second lowest rate from $\{b_1, b_2, C\}$, based on the rule of the second-price auction;

(b) When $b_2 \prec b_1$, then APO 2 is the winner, and leaves channel 2 to the LTE provider. The LTE provider works in the *cooperation mode*, fully occupies channel 2, and serves APO 2's users with a rate $r_{\text{pay}} = \min\{b_1, C\}$;

(c) When $b_1 = b_2 \neq \text{"N"}$, the LTE provider works in the *cooperation mode*, randomly chooses an APO with 0.5 probability to fully occupy its channel, and serves the APO's users with a rate $r_{\text{pay}} = b_1$;

(d) When $b_1 = b_2 = \text{"N"}$, the LTE provider works in the *competition mode*, randomly chooses one of the two channels with an equal probability, and shares the channel with the

[5]Intuitively, if the reserve rate C is very small, APO i is more likely to bid "N". In this case, APO i can achieve an expected data rate higher than that when onloading the users to the LTE provider.

corresponding APO.[6]

C. LTE provider's Payoff

Based on the summary of auction outcomes in the last subsection, we can write $r_{\rm pay}$ as a function of $\boldsymbol{b}$ and C:

$$r_{\rm pay}\left(\boldsymbol{b},C\right)=\begin{cases}0, & \text{if } b_1 = b_2 = \text{“N”},\\ \min\left\{\max\left\{b_1,b_2\right\},C\right\}, & \text{otherwise.}\end{cases} \tag{2}$$

We define the LTE provider's payoff as the data rate it can allocate to its own users, and compute it as:

$$\Pi^{\rm LTE}\left(\boldsymbol{b},C\right)=\begin{cases}\delta R_{\rm LTE}, & \text{if } b_1 = b_2 = \text{“N”},\\ R_{\rm LTE}-r_{\rm pay}\left(\boldsymbol{b},C\right), & \text{otherwise.}\end{cases} \tag{3}$$

There are two possible situations: (a) when both APOs bid “N”, the LTE provider works in the *competition mode*, and $\delta \in (0,1)$ captures the discount in the LTE provider's data rate due to the interference from the Wi-Fi APO in the same channel; (b) when at least one APO bids from $[0,C]$, the LTE provider works in the *cooperation mode*, fully occupies a channel, and obtains a total data rate of $R_{\rm LTE}$. Since the LTE provider needs to allocate a rate of $r_{\rm pay}\left(\boldsymbol{b},C\right)$ to the winning APO's users, its eventual payoff is $R_{\rm LTE}-r_{\rm pay}\left(\boldsymbol{b},C\right)$.

D. APOs' Payoffs and Allocative Externalities

We define APO i's ($i=1,2$) payoff as the data rate its users receive: when APO i cooperates with the LTE operator, these users are served by the LTE provider; otherwise, they are served by APO i. Based on the summary of auction outcomes in Section II-B and the definition of $r_{\rm pay}\left(\boldsymbol{b},C\right)$ in Section II-C, we summarize APO i's expected payoff as follows:

$$\Pi_i^{\rm APO}\left(b_i,b_j,C\right)=\begin{cases}r_{\rm pay}\left(\boldsymbol{b},C\right), & \text{if } b_i \prec b_j,\\ r_i, & \text{if } b_j \prec b_i,\\ \frac{1}{2}r_{\rm pay}\left(\boldsymbol{b},C\right)+\frac{1}{2}r_i, & \text{if } b_i = b_j \neq \text{“N”},\\ \frac{1+\eta}{2}r_i, & \text{if } b_i = b_j = \text{“N”}.\end{cases} \tag{4}$$

There are four possible situations: (a) when $b_i \prec b_j$, APO i wins the auction, and its users is served by the LTE provider with rate $r_{\rm pay}\left(\boldsymbol{b},C\right)$; (b) when $b_j \prec b_i$, APO j wins the auction, and the LTE provider fully occupies channel j. As a result, APO i can fully occupy its own channel i, and serve its users with rate r_i; (c) when $b_i = b_j \neq$ “N”, APO i and APO j become the winner with equal probabilities, and APO i's users receive rate $r_{\rm pay}\left(\boldsymbol{b},C\right)$ or rate r_i with equal probabilities; (d) when $b_i = b_j =$ “N”, there is no winner, and the LTE provider randomly chooses channel i and channel j with equal probabilities to coexist with the corresponding APO. With half probability, the LTE provider accesses channel j, and APO i has a data rate of r_i by fully occupying channel i; with half probability, the LTE provider accesses channel i, and APO i has a data rate of ηr_i. To conclude, the expected data rate that APO i's users receive is $\frac{1+\eta}{2}r_i$.

