Distributed Spectrum Access with Spatial Reuse

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***Abstract*—Efficient distributed spectrum sharing mechanism is crucial for improving the spectrum utilization. The spatial aspect of spectrum sharing, however, is less understood than many other aspects. In this paper, we generalize a recently proposed spatial congestion game framework to design efficient distributed spectrum access mechanisms with spatial reuse. We first propose a spatial channel selection game to model the distributed channel selection problem with fixed user locations. We show that the game is a potential game, and develop a distributed learning mechanism that converges to a Nash equilibrium only based on users' local observations. We then formulate the joint channel and location selection problem as a spatial channel selection and mobility game, and show that it is also a potential game. We next propose a distributed strategic mobility algorithm, jointly with the distributed learning mechanism, that can converge to a Nash equilibrium.**

***Index Terms*—Distributed spectrum sharing, spatial reuse, game theory, distributed learning, strategic mobility**

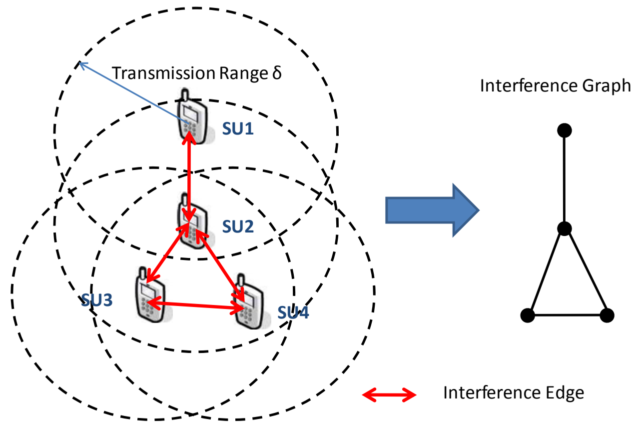

Fig. 1. Illustration of distributed spectrum access with spatial reuse

I. Introduction

DYNAMIC spectrum sharing is envisioned as a promising technique to alleviate the problem of spectrum underutilization [1]. It enables unlicensed wireless users (secondary users) to opportunistically access the licensed channels owned by legacy spectrum holders (primary users), and thus can significantly improve the spectrum efficiency [1].

A key challenge of dynamic spectrum sharing is how to resolve the resource competition by selfish secondary users in a decentralized fashion. If multiple secondary users transmit over the same channel simultaneously, it may lead to severe interference and reduced data rates for all users. Therefore, it is necessary to design efficient distributed spectrum sharing mechanism.

The competitions among secondary users for common spectrum resources have often been studied using noncooperative game theory (e.g., [2]–[5]). Nie and Comaniciu in [2] designed a self-enforcing distributed spectrum access mechanism based on potential games. Niyato and Hossain in [3] studied a price-based spectrum access mechanism for competitive secondary users. Flegyhzi *et al.* in [4] proposed a two-tier game framework for medium access control (MAC) mechanism design.

When not knowing spectrum information such as channel availabilities, secondary users need to learn the environment and adapt the spectrum access decisions accordingly. Maskery *et al.* in [6] used no-regret learning to solve this problem, assuming that the users' channel selections are common information. When users' channel selections are not observable, authors in [7], [8] designed multi-agent multi-armed bandit learning algorithms to minimize the expected performance loss of distributed spectrum access.

A common assumption of the above results is that secondary users are close-by and interfere with each other when they transmit on the same channel simultaneously. However, a critical feature of spectrum sharing in wireless communication is *spatial reuse*. If wireless users are located sufficiently far apart, then they can transmit in the same frequency band simultaneously without causing any performance degradation (see Figure 1 for an illustration). Such spatial effect on distributed spectrum sharing is less understood than many other aspects in existing literature [9], which motivates this study.

Recently, Tekin *et al.* in [10] proposed a novel spatial congestion game framework to take spatial relationship into account. The key idea is to extend the classical congestion game upon a general undirected graph, by assuming that a player's payoff depends on the number of its neighbors that choose the same resource (i.e., users are homogeneous in terms of channel contention). The homogeneous assumption follows from the set up of the classical congestion game (which only works on a fully connected graph). The application of such a homogeneous model, however, is quite restricted, since users typically have heterogenous channel contention probabilities in wireless systems. In this paper, we extend the spatial congestion game framework to formulate the random access based distributed spectrum sharing problem with spatial reuse, by taking users' heterogeneous channel contention probabilities into account. Such extension is highly

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non-trivial, and significantly expands possible applications of the model. Moreover, we propose distributed algorithms to achieve Nash equilibria of the generalized spatial games.

We consider two game models in this paper. In the first model, secondary users have fixed spectrum access locations, and each user selects a channel to maximize its own utility in a distributed manner. We model the problem as a spatial channel selection game. In the second more general model, users are mobile, and they are capable to select channels and spectrum access locations simultaneously in order to better exploit the gain of spatial reuse. We formulate the problem as a joint spatial channel selection and mobility game. The main results and contributions of this paper are as follows:

- *General game formulation*: We formulate the spatial channel selection problem and the joint channel and location selection problem as noncooperative games on general interference graphs, with heterogeneous channel available data rates depending on user and location.
- *Existence of Nash equilibrium and finite improvement property*: For both the spatial channel selection game and the joint spatial channel selection and mobility game, we show that they are potential games, and hence they always have at least one Nash equilibrium and possess the finite improvement property.
- *Distributed algorithms for achieving Nash equilibrium*: For the spatial channel selection game, we propose a distributed learning algorithm, which globally converges to a Nash equilibrium by only utilizing users' local observations. For the spatial channel selection and mobility game, we propose a distributed strategic mobility algorithm, which also converges to a Nash equilibrium, when jointly used with the distributed learning algorithm.

The rest of the paper is organized as follows. We introduce the system model and the spatial channel selection game in Sections II and III, respectively. We present the distributed learning mechanism for spatial channel selection in Section IV. Then we introduce the joint spatial channel selection and mobility game in Section V, and study the efficiency of Nash equilibrium in Section VI. We illustrate the performance of the proposed mechanisms through numerical results in Section VII, and finally conclude in Section VIII.

II. System Model

We consider a dynamic spectrum sharing network with a set $\mathcal{M} = \{1, 2, ..., M\}$ of independent and stochastically heterogeneous primary channels. A set $\mathcal{N} = \{1, 2, ..., N\}$ of secondary users try to access these channels in a distributed manner when the channels are not occupied by primary (licensed) transmissions.

To take the spatial relationship into account, we assume that the secondary users are located in a spatial domain $\triangle$, i.e., a finite set of possible spectrum access locations. We denote $d_n \in \triangle$ as the **location** of user n, and $\boldsymbol{d} = (d_1, .., d_N) \in \Pi \triangleq \triangle^N$ as **location profile** of all users. Each secondary user has a **transmission range** δ. Then given the location profile $\boldsymbol{d}$ of all users, we can obtain the **interference graph** $G_{\boldsymbol{d}} = \{\mathcal{N}, \mathcal{E}_{\boldsymbol{d}}\}$ to describe the interference relationship among users (see Figure 1 for an example). Here the vertex set $\mathcal{N}$ is the secondary user set, and the edge set $\mathcal{E}_{\boldsymbol{d}} = \{(i, j) : ||d_i, d_j|| \leq \delta, \forall i, j \neq i \in \mathcal{N}\}$ is the set of interference edges (with $||d_i, d_j||$ being the distance between locations d_i and d_j). If there is an interference edge between two secondary users, then they cannot successfully transmit their data on the same idle channel simultaneously due to collision. In the sequel, we also denote the set of interfering users with user n (i.e., user n's "neighbors") under the location profile $\boldsymbol{d}$ as $\mathcal{N}_n(\boldsymbol{d}) = \{i : (n, i) \in \mathcal{E}_{\boldsymbol{d}}, i \in \mathcal{N}\}$.

