2015 13th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)

ARM: Anonymous Rating Mechanism for Discrete Power Control

Shuqin Li*, Ziyu Shao+, and Jianwei Huang◇
* Bell Labs China, + ShanghaiTech University, ◇The Chinese University of Hong Kong
Email: Shuqin.Li@alcatel-sbell.com.cn, shaozy@shanghaitech.edu.cn, jwhuang@ie.cuhk.edu.hk

***Abstract*—Wireless interference management through continuous power control has been extensively studied in the literature. However, practical systems often adopt discrete power control with a limited number of power levels and MCSs (Modulation Coding Schemes). In general, discrete power control is NP-hard due to its combinatorial nature. To tackle this challenge, we propose an innovative approach of interference management: ARM (Anonymous Rating Mechanism). Inspired by the successes of the simple Anonymous Rating Mechanism in Internet and E-commerce, we develop ARM as distributed near-optimal algorithm for solving the discrete power control problem (*i.e.*, the joint scheduling, power allocation, and modulation coding adaption problem) under the physical interference model. We show that ARM achieves a close-to-optimal network throughput with a very low control overhead. We also characterize the performance gap of ARM due to the loss of rating information, and study the trade-off between such gap and the convergence time of ARM. Through comprehensive simulations under various network scenarios, we find that the optimality gap of ARM is small and such a small gap can be achievable with only a small number of power levels. Furthermore, the performance degradation is marginal if only limited local network information is available.**

I. Introduction

Power control is a key tool in the management of wireless interference. Most of the existing results for wireless interference management assume a continuous space of power levels [1]. Under such continuous power control setting, existing work can be divided into two main categories. One is concerned with achieving fixed Signal-to-Interference-plus-Noise-Ratio (SINR) targets [2] [3]. In this way, the link quality can be maintained at a desired target. A survey of related results can be found in [1]. The other is the joint SINR allocation and power control. Along this line, one more assumption is made: the link rate is a continuous function of the receiver SINR. A commonly used rate function is the Shannon capacity formula $r_l = B\log_2(1 + \text{SINR}_l)$, where r_l is the rate of a link l, and B is the bandwidth. Under such settings, both centralized algorithms (*e.g.,* [1] [4]) and distributed approximation algorithms (*e.g.,* [5]) have been proposed.

However, we have two important observations from the practical systems.

First, the common assumption of continuous mapping from link rate to SINR level in [1] [4] [5] implies that for each SINR level, there is a MCS available to achieve the corresponding capacity. This is not the case in practice, as there are only a limited number of MCSs available in practical wireless systems. For example, there are only four modulations in LTE: BPSK, QPSK, 16QAM, and 64QAM, with several coding rates [6]. Under the setting of limited MCSs, Zhou et al. in [7] proposed a distributed algorithm to solve the continuous power control problem. Such an algorithm induces high communication overheads, since it requires each link to obtain the global information for iterative updates. Furthermore, it is difficult to characterize the convergence time and impacts of various design parameters.

Second, and more importantly, we observe that only a limited number of power levels are used in practical systems [6]. For example, current 3GPP LTE standard of networks only supports discrete power control in the downlink via a user-specific data-to-pilot-power offset parameter [6], where a baseline of power and corresponding four fixed power offset parameters are chosen for each link [6]. There are two main advantages of discrete power control compared to continuous power control. One is the simplified design of the transmitter, and the other is the massive reduction of the information exchange overhead within system, which further simplifies the system design. In our simulation studies related to discrete power control, we also observe that using a small number of power levels is able to achieve a close-to-optimal throughput (comparing with the continuous power control benchmark) with light overhead of message exchanges, and further increasing the number of power levels can only have marginal improvements at the expense of a fast increasing implementation complexity.

Existing work on discrete power control can also be divided into two main categories. Most of them lie in the first category, which is concerned with achieving fixed Signal-to-Interference-plus-Noise-Ratio (SINR) targets by assuming either discrete power levels [8] or discrete power update step sizes [9]. A survey of these works can be found in [1]. The second category is our main focus, which is concerned with the joint optimization of link scheduling, power allocation, and modulation-coding adaptation under the setting of discrete power levels [10]–[12].

There are two main challenges of solving this joint optimization problem. One is the computational complexity. This discrete power control problem is a combinatorial optimization problem and shown to be NP-hard [10] [11], hence is hard to solve even in a centralized way. The other is the scalability requirement for algorithms in large-scale wireless networks. We need distributed or parallel implementations with low

978-3-9018-8274-6/15/$31.00 ©2015 IEEE

overhead of information exchanges.