[6]We consider a specific protocol where the LTE provider randomly chooses the channels with an equal probability in the *competition mode*.

TABLE I: Main Notations

r_i	APO i's private valuation (type)
$r_{\min}$, $r_{\max}$	Lower and upper bounds of r_i, $i=1,2$
$f\left(\cdot\right)$, $F\left(\cdot\right)$	PDF and CDF of r_i, $i=1,2$
$R_{\rm LTE}$	LTE provider's data rate without interference
η	APOs' data rate discounting factor
δ	LTE provider's data rate discounting factor
C	LTE provider's reserve rate (*decision variable*)
b_i	APO i's bid (*decision variable*)
$\Pi^{\rm LTE}\left(\boldsymbol{b},C\right)$	LTE provider's payoff
$r_{\rm pay}\left(\boldsymbol{b},C\right)$	Data rate LTE allocates to the winning APO
$\Pi_i^{\rm APO}\left(b_i,b_j,C\right)$	APO i's payoff

We note that APO i does not win the auction under case $b_j \prec b_i$ and case $b_i = b_j =$ “N”, but it achieves different payoffs: it obtains a payoff of r_i when $b_j \prec b_i$, and achieves a smaller payoff of $\frac{1+\eta}{2}r_i$ when $b_i = b_j =$ “N”. That is to say, even if APO i does not win the auction, it is more willing to see APO j winning rather than losing the auction. This shows the *positive allocative externalities* of the auction, which make our problem substantially different from standard auction problems. In the equilibrium of a standard second-price auction, bidders bid truthfully according to their private values, regardless of other bidders' valuations. With the allocative externalities in our problem, when APO i evaluates its payoff once it loses the auction, it needs to consider whether APO j wins the auction or not. Hence, the distribution of APO j's valuation (type) affects APO i's strategy. As we will show in the following sections, this leads to a special structure of APOs' bidding strategies in the equilibrium.

We summarize the main notations in Table I. For the parameters and distributions that characterize the APOs, r_i is APO i's private information, and the remaining information, *i.e.*, $r_{\min}, r_{\max}, f\left(\cdot\right), F\left(\cdot\right)$, and η, is publicly known to both APOs and the LTE provider. For the parameters that characterize the LTE provider, *i.e.*, $R_{\rm LTE}$ and δ, as we will see in the later sections, they will not affect the APOs' strategies. Therefore, they can be either known or unknown to the APOs.

In Section III and Section IV, we analyze the APOs' bidding strategies in the equilibrium under different values of the LTE provider's reserve rate C. Based on the analysis in Section III and Section IV, we will consider the LTE provider's optimal reserve rate C^* in Section V and Section VI.

III. Equilibrium Analysis I: $C \in [r_{\min}, r_{\max})$

In this section, we assume that the reserve rate C is given from $[r_{\min}, r_{\max})$, and analyze the APOs' strategies.

A. Definition of Symmetric Bayesian Nash Equilibrium

We focus on finding the symmetric Bayesian Nash equilibrium (SBNE), which is defined as follows.

Definition 1. *Under a reserve rate C, a bidding strategy function* $b^*\left(r,C\right)$, $r \in [r_{\min}, r_{\max}]$, *constitutes a symmetric Bayesian Nash equilibrium if*

$$\begin{aligned}&\mathbb{E}_{r_j}\left\{\Pi_i^{\rm APO}\left(\left(b^*\left(r_i,C\right),b^*\left(r_j,C\right)\right),C\right)|r_i\right\} \geq\\ &\mathbb{E}_{r_j}\left\{\Pi_i^{\rm APO}\left(\left(s_i,b^*\left(r_j,C\right)\right),C\right)|r_i\right\},\end{aligned} \tag{5}$$

for all $s_i \in [0, C] \cup \{\text{"N"}\}$*, all* $r_i \in [r_{\min}, r_{\max}]$*, and all* $i \neq j, i, j = 1, 2$.

Since it is the symmetric equilibrium, APO i and APO j apply the same bidding strategy function b^* in the equilibrium. The left hand side of inequality (5) stands for APO i's expected payoff when it bids $b^*(r_i, C)$. The expectation is taken with respect to APO j's type r_j, which is unknown to APO i. Inequality (5) implies that APO i ($i = 1, 2$) cannot improve its expected payoff by unilaterally changing its bid from $b^*(r_i, C)$ to any $s_i \in [0, C] \cup \{\text{"N"}\}$.

