Demand Response Management via Real-time Electricity Price Control in Smart Grids

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Abstract

Demand response management (DRM) based on real-time electricity price control is a key enabler of the future smart grid. A properly designed real-time pricing scheme may result in a "triple-win" solution: a flattened load demand enhances the robustness and lowers the generation costs for the power grid; a lower generation cost leads to a low wholesale price, which in turn increases the retailers' profit; users may reduce their electricity expenditures by responding to the time-varying prices. In this chapter, we discuss how to design a real-time pricing scheme to reduce the peak-to-average load ratio, to maximize each user's payoff, and to maximize the retailer's profit. In particular, we formulate the real-time pricing scheme as a two-stage optimization problem. In the lower stage, each user reacts to the prices and maximizes its payoff, which is the difference between the quality-of-usage and the payment to the retailer. In the upper stage, the retailer designs the real-time prices in response to the forecasted users' reactions to maximize its profit. We design an algorithmic framework of real-time pricing, in which users and the retailer interact with each other through a limited number of message exchanges to reach the optimal prices. Specifically, the proposed algorithm allows each user to optimally schedule its energy consumption either according to closed-form expressions or through an efficient iterative algorithm as a function of the prices. At the retailer side, we develop a simulated-annealing-based price control algorithm to solve the non-convex price optimization problem. Finally, we demonstrate the performance of the proposed real-time pricing scheme through extensive simulations based on realistic system parameters.

Contents

Chapter 1

Demand Response Management via Real-time Electricity Price Control in Smart Grids

1.1 Introduction

[1]Smart grid refers to an intelligent electricity generation, transmission, and delivery system enhanced with communication facilities and information technologies [3]. Smart grid is expected to have a higher efficiency and reliability comparing with today's power grid, and can relieve economic and environment issues caused by the traditional fossil-fueled power generation [4, 5]. Efficient demand response management and flexible exploitation of renewable energy are two fundamental features of smart grid. The real-time electricity price control has been proved effective in provisioning efficient demand response management. The transition from the traditional fossil-fueled power generation to the decarbonized power generation based on renewable resources (e.g., solar, wind, and geothermal resources) has been employed to cope with the rapid energy demand growth.

1.1.1 Background of real-time pricing based DRM in Smart Grids

In today's electric power grid, we often observe substantial hourly variations in the wholesale electricity price, and the spikes usually happen during peak hours due to the high generation costs. However, nowadays almost all end users are charged some flat-rate retail electricity price [9, 10], which does not reflect the

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actual wholesale price. With the flat-rate pricing, users often consume a large amount of electricity during peak hours, such as the time between late afternoon and bed time for residential users. This leads to a large fluctuation of electricity consumption between peak and off-peak hours. The high peak-hour demand not only induces high cost to the retailers due to the high wholesale prices, but also has a negative impact on the reliability of the power grid. Ideally, the retailer would like to have the electricity consumption evenly spread across different hours of the day through a proper demand response management.

In this chapter, we design an efficient real-time pricing scheme to reduce the peak-to-average load ratio[2] and maximize each user's payoff and the retailer's profit in the meantime, both with and without taking account of renewable energy integration. The key hurdle for achieving such a design objective lies in the asymmetry of information. For example, when the prices are announced before the energy scheduling horizon (i.e., ex-ante price), the retailer has to face the uncertainty of user responses and reimburse the wholesale cost based on the actual electricity consumption by the users. On the other hand, if the prices are determined after the energy is being consumed (i.e., ex-post price), the users have to bear the uncertainty, as they can only adjust their demand according to a (possible inaccurate) prediction of the actual price.

Our approach of solving the above issue is to rely on the communication infrastructure that is integrated into the smart grid. Suppose that each user is equipped with a smart meter that is capable of having two-way communications with the retailer through a communication network. Based on it, we introduce a

[2]The peak-to-average load ratio means the ratio of the hourly peak load to the average hourly load for the power grid.

novel ex-ante real-time pricing scheme for the future smart grid, where the real-time prices are determined at the beginning of each energy scheduling horizon. The contributions of this chapter can be summarized as follows:

- We formulate the real-time pricing scheme as a two-stage optimization problem with/without renewable energy integration. Each user reacts to the price and maximizes its payoff, which is the difference between the quality-of-usage and the payment. Meanwhile, the retailer designs the real-time prices in response to the forecasted user reactions to maximize its profit.

- The proposed algorithm allows each user to optimally schedule its energy consumption either according to simple closed-form expressions or through an efficient iterative algorithm. Furthermore, the users and the retailer interact with each other through a limited number of message exchanges to find the optimal price[3], which facilitates the elimination of cost uncertainty at the retailer side.

- We propose a real-time pricing algorithm based on the idea of simulated annealing, to reduce the peak-to-average load ratio in smart grid systems. For the practical implementation of the algorithm, we further study how to set the length of interaction period, so that the retailer is guaranteed to obtain the optimal price through communications with users.

1.1.2 State-of-art developments in the area of real-time pricing

It is often quite challenging to design a practical ex-ante real-time pricing scheme for the future smart grids. The main difficulties include realizing the "triple-win" target and dealing with the volatility of renewable energy. The study of designing "triple-win" real-time pricing schemes is a significant part of demand response management. There exists a rich body of literature on the real-time pricing without taking account of renewable energy supply, which can be divided into three main threads. The first thread is concerned with how users respond to the real-time price, hopefully in an automated manner, to achieve their desired levels of comfort with lower electricity bill payments (e.g., [10, 16, 17]). These results, however, did not mention how the real-time prices should be set. The second thread of work is concerned with setting the real-time prices at the retailer side (e.g., [18]), without taking into account users' potential responses to the forecasted prices. For example, the retailer may adjust the real-time retail electricity price through linking it closely to the wholesale electricity price in [18]. The last thread of work is concerned with setting the real-time retail electricity price based on the maximization of the aggregate surplus of users and

[3]In this chapter, we assume that the users and the retailer declare their information truthfully. The truthful information exchanges can guarantee the accurate decisions for end users and the retailer, without predicating the uncertainty of consumption scheduling decisions and real-time prices. The uncertainty predication is not the focus of this work, and thus it is not considered in this chapter.

retailers subject to the supply-demand matching (e.g., [19–23]). However, the price obtained in this way may not improve the retailer's profit. In principle, the retailer should be able to design real-time prices that maximizes its own profit by taking into account the users' potential responses to the prices (e.g., responses that maximize users' own payoffs). Ideally, the real-time pricing scheme should be able to achieve a "triple-win" solution that benefits the grid, the retailer, and the end users.

