Providing Long-Term Participation Incentive in Participatory Sensing

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Abstract—Providing an adequate long-term user participation incentive is important for a participatory sensing system to maintain enough number of active users (sensors), so as to collect a sufficient number of data samples and support a desired level of service quality. In this work, we consider the sensor selection problem in a general time-dependent and location-aware participatory sensing system, taking the long-term user participation incentive into explicit consideration. We study the problem systematically under different information scenarios, regarding both future information and current information (realization). In particular, we propose a Lyapunov-based VCG auction mechanism for the on-line sensor selection, which converges asymptotically to the optimal off-line benchmark performance, even with no future information and under asymmetry of current information. Extensive numerical results show that our proposed policy outperforms the state-of-art policies in the literature, in terms of both user participation (e.g., reducing the user dropping probability by 25% ~ 90%) and social performance (e.g., increasing the social welfare by 15% ~ 80%).

I. INTRODUCTION

A. Background and Motivations

The proliferation of mobile devices (e.g., smartphones) with rich embedded sensors has led to a revolutionary new sensing paradigm known as Participatory Sensing [1]–[3], in which mobile users voluntarily participate in and actively contribute to the sensing system, using their carrying smartphones. Due to the low deploying cost and high sensing coverage, this new paradigm has attracted a wide range of applications related to environment, infrastructure, and community monitoring (e.g., air pollution [4]–[6], wireless signal strengths [7]–[9], road traffic [10]–[12], and parking [13], [14]).

A typical participatory sensing system architecture usually consists of a service provider (also called service platform) residing in the cloud and a collection of mobile smartphone users [16]–[18]. The service provider launches a set of sensing tasks with different purposes and requirements, and mobile users actively subscribe to (participate in) the sensing tasks. In this work, we focus on the server-initiated sensing, where the service provider selects a specific set of participating smartphones to perform sensing tasks, depending on the spatio-temporal data requirements of sensing tasks, the geographical locations of users, and the sensing capabilities of users. Comparing with the user-initiated sensing scheme (where users actively decide when and where to sense), the server-initiated sensing scheme gives more control to the service provider to decide when and where to collect the data at what costs. Clearly, the success of such a sensing system strongly relies on the active participations of users, as well as their willingnesses to contribute resource to the sensing tasks.

Although many participatory sensing applications have been proposed in [4]–[14], they simply assume that users voluntarily participate in the system to perform sensing tasks. In reality, however, users may not be willing to participate in the sensing system, as this will incur extra operational cost (e.g., the battery energy expenditure and the transmission expense). Moreover, many sensing tasks are location-aware and time-dependent, and have unique spatio-temporal contexts. Sharing sensing data tagged with a spatio-temporal context may reveal a lot of personal and sensitive information, which poses potential privacy threats to the participating users [15].

All of these bring the incentive issue to the center of the participatory sensing system design.

Several recent works have been devoted to the incentive mechanism design issue in participatory sensing, mainly using pricing and auction [17]–[25]. Most of them focus on compensating the user’s direct sensing cost when performing a particular sensing task (e.g., in [17]–[23]), which we call the short-term sensing incentive. In practice, however, we find that the users participating in a sensing task may suffer some indirect cost even when not performing the sensing task. In this case, the short-term sensing incentive may not be enough to guarantee the long-term continuous participations of users. Intuitively, if a user is rarely selected as a sensor (hence rarely receives the short-term sensing incentive), the user may lose his interest in further participation and decide to drop out of the sensing system (e.g., shut down the sensing app on his smartphone). Without an adequate number of users participating in the system, however, the service provider may not be able to collect a sufficient number of sensing data to support a desired service quality.

To the best of our knowledge, [24] and [25] are the only results that explicitly study the long-term participation incentive in participatory sensing. To stimulate the continuous participation of users, Lee et al. in [24] and [25] introduce a virtual credit for lowering the bids of users who lost in the previous round of auction, hence increasing their winning probabilities in future auction rounds. However, they consider neither the truthfulness, nor the optimality of the proposed auction. In this work, we will study the long-term participatory incentive, joint with the short-term sensing incentive, with rigorous truthfulness and optimality analysis.

B. Solutions and Contributions

Specifically, we consider a general location-aware, time-dependent participatory sensing system, where the data in

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1 For example, in a location-aware sensing task, users need to periodically report their locations, and incur certain energy and transmission costs.
different time (slots) and/or locations may have different values for the sensing tasks. Each participating user has the potential to sense a specific region (at a certain sensing cost) in a specific time slot, depending on his location and mobility pattern. Fig. 1 illustrates a snapshot of such a sensing system (in a particular time slot), where the sensing region of each user is denoted by the shadow area around the user. In such a system, the service provider selects (allocates) users as sensors to perform sensing tasks slot by slot. We focus on the following sensor selection/allocation problem for the service provider:

- Which users should be selected as sensors in each time slot, aiming at maximizing the social welfare and ensuring the long-term participation incentive of users?

The problem is challenging to address due to the following reasons. First, the overlap of users’ sensing regions makes their sensing activities possibly redundant (hence partially “conflict” with each other). Second, the long-term participation incentive of users makes the sensor allocations in different time slots coupled. Based on the above, our model and problem formulation capture the following important features of a participatory sensing system: (i) long-term participation incentives, (ii) time-dependent and location-aware sensing requirements, and (iii) partial conflicting sensing activities. As far as we know, this is the first work that systematically studies a participatory sensing problem with all of the above features.

