Spectrum Investment with Uncertainty Based on Prospect Theory

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***Abstract*—We study a secondary wireless operator's spectrum investment problem under spectrum supply uncertainty using prospect theory. In order to meet the demands of its users, the secondary operator can either sense for the unused spectrum in a licensed band, or lease spectrum from a spectrum owner. Sensing is usually cheaper than leasing, but the amount of spectrum obtained by sensing is uncertain. We formulate such a hybrid spectrum investment problem as a two-stage optimization problem, and compute the optimal sensing and leasing decisions using backward induction. To model the realistic investment behaviors under uncertainty, we apply prospect theory to overcome the limitations of the widely adopted expected utility theory. We show that the investment decision model based on prospect theory leads to a non-convex optimization problem, which is challenging to solve in closed-form. However, we characterize the uniqueness of the optimal solution analytically, and compute it through a simple line search. Comparing with the expected utility theory benchmark, the analysis based on prospect theory shows that the operator will be more conservative in sensing, in order to reduce the investment risk and to avoid a large possible loss. In other words, the operator is both risk averse and loss averse.**

I. Introduction

A *secondary operator* in a cognitive radio network does not own any spectrum license, so it may acquire spectrum from a *spectrum owner* in order to provide wireless services to its own users. For example, Google (the secondary operator), who is interested in providing its own wireless services, can acquire spectrum from AT&T (the spectrum owner). There are two main approaches for a secondary operator to obtain spectrum, namely spectrum sensing and spectrum leasing. With *spectrum sensing*, a secondary operator detects the unused spectrum in a licensed band, and use it to provide services to its users without causing any harmful interferences to the existing primary (licensed) users. With *spectrum leasing*, a spectrum owner explicitly allows the secondary operator to tentatively operate in its licensed band with a leasing fee. In this paper, we consider that a secondary operator invests in the spectrum through both spectrum sensing and spectrum leasing.

Most prior studies of spectrum investment under *uncertainty* (e.g., [1]–[5] and several references therein) have used the *expected utility theory* (EUT). For example, Kasbekar and Sarkar in [1] studied the spectrum trading pricing competition from a game theoretical view under the uncertainty of the mixed strategies of the other decision makers. The same authors in [2] further considered a spectrum auction framework for access allocation, under the uncertainty of the number of secondary users. Gao *et al.* in [3] studied the spectrum contract between a primary spectrum owner and the secondary users with the uncertainty of the user types. In [4], Jin *et al.* presented an insurance-based spectrum trading problem between a primary spectrum owner and the secondary users, where the uncertainty is also the types of the users. Duan *et al.* in [5] considered the spectrum investment of a secondary operator under the spectrum sensing uncertainty. However, EUT assumes that a decision maker aims to maximize the weighted average of its utilities under different outcomes, which does not fully capture the realistic human decision process based on several well known psychological studies in the past few decades [6]–[8].

An alternative theory, the Nobel-prize-winning *prospect theory* (PT) [6]–[8], provides a psychologically more accurate description of the decision making under uncertainty than the EUT. There are three main characteristics of PT: (1) *Impact of reference point*: A decision maker evaluates an option based on the potential gains or losses with respect to a reference point (the zero point), and the choice of the reference point significantly affects the valuation. (2) *S-shaped asymmetrical value function*: A decision maker tends to be risk averse, since he strongly prefers avoiding losses than achieving gains. As a result, the value function is s-shaped and asymmetrical. More precisely, it is concave for gains, convex for losses, and steeper for losses than for gains (see Figure 1 later for a more concrete example). (3) *Probability distortion*: A decision maker tends to overreact to small probability events, but underreact to medium and large probability events. This characteristic is useful in explaining behaviors related to gambling and insurance [6]. As PT fits better into the reality than EUT based on many empirical studies, it has been applied in many areas, such as modeling the behavior of investment agents in finance [9] and the effort and wage levels of workers and firms in labor markets [10].

In this paper, we build upon PT to understand the realistic spectrum investment behavior of a secondary operator. We study the sensing and leasing decisions of a secondary operator, when it faces the risk of loss due to sensing *uncertainty*. Specifically, we formulate the decision problem as a two-stage sequential optimization. In Stage I, the operator determines the optimal amount of licensed spectrum to perform spectrum sensing. Due to the stochastic nature of the primary licensed users' traffic, the amount of idle spectrum that can be obtained through sensing is a random variable. If the spectrum obtained

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through sensing is not sufficient to satisfy its users' demand, the secondary operator will lease some additional spectrum in Stage II. To understand the operator's decisions under PT, we will compare with the benchmark decision under EUT (which is a special case of the PT model with proper choices of system parameters).