The integration of renewable energy into the traditional generation sector has been investigated from the perspectives of cost, reliability, and environment [24–26]. While renewable energy offers a cheaper and cleaner energy supply, it imposes great challenges on designing the real-time pricing scheme due to the stochastic nature of most renewable energy sources. As mentioned above, a properly designed real-time pricing scheme is dependent on the users' potential responses to the prices. However, the volatility of renewable energy makes it difficult for users to predicate the energy demand from the retailer, and such a problem has attracted a significant amount of research efforts in the past [25–29]. For example, [27, 28] proposed a high-computational-complexity algorithm to determine the optimal energy demand scheduling. The studies in [25, 26, 29] applied simulation-based approaches such as Monte Carlo simulation to evaluate the integration of renewable energy. Such high complexity algorithms may not be suitable for the large-scale commercial deployment, where the computations need to be done by autonomous energy-management units (e.g., smart meters) with limited computational capabilities. Therefore, it is crucial to design efficient user response algorithms when considering the renewable energy integration.

1.2 System model

We consider a microgrid (as shown in Fig. 1) with two types of participants: end users (i.e., customers) who might be integrated with renewable energy generation units (e.g., solar power panels), and a retailer from which end users purchase electricity to satisfy their additional energy needs (beyond local renewable energy generation). We assume that each user is equipped with a smart meter as shown in Fig. 1.2, and is allowed to sell redundant renewable energy to the retailer. The retailer determines the real-time retail and purchase electricity prices and informs users via a communication network (e.g., LAN). Let p_h and $\tilde{p}_h$ denote the retail and purchase prices in time slot h in the scheduling horizon $\mathcal{H} = \{1, ..., H\}$, respectively. A time slot can be, for example, one hour, and the scheduling horizon can be one day, i.e., $H = 24$ hours. At the beginning of every upcoming scheduling horizon, the retailer announces all prices p_h's and $\tilde{p}_h$'s used for the upcoming scheduling horizon, and each user u determines its optimal electricity consumption scheduling accordingly.

In this chapter, we consider time-varying prices, with the objective to reduce peak-to-average load ratio, increase the retailer's profit, and maximize the users' payoffs. In the following, we present the problems considered by users and retailer, respectively.

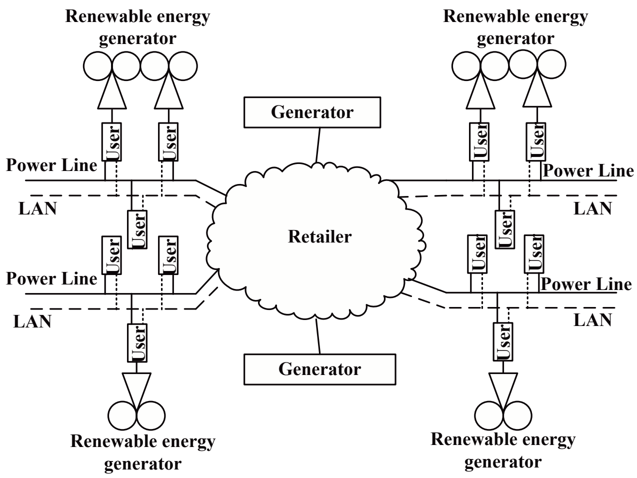

Figure 1.1: A simplified illustration of the retail electricity market.

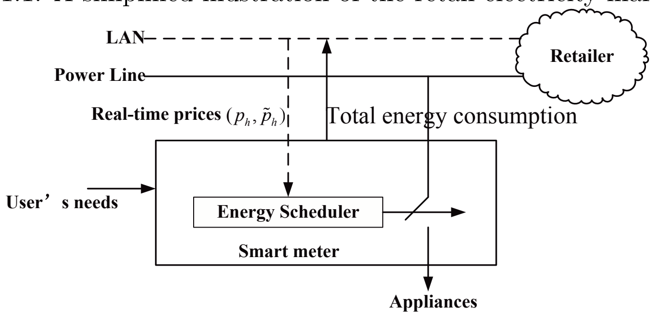

Figure 1.2: The operations of smart meter and retailer in our design.

1.2.1 Residential End Users

Let $\mathcal{U} = \{1, 2, \cdots, U\}$ denote the set of residential users. Assume that each user u has three types of appliances, denoted by $\mathcal{A}_u$, $\mathcal{B}_u$, and $\mathcal{C}_u$. The first category $\mathcal{A}_u$ includes *background appliances*, which consumes a fixed amount of energy per unit time during a fixed period of time. The background appliances are inelastic, in the sense that there is no flexibility to adjust the energy consumption across time. Examples of such appliances include lighting, refrigerator, and electric kettle. The second category $\mathcal{B}_u$ includes elastic appliances, which have a higher for more energy consumed per unit time (with a maximum consumption upper bound). The evaluation of quality-of-usage can be time-dependent for elastic appliances, as users may obtain a higher satisfaction to consume certain amount of energy during a certain time than in other time durations. Examples of such appliances include air conditioner, electric fan, and iron. The last category $\mathcal{C}_u$ includes semi-elastic appliances, which consume a fixed total energy within a preferred time period. This category is semi-elastic in the sense that there is flexibility to choose when to consume the energy within the preferred time period, but no flexibility to adjust the total energy consumption. Examples include washer/dryer, dishwasher, plug-in hybrid electric vehicle (PHEV), and electric geyser.

For each appliance a_u, we express its energy consumption over the scheduling horizon $\mathcal{H}$ by a scheduling vector $\boldsymbol{e}_{a_u}$ as follows:

$$\boldsymbol{e}_{a_u} = (e_{a_u,1}, \cdots, e_{a_u,H}), \tag{1.1}$$

where $e_{a_u,h}$ means the energy consumption of appliance a_u in time slot h. In what follows, we will introduce the energy consumption constraints of the three categories of appliances.

As a background appliance, each $a_u \in \mathcal{A}_u$ operates in a working period $H_{a_u} \in \mathcal{H}$, during which it consumes r_{a_u} energy per time slot. This can be mathematically described as

$$e_{a_u,h} = \begin{cases} r_{a_u,h}, & h \in \mathcal{H}_{a_u}, \\ 0, & \text{otherwise.} \end{cases} \tag{1.2}$$

We note that the time slots in $\mathcal{H}_{a_u}$ are allowed to be intermittent, i.e., a background appliance may not always consume energy in consecutive time slots. There is no flexibility to redistribute the load of this type of appliances. However, such appliances are ubiquitous in power systems and contribute a large percentage of the high demand during peak hours. It is thus important to properly schedule categories $\mathcal{B}_u$ and $\mathcal{C}_u$ appliances to avoid further overloading the peaks.

For each appliance $a_u \in \mathcal{B}_u$, user u obtains different levels of satisfaction for the same amount of energy consumed in different time slots. Suppose that the satisfaction is measured by a time-dependent quality-of-usage function $U_{a_u,h}(e_{a_u,h})$, which depends on who, when, and how much energy is consumed. For example, $U_{a_u,h}(e_{a_u,h})$ may be equal to zero during undesirable operation hours for any value of $e_{a_u,h}$. It is reasonable to assume that $U_{a_u,h}(e_{a_u,h})$ is a non-decreasing concave function of $e_{a_u,h}$ at any time slot $h \in \mathcal{H}$. Besides, for each appliance $a_u \in \mathcal{B}_u$, the energy consumption per time slot is subject to

$$0 \leq e_{a_u,h} \leq r_{a_u}^{\max}, \forall h \in \mathcal{H}, \tag{1.3}$$

where $r_{a_u}^{\max}$ is the maximum energy that can be consumed in the time slot when appliance a_u is working.