We solve the above sensor selection problem under different information scenarios, regarding both future information (i.e., complete, stochastic, or no future information) and current information (i.e., symmetric or asymmetric). Specifically, with complete or stochastic future information, we formulate and solve an off-line sensor selection problem as the benchmark (where we assume that the current information is always symmetric). With no future information, we formulate and solve an on-line sensor selection problem:2 (i) under information symmetry, we propose a Lyapunov-based on-line sensor selection policy (Policy 1), which converges to the optimal off-line benchmark asymptotically; and (ii) under information asymmetry, we propose a Lyapunov-based VCG auction policy (Policy 2), which is truthful, and meanwhile achieves the same asymptotically optimal performance as in Policy 1.

We would like to emphasize that the key contribution of this work is not on the Lyapunov optimization framework itself, but rather the novel problem formulation and solution techniques. For more clarity, we list the main results in Table I, and summarize the key contributions as follows.

- **Novel Model and Problem Formulation:** We study a general time-dependent and location-aware participatory sensing system, taking into consideration the important but under-explored issue of long-term user participation incentive. We propose a simple yet representative formulation based on the allocation probability of each user to capture such an incentive.

- **Multiple Information Scenarios:** We study the optimal sensor selection problem under different information scenarios. In particular, we propose on-line sensor selection policies that converge to the asymptotically optimal performance, even with no future information and under information asymmetry.

- **Performance Evaluations:** We compare the proposed on-line policies with the state-of-art policies, and show our proposed policies outperform the existing ones significantly, in terms of both user participation and social performance: (i) Comparing with the RAPD-VPC policy proposed in [24] [25], our policies can reduce the user dropping probability by \(25\% \sim 50\%\), and increase the social welfare by \(15\% \sim 40\%\); (ii) Comparing with the Greedy and Random policies widely used in existing systems (e.g., [10]), our policies can reduce the user dropping probability by \(70\% \sim 90\%\), and increase the social welfare by \(65\% \sim 80\%\).

### II. System Model

We consider a location-aware participatory sensing system, with a service provider (SP) and a set \(N = \{1, ..., N\}\) of mobile smartphone users (participating in the system). The SP wants to collect specific data in a certain area (via participating users’ smartphones) for specific tasks. Mobile users move randomly in and out of the desirable sensing area according to certain mobility patterns. As shown in Fig. 1, each user has the potential to sense a specific region in a certain period according to his location and mobility. The whole sensing area \(A\) is divided into a set \(\mathcal{T} = \{1, ..., I\}\) of grids.3 Each grid \(\mathcal{A}_i, i \in \mathcal{T}\), is associated with a weight \(w_i[t]\) for each time \(t\), denoting the value of the data in grid \(\mathcal{A}_i\). Obviously, such a data value is location-aware and time-dependent.

The SP requests data slot by slot, where each time slot ranges from several minutes to several hours, depending on the temporal data requirements of tasks. We consider the sensing operation in a long period consisting of a set \(T = \{1, ..., T\}\) of \(T\) slots. At the beginning of each time slot, the SP selects (allocates) a set of users to perform the sensing task in that time slot, depending on the user locations and the data values. Let \(x_{\tau}[t] \in \{0, 1\}\) denote whether a user \(n\) is selected as a sensor in slot \(t\), and \(x[t] = \{x_{\tau}[t], \forall \tau \in \mathcal{T}\}\) denote the sensor selection vector in slot \(t\). We further denote \(x_n = \{x_{\tau}[t], \forall \tau \in \mathcal{T}\}\) as the allocation vector of user \(n\) in all slots.

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2 Several recent studies also investigated the on-line policy for sensing task allocation, considering the uncertainty of user arrival [26], [27]. However, these studies did not consider the long-term user participation incentive.

3 A grid is the minimum unit of sensing area at a particular location, e.g., a square of 100m × 100m, associated with a single data in a particular time.
A. Mobile User Modeling

1) Sensing Region: Each mobile user has a certain sensing range in each time slot, mainly depending on his location and mobility pattern. In Fig. 1, the sensing region of each user is illustrated by the shadow area around the user. Let \( z_{n,t} \in \{0, 1\} \) denote whether a grid \( A_i \) is located in the sensing range of user \( n \) in slot \( t \). Then, the total sensing region of user \( n \) in each slot \( t \) can be defined as a vector: \( z_n[t] \triangleq \{ z_{n,i}[t], \forall i \in I \} \). Note that when user \( n \) moves out of the desirable sensing area in time slot \( t \), we can simply define: \( z_{n,t} = 0, \forall i \in I \). As mobile users move randomly, the sensing region \( z_n[t] \) of each user \( n \) also changes randomly across time slots.

2) Sensing Value: When a user \( n \) is selected as a sensor in slot \( t \), i.e., \( x_n[t] = 1 \), he performs the following sensing task: collect, process, and transmit all of the data within his sensing region \( z_n[t] \) to the SP. This generates a sensing value \( v_n[t] \) equal to the sum of weights of all grids within \( z_n[t] \):

\[
v_n[t] \triangleq x_n[t] \cdot \sum_{i \in I} z_{n,i}[t] \cdot w_i[t]. \tag{1}
\]

3) Sensing Cost: When performing sensing tasks, users incur extra operational cost (called sensing cost) due to, for example, the energy expenditure and the transmission expense. Let \( c_n[t] \) denote the total sensing cost of user \( n \) in slot \( t \) (including all potential expense used for collecting, processing, and transmitting the data within \( z_n[t] \) to the SP). Obviously, such a sensing cost is user-dependent and time-dependent.