Several works [10]–[12] attempted to address the above challenges. Gjendemsjø et al. in [10] proposed a binary power allocation (BPA) algorithm for maximizing the total throughput, where each subchannel can either be silent or transmitting at maximum power. This algorithm requires global information to compute the transmit power and user assignment. Zhang et al. in [11] proposed two distributed iterative algorithms to maximize the weighted sum-rate in multi-cell networks through discrete power allocation and coordinated scheduling. Both algorithms converge to some solutions without performance guarantees. Wang et al. in [12] applied an ant colony optimization technique to solve a discrete power allocation problem, obtaining a centralized solution without performance guarantees. What is more, all these works [10]–[12] made an impractical assumption of continuous mapping from link rate to SINR level.

In this paper, however, we adopt a more practical setting with a limited number of power levels and MCSs (Modulation Coding Schemes). To address the discrete power control problem with two main challenges under such a setting, we propose a distributed algorithm with provable near-optimal performance and light message exchange overhead. The key component in our algorithm is the Anonymous Rating Mechanism (ARM), which is inspired by the Anonymous Rating Mechanism widely used in today's online rating system. Examples of anonymous rating mechanisms include the "Like" of a post in Facebook (binary rating feedback), the voted score of a movie in IMDb (score rating from zero to ten), and the rating of a product in Amazon (one to five stars). They are simple but informative for users to learn from others, find the good, and avoid the bad. If we imagine that users can continuously improve their choices based on others' ratings, then it may lead to a network-wide performance improvement. Motivated by this, we propose the ARM framework for wireless interference management. The key idea is to allows links to continuously rate their changes of transmission rate caused by the change of interference from their neighboring links. For example, a link will provide a five-star rating if its transmission rate increases significantly due to its surrounding interference decreasing, or it will provide a one-star rating for a significant transmission rate drop. Based on all the feedback ratings from its neighbors, a link will smartly adapt its transmission power and MCS.

Our key results and contributions are summarized as follows:

- *Problem Formulation:* According to the best of our knowledge, this paper is the first to study and formulate the discrete power control problems (joint scheduling, power allocation and modulation-coding adaptation) under physical interference model with practical concerns of a limited number of power levels and MCSs.
- *Algorithm Design and Analysis:* We propose an ARM-based distributed algorithm to solve this problem with provable near-optimal performance guarantees. We characterize the key properties of the designed algorithm: approximation gap, perturbation error bound, convergence time, and trade-off between approximation gap and convergence time.
- *Robust Performance with Low Overhead:* Our algorithm allows distributed implementation with low message exchange overhead. Our algorithm is also robust to some rating information loss. We provide a bound of the induced optimality gap via perturbation analysis.
- *Simulation:* Extensive simulations based on practical system settings show that the performance gap between ARM and the optimal can be very low. Such a small gap can be achieved with only a small number of power levels, and the performance degradation is marginal if only limited local network information is available.

The remainder of this paper is organized as follows. We introduce the system model and problem formulation in Section II. Then we discuss design and analysis of the algorithm in Section III. Performance evaluation is conducted in Section IV. We conclude this paper in Section V. **Due to the page limitation, all proofs can be found in our technical report [13]**.

II. System Model and Problem Formulation

We consider a general wireless network consisting of a set of interfering *links* $\mathcal{L}$ (*i.e.,* each link is a pair of transmitter and receiver) sharing the same frequency. The size of the link set $\mathcal{L}$ is denoted as L. Such a model can cover both ad hoc networks and broadband cellular networks (*e.g.,* for the purpose of inter-cell interference management).

We denote the transmission power of a link $l \in \mathcal{L}$ as p_l, the value of which can only be chosen from a discrete power level set $\mathcal{P} = \{P_0, P_1, P_2, \dots, P_n\}$. For convenience, we set $P_0 \triangleq 0$, and let $P_n \triangleq P_{\max}$ denote the maximum transmission power.

Given $P_{\max}$ and the total number of power levels $n+1$, there are many choices of power level quantizations. Here we consider two types of discrete power level sets: *exponential power level set* and *uniform power level set*. Given the number of power levels $n + 1 \geq 2$, exponential power level set is denoted as $\{0\} \cup \{\frac{2^j}{2^{n-1}}, 0 \leq j \leq n-1\}$ and uniform power level set is denoted as $\{\frac{j}{n}, 0 \leq j \leq n\}$. Here each element in a power level set represents the ratio between the corresponding power level and the maximum transmission power. For example, the power level set $\{0, 1/4, 1/2, 1\}$ means that each link can select from four power levels: zero, a quarter, a half, and a full power.