As a semi-elastic appliance, each $a_u \in \mathcal{C}_u$ operates in a working period $\mathcal{H}'_{a_u} \subseteq \mathcal{H}$, during which it needs to consume E_{a_u} energy in total. Such a constraint can be written as

$$\sum_{h \in \mathcal{H}'_{a_u}} e_{a_u,h} = E_{a_u}, \tag{1.4}$$

for each appliance $a_u \in \mathcal{C}_u$. Noticeably, the period $\mathcal{H}'_{a_u}$ is consecutive from the beginning $\alpha_{a_u} \in \mathcal{H}$ to the end $\beta_{a_u} \in \mathcal{H}$. Thus, $\mathcal{H}'_{a_u}$ can be rewritten as $\mathcal{H}'_{a_u} = \{\alpha_{a_u}, \alpha_{a_u}+1, \cdots, \beta_{a_u}\}$. In practice, the choices of α_{a_u} and β_{a_u} depends on the habit (or preference) of user u. Moreover, the energy consumption per time slot is subject to the following constraint, i.e.,

$$0 \leq e_{a_u,h} \leq r_{a_u}^{\max}, \forall h \in \mathcal{H}'_{a_u}. \tag{1.5}$$

Furthermore, we have $e_{a_u,h} = 0$ for any $h \notin \mathcal{H}'_{a_u}$ as no operation (and hence energy consumption) is needed outside the working period $\mathcal{H}'_{a_u}$. For this type of appliance, there is flexibility to distribute the total load during the working period in response to the prices.

Let $w_{u,h}$ denote the amount of energy produced by the renewable energy generation unit of user u in time slot h. Suppose that end users are allowed to sell redundant renewable energy to the retailer, and we let $q_{u,h}$ denote the amount of renewable energy sold to the retailer by user u in time slot h. Apparently, we have

$$0 \le q_{u,h} \le w_{u,h}, \forall h \in \mathcal{H}, \tag{1.6}$$

because the amount of renewable energy sold cannot be larger than that of generation.

This leads to the following set of constraints on energy scheduling:

$$\sum_{a_u \in \mathcal{A}_u, \mathcal{B}_u, \mathcal{C}_u} e_{a_u,h} \le C_u^{\max} + w_{u,h} - q_{u,h}, \forall h \in \mathcal{H}. \tag{1.7}$$

In practice, such constraints are used to protect the total energy consumption from exceeding the grid capacity. To summarize, a valid energy consumption scheduling (of all users and all time slots) needs to satisfy the constraints (1.2)-(1.7).

At the user side, the energy scheduler in the smart meter optimizes the user's energy consumption scheduling according to the prices for the upcoming scheduling horizon. Noticeably, each user u has two contradicting goals, given that all its demands (i.e., constraints (1.2)-(1.7)) are met. The first goal is to maximize its overall satisfaction, given by

$$\sum_{h \in \mathcal{H}} \sum_{a_u \in \mathcal{B}_u} U_{a_u,h}(e_{a_u,h}). \tag{1.8}$$

The second goal is to minimize its electricity bill payment, obtained as

$$\sum_{h \in \mathcal{H}} p_h \left(\sum_{a_u \in \mathcal{A}_u, \mathcal{B}_u, \mathcal{C}_u} e_{a_u,h} - w_{u,h} + q_{u,h} \right) - \sum_{h \in \mathcal{H}} \tilde{p}_h q_{u,h} + \sum_{h \in \mathcal{H}} c_u w_{u,h}, \tag{1.9}$$

where p_h and $\tilde{p}_h$ respectively represent the retailer's retail and purchase prices in time slot h, and c_u means the user u's marginal-cost of renewable energy generation and is assumed to be zero in this chapter. A close look at (1.9) reveals that the electricity bill of user u includes three parts: the payment of purchasing electricity from the retailer, the profit of selling renewable energy to the retailer, and the cost of generating renewable energy. For simplicity, we assume that the marginal cost $c_u = 0$ in this chapter [4]. To balance the two objectives, each user u will schedule its energy consumption to maximize its payoff, which is the difference between the overall satisfaction and payment.

[4]The generation cost of renewable energy mainly comes from the construction and the maintenance, and thus the marginal-cost is considerably low.

This leads to the following optimization problem for each user u. i.e.,

$$
\begin{aligned}
\textbf{P1:}\ \text{maximize}\quad & \sum_{h\in\mathcal{H}}\sum_{a_u\in\mathcal{B}_u} U_{a_u,h}(e_{a_u,h}) + \sum_{h\in\mathcal{H}} \tilde{p}_h q_{u,h} \\
& - \sum_{h\in\mathcal{H}} p_h \Bigg(\sum_{a_u\in\mathcal{A}_u,\mathcal{B}_u,\mathcal{C}_u} e_{a_u,h} - w_{u,h} + q_{u,h} \Bigg) \\
\text{subject to}\quad & (1.2)-(1.7), \\
\text{variables}\quad & \boldsymbol{e}_{a_u}, \forall a_u \in \mathcal{B}_u, \mathcal{C}_u, \\
& q_{u,h}, \forall h \in \mathcal{H}.
\end{aligned}
$$

Note that through solving Problem **P1**, each user u can independently determine its optimal energy usage, based on the prices $\boldsymbol{p} = [p_1, \cdots, p_H]$ and $\tilde{\boldsymbol{p}} = [\tilde{p}_1, \cdots, \tilde{p}_H]$ forecasted by the retailer. For notational convenience, let $S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$ and $\hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$ denote user u's corresponding optimal total energy consumption and the optimal amount of energy sold to the grid in time slot h, respectively. Each user u can then communicate the values of $S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$'s and $\hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$'s in all time slots to the retailer via the communication infrastructure. With this information, the retailer can then efficiently calculate the demand response from each user for any price vector $\boldsymbol{p}$, thus removing the uncertainty of user responses.