Due to this direct sensing cost, users may be reluctant to perform sensing tasks without sufficient incentives. To avoid this, in each time slot, the SP will pay certain monetary or non-monetary compensation (which we call the short-term sensing incentive) to those users who are selected as sensors. Later we will show that this type of incentive can be easily addressed through, for example, a first-degree price discrimination [28] or a truthful auction [29] in each time slot.

4) Participatory Constraint: As discussed in Section I, users may suffer certain indirect cost even when not performing sensing tasks, induced by, for example, reporting location/mobility information or running sensing apps. Thus, if a user is rarely selected as a sensor (hence rarely receives the short-term sensing incentive), he may gradually lose the interest in further participation, and decide to no longer participate in the system (in this case, we say the user drops out of the system).

As shown in [24] and [25], such a long-term participation incentive strongly depends on the user’s Return on Investment (ROI). In this work, instead of directly estimating the total return and total investment, we use a simple yet representative indicator to reflect the user ROI: Allocation Probability,\(^4\) i.e., the probability of each user being selected as a sensor. Namely, we assume that each user \( n \) will drop out of the sensing system if his allocation probability (of being selected as a sensor) is smaller than a specific threshold \( D_n \), called the dropping threshold of user \( n \). Therefore, to ensure the active participation of users, the allocation probability of each user should be no smaller than his dropping threshold, which we call the user participatory constraint:

\[
D_n \leq d_n(x_n) \triangleq \frac{1}{T} \sum_{t \in T} x_n[t], \quad \forall n \in N, \tag{2}
\]

where \( d_n(x_n) \) is the time average allocation probability of user \( n \), depending on the allocations of user \( n \) in all slots.

B. Service Provider Modeling

Given the set \( N \) of mobile users participating in the system, the SP selects a subset of users as sensors in each time slot. We consider a non-commercial SP (e.g., a non-profit organization or a governmental department), whose primary goal is to maximize the total sensing value and minimize the total sensing cost in the entire time period, subjecting to the user participatory constraint in (2).

Given the allocation vector \( x[t] \triangleq \{ x_n[t], \forall n \in N \} \) in slot \( t \), the total sensing cost (in slot \( t \)) can be directly defined as the sum of all selected users’ sensing costs, i.e.,

\[
C[t] \triangleq \sum_{n \in N} x_n[t] \cdot c_n[t]. \tag{3}
\]

The total sensing value (in slot \( t \)), however, may not be the same as the aggregate sensing value of all selected users due to the overlap of their sensing regions. The key reason is that the same data collected by multiple users simultaneously can only generate value once. For convenience, let \( y_{i,t} \) denote whether a grid \( A_i \) is sensed by at least one mobile user, that is,

\[
y_{i,t} \triangleq \left\lceil \sum_{n \in N} x_n[t] \cdot z_{n,i}[t] \right\rceil^1, \tag{4}
\]

where \( \lceil x \rceil^1 = 1 \) if \( x \geq 1 \), and \( \lceil x \rceil^1 = x \) if \( x < 1 \). Then, the total sensing value (in slot \( t \)) can be defined as follows:

\[
V[t] \triangleq \sum_{i \in I} y_{i,t} \cdot w_i[t]. \tag{5}
\]

In the special case where the sensing regions of all selected users do not overlap, then \( y_{i,t} = \sum_{n \in N} x_n[t] \cdot z_{n,i}[t] \) and \( V[t] = \sum_{n \in N} v_n[t] \cdot x_n[t] \), i.e., the total sensing value is the sum of all selected users’ sensing values.

The social welfare generated in each slot \( t \) is the difference between the total sensing value and sensing cost, i.e.,

\[
S[t] \triangleq V[t] - C[t]. \tag{6}
\]

The overall (average) social welfare in all time slots is

\[
S(x) \triangleq \frac{1}{T} \sum_{t \in T} S[t] = \frac{1}{T} \sum_{t \in T} (V[t] - C[t]), \tag{7}
\]

where \( x \triangleq \{ x_n[t], \forall n \in N, t \in T \} \triangleq \{ x[t], \forall t \in T \} \).

C. Information Scenario

We will study the sensor selection problem in different network information scenarios. The network information consists of the weight (data value) of each grid, the sensing region and sensing cost of each user in each time slot. Formally, we define the network information in time slot \( t \) as:

\[
\theta[t] \triangleq \{ w_i[t], z_n[t], c_n[t], \forall i \in I, n \in N \}. \tag{8}
\]

Note that the sensing value \( v_n[t] \) of each user is not network information, as it is determined by \( w_i[t] \) and \( z_n[t] \).

\(^4\)Consider, for example, a user with an expected direct sensing cost \( c_1 \), an expected indirect sensing cost \( c_2 \), and an expected return \( r \) when being selected as a sensor. Then, an allocation probability \( \eta \) directly corresponds to the following expected ROI: \( \frac{\eta (c_1 + c_2) r e^{-\frac{r}{c_1} c_2}}{\eta (c_1 + c_2) r e^{-\frac{r}{c_1} c_2} + (1-\eta) e^{-\frac{r}{c_1} c_2}} \).
Regarding the future network information, we consider the scenarios of complete future information, stochastic future information, and no future information, depending on whether and how much the SP knows regarding the future network information. Regarding the current network information (realization), we consider the scenarios of information symmetry and asymmetry, depending on whether the SP can observe the private information of users (e.g., the sensing cost).