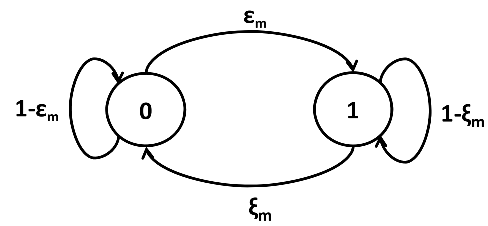

Fig. 2. Two states Markovian channel model

We consider a time-slotted system model as follows:

- *Channel state*: for each primary channel m, the channel state at time slot t is
$$S_m(t) = \begin{cases} 0 & \text{if channel } m \text{ is busy,} \\ 1 & \text{if channel } m \text{ is idle.} \end{cases}$$
- *Channel state changing*: the state of a channel changes according to a two-state Markovian process [11], [12] (see Figure 2). We denote the channel state probability vector of channel m at time t as $\boldsymbol{q}_m(t) \triangleq (Pr\{S_m(t) = 0\}, Pr\{S_m(t) = 1\})$, which forms a Markov chain as $\boldsymbol{q}_m(t) = \boldsymbol{q}_m(t-1)\Gamma_m, \forall t \geq 1$, with the transition matrix
$$\Gamma_m = \begin{bmatrix} 1 - \varepsilon_m & \varepsilon_m \\ \xi_m & 1 - \xi_m \end{bmatrix}.$$
Furthermore, the long run statistical channel availability $\theta_m \in (0, 1)$ of a channel m can be obtained from the stationary distribution of the Markov chain, i.e.,
$$\theta_m = \frac{\varepsilon_m}{\varepsilon_m + \xi_m}. \quad (1)$$
- *User-and-location specific channel throughput*: for each secondary user n at location d, its realized data rate $b^n_{m,d}(t)$ on an idle channel m in each time slot t evolves according to an i.i.d. random process with a mean $B^n_{m,d}$, due to users' heterogeneous transmission technologies and the local environmental effects such as fading [13]. For example, we can compute the data rate $b^n_{m,d}(t)$ according to the Shannon capacity as
$$b^n_{m,d}(t) = B_m \log_2 \left(1 + \frac{\zeta_n g^n_{m,d}(t)}{\omega^n_{m,d}}\right), \quad (2)$$
where B_m is the bandwidth of channel m, ζ_n is the fixed transmission power adopted by user n according to the requirements such as the primary user protection, $\omega^n_{m,d}$ denotes the background noise power, and $g^n_{m,d}(t)$ is the channel gain. In a Rayleigh fading channel environment, the channel gain $g^n_{m,d}(t)$ is a realization of a random variable that follows the exponential distribution [13].
- *Time slot structure*: each secondary user n executes the

following stages synchronously during each time slot:

- *Channel sensing*: sense one of the channels based on the channel selection decision generated at the end of previous time slot.
- *Channel contention*: we use persistence-probability-based random access mechanism[1], i.e., user n contends for an idle channel with probability $p_n \in \varrho \triangleq (p_{\min}, p_{\max})$, where $0 < p_{\min} < p_{\max} < 1$ denote the minimum and maximum contention probabilities. If multiple users contend for the same channel, a collision occurs and no user can transmit. Since each user (i.e., a wireless device) typically has limited battery power, to achieve a longer expected lifetime, we limit user's channel contention in a time slot as
$$\zeta_n p_n \leq \nu_n, \tag{3}$$
where ν_n denotes the energy constraint of user n.
- *Data transmission*: transmit data packets if the user is the only one contending for an idle channel (i.e., no collision is detected).
- *Channel selection*: choose a channel to access next time slot according to the distributed learning mechanism (introduced in Section IV).

Let $a_n \in \mathcal{M}$ be the channel selected by user n, $\boldsymbol{a} = (a_1, ..., a_N) \in \Lambda \triangleq \mathcal{M}^N$ be the channel selection profile of all users, and $\boldsymbol{p} = (p_1, ..., p_N)$ be the channel contention probability profile of all users. We can then obtain the *long run expected throughput* of each user n choosing channel a_n in location d_n as
$$Q_n(\boldsymbol{d}, \boldsymbol{a}, \boldsymbol{p}) = \theta_{a_n} B^n_{a_n,d_n} p_n \prod_{i \in \mathcal{N}_n^{a_n}(\boldsymbol{d},\boldsymbol{a})} (1 - p_i), \tag{4}$$
where $\mathcal{N}_n^{a_n}(\boldsymbol{d}, \boldsymbol{a}) \triangleq \{i : a_i = a_n \text{ and } i \in \mathcal{N}_n(\boldsymbol{d})\}$ is the set of interfering users that choose the same channel as user n. To take the fairness issue into account, we consider the proportional-fair utility [15] function in this study, i.e.,
$$U_n(\boldsymbol{d}, \boldsymbol{a}, \boldsymbol{p}) = \log Q_n(\boldsymbol{d}, \boldsymbol{a}, \boldsymbol{p}). \tag{5}$$
Other type of utility functions such as general alpha-fairness will be considered in a future work.

Equation (5) shows that user n's utility $U_n(\boldsymbol{d}, \boldsymbol{a}, \boldsymbol{p})$ is an increasing function of its contention probability p_n. This implies that, when a user is aggressive and does not care about the collisions, it can adopt the maximum possible channel contention probability p_n satisfying the energy constraint (3), i.e., $p_n = \min\{p_{\max}, \frac{\nu_n}{\zeta_n}\}$. When users take the cost of collisions into account, we can adopt the game theoretic framework for the contention control in [16]. Furthermore, a dynamic contention control scheme is proposed in [16] that converges to a stable channel contention probability profile such that no users can further improve unilaterally. In this paper, we hence assume that the channel contention probability p_n of each user is fixed and focus on the issues of distributed location and channel selections. For the sake of brevity, we also denote the utility of each user n as $U_n(\boldsymbol{d}, \boldsymbol{a})$, where the decision variables are location selections $\boldsymbol{d}$ and channel selections $\boldsymbol{a}$ only. Since our analysis is from the secondary users' perspective, we will use the terms "secondary user" and "user" interchangeably. Due to the space limit, all the proofs of the results can be found in [17].

[1]This model can also provide useful insights for the case that the contention-window-based random access mechanism is implemented, since the persistence probability p_n is related to the contention window size w_n according to $p_n = \frac{2}{w_n+1}$ [14].

III. Spatial Channel Selection

We first consider the case that all users' locations $\boldsymbol{d}$ are *fixed*, and each user tries to maximize its own utility by choosing a proper channel in a distributed manner. Given other users' channel selections a_{-n}, the problem faced by a user n is
$$\max_{a_n \in \mathcal{M}} U_n(\boldsymbol{d}, a_n, a_{-n}), \forall n \in \mathcal{N}. \tag{6}$$
The distributed nature of the spatial channel selection problem naturally leads to a formulation based on game theory, such that each user can self organize into a mutually acceptable channel selection (**Nash equilibrium**) $\boldsymbol{a}^* = (a_1^*, a_2^*, ..., a_N^*)$ with
$$a_n^* = \arg\max_{a_n \in \mathcal{M}} U_n(\boldsymbol{d}, a_n, a_{-n}^*), \forall n \in \mathcal{N}. \tag{7}$$

We next formulate the spatial channel selection problem as a game, and further show the existence of Nash equilibrium.