1.2.2 Retailer

When the retailer receives the information of $S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$'s and $\hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$'s from each user u, it will choose the prices to maximize its profit, which is the difference between revenue and cost. The revenue is given by

$$
R(\boldsymbol{p}, \tilde{\boldsymbol{p}}) = \sum_h \left(\sum_{u\in\mathcal{U}} S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}}) \right) p_h. \tag{1.10}
$$

The cost is twofold: the payment of purchasing renewable energy from users satisfying

$$
C_1(\boldsymbol{p}, \tilde{\boldsymbol{p}}) = \sum_h \left(\sum_{u\in\mathcal{U}} \hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}}) \right) \tilde{p}_h, \tag{1.11}
$$

and the cost of buying energy from the generators which is denoted as an increasing convex function of the load demand, i.e.,

$$
C_2(\boldsymbol{p}, \tilde{\boldsymbol{p}}) = \sum_{h\in\mathcal{H}} a \left(\sum_{u\in\mathcal{U}} S_{u,h}(\boldsymbol{p}, \tilde{p}_h) - \hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}}) \right)^2 + b \left(\sum_{u\in\mathcal{U}} S_{u,h}(\boldsymbol{p} - \hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})) \right)^3, \tag{1.12}
$$

where a and b are positive parameters describing the second-order and third-order marginal generation costs, respectively [30]. Mathematically, the retailer wants to solve the following optimization problem.

$$
\begin{aligned}
\textbf{P2:}\ \text{maximize}\ & L(\boldsymbol{p}, \tilde{\boldsymbol{p}}) = R(\boldsymbol{p}, \tilde{\boldsymbol{p}}) - w(C_1(\boldsymbol{p}, \tilde{\boldsymbol{p}}) + C_2(\boldsymbol{p}, \tilde{\boldsymbol{p}})) \\
\text{subject to}\quad & p^l \le p_h \le p^u, \quad \tilde{p}^l \le \tilde{p}_h \le \tilde{p}^u,\ \forall h \in \mathcal{H}, \\
\text{variables}\quad & \boldsymbol{p}, \tilde{\boldsymbol{p}}
\end{aligned}
$$

where (p^l, p^u) and $(\tilde{p}^l, \tilde{p}^u)$ denote the lower-bound and upper-bound of retail price and the lower-bound and upper-bound of purchase price due to regulation, respectively, and the coefficient w reflects the weight of cost in the net profit. Noticeably, the optimal solution of Problem **P2** depends on the forms of the total electricity consumption (i.e., $\sum_{u \in \mathcal{U}} S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$) and the amount of energy sold to the retailer (i.e., $\sum_{u \in \mathcal{U}} \hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$).

In the following sections, we would like to discuss the solution of the energy scheduling problem (i.e., Problem **P1**) and the profit maximization problem (i.e., Problem **P2**) with/without renewable energy generation, respectively.

1.3 Energy consumption scheduling

In this section, we will discuss the solution of Problem **P1** from two perspectives: within and without renewable energy generation.

1.3.1 Without renewable energy generation

In this subsection, we assume that an end user does not own renewable energy generation units. Thus, the amount of renewable energy generation $w_{u,h}$ and the renewable energy sold to the retailer $q_{u,h}$ are both equal to zero. Correspondingly, we simplify $S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$ as $S_{u,h}(\boldsymbol{p})$ and let $\hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$ equal to zero.

Due to the concavity of $U_{a_u,h}(e_{a_u,h})$, Problem **P1** is a convex optimization problem, and the optimal solution can be obtained by the primal-dual arguments [31]. That is, we can solve Problem **P1** through maximizing its Lagrangian and minimizing the corresponding dual function.

For the notational brevity, let $U'_{a_u,h}(e_{a_u,h})$ denote $\frac{\partial U_{a_u,h}(e_{a_u,h})}{\partial e_{a_u,h}}$ and $U'^{-1}_{a_u,h}(\cdot)$ denote the inverse function of $U'_{a_u,h}(\cdot)$. Let $\boldsymbol{\eta}_u = (\eta_{u,1}, \cdots, \eta_{u,H})$ be the Lagrange multiplier vector corresponding to the constraint (1.7). Likewise, let $E_{u,h}$ be the total energy consumed by background appliances in time slot h, satisfying

$$E_{u,h} = \sum_{\forall a_u \in \mathcal{A}_u : h \in \mathcal{H}_{a_u}} r_{a_u,h}. \tag{1.13}$$

For the time slot at which there is no semi-elastic appliance (i.e., $h \notin \hat{\mathcal{H}}_u = \bigcup_{a_u \in \mathcal{C}_u} \mathcal{H}'_{a_u}$), the following Theorem shows the optimal consumption of each appliance $a_u \in \mathcal{B}_u$.

Theorem 1. *In time slot $h \notin \hat{\mathcal{H}}_u$, the optimal energy consumption of each elastic appliance $a_u \in \mathcal{B}_u$ satisfies*[5]

$$e^*_{a_u,h}(p_h) = \begin{cases} \left[U'^{-1}_{a_u,h}(p_h) \right]_0^{r^{\max}_{a_u}}, & \text{if } \sum_{a_u \in \mathcal{B}_u} \left[U'^{-1}_{a_u,h}(p_h) \right]_0^{r^{\max}_{a_u}} \le C^{\max}_u - E_{u,h}, \\ \left[U'^{-1}_{a_u,h}(p_h + \eta^*_{u,h}) \right]_0^{r^{\max}_{a_u}}, & \text{otherwise.} \end{cases} \tag{1.14}$$

[5]The notation $[x]_a^b$ means $\max\{\min\{x, b\}, a\}$, and the notation $\lfloor x \rfloor$ returns the nearest integer that is less than or equal to x.

Here, the optimal Lagrange multiplier $\eta^*_{u,h}$ satisfies

$$\sum_{a_u \in \mathcal{B}_u} e^*_{a_u,h}(p_h) = C_u^{\max} - E_{u,h}, \tag{1.15}$$

and its value can be obtained through a bisection search due to the monotonicity of $U'^{-1}_{a_u,h}(p_h + \eta^*_{u,h})$.

Remark 1. By Theorem 1, the optimal total energy consumption of user u in time slot h satisfies

$$S_{u,h}(\boldsymbol{p}) = \min\left\{C_u^{\max}, E_{u,h} + \sum_{a_u \in \mathcal{B}_u} \left[U'^{-1}_{a_u,h}(p_h)\right]_0^{r^{\max}_{a_u}}\right\}, \tag{1.16}$$

as long as there is no energy consumption of semi-elastic appliances in this time slot.

Next, we focus on calculating the optimal total energy consumption in time slot h when there exist semi-elastic appliances. In particular, each user u can implement electricity consumption scheduling for given price vector $\boldsymbol{p}$ in time slot $h \in \hat{\mathcal{H}}$ as shown in Algorithm 1.

Remark 2. At each iteration of Algorithm 1, the energy consumption and the multiplier variables are updated according to closed-form expressions. Therefore, the complexity of Algorithm 1 is $\mathrm{O}(|\mathcal{B}_u|H + |\mathcal{C}_u|H + H)$ at each iteration.

With Algorithm 1, the smart meter can quickly calculate the optimal energy consumption of each appliance (and hence $S_{u,h}(\boldsymbol{p})$) in each time $h \in \hat{\mathcal{H}}_u$, in response to the price $\boldsymbol{p}$ forecasted by the retailer. This implies that the total energy consumption can be obtained by the smart meter through either Algorithm 1 or the closed-form expression (1.16) according to the forecasted price vector. The following example will illustrate both situations.