### III. Off-line Sensor Selection Benchmark

In this section, we study the sensor selection problem with complete future information and stochastic future information (as benchmarks). Note that in these benchmark cases, we assume information symmetry (regarding the current network information), where the SP is able to observe all of the network information realized in each time slot.

#### A. Complete Future Information

With complete future information, the SP is able to determine the sensor selections in all time slots jointly to maximize the overall social welfare. Thus, the SP’s problem is

\[
\text{max} \quad \frac{1}{T} \sum_{t \in T} (V[t] - C[t])
\]

s.t. (a) \( x_n[t] \in \{0, 1\}, \quad \forall n \in N, \forall t \in T, \)

(b) \( D_n \leq d_n(x_n), \quad \forall n \in N. \)

Obviously, (9) is an off-line allocation problem, and the solution presents the explicit allocation of each user in each time slot in advance. Note that (9) is a binary integer programming, and can be effectively solved by many classic methods, such as the branch-and-bound algorithm in [31].

It is easy to see that formulating and solving (9) requires the complete future information, which is obviously impractical. Hence, we will study another benchmark solution based on the stochastic information in the next subsection.

#### B. Stochastic Future Information

With stochastic information only, the SP cannot decide the explicit allocation of each user in each time slot in advance, due to the lack of complete future information. Hence, in this case, we will focus on the expected social welfare maximization based on the stochastic information.

Let \( x_n(\theta) \in \{0, 1\} \) denote whether a user \( n \) is selected as a sensor under a particular information realization \( \theta \), \( x(\theta) \triangleq \{x_n(\theta), \forall n \in N\} \) denote the allocation vector of all users under \( \theta \), and \( x_n(\theta, \theta_0 \in \Theta) \) denote the allocation of user \( n \) under all possible \( \theta \). Then, the expected social welfare maximization problem can be defined as follows:

\[
\text{max} \quad \int_{\Theta \in \Theta} (V(\theta) - C(\theta)) \cdot f(\theta) d\theta
\]

s.t. (a) \( x_n(\theta) \in \{0, 1\}, \quad \forall n \in N, \forall \theta \in \Theta, \)

(b) \( D_n \leq d_n(x_n), \quad \forall n \in N. \)

where

- \( C(\theta) = \sum_{n \in N} c_n(\theta) \cdot x_n(\theta) \) is the sensing cost under \( \theta \);
- \( V(\theta) = \sum_{i \in I} y_i(\theta) \cdot w_i(\theta) \) is the sensing value under \( \theta \);

\( \Theta \) is the feasible set of \( \theta \), i.e., the set of all possible network information realizations, and \( f(\theta) \) is the probability distribution function (pdf) of \( \theta \).

Similarly, (10) is an off-line problem, and the solution defines a contingency plan that specifies the allocation of each user under each possible information realization \( \theta \). Note that (10) is an integer programming with an infinite number of decision variables (as \( \theta \) is continuous), which is non-convex and NP-hard. Nevertheless, by the linear programming relaxation, we can easily transform (10) into a linear programming, and solve it by classic methods, e.g., the KKT analysis.\(^6\)

Next we analyze the gap between the maximum social welfare with complete information (denoted by \( S^{\infty} \)) derived from (9) and the maximum expected social welfare with stochastic information (denoted by \( S^{\ast} \)) derived from (10). Formally,

**Lemma 1.** If \( T \to \infty \), then \( S^{\ast} \to S^{\infty} \).

Lemma 1 indicates that as long as the total sensing period \( T \) is large enough, the social welfare loss induced by the loss of complete network information is negligible. Hence, both \( S^{\infty} \) and \( S^{\ast} \) will serve as the same benchmark for the on-line policies proposed in Sections IV and V.

It is notable that formulating and solving (10) still requires certain (stochastic) future information, which may not be available in practice. This motivates us to further study on-line policies that do not rely on any future information.

### IV. On-line Sensor Selection Policy

In this section, we study the sensor selection problem with no future information. We propose an on-line sensor selection policy based on the Lyapunov optimization framework [30], which relies only on the current network information and past sensor selection history, while not on any future information. Note that we also assume information symmetry (regarding the current information) in this section, and will further study the scenario of information asymmetry in the next section.

#### A. Lyapunov Optimization Technique

Lyapunov optimization [30] is a widely used technique for solving stochastic optimization problems with time average constraints, such as the social welfare maximization problem (9) in this work (with \( T \to \infty \)), where the user participatory constraint (b) is the time average constraint. Hence, we introduce the Lyapunov optimization technique to solve the sensor selection problem (9) with no future information.

1) **Queue Definition:** The key idea of Lyapunov optimization technique is to use the stability of queues to ensure that the time average constraints are satisfied. Following this idea, we first introduce a virtual queue \( (q_n) \) for each user \( n \). This virtual queue is used for buffering the virtual allocation request of each user. Here, we use the prefix “virtual” to denote that the request is not actually initiated by the user, but rather, it is used to reflect the requirement of the user participatory constraint. Namely, one virtual request represents that “to satisfy the user participatory constraint, the user should be selected as sensor

\(^5\) We leave the details in [31], as the method is standard. Moreover, solving the stochastic optimization problem is not the main contribution of this work.
in one additional time slot”. Hence, the backlog of a virtual queue denotes the total number of virtual requests in the queue (which may not be an integer), i.e., the total number of additional time slots that the user should be selected as a sensor (in order to meet his participatory constraint).

