Economics of Public Wi-Fi Monetization and Advertising

Haoran Yu*, Man Hon Cheung*, Lin Gao*†, and Jianwei Huang*
*Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong
†School of Electronic and Information Eng., Harbin Institute of Technology Shenzhen Graduate School, China
Email: yh012@ie.cuhk.edu.hk, mhcheung@ie.cuhk.edu.hk, gaolin@hitsz.edu.cn, jwhuang@ie.cuhk.edu.hk

***Abstract*—There has been a proliferation of public Wi-Fi hotspots that serve a significant amount of global mobile traffic today. In this paper, we propose a general *Wi-Fi monetization* model for public Wi-Fi hotspots deployed by venue owners (VOs), where VOs generate revenue from providing both the premium Wi-Fi access and the advertising sponsored Wi-Fi access to mobile users (MUs). With the premium access, MUs directly pay VOs for their Wi-Fi usage; while with the advertising sponsored access, MUs watch advertisements for the free usage of Wi-Fi. VOs sell their ad spaces to advertisers (ADs) via an ad platform, and share a proportion of the revenue with the ad platform. We formulate the economic interactions among the ad platform, VOs, MUs, and ADs as a three-stage Stackelberg game. By analyzing the equilibrium, we show that the ad platform's advertising revenue sharing policy affects a VO's Wi-Fi price but not the VO's advertising price. Moreover, we prove that a single term called *equilibrium indicator* determines whether a VO will fully rely on the premium access, or fully rely on the advertising sponsored access, or obtain revenue from both types of access. Numerical results show that the VO obtains a large revenue under a large advertising concentration level and a medium MU visiting frequency.**

I. Introduction

A. Motivations

Global mobile traffic grows unprecedentedly, and is expected to reach 24.3 exabytes per month by 2019 [1]. Facilitated by the recent technology development, Wi-Fi data offloading has become one of the main approaches to accommodate the mobile traffic explosion. According to the forecast of Cisco, 54% of the global mobile traffic will be offloaded to Wi-Fi and small cell networks by 2019 [1].

According to the report of Wireless Broadband Alliance [2], 50% of worldwide commercial Wi-Fi hotspots are owned by different *venues*, such as cafes, restaurants, hotels, and airports.[1] The venue owners (VOs) build public Wi-Fi for the access of mobile users (MUs), in order to enhance MUs' experiences and meanwhile provide location-based services (*e.g.*, shopping guides, navigation, billing) [3].

To compensate the Wi-Fi deployment and operational costs, VOs have been actively considering *monetizing* their hotspots.

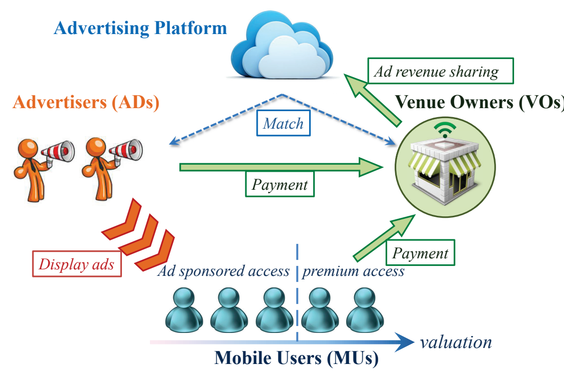

Fig. 1: Public Wi-Fi Monetization Ecosystem.

One conventional business model is that VOs directly charge MUs for their Wi-Fi usage. However, as most MUs prefer free Wi-Fi access, it is suggested that VOs should come up with new business models to create extra revenue streams [2]. Wi-Fi *advertising*, where VOs obtain revenue from advertisers (ADs) by broadcasting ADs' advertisements on their hotspots, has emerged as a promising monetization approach. It is especially attractive to ADs, as the accurate localization of Wi-Fi allows ADs to make location-aware advertising. Furthermore, with MUs' basic information collected by the hotspots,[2] ADs can efficiently find their targeted customers and deliver the personalized contents to them.

Nowadays, several companies, including SOCIFI (collaborated with Cisco) [4], Boingo [5], and Purple Wi-Fi [6], are providing the following technical supports for VOs and ADs on Wi-Fi advertising. First, they offer the devices and softwares which enable VOs to display selected advertisements on the Wi-Fi login pages and collect the statistics information (*e.g.*, number of visitors and click-through rates). Second, they manage the *ad platforms*, where VOs and ADs trade the ad spaces. Once ADs purchase the ad spaces, VOs and ad platforms share ADs' payment based on the sharing policy designed by ad platforms. Although Wi-Fi advertising has been emerging in practice, its influence on entities like VOs and MUs, as well as the detailed pricing and revenue sharing policies, have not been carefully studied in the existing literatures. This motivates our study in this work.

B. Contributions

We consider a general Wi-Fi monetization model, where VOs monetize their hotspots by providing two types of Wi-

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[1] Specifically, "Retails" and "Cafes & Restaurants" are the venues with the largest number of hotspots (4.5 and 3.3 million globally in 2015, respectively), followed by "Hotels", "Municipalities", and "Airports" [2].

[2] For instance, when MUs login the public hotspots with their social network accounts, SOCIFI collects customers' information such as age and gender [4].

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Fi access: *premium access* and *advertising sponsored access*. With the premium access, MUs pay VOs according to certain pricing schemes. With the advertising sponsored access, MUs are required to watch the advertisements, after which MUs use Wi-Fi for free during a certain period.[3] Depending on the VOs' pricing schemes, MUs with different valuations on Wi-Fi connection time will choose different types of access. For MUs choosing the advertising sponsored access, VOs sell the corresponding ad spaces to ADs through participating in the ad platform. Fig. 1 illustrates the Wi-Fi monetization ecosystem.

In this work, we will study such a Wi-Fi monetization system from two aspects.

1) Part 1: In the first part of our work, we model the economic interactions among different decision makers as a *three-stage Stackelberg game*, and study the game equilibrium systematically. Specifically, in Stage I, the ad platform designs an advertising revenue sharing policy for each VO, which indicates the fraction of advertising revenue that a VO needs to share with the ad platform. In Stage II, each VO announces its Wi-Fi price to MUs for the premium access, and its advertising price to ADs. In Stage III, MUs choose the access types (premium or advertising sponsored access), and ADs decide the number of ad spaces to purchase from the VO.

We analyze the equilibrium of the proposed Stackelberg game systematically. Our analysis shows that: (i) the VO's advertising price (to ADs) in Stage II is independent of the ad platform's advertising revenue sharing policy in Stage I, as a VO always charges the advertising price to maximize the total advertising revenue; (ii) the VO's Wi-Fi price (to MUs) in Stage II is set based on the ad platform's sharing policy in Stage I, since a VO will increase the Wi-Fi price to push more MUs to the advertising sponsored access if the VO can keep more advertising revenue.

2) Part 2: In the second part of our work, we define an *equilibrium indicator*, which intuitively describes the VO's relative benefit in providing the premium access, compared with providing the advertising sponsored access. The value of the equilibrium indicator determines whether a VO will fully rely on the premium access, or fully rely on the advertising sponsored access, or obtain revenue from both types of access.