Example 1 (User's response to the forecasted price vector $\boldsymbol{p}$): Assume that user u has six appliances, including two background appliances a_1 and a_2, two elastic appliances a_3 and a_4, and two semi-elastic appliances a_5 and a_6. Let the scheduling horizon $\mathcal{H}$ be $\{1, 2, \cdots, 8\}$. Specifically, the working period of appliance a_5 is $\{3, 4, 5, 6\}$, and the working period of appliance a_6 is $\{4, 5, 6, 7\}$. The total energy consumption of background appliances is $[4.0, 3.0, 3.0, 3.5, 2.5, 3.5, 3.5, 3.0]$ kWh in the scheduling horizon. The maximum allowable energy consumption $C_u^{\max}$ of user u is 40 kWh at each time slot. The maximum allowable energy consumptions of appliances a_3, a_4, a_5 and a_6 are 20 kWh, 20 kWh, 4 kWh, and 6 kWh at each time slot, respectively. The total energy consumptions of semi-elastic appliances a_5 and a_6 are 10 kWh and 10 kWh in the working period, respectively. Assume appliance a_3 and appliance a_4 have a quality-of-usage of $U_{a_3,h}(e_{a_3,h}) = 1.5w_{1,h}\log(m_{1,h} + e_{a_3,h})$ and a quality-of-usage of $U_{a_4,h}(e_{a_4,h}) = 1.5w_{2,h}\log(m_{2,h} + e_{a_4,h})$ in time slot h, respectively. The parameters in these two functions are given as follows,

$$\begin{pmatrix} w_{1,h} \\ w_{2,h} \end{pmatrix}_{h \in \mathcal{H}} = \begin{pmatrix} 6 & 8 & 6 & 8 & 6 & 10 & 8 & 6 \\ 6 & 8 & 10 & 8 & 10 & 6 & 10 & 8 \end{pmatrix}$$

Algorithm 1 Implementation of Electricity Consumption Scheduling for given $\boldsymbol{p}$ in $\hat{\mathcal{H}}_u$

1: **Initialization**; Randomly choose $\eta_{u,h}^{(1)} > 0, \forall h \in \hat{\mathcal{H}}_u$. Let $k = 1$, and the parameter δ be a very small positive number.

2: **repeat**

3: Sort the working period of every semi-elastic appliance (i.e., $\mathcal{H}'_{a_u}$) into the order $\{h_1, \cdots, h_i, \cdots, h_L\}$ such that $p_{h_i} + \mu_{u,h_i} \le p_{h_{i+1}} + \mu_{u,h_{i+1}}$ for all $h_i \in \mathcal{H}'_{a_u}$, where $L = |\mathcal{H}'_{a_u}|$.

4: Calculate the energy consumption of every elastic appliance as

$$\hat{e}_{a_u,h}(p_h, \eta_{u,h}^{(k)}) = \left[U'^{-1}_{a_u,h}(p_h + \eta_{u,h}^{(k)}) \right]_0^{r_{a_u}^{\max}}, \forall a_u \in \mathcal{B}_u, \forall h \in \mathcal{H}, \quad (1.17)$$

and the energy consumption of every semi-elastic appliance as

$$\begin{aligned} &\hat{e}_{a_u,h}(\boldsymbol{p}, (\eta_{u,h}^{(k)}, \forall h \in \hat{\mathcal{H}}_u)) \\ &= \begin{cases} r_{a_u}^{\max}, \text{ if } \exists i \in \{1, 2, \cdots, \lfloor \frac{E_{a_u}}{r_{a_u}^{\max}} \rfloor\} \text{ and } h = \mathcal{H}'_{a_u}(i) \\ E_{a_u} - r_{a_u}^{\max} \times \lfloor \frac{E_{a_u}}{r_{a_u}^{\max}} \rfloor, \text{if } h = \mathcal{H}'_{a_u}(\lfloor \frac{E_{a_u}}{r_{a_u}^{\max}} \rfloor + 1), \\ 0, \text{ otherwise.} \end{cases} \end{aligned} \quad (1.18)$$

5: Update the multiplier variables $\eta_{u,h}^{(k+1)}$'s according to

$$\begin{aligned} \eta_{u,h}^{(k+1)} =& \Bigg[\eta_{u,h}^{(k)} - \psi_{u,h}^{(k+1)} \Bigg(C_u^{\max} - E_{u,h} - \sum_{a_u \in \mathcal{B}_u} \hat{e}_{a_u,h}(p_h + \eta_{u,h}^{(k)}) \\ & - \sum_{a_u \in \mathcal{C}_u} \hat{e}_{a_u,h}(\boldsymbol{p}, (\eta_{u,h}^{(k)}, \forall h \in \hat{\mathcal{H}}_u)) \Bigg) \Bigg]^+, \forall h \in \hat{\mathcal{H}}_u, \end{aligned} \quad (1.19)$$

based on the sub-gradient update, where $\psi_{u,h}^{(k+1)}$'s are stepsizes at the kth iteration.

6: $k = k + 1$.

7: **until** $\sum_{h \in \hat{\mathcal{H}}_u} \Bigg(\sum_{a_u \in \mathcal{B}_u} (\hat{e}_{a_u,h}(p_h + \eta_{u,h}^{(k)}) - \hat{e}_{a_u,h}(p_h + \eta_{u,h}^{(k-1)}))^2 + \sum_{a_u \in \mathcal{C}_u} (\hat{e}_{a_u,h}(\boldsymbol{p}, (\eta_{u,h}^{(k)}, \forall h \in \hat{\mathcal{H}}_u)) - \hat{e}_{a_u,h}(\boldsymbol{p}, (\eta_{u,h}^{(k-1)}, \forall h \in \hat{\mathcal{H}}_u)))^2 \Bigg) \le \delta$, where δ is a small positive number.

8: $S_{u,h}(\boldsymbol{p}) = \sum_{a_u \in \mathcal{B}_u} \hat{e}_{a_u,h}(p_h + \eta_{u,h}^{(k)}) + \sum_{a_u \in \mathcal{C}_u} \hat{e}_{a_u,h}(\boldsymbol{p}, (\eta_{u,h}^{(k)}, \forall h \in \hat{\mathcal{H}}_u)) + E_{u,h}$ for all $h \in \hat{\mathcal{H}}_u$.

and

$$\begin{pmatrix} m_{1,h} \\ m_{2,h} \end{pmatrix}_{h \in \mathcal{H}} = \begin{pmatrix} 1.0 & 3.0 & 1.5 & 3.5 & 3.0 & 3.5 & 0.5 & 3.0 \\ 3.0 & 1.0 & 1.5 & 3.0 & 1.5 & 3.5 & 2.0 & 1.0 \end{pmatrix}.$$

Consider a given price vector $\boldsymbol{p} = \$[1.1, 1.0, 1.2, 1.2, 1.9,\ 1.4, 1.9, 1.0]$. In this case, the amount of consumed energy is calculated with the closed-form expression (1.16) in time slots $\{1, 2, 8\}$, since there is no semi-elastic appliance in these time slots. For other time slots, the smart meter needs to calculate the energy consumption according to Algorithm 1.