2) Queue Dynamics: With the above queue definition, each virtual request of user \( n \) will enter into the queue with a constant arrival rate of \( D_n \). Let \( x_n[t] \in \{0, 1\} \) denotes whether user \( n \) is selected as a sensor in time slot \( t \) (under certain sensor selection policy), and \( d_{n}^{1} \triangleq d_{n}(x_{n}^{1}) = \frac{1}{2} \sum_{t \in T} x_n[t] \) denote the average allocation probability of user \( n \). Intuitively, \( x_n[t] = 1 \) implies that one virtual request of user \( n \) leaves the queue at slot \( t \). Hence, the virtual request of user \( n \) will leave the queue with an average departure rate of \( d_{n}^{1} \).

Let \( q_n \) denote the queue backlog of user \( n \) in slot \( t \), and let \( q_n^t \triangleq \{q_n, \forall n \in N\} \) denote the queue backlog vector of all users. For each user \( n \), given the constant arrival of his virtual request and the allocation \( x_n[t] \) in each slot \( t \) (departure), we have the following dynamic equation for his virtual queue:

\[
q_n^{t+1} = (q_n^t - x_n^t[t])^\dagger + D_n,
\]

(11)

where \( x^\dagger = \max(x, 0) \).

Next, we show how to connect the queue stability condition with the user participatory constraint in our problem. We say a virtual queue \( q_n \) is rate stable if

\[
\lim_{t \to \infty} \frac{q_n^t}{t} = 0 \quad \text{with probability 1}.
\]

By the queue stability theorem \([30]\), a queue \( q_n \) is rate stable if and only if the arrival rate is no larger than the departure rate, i.e., \( D_n \leq d_n^1 \). This establishes the equivalence between the queue stability condition and the user participatory constraint. That is, to guarantee the user participatory constraint in our problem, we only need to ensure that the associated virtual queue is rate stable under the proposed policy.

3) Queue Stability: Now we study the queue stability using the Lyapunov drift. We first define the Lyapunov function:

\[
J[t] \triangleq \frac{1}{2} \sum_{n \in N} (q_n^t)^2.
\]

(12)

The Lyapunov drift in each slot \( t \) is defined as the change of Lyapunov function from one slot to the next, i.e.,

\[
\Delta[t] \triangleq J[t+1] - J[t].
\]

(13)

By the Lyapunov drift theorem (Th. 4.1 in \([30]\)), if a policy greedily minimizes the Lyapunov drift \( \Delta[t] \) in each slot \( t \), then all backlogs are consistently pushed towards a low level, which potentially maintains the stabilities of all queues (i.e., ensures the participatory constraints of all users).

4) Joint Queue Stability and Welfare Maximization: Next, we analyze the joint queue stability and objective optimization (i.e., expected social welfare maximization). By the Lyapunov optimization theorem (Th. 4.2 in \([30]\)), to stabilize the queues while optimizing the objective, we can use such an allocation policy that greedily minimizes the following drift-plus-penalty:

\[
\Pi[t] \triangleq \Delta[t] + \phi \cdot (C[t] - V[t]),
\]

(14)

where the (negative) social welfare, i.e., \( C[t] - V[t] \), is viewed as the penalty incurred on each slot \( t \); \( \phi \geq 0 \) is a non-negative control parameter that is chosen to achieve a desirable tradeoff between the optimality and queue backlog.

### Policy 1: Lyapunov-based Policy (Information Symmetry)

**Initialization:** \( q = q^0 \);
**for each time slot** \( t = 0, 1, ..., T \) **do**
**Allocation Rule:**

\[
x^t[t] = \arg \max_{x^t[t]} \left( V[t] - C[t] + \sum_{n \in N} \frac{q_n^t}{\phi} \cdot x_n[t] \right)
\]

**Updating Rule:**

\[
q_n^{t+1} = \left[ q^t - x_n^t[t] \right]^\dagger + D_n, \quad \forall n \in N
\]

We further notice that directly minimizing the drift-plus-penalty defined in (14) may be difficult (partly because \( \Delta[t] \) is a quadratic function). Hence, we will focus on minimizing a specific upper-bound of the drift-plus-penalty to achieve the joint stability and optimization.

Next, we give such an upper-bound. Notice that

\[
\Delta[t] \leq \frac{1}{2} \sum_{n \in N} (x_n^t[t]^2 + D_n^2 + 2 \cdot q_n^t \cdot (D_n - x_n^t[t]))
\]

\leq \sum_{n \in N} q_n^t \cdot (D_n - x_n^t[t]) + \phi \cdot (C[t] - V[t]),
\]

(15)

where \( \phi \triangleq \sum_{n \in N} \frac{1+D_n^2}{\phi^2} \) is a constant. Then, we have the following upper-bound for the drift-plus-penalty in (14):

\[
\Pi[t] \leq \sum_{n \in N} q_n^t \cdot (D_n - x_n^t[t]) + \phi \cdot (C[t] - V[t]).
\]

(16)

By the Lyapunov optimization theory, it is easy to show that minimizing the above upper-bound of the drift-plus-penalty is equivalent to minimizing the drift-plus-penalty itself, in terms of the queue stability and objective optimization.

**Remark.** Beyond following the standard Lyapunov optimization framework \([30]\), our own contributions in this part are two-fold. First, we explicitly define the virtual queue, and analytically connect the user participatory constraint and the queue stability. This is the basis of applying Lyapunov optimization in our problem. Second, we propose an upper-bound (16) for the drift-plus-penalty, which is problem-specific and does not have a generic form suitable for all problems. The later on-line policy is based on this upper-bound.