B. Unique Form of Symmetric Bayesian Nash Equilibrium

In this section, we show the unique form of bidding strategy that constitutes an SBNE for the reverse auction. We first introduce the following lemma.

Lemma 1. *The following equation admits at least one solution* r *in* $(C, r_{\max})$*:*

$$\frac{1}{2}\left(F(r) - F(C)\right)(C - r) + (1 - F(r))\left(C - \frac{1+\eta}{2} r\right) = 0, \tag{6}$$

where $F(\cdot)$ *is the CDF of random variable* r_i*,* $i = 1, 2$*. We denote the solutions* r *in* $(C, r_{\max})$ *as* $r_1^t, r_2^t, \ldots, r_M^t$*, where* $M = 1, 2, \ldots,$ *is the number of solutions.*

Based on the definition of $r_1^t, r_2^t, \ldots, r_M^t$ in Lemma 1, we introduce the following theorem.

Theorem 1. *Consider an* $r_T \in (C, r_{\max})$ *that belongs to the set of* $\{r_1^t, r_2^t, \ldots, r_M^t\}$*, then the following bidding strategy* b^* *constitutes an SBNE:*

$$b^*(r_i, C) = \begin{cases} \text{any value in } [0, r_{\min}], & \text{if } r_i = r_{\min}, \\ r_i, & \text{if } r_i \in (r_{\min}, C], \\ C, & \text{if } r_i \in (C, r_T), \\ C \text{ or "N"}, & \text{if } r_i = r_T, \\ \text{"N"}, & \text{if } r_i \in (r_T, r_{\max}], \end{cases} \tag{7}$$

where $i = 1, 2$.

We illustrate the structure of strategy b^* in Fig. 2, and find that:

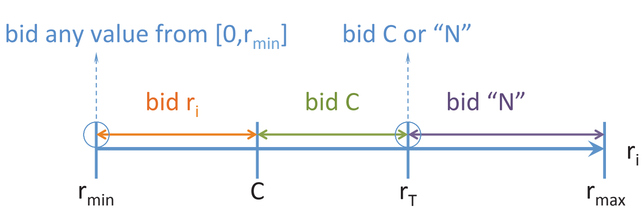

Fig. 2: Bidding Strategy Structure in SBNE when $C \in [r_{\min}, r_{\max})$.

(a) for APO i with type $r_i \in (r_{\min}, C]$, it bids r_i, *i.e.*, it requests the LTE provider to serve APO i's users with at least a rate that APO i can achieve by fully occupying channel i;

(b) for APO i with type $r_i \in (C, r_T)$, it bids C. Since $C < r_i$, the data rate APO i requests from the LTE provider is smaller than the rate that APO i achieves by fully occupying channel i. Recall that the feasible bid should be from $[0, C] \cup \{\text{"N"}\}$. If APO i bids "N", there is a chance that APO j also bids "N", which makes the LTE provider work in the *competition mode* and leads to a payoff of $\frac{1+\eta}{2} r_i$ to APO i based on (4). In order to avoid such a situation, APO i would bid C, and ensure that its payoff is at least C;[7]

(c) for APO i with type $r_i \in (r_T, r_{\max}]$, it bids "N". Similar as case (b), there is a chance that APO j also bids "N", and APO i obtains a payoff of $\frac{1+\eta}{2} r_i$. However, with $r_i \in (r_T, r_{\max}]$, value $\frac{1+\eta}{2} r_i$ is already large enough so that there is no need for APO i to lower its bid from "N" to any value from $[0, C]$.

There are two special points in (7):

(d) for APO i with $r_i = r_{\min}$, it has the same payoff if it bids any value from $[0, r_{\min}]$. This is because with probability one, APO i wins the auction.[8] From (4), APO i's payoff is $r_{\text{pay}}(\boldsymbol{b}, C)$, which equals $\min\{b_2, C\}$ and does not depend on APO i's bid;

(e) for APO i with $r_i = r_T$, it has the same expected payoff under bid C and bid "N".

It is easy to show that $b^*(r_i, C)$ in (7) is not a dominant strategy for the APOs. For example, if APO i's type $r_i \in (C, r_T)$ and APO j bids C, bidding "N" generates a larger payoff to APO i than bidding $b^*(r_i, C) = C$. This result is different from that of the standard second-price auction, where bidding the truthful valuation constitutes an equilibrium, and is also the weakly dominant strategy for the bidders.