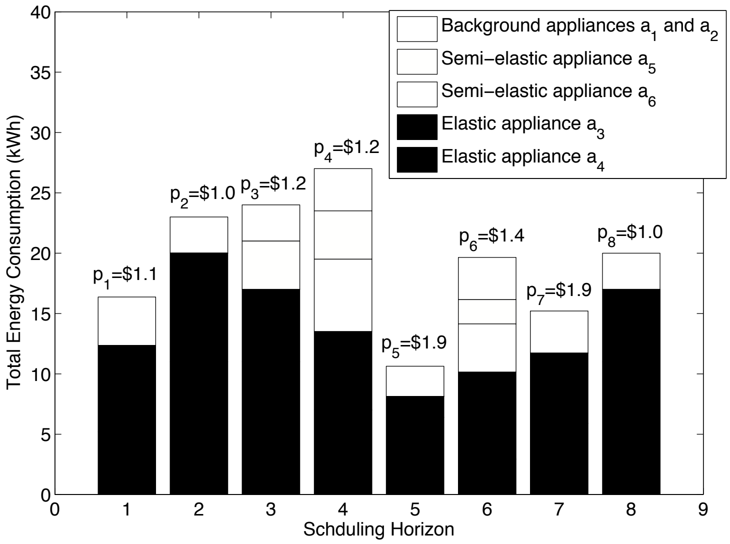

Figure 1.3: The scheduled energy consumption of each appliance in response to the forecasted price vector $\boldsymbol{p}$.

Fig. 1.3 shows the scheduled energy consumption of each appliance in response to the forecasted price vector $\boldsymbol{p}$. In this example, Algorithm 1 takes 4 iterations to converge to the desirable energy consumptions in time slots $\{3, 4, 5, 6, 7\}$. Recall Remark 2 that the complexity of Algorithm 1 is $O(|\mathcal{B}_u|H + |\mathcal{C}_u|H + H)$ at each iteration. Therefore, the algorithm can be implemented by the smart meters that have only low computation capability. It can be further seen from Fig. 1.3 that a larger price leads to a smaller energy consumption for semi-elastic appliances. This is because that the goal of a semi-elastic appliance is to consume a fixed amount of total energy with minimum payment. However, this is not always true for elastic appliances, which need to jointly consider the quality of usage and payment.

1.3.2 Within renewable energy generation

In this subsection, we assume that an end user is integrated with renewable energy generation units, and is capable of perfectly predicting the renewable energy generation[6].

[6]The short-run (day-ahead) renewable energy forecast can be quite accurate in practice [32, 33].

It is clear that when the retail price p_h is less than the purchase price $\tilde{p}_h$, each user u will receive a positive payoff if it first sells a amount of renewable energy to the retailer and then buys the same amount of energy from the retailer. This implies that if user u's total energy needs do not exceed the grid capacity $C_u^{\max}$ in each time slot, then the optimal amount of renewable energy sold to the retailer satisfies

$$\hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}}) = \begin{cases} w_{u,h}, & \text{if } p_h \le \tilde{p}_h, \\ 0, & \text{otherwise.} \end{cases} \tag{1.20}$$

Therefore, we further have the following result characterizing the optimal consumption of each appliance $a_u \in \mathcal{B}_u$ in time slot h without any semi-elastic appliance.

Theorem 2. *In time slot $h \notin \hat{\mathcal{H}}_u$, the optimal energy consumption of each elastic appliance $a_u \in \mathcal{B}_u$ satisfies*

$$e^*_{a_u,h}(p_h) = \begin{cases} \left[U'^{-1}_{a_u,h}(p_h)\right]_0^{r^{\max}_{a_u}}, & \text{if } \sum\limits_{a_u \in \mathcal{B}_u} \left[U'^{-1}_{a_u,h}(p_h)\right]_0^{r^{\max}_{a_u}} \le \hat{C}_{u,h} - E_{u,h}, \\ \left[U'^{-1}_{a_u,h}(p_h + \eta^*_{u,h})\right]_0^{r^{\max}_{a_u}}, & \text{otherwise,} \end{cases} \tag{1.21}$$

where $\hat{C}_{u,h}$ satisfies $\hat{C}_{u,h} = C_u^{\max} + w_{u,h} - \hat{q}_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$. Moreover, the optimal Lagrange multiplier $\eta^*_{u,h}$ satisfies

$$\sum_{a_u \in \mathcal{B}_u} e^*_{a_u,h}(p_h) = \hat{C}_{u,h} - E_{u,h}. \tag{1.22}$$

Remark 3. By Theorem 2, the optimal total energy consumption of user u satisfies

$$S_{u,h}(\boldsymbol{p}) = \min\left\{\hat{C}_{u,h}, E_{u,h} + \sum_{a_u \in \mathcal{B}_u} \left[U'^{-1}_{a_u,h}(p_h)\right]_0^{r^{\max}_{a_u}}\right\}, \tag{1.23}$$

as long as there is no energy consumption of semi-elastic appliances in time slot h.

On the other hand, when there exist semi-elastic appliances in time slot $h \in \hat{\mathcal{H}}$, each user u can determine electricity consumption scheduling through Algorithm 1, in which we replace $C_u^{\max}$ with $\hat{C}_{u,h}$.

1.4 Price control

In the section 1.3, we have characterized the users' electricity consumption as responses to electricity prices. In this section, we consider how the retailer optimizes the electricity prices according to users' responses.

1.4.1 Simulated annealing based price control (SAPC) algorithm

The retailer determines the optimal price vector for a certain time period (namely a scheduling horizon) right before the start of that time period. For example, the prices used for a day can be calculated during the last few minutes of the previous day. Due to the non-convexity of $S_{u,h}(\boldsymbol{p}, \tilde{\boldsymbol{p}})$, convex optimization methods cannot compute the optimal solution of the retailer side problem **P2**. Here, we propose a price control algorithm (referred to as SAPC) based on the Simulated Annealing (SA) method [34].

The sketch of the SAPC algorithm is as shown in Algorithm 2.

1.4.2 Computational complexity of SAPC

The following proposition discusses the convergence of the SA-based algorithms.

Proposition 1 ([35,36])**.** *The SAPC algorithm converges to the global optimal solution to Problem* ***P2****, as the control parameter T approaches to zero with $T = \frac{T_0}{\log(k)}$.*

For the practical implementation, we consider a solution very close to the global optimal solution[7] when $T < \epsilon$, where ϵ is a very small number.

In the SAPC algorithm, the number of rounds needed for convergence is $\exp(\frac{T_0}{\epsilon})$. Since H iterations are needed in one round, the total number of iterations needed when the SAPC algorithm converges is $\exp(\frac{T_0}{\epsilon})H$, where H is the number of time slots in the scheduling horizon. Therefore, we have the following Lemma.

Lemma 1. Given the initial temperature T_0 and the stopping criterion ϵ, the SAPC algorithm converges in $\exp(\frac{T_0}{\epsilon})H$ iterations[8].

As the retailer and users need to exchange pricing and response information during each iteration, the time needed for one iteration consists of the transmission time of packets, the computational time in response to the updated prices at each smart meter, and the computational time of updating the prices at the retailer side. The transmission time depends on the underlying communication technology. In practice, the transmission time of a packet with 32 bytes is at the order of $1 \sim 10^3$ ms per iteration over a broadband with a speed of 100 Mbps. One the other hand, the computational time depends on speeds of the

[7]Due to the non-convexity of Problem **P2**, there may exist several distinct global optimal solutions. The SAPC algorithm is guaranteed to find one such solution.