### B. On-line Allocation Policy

Based on the above theoretical analysis, we now design an on-line policy that aims at minimizing the drift-plus-penalty upper-bound in (16) in each time slot. We present such a Lyapunov optimization based policy in Policy 1.

1) Algorithm Design: The proposed Policy 1 consists of an allocation rule and an updating rule in each time slot. The allocation rule determines the sensor selection (allocation) \( x^t[t] \) in each slot \( t \), based on the current network information \( \theta[t] \) and the current queue backlogs \( q^t \), aiming at minimizing the upper-bound of drift-plus-penalty in (16). The updating rule updates the queue backlogs based on the current allocation result \( x^t[t] \) according to (11). It is easy to see that Policy 1 relies only on the current network information and the past sensor selection history (captured by the queue backlogs), while not on any complete or stochastic future information.

The first inequality follows because \( (q - x)^\dagger + D \leq q^2 + x^2 + D^2 + 2q \cdot (D - x) \). The second inequality follows because \( x_n^t[t]^2 \leq 1 \).
2) Optimality: Now we provide the optimality of Policy 1. Let $S^t[t]$ denote the social welfare generated in each slot $t$, and $S^*$ denote the maximum social welfare benchmark with the stochastic information (derived in Section III). Formally,

**Theorem 1 (Optimality).**

$$
\lim_{T \to \infty} \frac{1}{T} \sum_{t \in T} E(S^t[t]) \geq S^* - \frac{B}{\phi}.
$$

The proof follows the standard Lyapunov optimization theory [30]. By Theorem 1, we can find that Policy 1 converges to the maximum social welfare benchmark asymptotically, with a controllable approximation error bound $O(1/\phi)$.

Intuitively, in Policy 1, each virtual queue can be viewed as a regulation factor for lowering (regulating) the sensing cost of that user, and hence increasing the selection probability of that user. By the updating rule in Policy 1, we can further obtain the following approximation for the queue backlog\(^8\)

$$
q^t_n \approx q^0_n - \sum_{k=0}^{t-1} x^+_n[k] + t \cdot D_n.
$$

This implies that the time-attenuated queue backlog $q^t_n$ can be used to approximate the gap between the required allocation probability (i.e., $D_n$) and the actual allocation probability till slot $t$ to slot $t$ (i.e., $\sum_{k=0}^{t-1} x^+_n[k]$). Notice that the queue backlog $q^0_n$ is bounded, hence the gap goes to zero as $t \to \infty$.

V. A UCTION-BASED ON-LINE SENSOR SELECTION POLICY

In this section, we consider the scenario of information asymmetry, where the sensing cost of each user $n$ realized in each time slot $t$ (i.e., $c_n[t]$) is regarded as his private information, and cannot be observed by the SP. Obviously, without this private sensing cost, the SP cannot implement the allocation rule in Policy 1.

A. Auction Mechanism Design

We design an (reverse) VCG auction to address the credible information disclosure of users in each time slot, where the SP is the auctioneer (buyer), and users are the bidders (sellers). A standard VCG auction usually consists of an allocation rule (winner determination) and a payment rule. Our proposed auction mechanism involves a set of regulation factors (which are introduced for ensuring the user participatory constraint), hence includes an additional updating rule for the regulation factors. We present the detailed auction mechanism in Policy 2. Next we will explain these rules in details.

For convenience, we denote $c^t_n[t]$ as each user $n$’s bid (report) regarding his sensing cost in each slot $t$, and $\mu^t_n$ as the regulation factor (similar as the virtual queue in Section IV) associated with each user $n$ in each slot $t$.

1) **Allocation Rule**: The allocation rule aims at maximizing a regulated social welfare in each time slot:

$$
\tilde{S}[t] \triangleq V[t] - \sum_{n \in \mathcal{N}} x_n[t] \cdot \tilde{c}_n[t],
$$

**Policy 2: Auction-based Policy (Information Asymmetry)**

**Initialization**: $\mu = \mu^0$;

**for each time slot** $t = 0, 1, ..., T$ **do**

1. Denote $c^t_n[t]$ as the bid of each user $n$;
2. **Allocation Rule**: 
   
   $x^t_n = \arg \max_{x^t_n} V[t] - \sum_{n \in \mathcal{N}} x_n[t] \cdot (c^t_n[t] - \mu^t_n)$
   
3. **Payment Rule**: 
   
   $p_n[t] = x_n[t] \cdot (V^t[t] - C^t_{n^t}[t] - \tilde{S}_{\mathcal{N}}[t] + \mu^t_n)$
   
4. **Updating Rule**: 
   
   $\mu^t_{n+1} = \frac{1}{\phi} \cdot \left( [\phi \cdot \mu^t_n - x^+_n[t]] + D_n \right), \quad \forall n \in \mathcal{N}$

where $\tilde{c}_n[t] \triangleq c^t_n[t] - \mu^t_n$ is the regulated sensing cost of user $n$, depending on both the user bid and the regulator factor. For convenience, we denote $x^t_n \triangleq \{ x^+_n[k], \forall k \in \mathcal{N} \}$ as the allocation result in slot $t$ (i.e., that maximizes $\tilde{S}_n[t]$).