A. Spatial Congestion Game

We first review the spatial congestion game introduced in [10]. Spatial congestion games are a class of strategic games represented by $\Gamma = (\mathcal{N}, \mathcal{M}, \{\mathcal{N}_n(\boldsymbol{d})\}_{n\in\mathcal{N}}, \{U_n\}_{n\in\mathcal{N}})$. Specifically, $\mathcal{N}$ is the set of players, $\mathcal{M}$ is the set of resources, and $\mathcal{N}_n(\boldsymbol{d})$ is the set of players that can cause congestion to player n when they use the same resource. The payoff of player n for using resource $a_n \in \mathcal{M}$ is $U_n(\boldsymbol{a}) = f^n_{a_n}(C^n_{a_n}(\boldsymbol{a}))$, where $C^n_{a_n}(\boldsymbol{a}) = \sum_{i\in\mathcal{N}_n(\boldsymbol{d})} I_{\{a_i=a_n\}}$ denotes the number of players in the set $\mathcal{N}_n(\boldsymbol{d})$ that choose the same resource a_n as user n, and $f^n_{a_n}(\cdot)$ denotes some user-specific payoff function. Typically, $C^n_{a_n}(\boldsymbol{a})$ is also called the *congestion level*.

Note that the classical congestion games can be viewed as a special case of the spatial congestion games by setting the interference graph $G_{\boldsymbol{d}}$ as a complete graph, i.e., $\mathcal{N}_n(\boldsymbol{d}) = \mathcal{N}\backslash\{n\}$. For the classical congestion game, it is shown in [18] that it is an (exact) potential game, which is defined as

Definition 1 (**Potential Game** [18]). *A game is called a weighted potential game if it admits a potential function $\Phi(\boldsymbol{a})$ such that for every $n \in \mathcal{N}$ and $a_{-n} \in \mathcal{M}^{N-1}$,*
$$\Phi(a_n^{'}, a_{-n}) - \Phi(a_n, a_{-n}) = w_n \left(U_n(a_n^{'}, a_{-n}) - U_n(a_n, a_{-n})\right),$$
where $w_n > 0$ is some positive constant. Specifically, if $w_n = 1, \forall n \in \mathcal{N}$, then the game is also called an exact potential game.

Definition 2 (**Better Response Update** [18]). *The event where a player n changes to an action $a_n^{'}$ from the action a_n is a better response update if and only if $U_n(a_n^{'}, a_{-n}) > U_n(a_n, a_{-n})$.*

Definition 3 (**Finite Improvement Property** [18]). *A game has the finite improvement property if any asynchronous better response update process (i.e., no more than one player updates* *the strategy at any given time) terminates at a pure Nash equilibrium within a finite number of updates.*

An appealing property of the potential game is that it admits the finite improvement property, which guarantees the existence of a Nash equilibrium. When a general payoff function $f^n_{a_n}(\cdot)$ is considered, however, the spatial congestion game does not necessarily possess such a nice property [10]. We next extend the spatial congestion game framework for the random access mechanism in Section II, and show that the spatial channel selection problem in (6) with the payoff function given in (5) is a potential game.

B. Generalized Spatial Congestion Game Formulation

As mentioned, the spatial congestion game proposed in [10] assumes that a player's utility depends on the number of players in its neighbors that choose the same resource. For our case, however, a user's utility in (5) depends on who (instead of how many users) in its neighbors contend for the same channel, since users have heterogenous channel contention probabilities. We hence generalize the spatial congestion game framework for the random access mechanism in Section II by extending the definition of congestion level $C^n_{a_n}(\boldsymbol{a})$. According to (4) and (5), we have

$$U_n(\boldsymbol{d},\boldsymbol{a}) = \log\left(\theta_{a_n} B^n_{a_n,d_n} p_n\right) + \sum_{i\in\mathcal{N}^{a_n}_n(\boldsymbol{d},\boldsymbol{a})} \log(1-p_i).$$

We then extend the definition of $C^n_{a_n}(\boldsymbol{a})$ in the standard spatial congestion game by setting $C^n_{a_n}(\boldsymbol{a}) = \sum_{i\in\mathcal{N}^{a_n}_n(\boldsymbol{d},\boldsymbol{a})} \log(1-p_i)$. Here $C^n_{a_n}(\boldsymbol{a})$ is regarded as the generalized congestion level perceived by user n on channel a_n. When all users have the same channel contention probability $p_i = p$, we have $C^n_{a_n}(\boldsymbol{a}) = \log(1-p)\sum_{i\in\mathcal{N}^{a_n}_n(\boldsymbol{d},\boldsymbol{a})} I_{\{a_i=a_n\}}$, which degrades to the standard case. Then the user specific payoff function is $f^n_{a_n}(C^n_{a_n}(\boldsymbol{a})) = \log\left(\theta_{a_n} B^n_{a_n,d_n} p_n\right) + C^n_{a_n}(\boldsymbol{a})$. In the following, we refer to this game formulation as the spatial channel selection game. We show that

Lemma 1. *The spatial channel selection game on a general interference graph $G_{\boldsymbol{d}}$ is a weighted potential game, with the potential function as*

$$\Phi(\boldsymbol{d},\boldsymbol{a}) = \sum_{i=1}^{N} -\log(1-p_i) \times \left(\frac{1}{2}\sum_{j\in\mathcal{N}^{a_i}_i(\boldsymbol{d},\boldsymbol{a})} \log(1-p_j) + \log\left(\theta_{a_i} B^i_{a_i,d_i} p_i\right)\right), \quad (8)$$

and the weight $w_i = -\log(1-p_i)$.

The proof is given in Appendix A. It follows from Lemma 1 that

Theorem 1. *The spatial channel selection game on a general interference graph has a Nash equilibrium and the finite improvement property.*

By the finite improvement property, any asynchronous better response update sequence leads to a Nash equilibrium. However, implementing a better response update requires a user to know the strategies of other users, and then takes a better strategy to improve its payoff. This requires extensive information exchange among the users. The signaling overhead and energy consumption can be quite significant and even infeasible in some network scenarios. We next propose a distributed learning mechanism, which utilizes user's local observations only and converges to a Nash equilibrium.

IV. Distributed Learning Mechanism For Spatial Channel Selection

In this part, we introduce the distributed learning mechanism for spatial channel selection, and then show that it converges to a Nash equilibrium.

A. Distributed Learning Mechanism

Without information exchange, each user can only estimate the environment through local measurement. To achieve accurate estimations, a user needs to gather a large number of observation samples. This motivates us to divide the learning time into a sequence of decision periods indexed by $T(=1,2,...)$, where each decision period consists of K time slots (see Figure 3). During a single decision period, a user accesses the same channel in all K time slots. Thus the total number of users accessing each channel does not change within a decision period, which allows users to better learn the environment.

The key idea of distributed learning is to adapt a user's spectrum access decision based on its accumulated experiences. At the beginning of each period T, a user n chooses a channel $a_n(T) \in \mathcal{M}$ to access according to its mixed strategy $\boldsymbol{\sigma}_n(T) = (\sigma^n_m(T), \forall m \in \mathcal{M})$, where $\sigma^n_m(T)$ is the probability of choosing channel m. The mixed strategy is generated according to $\boldsymbol{Z}_n(T) = (Z^n_m(T), \forall m \in \mathcal{M})$, which represents its perceptions of choosing different channels based on local estimations. We map from the perceptions $\boldsymbol{Z}_n(T)$ to the mixed strategy $\boldsymbol{\sigma}_n(T)$ in the *proportional* way, i.e.,

$$\sigma^n_m(T) = \frac{Z^n_m(T)}{\sum_{i=1}^M Z^n_i(T)}, \forall m \in \mathcal{M}. \quad (9)$$

At the end of a decision period T, a user n computes its estimated expected payoff $U_n(T)$ based on the sample average estimation over K time slots in the period, i.e., $U_n(T) = \frac{\sum_{t=1}^K U_n(T,t)}{K}$ where $U_n(T,t)$ is the payoff received by user n in time slot t. Then user n adjusts its perceptions as ($\forall m \in \mathcal{M}$)

$$Z^n_m(T+1) = \frac{Z^n_m(T)}{\sum_{i=1}^M Z^n_i(T)} + \mu_T U_n(T) I_{\{a_n(T)=m\}}, \quad (10)$$

where μ_T is the smoothing factor and $I_{\{a_n(T)=m\}}$ is an indicator whether user n chooses channel m at period T. The user first normalizes the perception values (the first term on RHS of (10)) and then reinforces the perception of the channel just accessed (the second term on RHS of (10)). The purpose of normalization here is to bound the perception values. We summarize the distributed learning mechanism for spatial channel selection in Algorithm 1.