Notice that equation (6) may admit multiple solutions, *i.e.*, $M > 1$. Based on Theorem 1, each solution r_m^t, $m = 1, 2, \ldots, M$, corresponds to a strategy b^* defined in (7).

In the following theorem, we show the unique form of bidding strategy under an SBNE.

Theorem 2. *The strategy function in (7) is the unique form of bidding strategy that constitutes an SBNE.*

The sketch of the proof is as follows: first, we show the necessary conditions that a bidding strategy needs to satisfy to constitute an SBNE; second, we show that the function in (7) is the only function that satisfies all these conditions.

IV. Equilibrium Analysis II: $C \in [0, r_{\min}) \cup [r_{\max}, \infty)$

In this section, we analyze the APOs' bidding strategies under the situations where the LTE provider sets the reserve rate in one of the following intervals: $\left[0, \frac{1+\eta}{2} r_{\min}\right]$, $\left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$, or $[r_{\max}, \infty)$.

A. Equilibrium Analysis: $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$

We summarize the form of the bidding strategy in the SBNE in the following theorem.

Theorem 3. *When* $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right)$*, there is a unique SBNE, where* $b^*(r_i, C) = \text{"N"}$ $(i = 1, 2)$ *for all* $r_i \in [r_{\min}, r_{\max}]$*;*

[7] Specifically, based on (2), if APO i bids C and wins the auction, its payoff will be C; if APO i bids C but loses the auction, its payoff will be $r_i > C$.

[8] Notice that for APO j's type r_j, the probability that $r_j = r_{\min}$ is zero. In other words, with probability one, r_j is from set $(r_{\min}, r_{\max}]$. Based on (7), APO j bids from $(r_{\min}, C] \cup \{\text{"N"}\}$ and APO i wins the auction.

when $C = \frac{1+\eta}{2} r_{\min}$, a strategy function constitutes an SBNE if and only if it is in the following form ($i = 1, 2$):

$$b^* (r_i, C) = \begin{cases} \text{any value in } [0, C] \text{ or ``N''}, & \text{if } r_i = r_{\min}, \\ \text{``N''}, & \text{if } r_i \in (r_{\min}, r_{\max}]. \end{cases} \quad (8)$$

When $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$, the LTE provider only wants to allocate a limited data rate to the winning APO's users. In this case, the APOs bid "N" with probability one.[9]

B. Equilibrium Analysis: $C \in \left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$

We show that the bidding strategy that constitutes an SBNE has a unique form. First, we introduce the following lemma.

Lemma 2. *The following equation admits at least one solution r in $(r_{\min}, r_{\max})$:*

$$\frac{1}{2} F(r)(C - r) + (1 - F(r))\left(C - \frac{1+\eta}{2} r\right) = 0, \quad (9)$$

where $F(\cdot)$ is the CDF of random variable r_i, $i = 1, 2$. We denote the solutions r in $(r_{\min}, r_{\max})$ as $r_1^x, r_2^x, \ldots, r_K^x$, where $K = 1, 2, \ldots,$ is the number of solutions.

Based on the definition of $r_1^x, r_2^x, \ldots, r_K^x$ in Lemma 2, we introduce the following theorem.

Theorem 4. *When $C \in \left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$, consider an $r_X \in (r_{\min}, r_{\max})$ that belongs to the set of $\{r_1^x, r_2^x, \ldots, r_K^x\}$, then the following bidding strategy b^* constitutes an SBNE:*

$$b^* (r_i, C) = \begin{cases} C, & \text{if } r_i \in [r_{\min}, r_X), \\ C \text{ or ``N''}, & \text{if } r_i = r_X, \\ \text{``N''}, & \text{if } r_i \in (r_X, r_{\max}], \end{cases} \quad (10)$$

where $i = 1, 2$. Furthermore, such a bidding strategy b^* is the unique form of bidding strategy that constitutes an SBNE.

The bidding strategy in (10) is similar to that in (7), except that here it only has two regions instead of three regions. Specifically, here there are no APOs that bid their types r_i. This is because here the reserve rate C is smaller than $r_{\min}$, hence bidding any type $r_i \in [r_{\min}, r_{\max}]$ is not feasible. We illustrate the structure of strategy function b^* in Fig. 3.

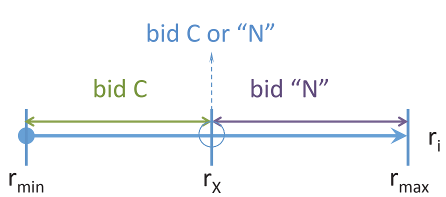

Fig. 3: Bidding Strategy Structure in SBNE when $C \in \left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$.