The key contributions of this work are as follows:

- *Novel Business Model for Wi-Fi Monetization*: To the best of our knowledge, this is the first work studying the advertising sponsored public Wi-Fi hotspots. We consider a general Wi-Fi monetisation model with both the premium access and the advertising sponsored access, which enable a VO to segment the market based on MUs' valuations, and maximize the VO's revenue.
- *Systematic Study of Public Wi-Fi Ecosystem*: We derive the optimal strategies for the ad platform, VOs, MUs, and ADs, and analyze the equilibrium of the three-stage game. We show that a single term called equilibrium indicator determines whether a VO only relies on the premium access, or only relies on the advertising sponsored access, or generates revenue from both types of access.
- *Simulation of Uniform Revenue Sharing Policy*: We construct simulations for the situation where the ad platform sets a uniform advertising revenue sharing ratio for all VOs. Simulation results show that under this uniform policy, the payoff of the VO with a large visiting frequency increases, and the payoff of the VO with a medium visiting frequency decreases.

[3]As an example, SOCIFI technically supports the premium access as well as the advertising sponsored access for its subscribed VOs [4].

C. Related Work

Several recent works have studied the business models related to Wi-Fi networks. Duan *et al.* in [7] and Musacchio *et al.* in [8] studied the pricing schemes of Wi-Fi owners. Yu *et al.* in [9] analyzed the optimal strategies for network operators and VOs to deploy public Wi-Fi networks cooperatively. Gao *et al.* in [10] and Iosifidis *et al.* in [11] investigated the Wi-Fi capacity trading problem, where cellular network operators lease third-party Wi-Fi offload their traffic. Manshaei *et al.* in [12] and Afrasiabi *et al.* in [13] analyzed the problem where Wi-Fi owners collaborate and share Wi-Fi access points. Different from these works, we study the monetization of public Wi-Fi through the Wi-Fi advertising, and focus on the economic interactions among different entities in the entire ecosystem.

A closely related work on Wi-Fi advertising is [14], where Bergemann *et al.* considered an advertising market where ADs have different market shares. Differences between [14] and our work are as follows. First, in [14], an MU is only interested in one AD's product, while in our model, an MU can be interested in multiple ADs' products. Second, in [14], authors analyzed the market with an infinite number of ADs, while in our work, we first analyze the problem with a finite number of ADs, and then consider the limiting asymptotic case with an infinite number of ADs. Moreover, in [15], [16], authors explored the influence of the targeting on the advertising market. However, none of the works [14]–[16] considered the ad platform and the associated advertising revenue sharing, which is a key focus of our study.

II. System Model

In this section, we define the strategies of four types of decision makers in the Wi-Fi monetization ecosystem: the ad platform, VOs, ADs, and MUs. We formulate their interactions as a three-stage Stackelberg game.

A. Ad Platform

The ad platform plays two major roles in the ecosystem. First, it offers the platform for VOs to find out ADs and sell their ad spaces to ADs. Second, it offers the necessary technical supports for VOs to display advertisements on their Wi-Fi hotspots.[4] To compensate its operational cost, the ad platform will share a fraction of the VOs' revenue collected from selling ad spaces to ADs.[5]

[4]SOCIFI Media Network is the ad platform managed by SOCIFI [4]. It collects visitors' data, provides the statistics such as the click-through rates, and supports the ad display in different formats (*e.g.*, website, video, message).

[5]As stated in [4], there is no cost for VOs to register SOCIFI Media Network, which earns profits from sharing revenue with VOs.

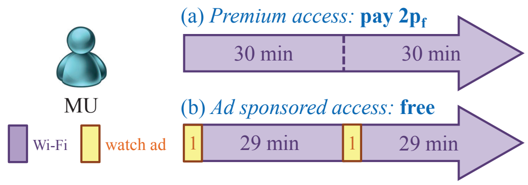

Fig. 2: Illustration of Wi-Fi Access.

Revenue Sharing Ratio δ: We consider a simple model, where there is no coupling among different venues or VOs (which is the case in most real situations). Hence, we can focus on a particular VO without loss of generality. Let $\delta \in [0,1]$ denote the ad platform's revenue sharing policy for the VO, which corresponds to the fraction of the advertising revenue that the VO needs to transfer to the ad platform. In Section VII, we will further discuss the coupling among VOs, and allow the ad platform to set a uniform $\delta_U \in [0,1]$ for all VOs.

B. VO's Pricing Decision

The VO provides two types of Wi-Fi access for MUs: the premium access and the advertising sponsored access.

Wi-Fi Price p_f: We assume that the VO charges the premium access based on a time segment structure: Each time segment has a fixed length (e.g., 30 minutes), and the VO charges p_f per time segment. Fig. 2 illustrates an example, where the length of one time segment is 30 minutes. If an MU chooses the premium access for two segments, it pays $2p_f$, and can use the Wi-Fi for the entire 60 minutes.

Advertising Price p_a: The MU can also use the Wi-Fi for free by choosing the advertising sponsored access. In this case, the MU has to watch an advertisement at the beginning of each time segment. To guarantee the fairness among the MUs that choose the advertising sponsored access, we assume that all advertisements have the same displaying time. Let p_a denote the advertising price for ADs (for showing one advertisement). In the example of Fig. 2, the ad display time is 1 minute. If an MU chooses the advertising sponsored access for two segments, it needs to watch 2 minutes of advertisements, and can use the Wi-Fi for the remaining 58 minutes free of charge. Meanwhile, the VO can obtain a revenue $2p_a$ from ADs.

C. MUs' Access Choices

MU's Payoff and Decision: We consider the operations in a certain period (*e.g.*, one week).[6] Let N denote the number of MUs visiting the VO during the period. We use $\theta \in [0,\theta_{\max}]$ to describe a particular MU's valuation on the Wi-Fi connection. We assume that θ follows the uniform distribution.[7]

Let $d \in \{0,1\}$ denote an MU's access choice, with $d=0$ denoting the ad sponsored access, and $d=1$ denoting the premium access. We normalize the length of each segment to 1, and define the payoff of a type-θ MU in *one* time segment as

$$\Pi^{\rm MU}(\theta,d) = \begin{cases} \theta(1-\beta), & \text{if } d=0, \\ \theta - p_f, & \text{if } d=1, \end{cases} \tag{1}$$

where $\beta \in [0,1]$ and term $1-\beta$ describes the discount of the MU's utility due to the inconvenience of watching advertisements.[8] When $d=0$, the MU's equivalent Wi-Fi usage time during each time segment is $1-\beta$; when $d=1$, the MU pays p_f to use the Wi-Fi during the whole segment.

Each MU will choose an access type that maximizes its payoff. Let $\varphi_f(p_f), \varphi_a(p_f) \in [0,1]$ denote the fractions of MUs choosing the premium access and the advertising sponsored access under price p_f, respectively.

MUs' Visiting Frequency λ: We further assume that the number of time segments that an MU demands at the venue within the considered period (say one week) is a random variable k with domain $\mathbb{N}$, and follows the Poisson distribution with parameter $\lambda > 0$. We assume that all MUs visiting the venue have the homogenous parameter λ. Since $\lambda = \mathbb{E}\{k\}$, λ reflects MUs' visiting frequency at the venue, and a larger λ implies that MUs visit the venue more often.