Algorithm 1 Distributed Learning For Spatial Channel Selection

initialization:
  set the initial perception value $\boldsymbol{Z}_n(1) = (\frac{1}{M}, ..., \frac{1}{M})$.
end initialization

loop for each decision period T and each user n in parallel:
  select a channel $a_n(T) \in \mathcal{M}$ according to the mixed strategy $\boldsymbol{\sigma}_n(T)$ by (9).
  for each time slot t in the period T **do**
    sense and **contend** to access the channel $a_n(T)$.
    record the realized utility $U_n(T,t)$
  end for
  calculate the average utility $U_n(T) = \frac{\sum_{t=1}^{K} U_n(T,t)}{K}$.
  update the perception values $\boldsymbol{Z}_n(T)$ according to (10).
end loop

B. Convergence of Distributed Learning

We now study the convergence of distributed learning mechanism. Based on the stochastic approximation theory [19], we focus on the analysis of the mean dynamics of distributed learning. To proceed, we define the mapping from the mixed strategies $\boldsymbol{\sigma}(T)$ to the expected payoff of user n choosing channel m as $V_m^n(\boldsymbol{\sigma}(T)) \triangleq E[U_n(T)|\boldsymbol{\sigma}(T), a_n(T) = m]$. Here the expectation $E[\cdot]$ is taken with respect to the mixed strategy profile $\boldsymbol{\sigma}(T)$ of all users. We show that

Lemma 2. *For the distributed learning mechanism for spatial channel selection, when smoothing factor μ_T satisfies $\sum_T \mu_T = \infty$ and $\sum_T \mu_T^2 < \infty$, then as T goes to infinity, the sequence $\{\boldsymbol{\sigma}(T), \forall T \geq 0\}$ converges to the limiting point of the differential equations ($\forall m \in \mathcal{M}, n \in \mathcal{N}$)*

$$\frac{d\sigma_m^n(T)}{dT} = \sigma_m^n(T)\left(V_m^n(\boldsymbol{\sigma}(T)) - \sum_{i=1}^{M}\sigma_i^n(T)V_i^n(\boldsymbol{\sigma}(T))\right). \tag{11}$$

The mean dynamics in (11) imply that for a user if a channel offers a better payoff than his current average payoff, then the user will choose that channel with a higher probability in future learning.

Based on the mean dynamics in (11), we can show that

Theorem 2. *When smoothing factor μ_T satisfies $\sum_T \mu_T = \infty$ and $\sum_T \mu_T^2 < \infty$, the distributed learning mechanism for spatial channel selection asymptotically converges to a Nash equilibrium.*

The key idea is to show that the time derivative of the expected potential function $L(\boldsymbol{\sigma}(T)) \triangleq E[\Phi(\boldsymbol{d}, \boldsymbol{a})|\boldsymbol{\sigma}(T)]$ is non-decreasing, i.e., $\frac{dL(\boldsymbol{\sigma}(T))}{dT} \geq 0$. Since $L(\boldsymbol{\sigma}(T))$ is bounded above, the learning dynamics must converge to an invariant set such that $\frac{dL(\boldsymbol{\sigma}(T))}{dT} = 0$, which corresponds to the set of Nash equilibria.

V. Joint Spatial Channel Selection and Mobility

Future mobile devices are envisioned to incorporate the intelligent functionality and will be capable of flexible spectrum access [20]. Most existing efforts (e.g., [2]–[8]), however, focus on spectrum sharing networks with stationary secondary users. How to better utilize the gain of spatial reuse in mobile cognitive radio networks is less understood. Due to the heterogeneous geo-locations of primary users, the spectrum availabilities can be very different over the spatial dimension. A secondary user can achieve higher throughput if it moves to a location with higher spectrum opportunities and fewer contending users. This motivates us to consider the throughput-driven mobility case that each user has the flexibility to change both its spectrum access location and channel.

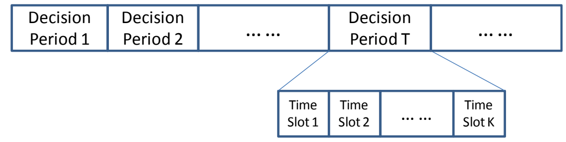

Fig. 3. Time structure of distributed learning

We note that the idea of strategic mobility is not necessarily applicable to all communication scenarios. For example, in vehicular ad-hoc networks, user's mobility is typically generated by user's driving plan, thus the idea of strategic mobility for better network throughput may not apply. However, there are some networking scenarios where strategic mobility can be very useful. For example, in areas of poor connectivity, cellular phone users often try to find a location with better connectivity by moving around and observing the signal strength bars. As another example, in many large academic conferences, a user often experiences poor Wi-Fi connections in a conference room with a lot of attendees. The connection gets much better when the user moves into the conference lobby just tens of meters away with much fewer users. To summarize, a user has the incentive to move if he has to complete an urgent communication task and the movement is within a reasonable distance.

A. Strategic Mobility Game with Fixed Channel Selection

We first study the case that the channel selection profile of all users is *fixed*, and users try to choose proper spectrum access locations to maximize their own payoffs in a distributed manner. Without loss of generality, we assume that the locations on the spatial domain $\triangle$ are connected[2], i.e., it is possible to get to any other locations from any location. We further introduce the user specific location selection space $\triangle_n \subseteq \triangle$ to characterize user heterogeneity in mobility preference. For example, if the user does not want to move, we have $\triangle_n = \{d_n\}$ where d_n is user n's fixed location. We then introduce the *strategic mobility game* $\Omega = (\mathcal{N}, \boldsymbol{d}, \{U_n\}_{n\in\mathcal{N}})$, where $\mathcal{N}$ is the set of users, $\boldsymbol{d} = (d_1, ...d_n) \in \Theta \triangleq \triangle_1 \times ... \times \triangle_N$ is the location profile of all users, and $U_n(\boldsymbol{d}, \boldsymbol{a})$ is the payoff of user n given the fixed channel selection profile $\boldsymbol{a}$ of all users. A location profile $\boldsymbol{d}^* = (d_n^*, d_{-n}^*)$ is a Nash equilibrium under a fixed $\boldsymbol{a}$ if and only if it satisfies that

$$d_n^* = \arg\max_{d_n \in \triangle_n} U_n(d_n, d_{-n}^*, \boldsymbol{a}), \forall n \in \mathcal{N}. \tag{12}$$

[2]For the case that the spatial domain is not connected, it can be partitioned into multiple connected sub-domains.