The only existing paper that applies PT to understand wireless networking is [11], where Li *et al.* compared the equilibrium strategies of a two-user random access game under EUT and PT. Since considering all the three characteristics in PT mentioned above is challenging, the authors only considered the a linear value function with probability distortion. They proved the existence of a unique Nash Equilibrium under PT. In this paper, we focus on understanding the impact of PT through modeling of the general s-shaped value function, which leads to a non-convex investment problem. Another key difference between [11] and our paper is that [11] focused on a user level network protocol design problem, while our paper focuses on an operator level investment decision.

Our key contributions are summarized as follows:

- *Behavior economical modeling of a wireless operator's investment decision under uncertainty*: We model a secondary operator's investment decisions under sensing uncertainty using PT, which captures the risk aversion and loss aversion of the operator.
- *Characterization of the unique optimal solution of the non-convex decision problem*: Despite the non-convexity of the spectrum sensing problem, we characterize the uniqueness of the optimal solution and compute it through a simple line search method.
- *Engineering insights based on comparison between EUT and PT*: We show that the operator under PT usually senses less to avoid a large possible loss by reducing the risk of sensing uncertainty, which indicates that the operator is both *loss averse* and *risk averse*.

The rest of this paper is organized as follows. In Section II, we introduce the spectrum investment model and formulate the sequential optimization problem. In Section III, we compute the global optimal solution of the non-convex optimization problem, and discuss various analysis insights. In Section IV, we use simulation results to evaluate the sensitivity of the optimal decision with respect to several model parameters. We conclude the paper in Section V.

II. System Model

A. Spectrum sensing and leasing tradeoff

We consider a cognitive radio network with a spectrum owner and a secondary operator. The spectrum owner divides its licensed spectrum into two parts, namely the service band and the transference band [5]. The owner only serves its primary users (PUs) in the service band. Due to the stochastic nature of PUs' traffic, the service band is not always fully utilized. The secondary operator can *sense* the service band for the idle (unused) spectrum at a particular time, and make a profit by providing services to its secondary users with such a spectrum resource. For the transference band, the spectrum owner does not use it for its PUs and can *lease* it to the secondary operator. As a more concrete (hypothetical) example, we consider the spectrum trading between Google and AT&T. AT&T is a spectrum owner, who provides wireless service to its PUs. However, it cannot fully utilize its spectrum in some rural areas, so it will divide its spectrum into the service band and transference band (at those under-utilized locations). Google wants to provide spectrum service to its own end-users but it does not own any spectrum. As a result, Google senses for spectrum holes in the service band, and pays AT&T for leasing its spectrum in the transference band. Since Google does not know the AT&T's users' activities before sensing, the amount of useful spectrum obtained through sensing is uncertain.

From the secondary operator's point of view, sensing is often a cheaper way to obtain spectrum than leasing, because the energy and time overhead involved in sensing is often much lower than the explicit cost of spectrum leasing [5]. However, the unused amount of spectrum obtained through sensing is uncertain. Therefore, the secondary operator needs to optimize the spectrum investment decisions in every time slot to obtain the best tradeoff between the cost and the risk. In this paper, we will only consider a single time slot, in which the operator makes both the sensing and leasing decisions only once. For the rest of the paper, we will use "operator" to denote "secondary operator", and "users" to denote "secondary users".

B. Problem Formulation

We formulate the operator's spectrum investment problem as a two-stage sequential optimization problem. The operator determines its sensing amount before leasing, because sensing is cheaper than leasing. The operator should lease only if sensing does not provide enough spectrum to satisfy the demand of its users.

In Stage I (the sensing stage), the operator determines its sensing amount B_s. For simplicity, we assume a linear sensing cost c_s per unit of sensed bandwidth, which represents the time and energy overhead for sensing [12]. Due to the stochastic nature of PUs' traffic, only $\alpha \in [0, 1]$ portion of the sensed spectrum is temporarily unused and can be sold to the secondary users. We assume that the operator knows the distribution of α through the sensing history.