Since the current Wi-Fi technology already achieves a large throughput, we assume that the capacity of the VO's Wi-Fi is not a bottleneck and can be considered as unlimited.[9]

D. ADs' Advertising Model

There are M ADs who seek to display advertisements at the venue. We assume that MUs have intrinsic interests on different ADs' products. An MU will purchase a particular AD's product, if and only if it is interested in that AD's product, and meanwhile sees the AD's advertisement at least once. This assumption reflects the *complementary perspective* of advertising [18], and has been widely used in the advertising literature [14]–[16]. Intuitively, this assumption means that the advertising does not change the consumers' preferences, but becomes a necessary condition to generate a purchase.[10]

AD's Popularity σ: We define the popularity of an AD as the percentage of MUs who are interested in the AD's product. Each AD's popularity at the venue is described by its type σ, which is uniformly distributed in $[0,\sigma_{\max}]$. We assume the popularity of a type-σ AD is

$$s(\sigma) \triangleq \gamma e^{-\gamma\sigma}, \tag{2}$$

[6]The time length of the period is chosen such that all the system parameters introduced in this paper can be well approximated by constants.

[7]The uniform distribution has been widely used to model MUs' valuations on the wireless service [7], [17]. The consideration of other distributions does not change the main conclusions in this paper.

[8]In Fig. 2's example, the time segment length is 30 minutes. If an MU chooses the advertising sponsored access and its utility is equivalent to the case where it directly uses Wi-Fi for 20 minutes (which we call *equivalent Wi-Fi usage time*) without watching advertisements, parameter β is computed as $1-\frac{20}{30}=\frac{1}{3}$. In this case, watching the advertisements generates significant discomfort than the actual length of ad display.

[9]The similar assumption on the unlimited Wi-Fi capacity has been made in reference [7]. Next we briefly discuss the problem with a limited Wi-Fi capacity. First, if the capacity is limited but MUs who choose $d=0$ and $d=1$ experience the same congestion level, then essentially it will not change our analysis. Second, if MUs with $d=0$ and $d=1$ experience different congestion levels but the difference in the congestion level is a constant, then the congestion difference can be easily factorized in our model, and also does not change the results. Third, if MUs with $d=0$ and $d=1$ experience different congestion levels and the difference is not a constant, the analysis will be more complicated, and we will discuss this in our future work.

[10]Besides the *complementary perspective*, reference [18] also mentioned the *persuasive perspective*, where the advertising alters consumers' preferences. We will study the persuasive perspective in our future work.

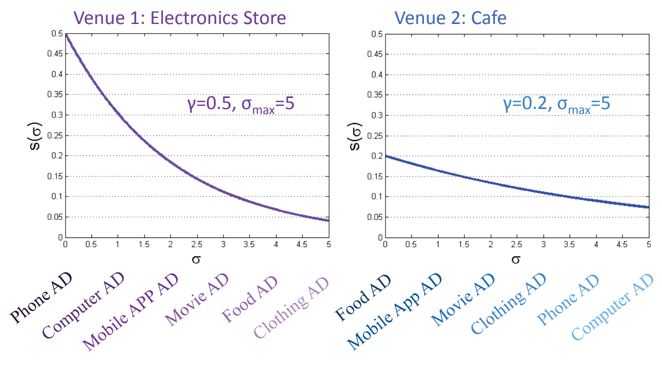

Fig. 3: Comparison of Venues with Different γ.

Stage I
The ad platform specifies the revenue sharing policy δ.

$\Downarrow$

Stage II
The VO specifies the Wi-Fi price p_f and ad price p_a.

$\Downarrow$

Stage III
Each MU with type $\theta \in [0, \theta_{\max}]$ makes access choice d; Each AD with type $\sigma \in [0, \sigma_{\max}]$ purchases m ad spaces.

Fig. 4: Three-Stage Stackelberg Game.

where $\gamma \in (0, 1]$.[11] It is easy to find that $s(\sigma)$ is decreasing in the type index σ and $s(\sigma) \leq 1$. The parameter γ measures the advertising *concentration level* at the venue, which is defined as the asymmetry of the popularities of ADs with different type σ. A large γ implies a high advertising concentration level, since those ADs with small values of γ have much higher popularities than other ADs.

In Fig. 3, we show different types of ADs' popularities at an electronics store and a cafe, respectively. Since the electronics store is more specialized and most visitors have interests on the electronics products, the phone AD and computer AD are much more popular than other types of ADs. Hence, the concentration level γ of the electronics store is high. On the contrary, the cafe is less specialized and has a lower concentration level than the electronics store.

Advertisement Display: Next we introduce the advertisement displaying setting. Recall that the number of time segments that an MU demands is Poisson distributed with an average of λ (segments/MU), and the proportion of MUs choosing the advertising sponsored access is $\varphi_a(p_f)$. Hence, the expected number of ad spaces that the VO has during the entire time period is $\lambda N \varphi_a(p_f)$. Let m be the number of advertisements that an AD decides to display at the venue. We assume when an MU chooses the advertising sponsored access, at the beginning of every time segment, the VO shows an advertisement from the AD to the MU with probability[12]

$$\frac{m}{\lambda N \varphi_a(p_f)}. \tag{3}$$

AD's Payoff: Next we study a type-σ AD's payoff. For an MU choosing the advertising sponsored access, the probability that it demands $k \in \mathbb{N}$ time segments is $\frac{e^{-\lambda}\lambda^k}{k!}$, and the probability that it does not see the AD's advertisement during these k time segments is $\left(1 - \frac{m}{\lambda N \varphi_a(p_f)}\right)^k$. Therefore, considering all possibilities of k, the probability for this MU to see the AD's advertisement *at least once* is

$$1 - \sum_{k=0}^{\infty} \left(\frac{e^{-\lambda}\lambda^k}{k!} \left(1 - \frac{m}{\lambda N \varphi_a(p_f)}\right)^k \right). \tag{4}$$

Based on the Maclaurin expansion of the exponential function, the probability above equals

$$1 - e^{-\frac{m}{N\varphi_a(p_f)}}, \tag{5}$$

which is an increasing and concave function of m.

Recall that an MU will purchase the AD's product, if and only if the MU is interested in the AD's product and sees its advertisement at least once. We use $\Pi^{\mathrm{AD}}(\sigma, m)$ to denote a type-σ AD's payoff, and have

$$\Pi^{\mathrm{AD}}(\sigma, m) = aN\varphi_a(p_f) s(\sigma) \left(1 - e^{-\frac{m}{N\varphi_a(p_f)}}\right) - p_a m. \tag{6}$$

The parameter a is the profit that an AD generates when an MU purchases its product,[13] $N\varphi_a(p_f)$ is the number of MUs choosing the advertising sponsored access, $s(\sigma)$ is the type-σ AD's popularity, and p_a is the VO's advertising price.

E. Three-Stage Stackelberg Game

We formulate the interactions among the ad platform, the VO, MUs, and ADs by a three-stage Stackelberg game, as illustrated in Fig. 4. From Section III to Section V, we analyze the three-stage game by backward induction.