Algorithm 2 Distributed Strategic Mobility Algorithm

initialization:
  set the temperature γ and the location update density τ_n.
end initialization

loop for each user n in parallel:
  generate a timer value following the exponential distribution with the mean equal to $\frac{1}{\tau_n|\triangle^n_{d_n}|}$, where d_n is the current location of the user and $|\triangle^n_{d_n}|$ is the number of feasible locations to move to next.
  count down until the timer expires.
  if the timer expires **then**
    record the payoff $U_n(\boldsymbol{d}, \boldsymbol{a})$.
    choose a new location $d_n^{'}$ randomly from the set $\triangle^n_{d_n}$.
    move to the new location $d_n^{'}$ and **record** the payoff $U_n(\boldsymbol{d}^{'}, \boldsymbol{a})$.
    stay in the new location $d_n^{'}$ with probability $\frac{e^{-\log(1-p_n)\gamma U_n(\boldsymbol{d}^{'},\boldsymbol{a})}}{e^{-\log(1-p_n)\gamma U_n(\boldsymbol{d},\boldsymbol{a})}+e^{-\log(1-p_n)\gamma U_n(\boldsymbol{d}^{'},\boldsymbol{a})}}$ OR **move back** to the original location d_n with probability $\frac{e^{-\log(1-p_n)\gamma U_n(\boldsymbol{d},\boldsymbol{a})}}{e^{-\log(1-p_n)\gamma U_n(\boldsymbol{d},\boldsymbol{a})}+e^{-\log(1-p_n)\gamma U_n(\boldsymbol{d}^{'},\boldsymbol{a})}}$.
  end if
end loop

We show that

Lemma 3. *The strategic mobility game Ω is a weighted potential game, with the same potential function as $\Phi(\boldsymbol{d}, \boldsymbol{a})$ in (8), and the weight $w_n = -\log(1-p_n)$.*

According to the property of the potential game, it follows that

Theorem 3. *The strategic mobility game Ω has a Nash equilibrium and the finite improvement property.*

Similarly to the spatial channel selection game, we can apply the distributed learning mechanism to achieve the Nash equilibrium. However, due to the cost of long distance traveling, it is often the case that each user only desires to move to a new location that is close enough to its current location in each single location update decision. Thus, we next propose a distributed strategic mobility algorithm that takes this local learning constraint into consideration.

B. Distributed Strategic Mobility Algorithm

We assume that each user has a traveling distance constraint ϑ_n, i.e., user n at location d_n can only move to a new location in the restricted set of locations $\triangle^n_{d_n} \triangleq \{d \in \triangle_n \backslash \{d_n\} : ||d, d_n|| \leq \vartheta_n\}$. Furthermore, we assume that each user n only has the information of its utility $U_n(\boldsymbol{d}, \boldsymbol{a})$ through local measurement[3].

Motivated by the distributed P2P streaming algorithm in [21], we design an efficient distributed strategic mobility algorithm by carefully coordinating users' asynchronous location updates to form a Markov chain (with the system state as the location profile $\boldsymbol{d}$ of all users). The details of the algorithm are given in Algorithm 2. Here users update their locations asynchronously according to a timer value that follows the exponential distribution with a rate of $\tau_n|\triangle^n_{d_n}|$, where the density τ_n describes how often a user n updates its location. Users with a higher QoS requirement may update its location more often (i.e., with a larger timer density), in order to achieve a higher data rate. Since the exponential distribution has support over $(0, \infty)$ and its probability density function is continuous, the probability that more than one users generate the same timer value and update their locations simultaneously equals zero. Furthermore, if a user n does not want to move, we have $|\triangle^n_{d_n}| = 0$ and hence the user n will not update its location according to the algorithm. If a user has a set of candidate locations $\triangle^n_{d_n}$ to move, it will have chances to update its location selection and hence improve its utility, which also improves the system potential $\Phi(\boldsymbol{d}, \boldsymbol{a})$ by the property of potential game. In the algorithm, we will use a temperature parameter γ to control the randomness of users' location selections. As γ increases, a user will choose a location of a higher utility with a larger probability. As an example, the system state transition diagram of the distributed strategic mobility Markov chain by two users is shown in Figure 4.

We show in Lemma 4 that the distributed strategic mobility Markov chain is time reversible. Time reversibility means that when tracing the Markov chain backwards, the stochastic behavior of the reverse Markov chain remains the same. A nice property of a time reversible Markov chain is that it always admits a unique stationary distribution, which guarantees the convergence of the distributed strategic mobility algorithm.

Lemma 4. *The distributed strategic mobility algorithm induces a time-reversible Markov chain with the unique stationary distribution*

$$Pr(\boldsymbol{d}, \boldsymbol{a}) = \frac{e^{\gamma\Phi(\boldsymbol{d},\boldsymbol{a})}}{\sum_{\tilde{\boldsymbol{d}}\in\Theta} e^{\gamma\Phi(\tilde{\boldsymbol{d}},\boldsymbol{a})}}, \forall \boldsymbol{d} \in \Theta, \tag{13}$$

where $Pr(\boldsymbol{d}, \boldsymbol{a})$ is the probability that the location profile $\mathbf{d}$ is chosen by all users under the fixed channel selection strategy profile $\boldsymbol{a}$.

The key of the proof is to verify that the distribution in (13) satisfies the detailed balance equations of the distributed strategic mobility Markov chain, i.e., $Pr(\boldsymbol{d}, \boldsymbol{a})q_{\boldsymbol{d},\boldsymbol{d}'} = Pr(\boldsymbol{d}^{'}, \boldsymbol{a})q_{\boldsymbol{d}',\boldsymbol{d}}$. Let $\Phi^*(\boldsymbol{a}) = \max_{\boldsymbol{d}\in\Theta} \Phi(\boldsymbol{d}, \boldsymbol{a})$ be the maximum of the potential function of the game, and $\bar{\Phi}(\boldsymbol{a})$ be the expected performance by the distributed strategic mobility algorithm. We have

Theorem 4. *For the distributed strategic mobility algorithm, as the temperature $\gamma \to \infty$, the expected performance $\bar{\Phi}(\boldsymbol{a})$ approaches to $\Phi^*(\boldsymbol{a})$, and the distributed strategic mobility algorithm converges to a Nash equilibrium.*

Note that in practice we can only implement a finite value of the temperature γ. The value of the temperature γ is bounded such that the potential $e^{\gamma\Phi(\boldsymbol{d},\boldsymbol{a})}$ does not exceed the range of the largest predefined real number on a personal computer.

[3]Users can adopt the similar sample average estimation approach as in distributed learning mechanism in Section IV-A.

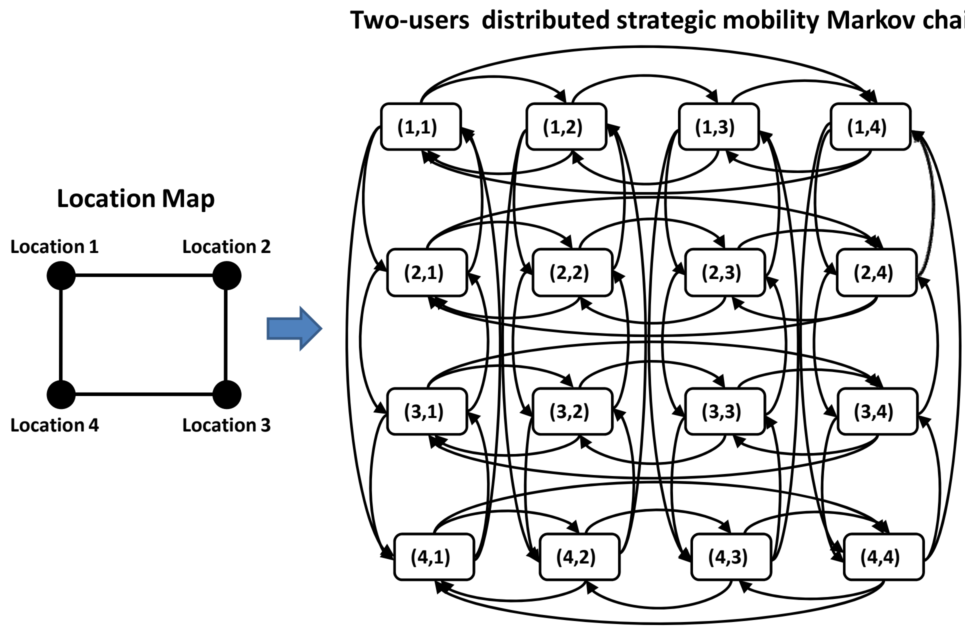

Fig. 4. System state transition diagram of the distributed strategic mobility Markov chain by two users. In the location map on the left hand-side, one location is reachable directly from another location if these two locations are connected by an edge. In the transition diagram of the Markov chain on the right hand-side, (d_1, d_2) denotes the system state with d_1 and d_2 being locations of user 1 and 2, respectively. The transition between two system states is feasible if they are connected by a link.