In Stage II (the leasing stage), the operator determines the leasing amount B_l after observing the available spectrum through sensing, $B_s\alpha$. The linear leasing cost c_l is determined through negotiation between the operator and spectrum owner, and is assumed to be larger than c_s[1].

We assume that the operator is a price-taker in the market, as there can be many operators in the same market due to market deregulation. Under a fixed usage-based price π, we

[1]It is easy to see that if the leasing cost c_l is smaller than the sensing cost c_s, then it is optimal for the secondary operator to lease only without considering sensing. The spectrum investment problem hence becomes trivial.

assume that the secondary users' maximum bandwidth demand for this operator is fixed at D. The profit of the operator is

$$R = \min\{\pi D, \pi(B_l + B_s\alpha)\} - (B_s c_s + B_l c_l). \quad (1)$$

Here, the revenue depends on the minimum of spectrum supply $(B_l + B_s\alpha)$ and demand D, and the cost depends on both the sensing decision B_s and leasing decision B_l.

In the next section, we will study the operator's optimal sensing and leasing decisions that maximize its profit.

III. Solving the Two-stage Optimization Problem

Next, we use backward induction [13] to solve the two-stage sequential optimization problem. First, we derive the operator's optimal leasing decision in Stage II. Then, we obtain the operator's optimal sensing decision in Stage I under sensing uncertainty using PT.

A. Optimal leasing decision in Stage II

By observing the profit expression in (1), it is clear that the operator will not lease any spectrum if $c_l \geq \pi$ (i.e., when the leasing cost is too high). For the rest of the paper, we will focus on the nontrivial case where $c_l < \pi$, so that it is beneficial for the operator to lease the spectrum and sell in the retail market (if there is a demand for it). Under such an assumption, we can show that the optimal leasing amount satisfies

$$B_l^* = \max\{D - B_s\alpha, 0\}, \quad (2)$$

which is the difference between the total demand D and the successful sensing amount $B_s\alpha$. If the amount of spectrum $B_s\alpha$ obtained by spectrum sensing in Stage I exceeds the total demand D, then $B_l^* = 0$.

B. Optimal sensing decision in Stage I

In order to use PT to model the operator's decision under spectrum sensing uncertainty, we first need to determine the *reference point*. In this paper, we choose the maximum profit that the operator can achieve without using sensing (hence it does not need to face any uncertainty) as the reference point. To achieve such a risk-free reference point, the operator will lease a bandwidth B_l equal to D and achieve a profit of

$$R_p = D(\pi - c_l). \quad (3)$$

The operator considers a profit R lower than the reference point R_p as a loss, and a profit R larger the reference point R_p as a gain, and makes a different decision when facing a gain or a loss in PT. For simplicity, we assume the sensing realization factor α is a uniformly distributed random variable. Therefore, the profit in (1) is a continuous random variable. We apply PT for continuous outcomes [14], and obtain the expected utility of the operator as

$$U(B_s) = \int_R v(R - R_p) dF(R), \quad (4)$$

where $F(R)$ is the cumulative distribution function that the profit is less than or equal to R (shown in (7) in Appendix A), and $v(x)$ is the value function that is central in PT [6].

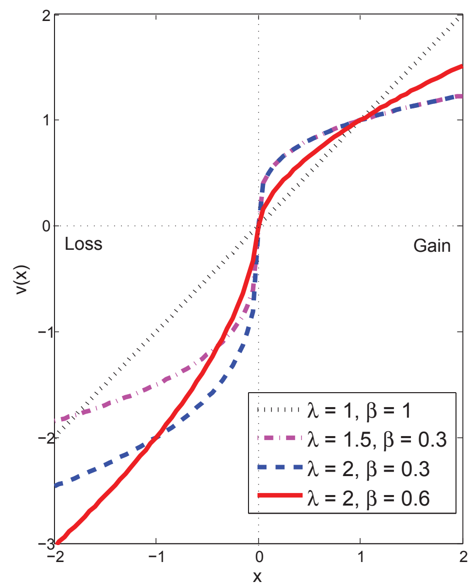

Fig. 1. The S-shaped asymmetrical value function $v(x)$ in PT.