III. Stage III: MUs' Access and ADs' Advertising

In this section, we analyze MUs' optimal access strategies and ADs' optimal advertising strategies in Stage III, given the ad platform's revenue sharing policy δ in Stage I, and the VO's pricing decisions p_f and p_a in Stage II.

A. MUs' Optimal Access

Equation (1) characterizes an MU's payoff for one time segment. Since an MU's payoff for multiple time segments is simply the summation of its payoff for each time segment, an MU's access choice only depends on its type θ and is independent of the number of time segments it demands. Equation (1) suggests that a type-θ MU will choose $d = 1$

[11]Reference [14] used a similar exponential function to model the market share of a particular AD. However, reference [14] considered a model with an infinite number of ADs, and directly made assumptions on an AD's market share. In our work, we model a finite number of ADs, and use a randomly distributed parameter σ to describe an AD's popularity. In Section IV, we first analyze the VO's optimal pricing for a finite number of ADs, and then focus on the limiting asymptotic case with an infinite number of ADs. Therefore, compared with [14], our model and analysis are different and more reasonable.

[12]As shown in the later analysis, the VO will set p_a large enough so that the total number of displayed advertisements does not exceed $\lambda N \varphi_a(p_f)$. Hence, the summation of (3) over all ADs will not be greater than 1.

[13]Since our work focuses on studying the heterogeneity of ADs' popularities, we assume a is homogeneous for all ADs at the venue.

if $\theta - p_f \geq \theta(1-\beta)$, and $d = 0$ otherwise. Therefore, the optimal access choice of a type-θ MU is

$$d^*(\theta, p_f) = \begin{cases} 1, & \text{if } \theta \geq \theta_T(p_f), \\ 0, & \text{if } \theta < \theta_T(p_f), \end{cases} \tag{7}$$

where $\theta_T(p_f) \triangleq \min\left\{\frac{p_f}{\beta}, \theta_{\max}\right\}$ is the *threshold user type*. Intuitively, MUs with high valuations on the Wi-Fi connection will pay and use the Wi-Fi for the whole time segment, while MUs with low valuations will watch advertisements to have the free Wi-Fi consumptions.

Since θ follows the uniform distribution, under price p_f, the fractions of MUs choosing different types of access are

$$\varphi_a(p_f) = \frac{\theta_T(p_f)}{\theta_{\max}} \text{ and } \varphi_f(p_f) = 1 - \frac{\theta_T(p_f)}{\theta_{\max}}. \tag{8}$$

B. ADs' Optimal Advertising

According to (6), a type-σ AD's problem is as follows.

Problem 1. *The type-σ AD determines the optimal number of ad displays m^* by solving*

$$\max\ aN\varphi_a(p_f)s(\sigma)\left(1 - e^{-\frac{m}{N\varphi_a(p_f)}}\right) - p_a m \tag{9}$$

$$\text{var.} \quad m \geq 0, \tag{10}$$

where $s(\sigma)$ is the type-σ AD's popularity defined in (2).

Hence, the type-σ AD's optimal advertising strategy is:

$$m^*(\sigma, p_f, p_a) = \begin{cases} N\varphi_a(p_f)\left(\ln\left(\frac{a\gamma}{p_a}\right) - \gamma\sigma\right), & \text{if } 0 \leq \sigma \leq \sigma_T(p_a), \\ 0, & \text{if } \sigma_T(p_a) < \sigma \leq \sigma_{\max}, \end{cases} \tag{11}$$

where $\sigma_T(p_a) \triangleq \min\left\{\frac{1}{\gamma}\ln\left(\frac{a\gamma}{p_a}\right), \sigma_{\max}\right\}$ is the *threshold AD type*, indicating whether an AD places advertisements.

It is easy to see that $m^*(\sigma, p_f, p_a)$ is non-increasing in the AD's type σ. The reason is that an AD's popularity $s(\sigma)$ decreases with its type σ. Only for ADs with high popularities, the benefit of advertising can compensate the cost of purchasing the ad spaces from the VO.

Moreover, $m^*(\sigma, p_f, p_a)$ increases with the number of MUs choosing the advertising sponsored access, $N\varphi_a(p_f)$. However, the threshold $\sigma_T(p_a)$ is independent of $N\varphi_a(p_f)$. When $N\varphi_a(p_f)$ increases, the number of MUs that both choose the advertising sponsored access and like the product from an AD with type $\sigma = \sigma_T(p_a)$ indeed increases. While expression (5) implies that since there are more MUs, the probability for an MU to see the advertisements from the AD with type $\sigma = \sigma_T(p_a)$ decreases. As a result, the change of $N\varphi_a(p_f)$ does not affect the number of ADs who choose to display advertisements at the venue.

IV. Stage II: VO's Wi-Fi and Advertising Pricing

In this section, we study the VO's advertising pricing p_a and Wi-Fi pricing p_f in Stage II, given the ad platform's revenue sharing policy δ in Stage I, and considering the prediction of MUs and ADs' strategies in Stage III.

A. VO's Optimal Advertising Price p_a^*

We first assume that p_f is fixed and optimize the VO's advertising price p_a. In the next subsection, we will optimize p_f, by considering the result in this subsection.

Let $Q(p_a)$ denote the expected total number of sold ad spaces to all ADs. According to (11), if $p_a > a\gamma$, no AD will purchase the ad spaces and $Q(p_a) = 0$; if $0 \leq p_a \leq a\gamma$, we compute $Q(p_a)$ as follows:

$$\begin{aligned} Q(p_a) &= M\int_0^{\sigma_T(p_a)} \frac{1}{\sigma_{\max}} N\varphi_a(p_f)\left(\ln\left(\frac{a\gamma}{p_a}\right) - \gamma\sigma\right) d\sigma \\ &= \frac{MN\varphi_a(p_f)}{\sigma_{\max}}\left(\ln\left(\frac{a\gamma}{p_a}\right)\sigma_T(p_a) - \frac{\gamma}{2}\sigma_T^2(p_a)\right), \end{aligned} \tag{12}$$

where M is the number of ADs, and $\frac{1}{\sigma_{\max}}$ is the probability density function for an AD's type σ.

We define $\Pi_a^{\mathrm{VO}}(p_f, p_a, \delta)$ as the VO's expected advertising revenue. It is easy to obtain

$$\Pi_a^{\mathrm{VO}}(p_f, p_a, \delta) = \begin{cases} (1-\delta)p_a Q(p_a), & \text{if } 0 \leq p_a \leq a\gamma, \\ 0, & \text{if } p_a > a\gamma. \end{cases} \tag{13}$$

where $1-\delta$ denotes the fraction of advertising revenue that the VO keeps under the ad platform's policy. Based on (13), we formulate the VO's advertising pricing problem as follows.

Problem 2. *The VO determines p_a^* by solving*

$$\max \quad (1-\delta)p_a Q(p_a) \tag{14}$$

$$\text{s.t.} \quad Q(p_a) \leq \lambda N\varphi_a(p_f), \tag{15}$$

$$\text{var} \quad 0 \leq p_a \leq a\gamma, \tag{16}$$

where constraint (15) means that the VO can sell at most $\lambda N\varphi_a(p_f)$ ad spaces.