Numerical results show that the algorithm with a large enough feasible γ can converge to a near-optimal solution such that $\bar{\Phi}(\boldsymbol{a})$ is close to $\Phi^*(\boldsymbol{a})$.

C. Joint Channel Selection and Strategic Mobility

We now consider the case that each user has the flexibility to choose its location and channel simultaneously. Similarly to Section V-A, we formulate the problem as a joint spatial channel selection and mobility game $\Upsilon = (\mathcal{N}, (\boldsymbol{d}, \boldsymbol{a}), \{U_n\}_{n\in\mathcal{N}})$. A location and channel profile $(\boldsymbol{d}^*, \boldsymbol{a}^*)$ is a Nash equilibrium if and only if it satisfies that $(\forall n \in \mathcal{N})$

$$(d_n^*, a_n^*) = \arg \max_{d_n\in\triangle_n, a_n\in\mathcal{M}} U_n(d_n, d_{-n}^*, a_n, a_{-n}^*). \quad (14)$$

We show that the game Υ is also a weighted potential game.

Lemma 5. *The joint spatial channel selection and mobility game Υ is a weighted potential game, with the same potential function as $\Phi(\boldsymbol{d}, \boldsymbol{a})$ in (8), and the weight $w_n = -\log(1-p_n)$.*

Lemma 5 implies the following key result.

Theorem 5. *The joint spatial channel selection and mobility game has a Nash equilibrium and the finite improvement property.*

To reach a Nash equilibrium of the joint spatial channel selection and mobility game, we can run the distributed learning mechanism for channel selection and distributed strategic mobility algorithm together. According to the numerical results, the distributed learning mechanism can converge to a Nash equilibrium in less than one minute ($<300 \times 100$ time slots, and each time slot is assumed to be 2 milliseconds, which is longer than one normal time-slot in the standard GSM system). Thus, we can implement the distributed strategic mobility algorithm at a larger time-scale (say every few minutes), and implement the distributed learning for channel selection at a smaller time scale (say every few milliseconds). Under such separation of time scales, it is reasonable to assume that the distributed learning mechanism operating at the small time scale achieves convergence between two updates at the large time scale. We show that

Theorem 6. *With the separation of time-scales, the joint distributed learning mechanism and strategic mobility algorithm converges to a Nash equilibrium of the joint spatial channel selection and mobility game as the temperature $\gamma \to \infty$.*

The key idea of the proof is that the distributed learning mechanism globally maximizes the potential function $\Phi(\boldsymbol{d}, \boldsymbol{a})$ in decision variable $\boldsymbol{a}$ given the fixed location profile $\boldsymbol{d}$, i.e., $\max_{\boldsymbol{a}} \Phi(\boldsymbol{d}, \boldsymbol{a})$. Then the strategic mobility algorithm at the larger timescale also maximizes the potential function $\Phi(\boldsymbol{d}, \boldsymbol{a})$ in terms of decision variable $\boldsymbol{d}$ given that the channel selections are $\boldsymbol{a}_{\boldsymbol{d}}^* = \arg\max_{\boldsymbol{a}} \Phi(\boldsymbol{d}, \boldsymbol{a})$. That is, the algorithm will converge to the equilibrium such that the best location profile $\boldsymbol{d}^*$ with the maximum potential $\Phi(\boldsymbol{d}^*, \boldsymbol{a}_{\boldsymbol{d}^*}^*)$ will be selected. And a maximum point to the potential function is also a Nash equilibrium of the potential game [18].

VI. Price of Anarchy

In previous sections, we have considered the existence of Nash equilibrium and proposed distributed algorithms for achieving the equilibrium. We will further explore the efficiency of the Nash equilibrium, which can offer more useful insights for the game theoretic approach for distributed spectrum sharing with spatial reuse.

Following the definition of price of anarchy (PoA) in game theory [5], we will quantify the efficiency ratio of the worst-case Nash equilibrium over the centralized optimal solution.

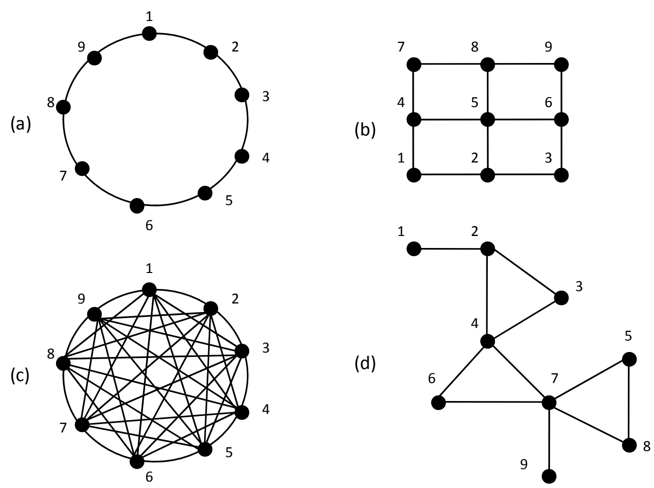

Fig. 5. Interference graphs

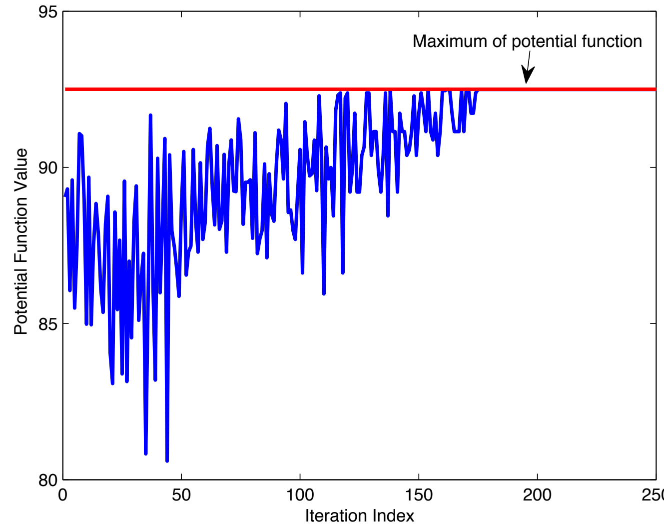

Fig. 6. Learning dynamics of potential function value

We first consider the spatial channel selection game with a fixed spectrum access location profile $\boldsymbol{d}$. Let Ξ be the set of Nash equilibria of the game. Then the PoA is defined as

$$\text{PoA} = \frac{\min_{\boldsymbol{a}\in\Xi}\sum_{n\in\mathcal{N}}U_n(\boldsymbol{d},\boldsymbol{a})}{\max_{\boldsymbol{a}\in\mathcal{M}^N}\sum_{n\in\mathcal{N}}U_n(\boldsymbol{d},\boldsymbol{a})},$$

which is always not greater than 1. A larger PoA implies that the set of Nash equilibrium is more efficient (in the worst-case sense) using the centralized optimum as a benchmark. Let $\varpi = \max_{n\in\mathcal{N}}\{-\log(1-p_n)\}$, $E(\boldsymbol{d}) = \min_{n\in\mathcal{N}}\max_{m\in\mathcal{M}}\left\{\log\left(\theta_m B^n_{m,d_n} p_n\right)\right\}$, and $K(\boldsymbol{d}) = \max_{n\in\mathcal{N}}\{|\mathcal{N}_n(\boldsymbol{d})|\}$. We can show that

Theorem 7. *For the spatial channel selection game with a fixed spectrum access location profile* $\mathbf{d}$*, the PoA is no less than* $1-\frac{K(\boldsymbol{d})\varpi}{E(\boldsymbol{d})}$.