As shown in Fig. 1, the value function $v(x)$ is an s-shaped function, which is concave for a positive argument (gain), and is convex for a negative argument (loss). Moreover, the impact of loss is usually larger than the gain of the same absolute value. A commonly chosen value function [7], [8], [15] is

$$v(x) = \begin{cases} v^+(x) = x^\beta, & x \geq 0, \\ v^-(x) = -\lambda(-x)^\beta, & x < 0, \end{cases} \quad (5)$$

where $\lambda > 1$ and $0 < \beta < 1$. λ is the loss penalty parameter, where a larger λ indicates that the operator is more *loss averse*. β is the risk aversion parameter, where the value function is more concave when β approaches zero, meaning that the operator is more *risk averse*. We note that EUT is a special case when $\lambda = 1$ and $\beta = 1$. However, empirical studies show that choosing $\lambda = 2.25$ and $\beta = 0.88$ often matches the reality well [8].

The operator's spectrum sensing problem in Stage I with sensing uncertainty is

$$\max_{B_s \geq 0} \quad U(B_s). \quad (6)$$

Problem (6) is a *non-convex* optimization problem, because it involves the s-shaped value function in (5). Hence, it is challenging to analytically characterize the closed-form optimal solution. In general, a non-convex optimization problem might have multiple local and global optimal solutions. For our problem, however, we can show that there exists a single local optimal (hence global optimal) solution by exploiting the special structure of the problem.

To solve problem (6), we divide the feasible range of B_s into three intervals, $[0, D]$, $(D, Dc_l/c_s)$, and $[Dc_l/c_s, \infty)$, and we analyze the optimal decision B_s^* within each interval. We summarize the key result in Theorem 1. The detailed proof of Theorem 1 is given in Appendix A.

TABLE I
OPTIMAL SENSING AMOUNT AND LEASING AMOUNT UNDER PT

Condition	Optimal Sensing Amount B_s^*	Optimal Leasing Amount B_l^*
$c_l/c_s \geq 1+\lambda^{\frac{1}{1+\beta}}$	$B_s^* \in [D, Dc_l/c_s)$	$B_l^* = 0$
$c_l/c_s < 1+\lambda^{\frac{1}{1+\beta}}$	$B_s^* = 0$	$B_l^* = D$

TABLE II
OPTIMAL SENSING AMOUNT, LEASING AMOUNT, AND EXPECTED UTILITY UNDER EUT

Condition	Optimal Sensing Amount B_s^*	Optimal Leasing Amount B_l^*	Expected Utility $U(B_s^*)$
$c_l/c_s \geq 2$	$B_s^* = \sqrt{\frac{c_l}{2c_s}}D$	$B_l^* = 0$	$U(B_s^*) = (\pi - \sqrt{2c_l c_s})D$
$c_l/c_s < 2$	$B_s^* = 0$	$B_l^* = D$	$U(B_s^*) = (\pi - c_l)D$

Theorem 1. *The optimal sensing amount B_s^* and leasing amount B_l^* are summarized in Table I.*

The proof of Theorem 1 is given in Appendix A. The results in Table I depend on the value of the ratio c_l/c_s. For the case $c_l/c_s \geq 1+\lambda^{\frac{1}{1+\beta}}$ (i.e., leasing is quite expensive), we have $B_s^* \in [D, Dc_l/c_s)$. From Proposition 1 in Appendix A, we establish that there exists at most one local (and hence global) maximum point for $U(B_s)$ under such a cost setting. Thus, we can compute B_s^* by using a line search algorithm [16]. When $c_l/c_s < 1+\lambda^{\frac{1}{1+\beta}}$, where leasing is relatively cheap, the operator will choose to lease only. Notice that c_l can still be larger than c_s in this range, but the uncertainty makes the "effective cost" of sensing higher than that of leasing.

Next, we further characterize the impact of the loss penalty parameter λ on the optimal sensing decision B_s^*.

Theorem 2. *When $c_l/c_s \geq 1+\lambda^{\frac{1}{1+\beta}}$, then the optimal sensing amount B_s^* is monotonically decreasing with λ.*

The proof of Theorem 2 is given in our technical report [17]. Theorem 2 indicates that if a operator is more loss averse (hence the loss penalty parameter λ is larger), it will sense less.