2) **Payment Rule**: The payment to user $n$ in each time slot $t$ is: (i) $p_n[t] = 0$ if user $n$ is not selected, i.e., $x^+_n[t] = 0$, or (ii) if user $n$ is selected, i.e., $x^+_n[t] = 1$, then

$$
p_n[t] = V[t] - C_{n^t}[t] - \tilde{S}_{\mathcal{N}}[t] + \mu^t_n,
$$

where $V[t]$ is the total sensing value under $x_n[t]$, $C_{n^t}[t] = \sum_{k \neq n} x^+_k[t] \cdot \tilde{c}_k[t]$ is the total sensing cost except that of user $n$ under $x_n[t]$, and $\tilde{S}_{\mathcal{N}}[t]$ is the maximum achievable social welfare when excluding user $n$ in the system. The first 3 terms correspond to the payment in a standard VCG auction. The last term is used to compensate the user cost regulation.

3) **Updating Rule**: Inspired by Policy 1, we have:

$$
\mu^t_{n+1} = \frac{1}{\phi} \cdot \left( [\phi \cdot \mu^t_n - x^+_n[t]] + D_n \right), \quad \forall n \in \mathcal{N}.
$$

The above updating rule is exactly same as that in Policy 1, by simply viewing $\phi \cdot \mu^t_n$ as $q^0_n$. Obviously, if users are truthful, then the above allocation/updating rule achieves the exactly same allocation and performance as in Policy 1.

B. Truthfulness and Optimality

Now we study the truthfulness and optimality of Policy 2.

**Theorem 2 (Truthfulness).** The auction in Policy 2 is truthful.

**Proof:** Due to the space limit, we only show that each user $n$ has no incentive to report (bid) a cost higher than his true cost.\(^9\) There are 4 possible outcomes:

(a) \{loss, loss\}: user $n$ loses when bidding both truthfully and non-truthfully. He receives a zero payment in both strategies.

(b) \{win, loss\}: user $n$ wins (loses) when bidding truthfully (non-truthfully). He receives a smaller payment (i.e., zero) when bidding non-truthfully.

(c) \{loss, win\}: user $n$ loses (wins) when bidding truthfully (non-truthfully). This is practically impossible, as a user losing with a lower cost will never win when submitting a higher cost.

\(^9\)The proof for “users are not willing to report costs lower than the true values” is similar. Please refer to [31] for details.
The whole area is divided into 2500 grids, each corresponding to a square of 200m × 200m. The sensing region of each user in each time slot is simply defined as a disk, centered at his location, with a radius of 800m. Users move according to the random walk model: in each time slot, each user moves to a location (grid) randomly according to certain probability distribution. The sensing region of each user in each time slot is defined as a disk, centered at its location, with a radius randomly picked from [400m, 800m]. We run the system in a period of 10,000 time slots, which is long enough for obtaining stable outcomes under our adopted policies.

We consider two different simulation scenarios (a) and (b), depending on the different data value distributions in different areas, as shown in Fig. 2. In scenario (a), there is no hotspot, and all grids are of the similar importance. Hence, the data value in different areas follows an i.i.d. distribution. In scenario (b), there is one hotspot, and the grids near to the centre of the hotspot are more important than those far from the hotspot, and hence have larger data values. Note that a scenario with multiple hotspots can be viewed as an intermediate case between (a) and (b). For fair comparison, we set the average data value in the whole area as 0.5 for both scenarios.

### A. Performance Comparisons

Now we compare the performance of our proposed policy with the RADP-VPC policy proposed in [24] and [25], a well-known policy that considers the participation incentive. To draw a more convincing conclusion, we also compare our policy with those not considering the participation incentive, e.g., Random selection and Greedy selection (both are widely used in practical applications such as Waze [10]).

1) Dropping Probability: We first compare the user dropping probability under different policies. In this simulation, we set the dropping threshold as 0.5 for all users. Namely, if the allocation probability of a user is less than 0.5, the user will drop out of the system. Fig. 3 illustrates the dynamics of user allocation probabilities as well as the dropping of users. We can see that, in scenario (a) (the first row), more than 70% of users drop under the Greedy or Random sensor selection policy, and around 25% of users drop under the RADP-VPC policy (α = 1); and in scenario (b) (the second row), more than 90% of users drop under the Greedy or Random sensor selection policy, and more than 50% of users drop under the RADP-VPC policy (α = 1). Our proposed policy, however, retains all users in both scenarios.

We can further see that under the same policy (except our proposed one), more users drop in the scenario (b) than in scenario (a). The reason is as follows. In scenario (b) with one hotspot, most of the data value is concentrated in the hotspot area, and hence a large total sensing value can potentially be collected by a small number of users (located in the hotspot area). In scenario (a) with no hotspot, however, the data value is uniformly distributed in all areas, and hence a large total sensing value can be collected only by a large enough number of users (distributed in the whole area). Hence, to achieve the same level of sensing value, the number of sensors needed in scenario (b), on average, is smaller than that needed in scenario (a). Accordingly, the user allocation probability is lower, and hence more users drop, in scenario (b).