Intuitively, when users are less aggressive in channel contention (i.e., ϖ is smaller) and users are more homogeneous in term of channel utilization (i.e., $E(\boldsymbol{d})$ is larger), the worst-case Nash equilibrium is closer to the centralized optimum and hence the PoA is larger. Moreover, Theorem 7 implies that we can increase the efficiency of spectrum sharing by better utilizing the gain of spatial reuse (i.e., reducing the interference edges $K(\boldsymbol{d})$ on the interference graph). Similarly, by defining that $\eta = \max_{\boldsymbol{d}\in\Theta}\{\frac{K(\boldsymbol{d})}{E(\boldsymbol{d})}\}$, we see that the PoA of the joint spatial channel selection and mobility game is no less than $1-\eta\varpi$.

VII. Numerical Results

We now evaluate the proposed algorithms by simulations. We consider a Rayleigh fading channel environment. The data rate of secondary user n on an idle channel m at location d is given as $b^n_{m,d} = h_d b^n_m$. Here h_d is a location dependent parameter. Parameter b^n_m is the data rate computed according to the Shannon capacity, i.e., $b^n_m = B_m\log_2(1+\frac{\zeta_n g^n_m}{\omega^n_{m,d}})$, where B_m is the bandwidth of channel m, ζ_n is the power adopted by user n, $\omega^n_{m,d}$ is the noise power, and g^n_m is the channel gain (a realization of a random variable that follows the exponential distribution with the mean $\bar{g}^n_m$). In the following simulations, we set $B_m = 10$ MHz, $\omega^n_{m,d} = -100$ dBm, and $\zeta_n = 100$ mW. By choosing different location parameter h_d and mean channel gain $\bar{g}^n_m$, we have different mean data rates $E[b^n_{m,d}] = B^n_{m,d} = h_d E[b^n_m] = h_d B^n_m$ for different channels, locations, and users. For simplicity, we set the channel availabilities $\theta_m = 0.5$.

A. Distributed Learning For Spatial Channel Selection

We first evaluate the distributed learning algorithm for channel selection with fixed user locations. For the distributed learning algorithm initialization, we set the length of each decision period $K = 100$, which can achieve a good estimation of the expected payoff. For the smoothing factor μ_T, a higher value can lead to a faster convergence. We hence set $\mu_T = \frac{1}{T}$, which has the fastest convergence while satisfying the convergence condition in Theorem 2.

Since locations are fixed, we set the location parameter $h_d = 1$. We consider a network of $M = 5$ channels and $N = 9$ users with four different interference graphs (see Figure 5). Graphs (a), (b), and (c) are the commonly-used regular interference graphs, and Graph (d) is a randomly-generated non-regular interference graph. Let $\boldsymbol{B}_n = (B^n_1, ..., B^n_M)$ be the mean data rate vector of user n on M channels. We set $\boldsymbol{B}_1 = \boldsymbol{B}_2 = \boldsymbol{B}_3 = (0.1, 0.3, 0.8, 1.0, 1.5)$ Mbps, $\boldsymbol{B}_4 = \boldsymbol{B}_5 = \boldsymbol{B}_6 = (0.2, 0.6, 1.6, 2.0, 3.0)$ Mbps, and $\boldsymbol{B}_7 = \boldsymbol{B}_8 = \boldsymbol{B}_9 = (0.5, 1.5, 4.0, 5.0, 7.5)$ Mbps. The fixed channel contention probabilities p_n of the users are randomly assigned from the set $\{0.1, 0.2, ..., 0.9\}$.

Let us first look at the convergence dynamics, using graph (d) in Figure 5 as an example. Figure 6 shows the learning dynamics of the potential function value Φ. We see that the distributed learning algorithm can lead the potential function of the spatial channel selection game to the maximum point, which is a Nash equilibrium according to the property of potential game.

To benchmark the performance of the distributed learning algorithm, we compare it with the solution obtained by the centralized global optimization of $\max_{\boldsymbol{a}}\sum_{n\in\mathcal{N}}U_n(\boldsymbol{d},\boldsymbol{a})$ on all the interference graphs. The results are shown in Figure 7.

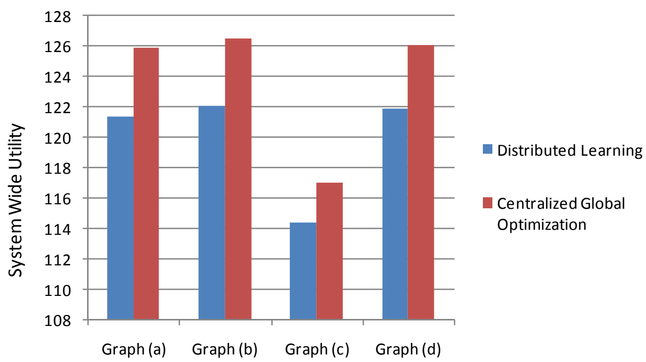

Fig. 7. Comparison of distributed learning and global optimization

We see that the performance loss of the distributed learning is less than 5% in all cases.

B. Joint Distributed Learning and Strategic Mobility

We next study the joint distributed learning and strategic mobility algorithm. We consider a location map as shown in graph (a) of Figure 8. Black cells are obstacles, and no users can move there. Each user in a cell can interfere with those users within the same cell and the ones in neighboring cells (along the line and diagonal). Each user initially locates in the same cell in the bottom left corner, and is allowed to move to the neighboring cells once it gets the chance to update its location. Each cell is randomly assigned with a location parameter h_d from the set $\{0.5, 1.0, 2.0\}$, and each user has different mean data rates B_m^n as specified in Section VII-A.

We implement the joint algorithm with the temperature $\gamma = 10$, 20, and 50, respectively. The location update process follows the exponential distribution with a mean of 10. We show in Figure 8 users' locations and channel selections at the iteration step $t = 2, 50$, and 100, respectively (with the temperature $\gamma = 50$). We observe that users try to spread out in terms of physical locations and meanwhile choose channels with higher data rates, in order to maximize their payoffs. From Figure 9, we see that the performance of the algorithm improves as the temperature γ increases, and the convergence time also increases accordingly. When $\gamma = 50$, the performance loss of the joint algorithm is less than 6%, compared with the global optimal solution, i.e., $\max_{\boldsymbol{d},\boldsymbol{a}} \sum_{n\in\mathcal{N}} U_n(\boldsymbol{d},\boldsymbol{a})$. This shows the efficiency of the Nash equilibrium. When users are static (without strategic mobility) and close-by, the performance loss of the distributed learning for channel selection can be as high as 18%, which justifies the motivations for the strategic mobility design.

VIII. Conclusion

In this paper, we generalize the spatial congestion game framework for distributed spectrum access mechanism design with spatial reuse. We consider both the spatial channel selection game and the joint spatial channel selection and mobility game, and propose distributed algorithms using users' local information that converge to the Nash equilibria for both games. Numerical results verify that Nash equilibria are quite efficient and have less than 6% performance loss, compared with the centralized optimal solutions.