[8]In each iteration of the SAPC algorithm, all users need to independently run Algorithm 1 once and calculate (25) (H-$|\hat{\mathcal{H}}_u|$) times, where $|\hat{\mathcal{H}}_u|$ is the number of time slots in which there exist semi-elastic appliances for user u.

Algorithm 2 The SAPC Algorithm

1: **Initialization**: The retailer initializes price vectors $\{\boldsymbol{p}, \tilde{\boldsymbol{p}}\}$ as the price vectors used in the current scheduling horizon, and broadcasts the initial price vectors encapsulated in a packet to users' smart meters via the communication network. Let the parameter ϵ be a very small positive number. Set $T = T_0$ and $k = 1$.

2: **repeat**

3: **for** each time slot $h \in \{1, 2, \cdots, H\}$ **do**

4: The retailer randomly picks $p'_h \in [p^l, p^u]$ and $\tilde{p}'_h \in [\tilde{p}^l, \tilde{p}^u]$, and broadcasts $\{p'_h, p_{h-1}\}$ and $\{\tilde{p}'_h, \tilde{p}_{h-1}\}$ encapsulated in a packet to users' smart meters via the communication network.

5: After receiving the packet from the retailer, the smart meter updates the tentative price vectors as $(p'_h, \boldsymbol{p}_{-h})$ and $(\tilde{p}'_h, \tilde{\boldsymbol{p}}_{-h})$ in its memory.

6: The smart meter calculates the response to $(p'_h, \boldsymbol{p}_{-h})$ and $(\tilde{p}'_h, \tilde{\boldsymbol{p}}_{-h})$ by either (1.16), (1.23) or Algorithm 1.

7: The smart meter informs the retailer of the total energy consumption in each time slot encapsulated in a packet.

8: After receiving feedback from the users including the energy consumption information $S_{u,h}(p'_h, \boldsymbol{p}_{-h}, \tilde{p}'_h, \tilde{\boldsymbol{p}}_{-h},)$'s and $\hat{q}_{u,h}(p'_h, \boldsymbol{p}_{-h}, \tilde{p}'_h, \tilde{\boldsymbol{p}}_{-h})$'s, the retailer calculates $L(p'_h, \boldsymbol{p}_{-h}, \tilde{p}'_h, \tilde{\boldsymbol{p}}_{-h})$ (i.e., the objective function in Problem **P2**) according to the received $S_{u,h}(\cdot)$'s and $\hat{q}_{u,h}(\cdot)$'s.

9: The retailer computes $\Delta = L(\boldsymbol{p}_{-h}, p'_h, \tilde{\boldsymbol{p}}_{-h}, \tilde{p}'_h) - L(\boldsymbol{p}, \tilde{\boldsymbol{p}})$, and let $p_h = p'_h$ and $\tilde{p}_h = \tilde{p}'_h$ with the probability 1 if $\Delta \geq 0$. Otherwise, let $p_h = p'_h$ and $\tilde{p}_h = \tilde{p}'_h$ with the probability $\exp(\frac{\Delta}{T})$, and do not change the values of p_h and $\tilde{p}_h$ with the probability $(1 - \exp(\frac{\Delta}{T}))$.

10: The retailer updates the memory of $L(\boldsymbol{p}, \tilde{\boldsymbol{p}})$ with the latest price information.

11: **end for**

12: $k = k + 1$.

13: $T = \frac{T_0}{\log(k)}$.

14: **until** $T < \epsilon$.

15: The retailer broadcasts the updated price vectors $\boldsymbol{p}$ and $\tilde{\boldsymbol{p}}$ encapsulated in a packet to the smart meters at the beginning of the following scheduling horizon.

16: The smart meter makes consumption scheduling decisions by either (1.16), (1.23) or Algorithm 1 according to $(p'_h, \boldsymbol{p}_{-h})$ and $(\tilde{p}'_h, \tilde{\boldsymbol{p}}_{-h})$.

processors of the retailer and smart meters, and the number of appliances of each user. Let the transmission time and the computational time be T_t time units and T_c time units, respectively. Then, by Lemma 1, the SAPC algorithm takes $(T_t+T_c)H\exp(\frac{T_0}{\epsilon})$ time units. This implies that the retailer needs a total of $(T_t+T_c)H\exp(\frac{T_0}{\epsilon})$ time units for computing the proper prices for the next scheduling horizon.

1.5 Simulation results

In this section, we conduct simulations to illustrate the effectiveness of the proposed real-time pricing scheme.

Example 2: We consider a smart grid with 100 users, where each user u has four elastic appliances (u_1, u_2, u_3 and u_4) and two semi-elastic appliances (u_5 and u_6). Assume that each user u has a quality-of-usage of $U_{u_i,h}(e_{u_i,h}) = -a_{u_i,h}(e_{u_i,h}+b_{u_i,h})^{-1}$ for each elastic appliance u_i in each time slot h. Specifically, each parameter $a_{u_i,h}$ is chosen from the uniform distribution on $[10,20]$, and each parameter $b_{u_i,h}$ is chosen from the uniform distribution on $[2,5]$. The time scheduling horizon is $\mathcal{H}=\{1,2,\cdots,12\}$. The maximum allowable energy consumption of each user follows the uniform distribution on $[10,15]$ kWh in each time slot. The maximum allowable energy consumptions of each appliance follows the uniform distribution on $[1.0,2.0]$ kWh at each time slot. The total energy consumption of each semi-elastic appliance follows the uniform distribution on $[4,6]$ kWh in the working period. Assume that the total energy consumption of background appliances follows the uniform distribution on [1,2] kWh at each time slot for each user. Let the time scheduling horizon be $\mathcal{H}=\{1,2,\cdots,12\}$. Let the working period of each semi-elastic appliance be consecutive time slots with the beginning α_{u_i} and the end β_{u_i}, where α_{u_i} and β_{u_i} are randomly chosen from $\mathcal{H}$.

Finally, let $w=1$, $a=10^{-4}$, and $b=2\times10^{-5}$ in (1.13).

We first compare the total energy consumption at each time slot under different choices of prices in Fig. 1.4. Here, both the optimal flat-rate price and the optimal real-time price are computed by the proposed SAPC algorithm. When the proposed SAPC algorithm is used for computing the optimal flat-rate price, all elements in the price vector are simultaneously updated to the same value at each round. From Fig. 1.4, we can see that in any time slot, the increase of electricity price leads to the reduction of total energy consumption in the time slot, regardless of the prices setting in other time slots. For example, in time slot 5, the most energy consumption happens when p_5=\$0.58, while the least energy consumption happens when p_5=\$1.50. For the flat-rate scheme, this implies that when the retailer increases the price in each time slot, all load demands are reduced in the scheduling horizon accordingly, which can be also found in Fig. 1.4 from the two flat-rate schemes. The reduction of load demand further leads to the reduction of peak demand. Furthermore, Fig. 1.4 shows that compared to all three flat-rate pricing schemes, the optimal real-time pricing scheme flattens the load demand curve and reduces the peak-to-average load ratio. In particular, the optimal real-time pricing scheme reduces the peak-to-average load ratio by about 20% compared to the optimal flat-rate pricing scheme.