The solution to Problem 2 is summarized in the following proposition.

Proposition 1 (Advertising price)**.** *The VO's optimal advertising price p_a^* is independent of the ad platform's advertising revenue sharing policy δ, and is given by*

$$p_a^* = \begin{cases} a\gamma e^{-\sqrt{\frac{2\lambda\gamma\sigma_{\max}}{M}}}, & \text{if } \frac{\lambda}{M} \leq \min\left\{\frac{\gamma\sigma_{\max}}{2}, 1, \frac{2}{\gamma\sigma_{\max}}\right\}, \\ a\gamma e^{-\left(\frac{\gamma\sigma_{\max}}{2} + \frac{\lambda}{M}\right)}, & \text{if } \frac{\gamma\sigma_{\max}}{2} \leq \frac{\lambda}{M} \leq 1, \\ a\gamma e^{-\left(\frac{\gamma\sigma_{\max}}{2} + 1\right)}, & \text{if } \frac{\gamma\sigma_{\max}}{2} \leq 1 \leq \frac{\lambda}{M}, \\ a\gamma e^{-2}, & \text{other cases.} \end{cases} \tag{17}$$

We observe that the expression of p_a^* is sensitive to the number of ADs M, and the parameter of ADs' popularity distribution $\sigma_{\max}$. To facilitate the later analysis, we focus on a large advertising market in the rest of the paper with the following assumption.[14]

Assumption 1. *There are infinitely many ADs in the advertising market, i.e., $M \to \infty$, and the lowest popularity among all types of ADs is zero, i.e., $\sigma_{\max} \to \infty$.*

[14] Assumption 1 is for the sake of presentations. Without Assumption 1, there will be seven different regimes (which are divided based on the relations among $\frac{\lambda}{M}$, $\frac{\gamma\sigma_{\max}}{2}$, $\frac{2}{\gamma\sigma_{\max}}$, and 1) that we need to discuss (and we can solve), and we will not be able to include the full analysis here due to the space limit. The consideration of finite systems without Assumption 1 does not change the main results in the later sections. In reference [14], the authors directly modeled and analyzed the advertising market with an infinite number of ADs.

We define p_a^∞ as the VO's optimal advertising price under Assumption 1. According to Proposition 1, we conclude p_a^∞ in the following proposition.[15]

Proposition 2 (Advertising price with Assumption 1)**.** *Under Assumption 1, The VO's optimal advertising price p_a^∞ is*

$$p_a^\infty = \begin{cases} a\gamma e^{-\sqrt{\frac{2\lambda\gamma}{\eta}}}, & \text{if } 0<\lambda\le\frac{2\eta}{\gamma}, \\ a\gamma e^{-2}, & \text{if } \lambda>\frac{2\eta}{\gamma}, \end{cases} \tag{18}$$

where $\eta \triangleq \lim\limits_{M,\sigma_{\max}\to\infty} \frac{M}{\sigma_{\max}}$ and takes a value in $[0,\infty)$.

Next we explain the physical meaning of η. Under Assumption 1, if we randomly pick an MU, the expected number of ADs that the MU likes is computed as

$$\lim_{M,\sigma_{\max}\to\infty} M\int_0^{\sigma_{\max}} \frac{s(\sigma)}{\sigma_{\max}}d\sigma = \lim_{M,\sigma_{\max}\to\infty}\frac{M}{\sigma_{\max}}, \tag{19}$$

which is η. Hence, η describes the popularity of the ad market.

Next we discuss the VO's advertising price p_a^∞ with $\lambda \in \left(0,\frac{2\eta}{\gamma}\right]$ and $\lambda\in\left(\frac{2\eta}{\gamma},\infty\right)$, respectively.

1) Small λ with $\lambda\in\left(0,\frac{2\eta}{\gamma}\right]$: The advertising price p_a^∞ decreases with λ. This is because MUs' small demand rate λ leads to a limited number of ad spaces. When λ increases, the VO has more ad spaces to sell, and will decrease p_a^∞ to attract more ADs. We can verify that the number of sold ad spaces is $\lambda N\varphi_a(p_f)$, *i.e.*, the VO always sells out all of the spaces. We call ADs that purchase the ad spaces as *active* ADs, and the number of active ADs is

$$M\frac{\sigma_T(p_a^\infty)}{\sigma_{\max}} = \sqrt{\frac{2\lambda\eta}{\gamma}}. \tag{20}$$

Moreover, the VO's expected advertising revenue is

$$\Pi_a^{\mathrm{VO}}(p_f,p_a^\infty,\delta) = (1-\delta)\,aN\varphi_a(p_f)\,\lambda\gamma e^{-\sqrt{\frac{2\lambda\gamma}{\eta}}}. \tag{21}$$

Both (20) and (21) increase with λ for the small λ value.

2) Large λ with $\lambda\in\left(\frac{2\eta}{\gamma},\infty\right)$: The advertising price p_a^∞ is independent of λ. The reason is that, under MUs' large demand rate λ, the VO has sufficient ad spaces to sell, so it can directly set p_a^∞ to maximize the objective function (14) while guaranteeing the capacity constraint (15) satisfied. We can verify that the number of sold ad spaces $Q(p_a^\infty)$ is $\frac{2\eta}{\gamma}N\varphi_a(p_f)$, which is smaller than the capacity $\lambda N\varphi_a(p_f)$.[16] Furthermore, the expected number of active ADs is

$$M\frac{\sigma_T(p_a^\infty)}{\sigma_{\max}} = \frac{2\eta}{\gamma}, \tag{22}$$

and the VO's expected advertising revenue is

$$\Pi_a^{\mathrm{VO}}(p_f,p_a^\infty,\delta) = 2(1-\delta)\,aN\varphi_a(p_f)\,\eta e^{-2}. \tag{23}$$

Both (22) and (23) are independent of λ.

[15]In Section VII, we show that even without Assumption 1, the p_a^∞ derived in Proposition 2 is the optimal advertising price for most parameter settings.

[16]In this case, the VO can fill the unsold ad spaces with other contents, *e.g.*, its own business promotion, to guarantee the fairness among MUs choosing the advertising sponsored access.