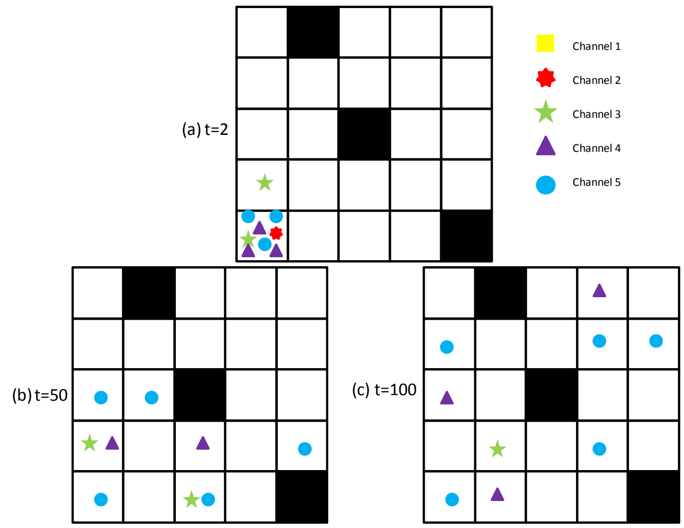

Fig. 8. Dynamics of users' locations and channel selections with the temperature $\gamma = 50$

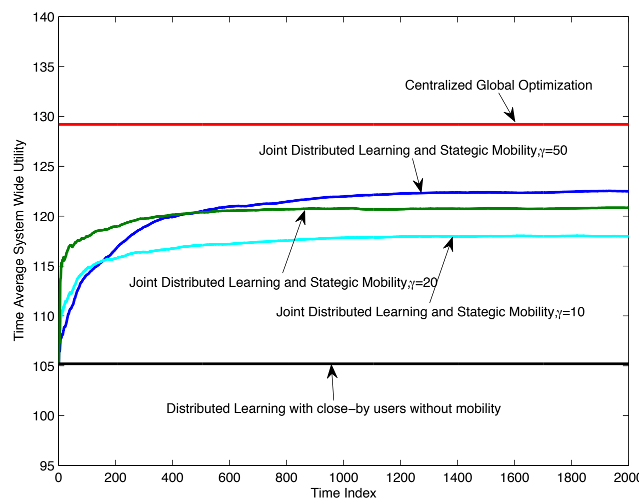

Fig. 9. Dynamics of time average system utility with location update process following the exponential distribution and the temperature $\gamma = 10$, 20, and 50, respectively

For the future work, we are going to investigate the distributed spectrum sharing mechanism design with spatial reuse that can achieve the centralized optimal solution.

Appendix

A. Proof of Lemma 1

For the ease of exposition, we first define $\rho_i \triangleq \log(1-p_i)$, $\xi_{m,d}^i \triangleq \log(\theta_m B_{m,d}^i p_i)$, and

$$\Phi_i^m(\boldsymbol{d},\boldsymbol{a}) = -\rho_i \left(\frac{1}{2} \sum_{j\in\mathcal{N}_i^m(\boldsymbol{d},\boldsymbol{a})} \rho_j + \xi_{m,d_i}^i \right) I_{\{a_i=m\}}.$$

Thus, we have $\Phi(\boldsymbol{d},\boldsymbol{a}) = \sum_{i=1}^N \sum_{m=1}^M \Phi_i^m(\boldsymbol{d},\boldsymbol{a})$.

Now suppose that a user k unilaterally changes its strategy a_k to $a_k^{'}$. Let $\boldsymbol{a}' = (a_1, ..., a_{k-1}, a_k^{'}, a_{k+1}, ..., a_N)$ be the new strategy profile. Thus, the change in potential Φ from $\boldsymbol{a}$ to $\boldsymbol{a}^{'}$ is given by

$$\begin{aligned}
&\Phi(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi(\boldsymbol{d},\boldsymbol{a})=\sum_{i=1}^{N}\sum_{m=1}^{M}\Phi_i^m(\boldsymbol{d},\boldsymbol{a}^{'})\\
&\quad-\sum_{i=1}^{N}\sum_{m=1}^{M}\Phi_i^m(\boldsymbol{d},\boldsymbol{a})\\
&=\sum_{m=1}^{M}\Phi_k^m(\boldsymbol{d},\boldsymbol{a}^{'})-\sum_{m=1}^{M}\Phi_k^m(\boldsymbol{d},\boldsymbol{a})\\
&\quad+\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}\sum_{m=1}^{M}\Phi_i^m(\boldsymbol{d},\boldsymbol{a}^{'})-\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}\sum_{m=1}^{M}\Phi_i^m(\boldsymbol{d},\boldsymbol{a})\\
&=\left(\sum_{m=1}^{M}\Phi_k^m(\boldsymbol{d},\boldsymbol{a}^{'})-\sum_{m=1}^{M}\Phi_k^m(\boldsymbol{d},\boldsymbol{a})\right)\\
&\quad+\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}\left(\Phi_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a})\right)\\
&\quad+\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}\left(\Phi_i^{a_k}(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi_i^{a_k}(\boldsymbol{d},\boldsymbol{a})\right). \qquad (15)
\end{aligned}$$

Equation (15) consists of three parts. Next we analyze each part separately. For the first part, we have

$$\begin{aligned}
&\sum_{m=1}^{M}\Phi_k^m(\boldsymbol{d},\boldsymbol{a}^{'})-\sum_{m=1}^{M}\Phi_k^m(\boldsymbol{d},\boldsymbol{a})\\
=&\Phi_k^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi_k^{a_k}(\boldsymbol{d},\boldsymbol{a})=-\rho_k\left(\frac{1}{2}\sum_{j\in\mathcal{N}_k^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})}\rho_j+\xi^k_{a_k^{'},d_k}\right)\\
&+\rho_k\left(\frac{1}{2}\sum_{j\in\mathcal{N}_k^{a_k}(\boldsymbol{d},\boldsymbol{a})}\rho_j+\xi^k_{a_k,d_k}\right). \qquad (16)
\end{aligned}$$

For the second part in (15),

$$\begin{aligned}
&\Phi_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a})\\
&=-\frac{1}{2}\rho_i\left(\sum_{j\in\mathcal{N}_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})}\rho_j-\sum_{j\in\mathcal{N}_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a})}\rho_j\right)I_{\{a_i=a_k^{'}\}}\\
&=-\frac{1}{2}\rho_i\rho_k I_{\{a_i=a_k^{'}\}}.
\end{aligned}$$

This means

$$\begin{aligned}
&\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}\left(\Phi_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi_i^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a})\right)\\
&=\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}-\frac{1}{2}\rho_i\rho_k I_{\{a_i=a_k^{'}\}}=-\frac{1}{2}\rho_k\sum_{i\in\mathcal{N}_k^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a}^{'})}\rho_i. \qquad (17)
\end{aligned}$$

For the third term in (15), we can similarly get

$$\sum_{i\in\mathcal{N}_k(\boldsymbol{d})}\left(\Phi_i^{a_k}(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi_i^{a_k}(\boldsymbol{d},\boldsymbol{a})\right)=\frac{1}{2}\rho_k\sum_{i\in\mathcal{N}_k^{a_k^{'}}(\boldsymbol{d},\boldsymbol{a})}\rho_i. \qquad (18)$$

Substituting (16), (17), and (18) into (15), we obtain

$$\Phi(\boldsymbol{d},\boldsymbol{a}^{'})-\Phi(\boldsymbol{d},\boldsymbol{a}=-\rho_k\left(U_k(\boldsymbol{d},\boldsymbol{a}^{'})-U_k(\boldsymbol{d},\boldsymbol{a})\right). \qquad (19)$$

Since $0<p_k<1$ and hence $-\log(1-p_k)>0$, we can conclude that $\Phi(\boldsymbol{d},\boldsymbol{a})$ defining in (8) is a weighted potential function with the weight $-\log(1-p_k)$. □

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