C. The EUT benchmark

For comparison, we also consider the operator's optimal sensing and leasing decisions under the EUT model. As mentioned above, EUT model is a special case of the PT model with $\beta = 1$ and $\lambda = 1$. Under the EUT model, we can obtain the solution of problem (6) in *closed-form*.

Theorem 3. *The optimal sensing amount B_s^*, leasing amount B_l^*, and the maximum expected utility $U(B_s^*)$ under EUT are summarized in Table II.*

The proof of Theorem 3 is given in our technical report [17]. The results in Table II also depend on the ratio c_l/c_s. For the case $c_l/c_s \geq 2$ (hence leasing is significantly more expensive), we have $B_s^* = \sqrt{\frac{c_l}{2c_s}}D$. When $c_l/c_s < 2$, the leasing is cheap enough so that the operator will choose to lease only. The threshold value 2 is due to the assumption of uniform distribution of α, as the expected "effective cost" of getting one unit of idle spectrum through sensing is $2c_s$.

IV. NUMERICAL EXAMPLES

In this section, we illustrate the operator's optimal sensing decision under various system parameters (λ and β) through numerical examples. We compare it with the EUT benchmark, where $\lambda = \beta = 1$. The key insights under the PT modeling include: (1) The operator is *risk averse* and prefers a smaller profit with a lower risk in practical settings, and (2) The operator is *loss averse* and prefers avoiding losses than achieving gains.

First, in Fig. 2, we plot the optimal sensing amount B_s^* against the loss penalty parameter λ for different values of the risk aversion parameter β. The other system parameters are fixed at $c_l/c_s = 10$ and $D = 10$. It is clear that for a fixed value of β, the optimal sensing amount B_s^* is decreasing in λ as stated in Theorem 2. The intuition is that when λ is larger, the operator is more conservative and loss averse, so it will sense less to avoid a potential loss.

Fig. 2 also illustrates how B_s^* changes with β for a fixed value of λ. In fact, sensing involves uncertainty (i.e. risk), but leasing does not. If the operator senses less under PT than under EUT, it means that it does not want to "gamble" too much by sensing, so it is *risk-averse*. Otherwise, it is *risk seeking*. We observe both risk seeking and risk aversion behaviors in Fig. 2, and which one dominates depends heavily on the value of λ:

- When λ is small (e.g., $\lambda = 1.5$), B_s^* is decreasing with β. As a result, B_s^* under PT (for a smaller value of β) is larger than that under EUT (i.e., the point with $\lambda = 1$ and $\beta = 1$). The intuition is that when the loss penalty is small, the perceived value of possible loss (i.e., $v(x)$ in (5)) is less than its actual value (i.e., x in (5)) due to the risk aversion parameter β, so that the operator tends to be *risk seeking* and takes a larger B_s^*.
- When λ is large (e.g., $\lambda = 4$), the operator will try to avoid the possibility of getting a large loss, so the operator under PT is *risk averse* and senses less comparing with EUT (i.e., the point with $\lambda = 1$ and $\beta = 1$).

From psychological experiments, a large λ (i.e., loss aversion) often fits into the reality better [6]. As a result, in a practical setting, we expect the operator to sense less comparing with the EUT benchmark. In other words, risk aversion dominates the operator's decision.

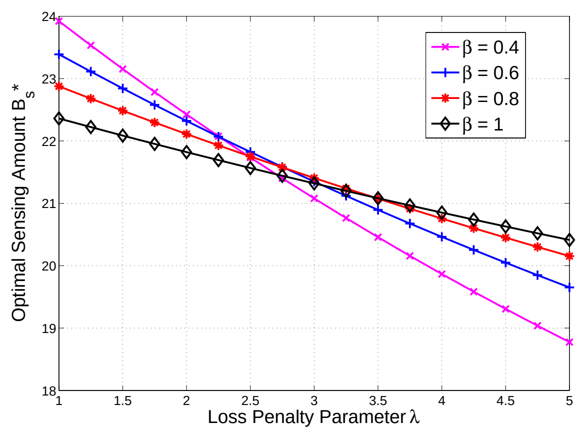

Fig. 2. Optimal sensing amount B_s^* versus loss penalty parameter λ for different values of β. Other parameters are $c_l/c_s = 10$ and $D = 10$.