B. *VO's Optimal Wi-Fi Price p_f^**

Now we analyze the VO's Wi-Fi pricing. We define $\Pi_f^{\mathrm{VO}}(p_f)$ as the VO's revenue in providing the premium access. Since there are $N\varphi_f(p_f)$ MUs choosing the premium access and the expected number of time segments that an MU demands is λ, we have

$$\Pi_f^{\mathrm{VO}}(p_f) = \lambda p_f N\varphi_f(p_f). \tag{24}$$

Based on (21), (23), and (24), we find that p_f affects the VO's revenue in providing both types of access. The VO's total revenue is computed as

$$\begin{aligned}&\Pi^{\mathrm{VO}}(p_f,\delta)\\ &= \lambda p_f N\varphi_f(p_f) + (1-\delta)\,aNg(\lambda,\gamma,\eta)\,\varphi_a(p_f),\end{aligned} \tag{25}$$

where

$$g(\lambda,\gamma,\eta) \triangleq \begin{cases} \lambda\gamma e^{-\sqrt{\frac{2\lambda\gamma}{\eta}}}, & \text{if } 0<\frac{\lambda\gamma}{\eta}\le 2, \\ 2\eta e^{-2}, & \text{if } \frac{\lambda\gamma}{\eta}>2. \end{cases} \tag{26}$$

By checking $\varphi_f(p_f)$ and $\varphi_a(p_f)$ in (8), it is easy to find that $\Pi^{\mathrm{VO}}(p_f,\delta)$ does not change with p_f for $p_f \in [\beta\theta_{\max},\infty)$. This is because all MUs will choose the advertising sponsored access if $p_f\ge\beta\theta_{\max}$, and increasing p_f will no longer have an impact on $\Pi^{\mathrm{VO}}(p_f,\delta)$. Therefore, we only need to consider optimizing $\Pi^{\mathrm{VO}}(p_f,\delta)$ over $p_f\in[0,\beta\theta_{\max}]$, and formulate the optimal Wi-Fi pricing problem as follows.

Problem 3. *The VO determines p_f^* by solving*

$$\max\ \lambda p_f N\left(1-\frac{p_f}{\beta\theta_{\max}}\right) + (1-\delta)aNg(\lambda,\gamma,\eta)\frac{p_f}{\beta\theta_{\max}} \tag{27}$$

$$\text{var}\quad 0\le p_f\le\beta\theta_{\max}. \tag{28}$$

Solving Problem 3, we conclude the VO's optimal Wi-Fi pricing in the following proposition.

Proposition 3 (Wi-Fi price)**.** *Given the ad platform's fixed sharing policy δ, the VO's optimal Wi-Fi price $p_f^*(\delta)$ lies in the interval $\left[\frac{\beta\theta_{\max}}{2},\beta\theta_{\max}\right]$, and is given by*

$$p_f^*(\delta) = \frac{\beta\theta_{\max}}{2} + \min\left\{\frac{(1-\delta)\,a}{2\lambda}g(\lambda,\gamma,\eta),\frac{\beta\theta_{\max}}{2}\right\}. \tag{29}$$

When $\delta=1$, *i.e.*, the ad platform takes away all the advertising revenue, the VO simply chooses $p_f^*(1)=\frac{\beta\theta_{\max}}{2}$ to maximize its revenue in providing the premium access. When δ decreases, *i.e.*, the fraction of advertising revenue left to the VO increases, the VO increases its Wi-Fi price $p_f^*(\delta)$ to push more MUs to choose the advertising sponsored access.

V. Stage I: Ad Platform's Revenue Sharing

In this section, we study the ad platform's sharing policy in Stage I, considering the prediction of the VO's pricing in Stage II, and MUs and ADs' strategies in Stage III.

The ad platform obtains $\delta\in[0,1]$ fraction of the total advertising revenue from the VO. Based on (21) and (23), the ad platform's payoff $\Pi^{\mathrm{APL}}(\delta)$ is given by

$$\Pi^{\mathrm{APL}}(\delta) = \delta aNg(\lambda,\gamma,\eta)\frac{p_f^*(\delta)}{\beta\theta_{\max}}, \tag{30}$$

where $p_f^*(\delta)$ is the VO's optimal Wi-Fi price under policy δ, and has been given in Proposition 3.

Before computing the optimal δ^*, we introduce an *equilibrium indicator* Ω, based on which δ^* has different expressions. We define Ω as

$$\Omega \triangleq \frac{\lambda\beta\theta_{\max}}{ag(\lambda,\gamma,\eta)}. \tag{31}$$

The physical meaning of Ω can be interpreted as follows. Based on (8) and (24), the VO's revenue in providing the premium access can be written as

$$\lambda\beta\theta_{\max}N\varphi_f^2(p_f). \tag{32}$$

Based on (8), (21), (23), and (30), the total advertising revenue obtained by the ad platform and the VO can be written as

$$ag(\lambda,\gamma,\eta)N(1-\varphi_f(p_f)). \tag{33}$$

Since terms $\lambda\beta\theta_{\max}$ and $ag(\lambda,\gamma,\eta)$ act as the coefficients for (32) and (33), respectively, Ω intuitively describes the VO's relative benefit in providing the premium access, compared with providing the advertising sponsored access.

Based on the indicator Ω, we summarize the ad platform's optimal policy δ^* and payoff $\Pi^{\mathrm{APL}}(\delta^*)$ as follows.

Proposition 4 (Revenue sharing policy)**.** *The ad platform's optimal advertising revenue sharing policy δ^* is given by*

$$\delta^* = \begin{cases} 1-\Omega, & \text{if } 0 \le \Omega \le \frac{1}{3}, \\ \frac{1}{2}+\frac{\Omega}{2}, & \text{if } \frac{1}{3} < \Omega < 1, \\ 1, & \text{if } \Omega \ge 1. \end{cases} \tag{34}$$

Hence, $\delta^* \ge \frac{2}{3}$, i.e., the ad platform takes away at least two-third of the total advertising revenue. The ad platform's corresponding payoff is

$$\Pi^{\mathrm{APL}}(\delta^*) = \begin{cases} (ag(\lambda,\gamma,\eta)-\lambda\beta\theta_{\max})N, & \text{if } 0\le\Omega\le\frac{1}{3}, \\ \frac{N}{8\lambda\beta\theta_{\max}}(\lambda\beta\theta_{\max}+ag(\lambda,\gamma,\eta))^2, & \text{if } \frac{1}{3}<\Omega<1, \\ \frac{1}{2}ag(\lambda,\gamma,\eta)N, & \text{if } \Omega \ge 1. \end{cases} \tag{35}$$

VI. Three-Region Structure of Equilibrium

We analyze the equilibrium outcomes under different values of Ω in Propositions 5, 6, and 7, respectively.

Proposition 5. *When $\Omega \ge 1$, we have:*

- *The ad platform takes all advertising revenue, i.e., $\delta^* = 1$;*
- *The VO charges the lowest Wi-Fi price, i.e., $p_f^* = \frac{\beta\theta_{\max}}{2}$;*[17]
- *The VO only obtains revenue from the premium access, i.e., $\Pi^{\mathrm{VO}} = \Pi_f^{\mathrm{VO}}$ and $\Pi_a^{\mathrm{VO}} = 0$.*

Condition $\Omega \ge 1$ implies that providing the premium access is more beneficial to the VO. Hence, the VO charges the lowest Wi-Fi price $p_f^* = \frac{\beta\theta_{\max}}{2}$ to attract most MUs to choose the premium access.[18] In this case, the ad platform sets $\delta^* = 1$ and takes away all the advertising revenue.[19]

[17]Here, p_f^* is the VO's Wi-Fi price in the equilibrium, which is obtained from plugging the value of δ^* into the expression of $p_f^*(\delta)$ in (29).

[18]According to Proposition 3, VO's optimal Wi-Fi price lies in interval $\left[\frac{\beta\theta_{\max}}{2}, \beta\theta_{\max}\right]$, so $p_f^* = \frac{\beta\theta_{\max}}{2}$ is the lowest Wi-Fi price.