Similar as the equilibrium analysis for $C \in [r_{\min}, r_{\max})$, here equation (9) may admit multiple solutions, *i.e.*, $K > 1$, in which case each solution r_k^x, $k = 1, 2, \ldots, K$, corresponds to a strategy b^* defined in (10).

[9] In Theorem 3, when $C = \frac{1+\eta}{2} r_{\min}$, the APO with type $r_{\min}$ can bid any value. However, the probability for an APO to have the type $r_{\min}$ is zero due to the continuous distribution of r.

C. Equilibrium Analysis: $C \in [r_{\max}, \infty)$

We show the unique form of bidding strategy that constitutes an SBNE in the following theorem.

Theorem 5. *When $C \in [r_{\max}, \infty)$, a strategy function constitutes an SBNE if and only if it is in the following form ($i = 1, 2$):*

$$b^* (r_i, C) = \begin{cases} \text{any value in } [0, r_{\min}], & \text{if } r_i = r_{\min}, \\ r_i, & \text{if } r_i \in (r_{\min}, r_{\max}), \\ r_{\max} \text{ or ``N''}, & \text{if } r_i = r_{\max}. \end{cases} \quad (11)$$

When $C \in [r_{\max}, \infty)$, the LTE provider is willing to allocate a large data rate to the winning APO's users. Based on (11), the APOs bid values from $[0, C]$ with probability one.[10]

V. LTE Provider's Payoff Maximization

In Section III and Section IV, we have shown that there is a unique form of APO i's bidding strategy $b^* (r_i, C)$ in the SBNE for any reserve rate $C \in [0, \infty)$. In this section, we first derive the LTE provider's expected payoff as a function of C, and then formulate its payoff maximization problem.

We first make the following assumption on the cumulative distribution function of an APO's type.

Assumption 1. *Under the cumulative distribution function $F(\cdot)$, (a) equation (6) has a unique solution in $(C, r_{\max})$, i.e., $M = 1$, and (b) equation (9) has a unique solution in $(r_{\min}, r_{\max})$, i.e., $K = 1$.*

Assumption 1 implies that r_T and r_X are unique. Such an assumption is mild. First, we can show that it is satisfied for any uniform distribution. Second, through the simulation, we have observed that it is also satisfied for both the truncated normal distribution and the truncated power-law distribution.

Based on Theorem 1 and Theorem 2, the uniqueness of r_T implies the unique expression of APOs' bidding strategy b^* for $C \in [r_{\min}, r_{\max})$. Similarly, from Theorem 4, the uniqueness of r_X implies the unique expression of strategy b^* for $C \in \left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$.

We define the LTE provider's expected payoff as

$$\bar{\Pi}^{\rm LTE}(C) \triangleq \mathbb{E}_{r_1, r_2} \left\{\Pi^{\rm LTE}\left(\left(b^*(r_1, C), b^*(r_2, C)\right), C\right)\right\}, \quad (12)$$

where $b^* (r_i, C)$, $i = 1, 2$, is given in (7), (8), (10), and (11) in the different regions of C. Next we characterize $\bar{\Pi}^{\rm LTE}(C)$ in the different regions of C.

A. LTE Provider's Payoff: $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$

When $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$, the APOs submit their bids according to strategy b^* in (8). It is easy to find that $b^* (r_i, C) =$ "N" ($i = 1, 2$) with probability one, and hence the LTE provider always works in the *competition mode*. Based on (3), we can compute $\bar{\Pi}^{\rm LTE}(C)$ as

$$\bar{\Pi}^{\rm LTE}(C) = \delta R_{\rm LTE}. \quad (13)$$

[10] Notice that the probability for an APO to have the type $r_{\max}$ is zero.

B. LTE Provider's Payoff: $C \in \left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$

When $C \in \left(\frac{1+\eta}{2} r_{\min}, r_{\min}\right)$, the APOs' bidding strategy is summarized in (10). Hence, the probabilities for an APO with a random type to bid C and "N" are $F(r_X)$ and $1 - F(r_X)$, respectively. Therefore, we can compute $\bar{\Pi}^{\mathrm{LTE}}(C)$ as[11]

$$\bar{\Pi}^{\mathrm{LTE}}(C) = (1 - F(r_X))^2 \delta R_{\mathrm{LTE}} + \left(1 - (1 - F(r_X))^2\right)(R_{\mathrm{LTE}} - C). \quad (14)$$

That is to say: (a) when both APOs bid "N", the LTE provider works in the *competition mode*, and obtains a payoff of δR_{LTE}; (b) when at least one APO bids C, the LTE provider works in the *cooperation mode*, and allocates a rate of C to the winning APO's users.