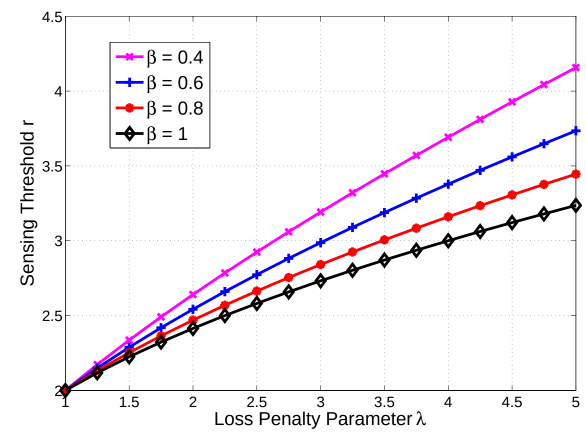

Fig. 3. Sensing threshold r versus loss penalty parameter λ for different values of β.

In Fig. 3, we plot the sensing threshold r $(= 1 + \lambda^{\frac{1}{1+\beta}}.)$ against the loss penalty parameter λ for different values of β. For the interpretation of sensing threshold r in Table I, when the leasing and sensing cost ratio $c_l/c_s \geq r$, meaning that leasing is expensive, the operator will choose to sense a positive amount. Otherwise, when $c_l/c_s < r$, the operator will lease only, and will not perform any sensing. We can see in Fig. 3 that the threshold r for sensing under PT is higher than that under EUT (i.e., the point with $\lambda = 1$ and $\beta = 1$ at the bottom left corner). In other words, the range of cost parameters under which the sensing happens under PT is smaller than that under EUT. And such range becomes smaller when the the parameters deviate more from EUT's parameters ($\lambda = 1$, $\beta = 1$).

V. Conclusions and Future Work

In this paper, we considered a spectrum investment problem with sensing uncertainty, where an operator decides its spectrum sensing and leasing amounts to maximize its profit based on a behavioral economics model using prospect theory. We compared and contrasted the optimal decisions under prospect theory and the more widely used expected utility theory, and highlighted the key insights. Under prospect theory, our results suggested that the operator puts more weight on avoiding loss than achieving gain. More precisely, it prefers a smaller possible gain with a lower risk than a larger possible gain with a higher risk.

This study demonstrated that a more realistic modeling based on prospect theory is important in the study of decisions in the wireless industry. On the other hand, this study is only a small first step, as we have only considered the s-shaped value function. Our next step is to further understand how the probability distortion in prospect theory affects the wireless operator's decision making, and we will study it in our future work. The other direction for future study is to consider the operator's decision in multiple time slots with a possible shift of reference point. The issue of dynamic reference point is a recent active research field in prospect theory [9], [18].

Appendix

A. Proof of Theorem 1

In the proof, we divide the feasible range of B_s into three intervals, $[0, D]$, $(D, Dc_l/c_s)$, and $[Dc_l/c_s, \infty)$, and we analyze the optimal decision B_s^* within each interval. This division guarantees $B_l = D - B_s\alpha$ in the range $B_s \in [0, D]$ thus eases our analysis. Another advantage of this division is that it allows us to avoid complicated analysis of the range $B_s \geq Dc_l/c_s$.

• Case I: $B_s \in [0, D]$. In this case, the sensing amount B_s is smaller than the demand D. In order to satisfy the demand, the optimal leasing amount $B_l = D - B_s\alpha > 0$ and the revenue is $R = (\pi - c_l)D - B_s c_s + B_s c_l \alpha$.

Since α is a uniformly distributed random variable in [0,1], the revenue R is uniformly distributed in the interval $[(\pi - c_l)D - B_s c_s, (\pi - c_l)D - B_s c_s + B_s c_l]$. The cumulative distribution function (CDF) of the profit is shown in (7) on the top of the next page. The utility under PT is:

$$
\begin{aligned}
U(B_s) &= \int_{R=R_P}^{\infty} v^{+}(R - R_P)dF(R) + \int_{R=-\infty}^{R_P} v^{-}(R - R_P)dF(R) \\
&= \int_{0}^{c_s/c_l} -\lambda(B_s c_s - B_s c_l \alpha)^{\beta} d\alpha + \int_{c_s/c_l}^{1} (B_s c_l \alpha - B_s c_s)^{\beta} d\alpha \\
&= \frac{[(c_l - c_s)^{\beta+1} - \lambda(c_s)^{\beta+1}]}{(\beta+1)c_l} \cdot (B_s)^{\beta}. \qquad (8)
\end{aligned}
$$

Here, we plug (1), (5), and (7) into (4) to obtain (8). From (8) we find that $U(B_s)$ is a monotone function of B_s. If $c_l \geq (1 + \lambda^{\frac{1}{1+\beta}})c_s$, then $U(B_s)$ is monotone increasing in B_s. If $c_l < (1 + \lambda^{\frac{1}{1+\beta}})c_s$, $U(B_s)$ is monotone decreasing in B_s.