We then compare the revenue, the cost, and the profit under different settings

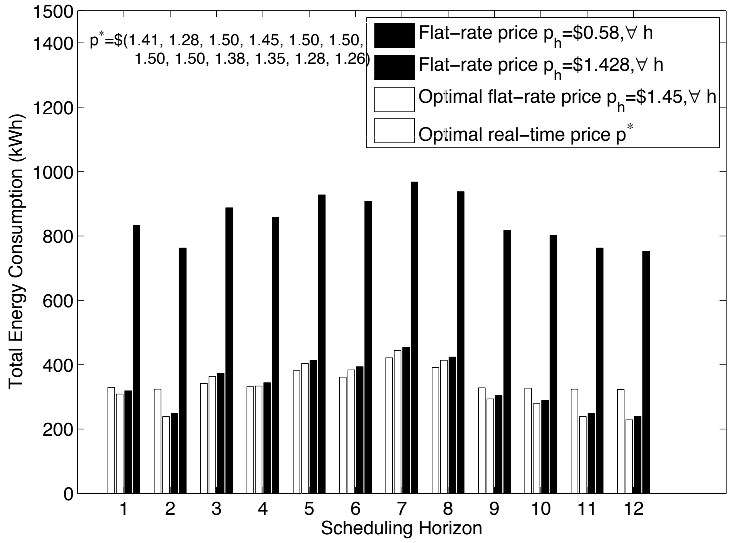

Figure 1.4: The total energy consumption under different choices of prices.

of price in Table 1.1. Specifically, these performances are evaluated at the retailer side. Comparing with the optimal real-time pricing, we can see that (i) to achieve the same total payment from users (i.e., \$5923.6), the cost under the flat-rate pricing scheme (i.e., \$6930.7) is much higher due to the increase of peak demand (as shown in Fig.1.4); (ii) to achieve the same cost (i.e., \$ 1191.2), the retailer has to set the flat rate to be high enough to compensate the peak cost; (iii) the maximum profit under the optimal flat-rate pricing scheme is \$4600.6, which is lower than that achieved by the optimal real-time pricing scheme.

Example 3: In this example, we conduct an experiment to test the total time needed for obtaining the optimal price vector with the SAPC algorithm. Our experiment emulates the retailers and users by computers that are connected to the public network through sub-networks from different service providers. The retailer computer is located in The Chinese University of Hong Kong, and the user computers are scattered throughout the entire Hong Kong. We consider a network of N user computers, where N is from 100 to 1000. Each user has six elastic appliances, four semi-elastic appliances, and several background appliances. We implement the retailer side procedure and the user side procedure in the SAPC algorithm with MATLAB[9]. The scheduling horizon is $\mathcal{H} = \{1, 2, \cdots, 24\}$ hours. The initial price vector is randomly picked.

Fig. 1.5 shows the total time needed for computing the optimal price vector. It can be seen that both the total time and the time spent on two-way communications do not significantly increase with the number of users. For example, it only takes 580 seconds for the retailer to communicate with the users for the purpose of probing $S_{u,h}(\boldsymbol{p})$'s, even with 1000 users. Even if we include the MATLAB execution time for the computations at the users and retailer side, the

[9]In this paper, MATLAB with version R2010b is used on a HP Compaq dx7300 desktop with 3.6GHz processors and 1Gb of RAM.

Table 1.1: Users' and retailer's behaviors under different price setting

Flat-rate pricing			
Price setting	Total Payment/Revenue	Cost	Profit
$p_h = \$0.58, \forall h$	\$5923.6	\$6930.7	\$-1007.1
$p_h = \$1.428, \forall h$	\$5787	\$1191.2	\$4595.8
Optimal flat-rate prices $p_h = \$1.45, \forall h$	\$5698.1	\$1097.5	\$4600.6
Optimal real-time pricing (achieved by SAPC)			
Optimal real-time prices	Total Payment/Revenue	Cost	Profit
$\boldsymbol{p}^* =$ \$(1.41, 1.28, 1.50, 1.45, 1.50, 1.50, 1.50 1.50, 1.38, 1.35, 1.28, 1.26)	\$5923.6	\$1191.2	\$4732.4

* The total payment (from the users) equals to the retailer's revenue. The cost and profit are computed from the retailer's point of view.

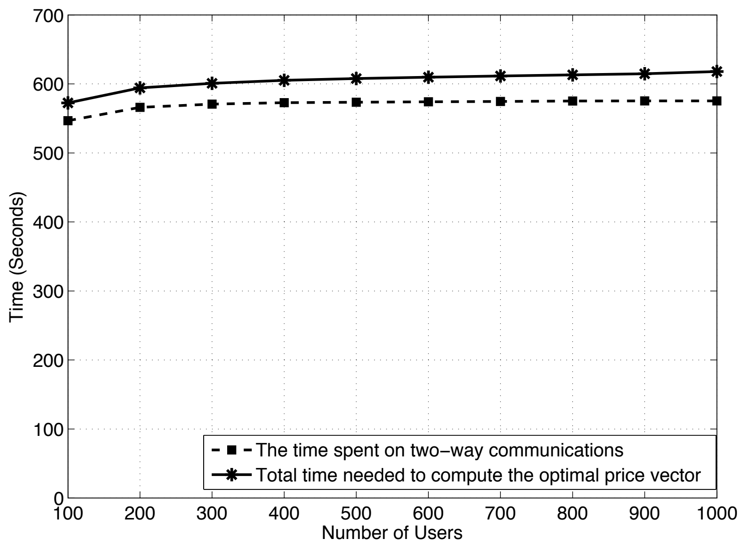

Figure 1.5: Total time needed for obtaining the price vector VS number of users

total time just goes up a bit to 620 seconds. Note that the computational time in real-system deployment can be much shorter with special-purpose FPGAs. This result is very encouraging, as it implies that the retailer only needs a few minutes before midnight to determine the optimal price vector for the next day (24 hours). More importantly, the result implies that the algorithm is rather scalable with the number of users, as the time cost does not increase much when the number of users becomes large. This is mainly due to the parallel computation of demand responses at the users' side.

1.6 Conclusions

In this chapter, we proposed a real-time pricing scheme for the purpose of peak-to-average load ratio reduction in smart grid. The proposed scheme solves a two-stage optimization problem. At the users' side, we obtain the optimal energy consumption schedules that maximize the users' quality-of-usage with minimum electricity payments. Such optimal energy consumption can be computed either according to the closed-form expressions or through an efficient iterative algorithm. At the retailer side, we used a Simulated-Annealing-based Price Control (SAPC) algorithm to compute the optimal real-time price that maximizes the retailer's profit. Simulation results showed that our proposed algorithm can lead to performance improvement for both the retailer and users with minimum communication and computational overhead.

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