C. LTE Provider's Payoff: $C \in [r_{\min}, r_{\max})$

When $C \in [r_{\min}, r_{\max})$, the APOs' strategy is given in (7). We can compute $\bar{\Pi}^{\mathrm{LTE}}(C)$ as[12]

$$\bar{\Pi}^{\mathrm{LTE}}(C) = -\bar{r}_{\mathrm{pay}}(C) + (1 - F(r_T))^2 \delta R_{\mathrm{LTE}} + \left(1 - (1 - F(r_T))^2\right) R_{\mathrm{LTE}}. \quad (15)$$

Here, $\bar{r}_{\mathrm{pay}}(C)$ is defined as the expected data rate that the LTE provider allocates to the winning APO's users, and is computed as:

$$\begin{aligned}\bar{r}_{\mathrm{pay}}(C) &\triangleq \mathbb{E}_{r_1, r_2}\left\{r_{\mathrm{pay}}\left(\left(b^*(r_1, C), b^*(r_2, C)\right), C\right)\right\} \\ &= 2\int_{r_{\min}}^{C} F(x) f(x)\, x dx + 2CF(r_T) \\ &\quad - CF^2(r_T) - CF^2(C). \end{aligned} \quad (16)$$

D. LTE Provider's Payoff: $C \in [r_{\max}, \infty)$

Based on (11), when $C \in [r_{\max}, \infty)$, the APOs bid values from $[0, C]$ with probability one, and the LTE provider always works in the *cooperation mode*. Then we can compute $\bar{\Pi}^{\mathrm{LTE}}(C)$ as

$$\bar{\Pi}^{\mathrm{LTE}}(C) = R_{\mathrm{LTE}} - 2\int_{r_{\min}}^{r_{\max}} F(x) f(x)\, x dx. \quad (17)$$

E. LTE Provider's Payoff Maximization Problem

Based on $\bar{\Pi}^{\mathrm{LTE}}(C)$ derived in Section V-A to Section V-D, the LTE provider determines the optimal reserve rate by solving

$$\begin{aligned} &\max \ \bar{\Pi}^{\mathrm{LTE}}(C) \\ &\text{s.t.} \quad b_{\max}(C) \le R_{\mathrm{LTE}}, \\ &\text{var.} \quad C \in [0, \infty), \end{aligned} \quad (18)$$

where we define

$$b_{\max}(C) \triangleq \max\left\{b^*(r_i, C) \in [0, C] : r_i \in [r_{\min}, r_{\max}]\right\}, \quad (19)$$

which is the maximum possible bid (except "N") from the APOs in the SBNE under C. Constraint $b_{\max}(C) \le R_{\mathrm{LTE}}$ ensures that the LTE provider has enough capacity to satisfy the bid from the winning APO. Next we solve problem (18).

[11]Notice that r_X is the solution to (9) and is also a function of C.

[12]Notice that r_T is the solution to (6) and is also a function of C.

VI. LTE Provider's Optimal Reserve Rate

In this section, we characterize the properties of the optimal reserve rate C^* for a general distribution function $F(\cdot)$ that satisfies Assumption 1. We first show the properties of $\bar{\Pi}^{\mathrm{LTE}}(C)$ in the following lemmas.

Lemma 3. *$C = r_{\max}$ is a local minimum of $\bar{\Pi}^{\mathrm{LTE}}(C)$.*

We can show that $\frac{d\bar{\Pi}^{\mathrm{LTE}}(C)}{dC} = 0$ for $C \in [r_{\max}, \infty)$, and there exists an $\epsilon > 0$ such that $\frac{d\bar{\Pi}^{\mathrm{LTE}}(C)}{dC} < 0$ for $C \in (r_{\max} - \epsilon, r_{\max})$. This implies that the LTE provider will not choose the optimal reserve rate C^* from set $[r_{\max}, \infty)$. To maximize $\bar{\Pi}^{\mathrm{LTE}}(C)$, the LTE provider will not choose a very large reserve rate, in which case the APOs are willing to cooperate with probability one.