Intuitively, when the leasing cost c_l is high, the utility increases with more sensing (and hence less leasing). When c_l is low, the utility decreases with more sensing.

$$F(R)=\begin{cases}0, & \text{if } R<(\pi-c_l)D-B_sc_s.\\ \dfrac{R-[(\pi-c_l)D-B_sc_s]}{B_sc_l}, & \text{if } (\pi-c_l)D-B_sc_s\le R\le(\pi-c_l)D-B_sc_s+B_sc_l.\\ 1, & \text{if } R>(\pi-c_l)D-B_sc_s+B_sc_l.\end{cases} \tag{7}$$

• Case II: $B_s \in (D, Dc_l/c_s)$. In this case, profit R is a random variable that is partially discrete and partially continuous: It is equal to $\pi D - B_s c_s$ with probability $(1-\frac{D}{B_s})$, and is uniformly distributed in the interval $(\pi D - B_s c_s - c_l D, \pi D - B_s c_s)$ with a probability density $\frac{D}{B_s}$ based on (7). We plug (1), (5), and (7) into (4), and the utility is

$$\begin{aligned}U(B_s)=&\int_{R>R_P} v^+(R-R_P)dF(R)+\int_{R<R_P} v^-(R-R_P)dF(R)\\ &+(1-\frac{D}{B_s})\cdot(c_lD-B_sc_s)^\beta\\ =&\int_{c_s/c_l}^{D/B_s}(B_s\alpha c_l-B_sc_s)^\beta d\alpha+\int_0^{c_s/c_l}-\lambda(-B_s\alpha c_l+B_sc_s)^\beta d\alpha\\ &+(1-\frac{D}{B_s})(c_lD-B_sc_s)^\beta\\ =&\frac{1}{(\beta+1)(B_sc_l)}[(Dc_l-B_sc_s)^{\beta+1}-\lambda(B_sc_s)^{\beta+1}]\\ &+(1-\frac{D}{B_s})(c_lD-B_sc_s)^\beta. \end{aligned}\tag{9}$$

Proposition 1. *There is at most one local maximum point of $U(B_s)$ in the range $B_s \in (D, Dc_l/c_s)$. When $c_l/c_s < 1+\lambda^{\frac{1}{1+\beta}}$, the local maximum point is at the left boundary point ($B_s = D^+$)[2]. When $c_l/c_s \ge 1+\lambda^{\frac{1}{1+\beta}}$, the local maximum point is a strictly interior point $B_s^* \in (D, Dc_l/c_s)$.*

The proof of Proposition 1 is given in our technical report [17].

• Case III: $B_s \in [Dc_l/c_s, \infty)$. In this case, $Dc_l - B_s c_s \le 0$. From (1) and (3), we know $R - R_p = Dc_l - B_s c_s - \max\{D - B_s\alpha, 0\} \le 0$, which means the total cost of sensing $B_s c_s$ will be larger than or equal to the cost of only using spectrum leasing Dc_l. However, the amount of revenue $D\pi$ produced by the two methods is the same since the total demand is limited. Hence it is possible to choose the optimal B_s^* in this range to maximize the utility.

To summarize the analysis of the above three cases, we obtain the following results:

(1) When $c_l \ge 1+\lambda^{\frac{1}{1+\beta}} c_s$, the optimal solution is $B_s^* \in (D, Dc_l/c_s)$.

(2) When $c_l < 1+\lambda^{\frac{1}{1+\beta}} c_s$, the optimal solution is $B_s^* = 0$.

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[2]Here, we use D^+ to denote $D+\epsilon$ and D^- to denote $D-\epsilon$, where ϵ is a small positive number approaching zero (i.e. $\epsilon \to 0^+$).