[19]Although the VO charges a low Wi-Fi price, there are still MUs choosing the advertising sponsored access. Hence, the advertising revenue obtained by the ad platform is not zero.

Proposition 6. *When $\frac{1}{3} < \Omega < 1$, we have:*

- *The ad platform chooses $\delta^* = \frac{1}{2} + \frac{\Omega}{2}$;*
- *The VO sets the Wi-Fi price as $p_f^* = \frac{ag(\lambda,\gamma,\eta)}{4\lambda} + \frac{\beta\theta_{\max}}{4}$;*
- *The VO obtains revenue from both types of access, i.e., $\Pi_a^{\mathrm{VO}}, \Pi_f^{\mathrm{VO}} \ne 0$.*

Under condition $\frac{1}{3} < \Omega < 1$, the VO generates revenue from both types of access and charges $p_f^* \in \left(\frac{\beta\theta_{\max}}{2}, \beta\theta_{\max}\right)$.

Proposition 7. *When $0 \le \Omega \le \frac{1}{3}$, we have:*

- *The ad platform chooses $\delta^* = 1 - \Omega$;*
- *The VO charges the highest Wi-Fi price, i.e., $p_f^* = \beta\theta_{\max}$;*
- *The VO only obtains revenue from the advertising sponsored access, i.e., $\Pi^{\mathrm{VO}} = \Pi_a^{\mathrm{VO}}$ and $\Pi_f^{\mathrm{VO}} = 0$.*

Condition $0 \le \Omega \le \frac{1}{3}$ implies that providing the advertising sponsored access is more beneficial to the VO. Hence, the VO charges the highest Wi-Fi price $p_f^* = \beta\theta_{\max}$ to push all MUs to choose the advertising sponsored access. In this case, the premium access revenue Π_f^{VO} is zero.

VII. Numerical Results

In this section, we first study the performance of advertising price p_a^∞ without Assumption 1. Then we compare the payoffs of the ad platform, the VO, and ADs at venues with different values of γ and λ. Finally, we investigate a situation where the ad platform sets a uniform sharing policy for multiple VOs.

A. Performance of p_a^∞ without Assumption 1

In Proposition 2, we have shown that p_a^∞ in (18) is the optimal solution of Problem 2, assuming both M and $\sigma_{\max}$ going to ∞ (Assumption 1). Now we numerically demonstrate that price p_a^∞ computed based on (18) is optimal to Problem 2 for most finite values of M and $\sigma_{\max}$ as well.

We choose $\delta \sim U\left[\frac{2}{3}, 1\right]$, $N \sim U[50, 300]$, $\gamma \sim U[0, 01, 1]$, $\lambda \sim U[0.1, 5]$, $a \sim U[1, 3]$, and $\varphi_a \sim U[0.5, 1]$, where $U[\cdot]$ denotes the uniform distribution. We change M and $\sigma_{\max}$ from 1 to 15. For each $(M, \sigma_{\max})$-pair, we compute p_a^∞ by (18), and check whether it also optimally solves Problem 2.[20] We run the experiment $10,000$ times for each $(M, \sigma_{\max})$-pair.

In Fig. 5, we plot the percentage of times that p_a^∞ is the optimal solution to Problem 2. We observe that the percentage increases in both M and $\sigma_{\max}$. Furthermore, when $M, \sigma_{\max} \ge 6$, the percentage is always above 99%.

Observation 1. *Without Assumption 1, the advertising price computed based on p_a^∞ in (18) is still optimal for the VO under most parameter settings, even when M and $\sigma_{\max}$ are reasonably small.*

[20]Recall that for p_a^∞ in (18), parameter η is defined as $\lim_{M,\sigma_{\max}\to\infty} \frac{M}{\sigma_{\max}}$. Now we simply choose $\eta = \frac{M}{\sigma_{\max}}$ to compute p_a^∞ for the finite M and $\sigma_{\max}$ situation. In Proposition 1 we have derived the optimal p_a^* to Problem 2. To verify the optimality of p_a^∞, we just need to check whether p_a^∞ equals p_a^*.

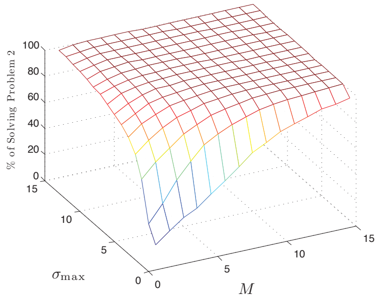

Fig. 5: Performance of p_a^∞ without Assumption 1.

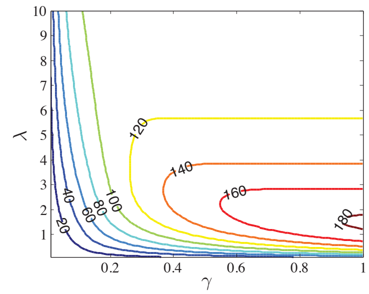

Fig. 6: Ad Platform's Payoff Π^{APL}.

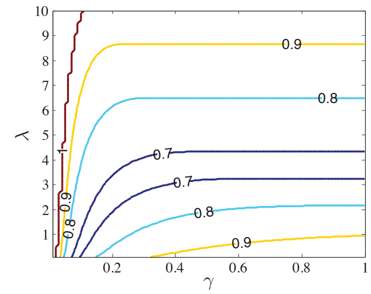

Fig. 7: Ad Revenue Sharing Policy δ^*.

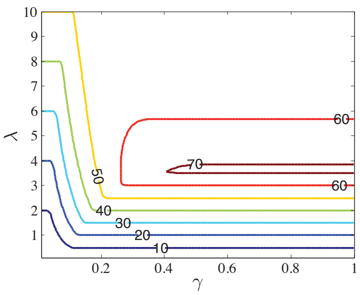

Fig. 8: VO's Total Revenue Π^{VO}.

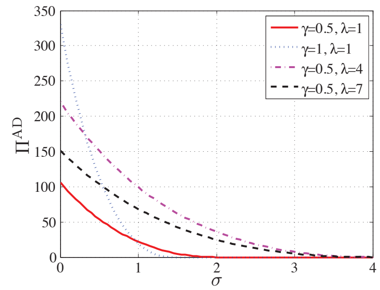

Fig. 9: AD's Payoff Π^{AD}.

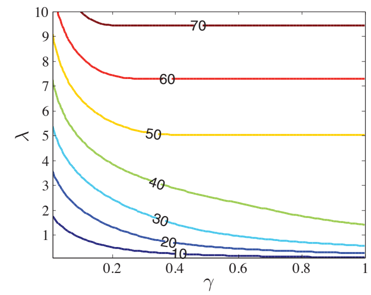

Fig. 10: Uniform Ad Revenue Sharing Policy: VO's Total Revenue Π^{VO}

B. *Ad Platform's Payoff and δ^* with Different (γ, λ)*

Next we compare the ad platform's payoff and revenue sharing policy for VOs with different values of advertising concentration level γ and MUs' visiting frequency λ. We choose $N = 200$, $\theta_{\max} = 1$, $\beta = 0.1$, $\eta = 1$, and $a = 4$.