Lemma 4. *(a) When $R_{\mathrm{LTE}} > \frac{1+\eta}{2(1-\delta)} r_{\min}$, $C = \frac{1+\eta}{2} r_{\min}$ is a local minimum of $\bar{\Pi}^{\mathrm{LTE}}(C)$; (b) when $R_{\mathrm{LTE}} \le \frac{1+\eta}{2(1-\delta)} r_{\min}$, $C = \frac{1+\eta}{2} r_{\min}$ is a global maximum of $\bar{\Pi}^{\mathrm{LTE}}(C)$.*

When we have $R_{\mathrm{LTE}} > \frac{1+\eta}{2(1-\delta)} r_{\min}$, we can show that $\lim_{C \downarrow \frac{1+\eta}{2} r_{\min}} \frac{d\bar{\Pi}^{\mathrm{LTE}}(C)}{dC} > 0$.[13] Furthermore, $\bar{\Pi}^{\mathrm{LTE}}(C)$ is a constant for $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$ based on (13). Hence, it is easy to find that $C = \frac{1+\eta}{2} r_{\min}$ is a local minimum point, and the LTE provider will not choose C^* from set $\left[0, \frac{1+\eta}{2} r_{\min}\right]$. This is because $R_{\mathrm{LTE}} > \frac{1+\eta}{2(1-\delta)} r_{\min}$ is equivalent to $(1-\delta) R_{\mathrm{LTE}} > \frac{1+\eta}{2} r_{\min}$. Here, $(1-\delta) R_{\mathrm{LTE}}$ stands for the additional increase in the LTE network's capacity when it works in the *cooperation mode*. Based on (7), (8), (10), and (11), $\frac{1+\eta}{2} r_{\min}$ is the lower bound of the data rate that any APO with type in $(r_{\min}, r_{\max}]$ may request from the LTE provider. When $(1-\delta) R_{\mathrm{LTE}} > \frac{1+\eta}{2} r_{\min}$, the LTE provider benefits from cooperating with the APOs that request small data rates. Therefore, it chooses C^* greater than $\frac{1+\eta}{2} r_{\min}$ so as to accept the bids from these APOs.

When $R_{\mathrm{LTE}} \le \frac{1+\eta}{2(1-\delta)} r_{\min}$, we can show that the value of $\bar{\Pi}^{\mathrm{LTE}}(C)$ under $C \in \left(\frac{1+\eta}{2} r_{\min}, \infty\right)$ is smaller than that under $C = \frac{1+\eta}{2} r_{\min}$. Since $\bar{\Pi}^{\mathrm{LTE}}(C)$ is a constant for $C \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$, point $C = \frac{1+\eta}{2} r_{\min}$ is a global maximum. The reason is that, when $(1-\delta) R_{\mathrm{LTE}} \le \frac{1+\eta}{2} r_{\min}$, the additional gain in the LTE network's capacity cannot cover the request from the APOs. Hence, the LTE provider works in the competition mode by choosing $C = \frac{1+\eta}{2} r_{\min}$.

Based on Lemma 3 and Lemma 4, we characterize the optimal C^* that solves problem (18) in the following theorem.

Theorem 6. *We have the following three situations:*
(1) When $R_{\mathrm{LTE}} \le \frac{1+\eta}{2(1-\delta)} r_{\min}$, the optimal C^* can be any value from $\left[0, \frac{1+\eta}{2} r_{\min}\right]$;
(2) When $\frac{1+\eta}{2(1-\delta)} r_{\min} < R_{\mathrm{LTE}} \le r_{\max}$, the optimal C^* lies in the range of $\left(\frac{1+\eta}{2} r_{\min}, R_{\mathrm{LTE}}\right]$;
(3) When $R_{\mathrm{LTE}} > \max\left\{r_{\max}, \frac{1+\eta}{2(1-\delta)} r_{\min}\right\}$, the optimal C^* lies in the range of $\left(\frac{1+\eta}{2} r_{\min}, r_{\max}\right)$.

When $R_{\mathrm{LTE}} \le \frac{1+\eta}{2(1-\delta)} r_{\min}$, the LTE provider does not have enough capacity to satisfy the APOs' requests. By setting

[13]The downward arrow $\downarrow$ corresponds to the right-sided limit.