2) Social Welfare: We then compare the average social welfare under different policies in Fig. 4. Curve (1) is the maximum social welfare with no participatory constraint, and serves as an upper-bound of the maximum achievable social welfare with the participatory constraint. Curve (2) is the maximum social welfare benchmark (with the participatory constraint) with complete or stochastic future information derived in Section III. The gap between curves (1) and (2) is called the incentive cost, which is used to guarantee the user long-term

\[ \text{In the RADP-VPC policy, each user } n \text{’s cost is regulated by a virtual credit } v_n, \text{ and the virtual credit } v_n \text{ is updated in the following way: (i) } v_n = v_n + \alpha \text{ if user } n \text{ is not selected as sensor in the previous slot, and (ii) } v_n = 0 \text{ if user } n \text{ is selected as sensor in the previous slot, where } \alpha > 0 \text{ is a controllable parameter. Intuitively, a larger } \alpha \text{ can better satisfy the user participatory constraint, but may reduce the generated social welfare.} \]

\[ \text{In the Greedy (Random) policy, users are selected one by one in a descending (random) order of their generated social welfare.} \]

\[ \text{To reduce the “start effect” where a user may mistakenly drop in the first few slots (due to the low allocation probability in these slots), we assume that all users will be selected as sensor in the first 40 time slots.} \]
RADP-VPC largely depends on the choice of parameter $\alpha$, e.g., {mark asymptotically, with very small approximation errors, $\alpha = 1$}. Due to the higher dropping probability, the incentive cost is higher in scenario (b) than (a). Namely, the incentive cost is higher in scenario (b) than (a) due to the higher dropping probability.

Curves (3a)-(3c) denote the social welfares achieved by our proposed Lyapunov-based Policy 1 or 2 (with $\phi =$ 20, 10, and 5, respectively). Our policy converges to the optimal benchmark asymptotically, with very small approximation errors, e.g., \{1%, 2%, 3%\} in scenario (a) and \{1.5%, 3%, 4.5%\} in scenario (b). Note that the approximation error bound is controllable, via choosing different values of $\phi$. We can further see that the benchmark (i.e., the maximum social welfare) is higher in scenario (b), as the same amount of sensing value can potentially be collected by fewer users in scenario (b) than in scenario (a). Accordingly, our proposed policy can achieve a higher social welfare in scenario (b).

Curves (4a)-(4c) denotes the social welfares achieved by the RADP-VPC policy (with $\alpha =$ 1, 0.5, and 0.2, respectively) proposed in [24] and [25]. Obviously, the performance of RADP-VPC largely depends on the choice of parameter $\alpha$. In scenario (a), the social welfare gap between the RADP-VPC policy and our policy ranges from 15% (when $\alpha =$ 1) to 50% (when $\alpha =$ 0.2). In scenario (b), this gap increases to 40% and 75%. In fact, different from our policy or benchmark, the RADP-VPC policy achieves a worse performance in scenario (b), due to the higher dropping probability in scenario (b). This illustrates the importance of considering the long-term participatory incentive in a sensing system.

Finally, Curves (5) and (6) denotes the social welfares achieved by the Random and Greedy sensor selection policies. Neither policy considers the long-term participation incentive, hence users drop quickly (see Fig. 3) and the social welfare decreases dramatically. The social welfare gap between the these two policies and our policy is larger than 60% in scenario (a) and 85% in scenario (b). Similarly to the RADP-VPC policy, these two policies both achieves a worse performance in scenario (b) than in (a), due to the higher dropping probability in scenario (b). Counter-intuitively, the Greedy policy achieves a worse performance than the Random policy, due to the higher user dropping probability in the Greedy policy. This also illustrates the importance of considering the long-term participatory incentive in a sensing system.

B. Impact of Participatory Constraint

So far, we have shown in Fig. 4 that our policy converges to the maximum social welfare benchmark asymptotically. Next, we show in Figs. 5-6 that how the participatory constraint affects this benchmark. We provide the results in scenario (a) only, as those in scenario (b) is similar.

Fig. 5 illustrates the user allocation probability vs the number of participating users, under different sensing costs (where $C/W$ denotes the average ratio of unit sensing cost and unit data value). Obviously, the allocation probability decreases with both the sensing cost and the number of users (due to the partial conflict of their sensing activities). Note that in this result, there is no participatory constraint. Namely, users never
Fig. 5. Allocation Probability vs User Number

Fig. 6. Social Welfare vs User Number

drop, and in each time slot they will be selected based on the realized costs. This result is useful for explaining the different impacts of participatory constraint discussed later.

Fig. 6 illustrates the maximum social welfare (benchmark) vs the number of participating users $N$, under different dropping thresholds. We can see that when the dropping threshold is small (e.g., $D_n \leq 0.35$), the maximum social welfare always increases with the number of users, and the increase rate becomes larger with a smaller dropping threshold. When the dropping threshold is large (e.g., $D_n \geq 0.4$), the maximum social welfare first increases with the number of users, and then decreases with the number of users. This implies that in a sensing system with a mild or no participatory constraint (e.g., a small or zero dropping threshold), we can always increase the social welfare by involving more users into the sensing system. With a stringent participatory constraint (e.g., a large dropping threshold), however, involving more users may not always increase the social welfare, due to the high incentive cost to retain users in the system. We further notice that given the number of participating users, the maximum social welfare decreases with $D_n$ when $D_n \geq 0.2$, while keeps unchanged when $D_n \leq 0.2$. This implies that a very small dropping threshold (i.e., a very loose participatory constraint) does not affect the system performance significantly.

VII. CONCLUSION

In this work, we studied the optimal sensor selection problem in a general time-dependent and location-aware participatory sensing system with the user long-term participatory constraint. We proposed Lyapunov based on-line sensor selection (auction) policies, which do not rely on future information and achieve the optimal off-line benchmark performance asymptotically. There are several possible extensions in the future work. An interesting one is to study the truthful mechanism when users are not myopic and can somehow anticipate the impact of their activities on the future time slots.

REFERENCES