Fig. 6 is a contour plot illustrating the ad platform's revenue. The horizontal axis corresponds to parameter γ, and the vertical axis corresponds to parameter λ. The values on the contour curves are the ad platform's payoff, Π^{APL}, obtained from venues with different (γ, λ) pairs. We observe that the ad platform obtains a large Π^{APL} from the venue with a large γ ($\gamma > 0.9$) and a small λ ($1.2 < \lambda < 1.8$). This is because such a venue has a small equilibrium indicator Ω. According to Proposition 7, in this case, all MUs choose the advertising sponsored access, and the total advertising revenue is large. Furthermore, the small indicator Ω implies that the VO can only rely on the advertising sponsored access, hence the ad platform sets a large δ^* ($= 1 - \Omega$ based on Proposition 7) to extract most of the advertising revenue from the VO.

Fig. 7 is a contour plot illustrating the ad platform's revenue sharing ratio δ^*. There are two potential approaches for the ad platform to maximize its revenue: (a) reducing δ^* to motivate the VO to push more MUs towards the advertising sponsored access, at the expense of a smaller revenue per ad display; (b) increasing δ^* to improve the revenue per ad display, at the expense of making the advertising sponsored access less attractive to the VO. In Fig. 7, ratio δ^* first decreases with λ, then increases with λ, which means approach (a) is more effective when λ is small and approach (b) is more effective when λ is large. This is because a large λ leads to a large indicator Ω, which means that the VO prefers the premium access, even if the ad platform leaves a large proportion of the advertising revenue to the VO.

We summarize the observations in Fig. 6 and 7 as follows.

Observation 2. *The ad platform obtains a large Π^{APL} at the venue with a large γ and a small λ, and its optimal revenue sharing ratio δ^* first decreases and then increases with λ.*

C. *VO's Total Revenue and ADs' Payoffs with Different (γ, λ)*

We apply the same parameter settings as Section VII-B, and investigate the VO's total revenue and ADs' payoffs at venues with different (γ, λ) pairs.

Fig. 8 is a contour plot illustrating the VO's total revenue Π^{VO}, obtained from the advertising sponsored access and the premium access. We find that the VO with a large γ ($\gamma > 0.4$) and a medium λ ($3.5 < \lambda < 3.9$) has a large Π^{VO}. The reason is that the ad platform's revenue sharing ratio is relatively small (*i.e.*, $\delta^* < 0.7$ as shown in Fig. 7). We also observe that when both γ and λ are large ($\gamma > 0.2$ and $\lambda > 6$), the total revenue Π^{VO} is not very sensitive to γ and λ. First, based on (25) and (26), Π^{VO} is independent of γ for $\gamma \geq \frac{2\eta}{\lambda}$. Second, when λ increases, the VO's revenue from the premium access Π_f^{VO} increases, while the VO's revenue from the advertising sponsored access Π_a^{VO} decreases. Hence, the total revenue Π^{VO} does not significantly change with λ.

In Fig. 9, we plot the ADs' payoffs Π^{AD} against the AD type σ under different values of γ and λ. It is easy to see that the ADs with higher popularities (*i.e.*, smaller σ) have higher payoffs. When comparing curves with the same $\lambda = 1$ and different values of γ (0.5 and 1), we find that the increase of the concentration level γ makes ADs with small values of σ even more popular, and hence increases their payoffs. ADs with large values of σ will have smaller payoffs accordingly. When comparing curves with the same $\gamma = 0.5$ and different values of λ (1, 4, and 7), we observe that ADs' payoffs first increase and then decrease with λ. According to (6), the increase of visiting frequency λ affects Π^{AD} in two aspects: (a) from (18), the advertising price p_a^{∞} becomes cheaper, which potentially increases Π^{AD}; (b) the VO decreases the Wi-Fi price p_f^* to attract MUs to the premium access, hence the proportion of MUs choosing the advertising sponsored access, φ_a^*, becomes smaller, which potentially decreases Π^{AD}. In Fig. 9, impact (a) dominates when λ increases from 1 to 4, and impact (b) dominates when λ increases from 4 to 7.

We summarize key observations in Fig. 8 and 9 as follows.

Observation 3. *(i) The VO with a large γ and a medium λ obtains a large total revenue Π^{VO}; (ii) ADs obtain large payoffs Π^{AD} at the venue with a medium λ, and their payoffs decrease in the type index σ.*

D. Uniform Advertising Revenue Sharing Policy δ_U

In Section II-A, we assumed that the ad platform can set different advertising revenue sharing ratios for different VOs. This, however, may not be desirable in practice due to fairness considerations. In Fig. 10, we consider the more practical case where the ad platform chooses a uniform advertising revenue sharing ratio $\delta_U \in [0,1]$ for all VOs.

We assume VOs have uniformly distributed γ and λ ($\gamma \sim U[0.01,1]$, $\lambda \sim U[0.1,10]$), and are identical in other parameters. We formulate the ad platform's problem as follows.

Problem 4. *The ad platform decides δ_U^* by solving*

$$\max \mathbb{E}_{\gamma,\lambda}\left\{\delta_U a N g\left(\lambda,\gamma,\eta\right)\frac{p_f^*\left(\delta_U\right)}{\beta\theta_{\max}}\right\} \tag{36}$$

$$\text{var.} \quad \delta_U \in [0,1], \tag{37}$$

where $p_f^*\left(\delta_U\right)$ is the VO's optimal Wi-Fi pricing response under revenue sharing ratio δ_U, and is given in (29).

We consider $10,000$ VOs, and solve Problem 4 numerically under the same parameter settings as Section VII-B. We obtain the optimal $\delta_U^* = 0.81$.

Fig. 10 is a contour figure illustrating the VO's total revenue Π^{VO} with different values of γ and λ under $\delta_U^* = 0.81$. Comparing the results with the VO-specific revenue sharing case in Fig. 8, we observe that here a VO with a medium λ has a smaller revenue, while a VO with a large λ has a larger revenue. This is consistent with the comparison between δ^* in the VO-specific revenue sharing case (as shown in Fig. 7) and δ_U^* here. For those VOs with $\delta^* < \delta_U^*$, their revenues decrease in the uniform revenue sharing case. Otherwise, their revenues increase. Therefore, we conclude the following observation.

Observation 4. *Compared with the VO-specific revenue sharing policy, the ad platform's uniform revenue sharing policy increases the payoff of the VO with a large λ, and decreases the payoff of the VO with a medium λ.*

VIII. Conclusion

In this work, we studied the public Wi-Fi monetization problem, and analyzed the economic interactions among the ad platform, VOs, MUs, and ADs. Our analysis addressed several important observations: (i) the ad platform's advertising revenue sharing policy affects a VO's Wi-Fi price but not the VO's advertising price; (ii) the ad platform obtains a large payoff at the venue with a large γ and a small λ; and (iii) a VO with a large γ and a medium λ generates a large total revenue. In our future work, we will consider VOs that provide the premium access with the quality-of-service guarantee, and investigate the corresponding influence on the equilibrium outcomes. Moreover, we will study the problem under more general settings, by relaxing some important assumptions (such as the unlimited Wi-Fi capacity, and the uniform distribution of MU type and AD type).

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