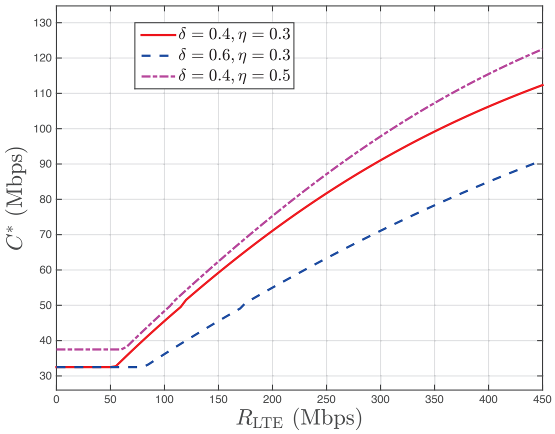

Fig. 4: Optimal Reserve Rate C^* vs. R_{LTE} (Uniform Distribution).

$C^* \in \left[0, \frac{1+\eta}{2} r_{\min}\right]$, the LTE provider eventually works in the *competition mode*. When $\frac{1+\eta}{2(1-\delta)} r_{\min} < R_{\text{LTE}} \leq r_{\max}$, the additional gain in the LTE network's capacity under cooperation can cover the requests from the APOs that bid small values. Hence, the LTE provider chooses C^* above $\frac{1+\eta}{2} r_{\min}$ to accept these APOs' bids. Meanwhile, the LTE provider has to choose C^* no larger than R_{LTE}, because otherwise it does not have enough capacity to satisfy the requests from the APOs that bid large values. When $R_{\text{LTE}} > \max\left\{r_{\max}, \frac{1+\eta}{2(1-\delta)} r_{\min}\right\}$, since the maximum possible bid from the APOs is $r_{\max}$, the LTE provider always has enough capacity to satisfy the APOs' requests. In this case, the LTE provider chooses C^* from $\left(\frac{1+\eta}{2} r_{\min}, r_{\max}\right)$, and C^* is no longer constrained by R_{LTE}.

Notice that the analysis above holds for a general cumulative distribution function $F(\cdot)$. However, it is difficult to characterize the closed form of C^* even under a specific function $F(\cdot)$. For example, with a uniform distribution function $F(\cdot)$, the corresponding expression of $\bar{\Pi}^{\text{LTE}}(C)$ is so complicated that the closed form of C^* cannot be obtained. Hence, we study C^* numerically in the next section.

VII. Numerical Results

In this section, we investigate the relations between the optimal reserve rate C^* and some system parameters through the simulation. We choose $r_i \sim U[50 \text{ Mbps}, 200 \text{ Mbps}]$ $(i = 1, 2)$, and consider three pairs of data rate discounting factors: $(\delta, \eta) = (0.4, 0.3), (0.6, 0.3)$, and $(0.4, 0.5)$. For each pair of (δ, η), we change R_{LTE} from 0 Mbps to 450 Mbps, and determine the corresponding C^* numerically. In Fig. 4, we plot C^* against R_{LTE} under the different pairs of (δ, η).

We observe that C^* is independent of R_{LTE} when R_{LTE} is very small. In this case, the LTE provider does not have enough capacity to satisfy the APOs' requests. Therefore, it chooses a small reserve rate, and works in the *competition mode*. When R_{LTE} is above $\frac{1+\eta}{2(1-\delta)} r_{\min}$, C^* becomes increasing in R_{LTE}. This is because with a larger throughput R_{LTE}, the LTE provider is more willing to cooperate with the APOs, and thus increases the reserve rate to attract the APOs.

With $\eta = 0.3$, we find that C^* under $\delta = 0.4$ is always no smaller than that under $\delta = 0.6$. Under a smaller δ, the LTE provider is more heavily affected by the interference from Wi-Fi. In this case, the LTE provider chooses a larger reserve rate to motivate the cooperation with the APOs.

With $\delta = 0.4$, we find that C^* under $\eta = 0.5$ is always larger than that under $\eta = 0.3$. This is because under a larger η, the APOs are less heavily interfered by the LTE provider, and hence are less willing to cooperate with the LTE provider. As a result, the LTE provider needs to increase its reserve rate to attract the APOs.

We summarize the main observations as follows.

Observation 1. *The optimal reserve rate C^* is non-decreasing in R_{LTE}, non-increasing in δ, and increasing in η.*

VIII. Conclusion

In this paper, we proposed a framework for LTE's coopetition with Wi-Fi in the unlicensed spectrum. We designed a reverse auction for the LTE provider to fully obtain the channel from the APOs by onloading their traffic. The analysis is quite challenging as the designed auction involves the positive allocative externalities. We characterized the unique bidding strategies of the APOs, and analyzed the optimal reserve rate of the LTE provider. In our future work, we plan to extend the analysis to a more general situation where there are more than two APOs, and different APOs can share the same channel.

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