The transmission rate of each link $l \in \mathcal{L}$ is denoted as $R_l = f(\text{SINR}_l)$, which is determined by the SINR at its receiver

$$\text{SINR}_l \triangleq \frac{h_{ll} p_l}{N_0 + \sum_{k \neq l} h_{lk} p_k}, \tag{1}$$

where h_{lk} denotes the channel gain from the link k's transmitter to link l's receiver. According to different SINR levels, links adjust the data rate by choosing different MCSs. When the channel condition significantly improves, higher order modulations will be chosen for higher data rates. Thus the data rate can be modeled as the following step function:

$$f(\text{SINR}_l) = \begin{cases} 0, & \text{if } \text{SINR}_l < \gamma_0, \\ r_1, & \text{if } \gamma_0 \le \text{SINR}_l < \gamma_1, \\ \vdots & \\ r_m, & \text{if } \text{SINR}_l \ge \gamma_m. \end{cases} \quad (2)$$

In Table I, we show one example of the step function coming from the practical system setting [14] . There are 11 available MCSs. For each MCS, Table II lists the corresponding spectrum efficiency and required SINR level.

TABLE I
EXAMPLE OF MODULATION-CODING-SCHEMES(MCSS)

MCS	Spectrum efficiency (bit/s/Hz)	Required SINR (dB)
QPSK 1/2	1.0	6.3
QPSK 5/8	1.25	8.5
QPSK 3/4	1.5	11
QPSK 5/6	1.67	13.8
16QAM 1/2	2.0	11.8
64QAM 1/2	3.0	16
16QAM 3/4	3.0	17
64QAM 5/8	3.75	19.3
64QAM 2/3	4	20.5
64QAM 3/4	4.5	22
64QAM 5/6	5	24.5

We suppose that once each link decides a power level, it also selects the "best MCS" for the current transmission, which achieves the highest spectrum efficiency (*i.e.,* the largest transmission rate) that can be supported by its current SINR level. This simplifies our description, *i.e.,* we can simply represent a joint power control and MCS adaptation decision by specifying its power level selection.

Now we define the *power configuration* as the vector form of the power level of each link, *i.e.,* $\boldsymbol{p} \triangleq (p_1, p_2, \ldots, p_L)$. Given a particular power configuration $\boldsymbol{p}$, let $p_l(\boldsymbol{p})$ and $R_{l\boldsymbol{p}}$ denote the power level and rate of link l respectively. For each link $l \in \mathcal{L}$, its transmission power can select a value from power level set $\mathcal{P}$, resulting to many different power configurations. We denote $\boldsymbol{\mathcal{P}}$ as the set of all possible power configurations. Our objective is to choose some power configurations from set $\boldsymbol{\mathcal{P}}$ to maximize the total transmission rate of all links. We formulate it as a combinatorial optimization problem, shown as follows:

$$\max_{\boldsymbol{p} \in \boldsymbol{\mathcal{P}}} \sum_{l \in \mathcal{L}} R_{l\boldsymbol{p}}. \quad (3)$$

Such a rate maximization problem is NP-hard [15], which means that there is no efficient algorithm to solve it in a centralized way. Next we will show how we can solve it near-optimally in a distributed way. By adopting such a solution, it is straightforward to see that we can also solve a more general combinatorial optimization problem:

$$\max_{\boldsymbol{p} \in \boldsymbol{\mathcal{P}}} \sum_{l \in \mathcal{L}} U_l(R_{l\boldsymbol{p}}). \quad (4)$$

where $U_l(\cdot)$ is a general utility function associated with link l and it can be of any form, not necessary concave.

III. ALGORITHM DESIGN AND ANALYSIS

We design a distributed approximation algorithm via the distributed MCMC method [16]. There are two key steps for such approach: first transform the original combinatorial problem to an equivalent sampling problem with a given probability distribution, then sample (or approximately sample) the given probability distribution by constructing a Markov chain distributedly with the desirable distribution as its unique stationary distribution.

We denote the set of optimal solutions for problem (3) as $\boldsymbol{\mathcal{P}}^o$ and denote the size of set $\boldsymbol{\mathcal{P}}^o$ as $|\boldsymbol{\mathcal{P}}^o|$. Then we have:

$$\boldsymbol{\mathcal{P}}^o = \arg\max_{\mathbf{p} \in \boldsymbol{\mathcal{P}}} \sum_{l \in \mathcal{L}} R_{l\boldsymbol{p}}. \quad (5)$$

We associate each power configuration $\mathbf{p} \in \boldsymbol{\mathcal{P}}$ with a probability $\pi_{\boldsymbol{p}}$. Then solving problem (3) is equivalent to sampling the space of power configurations $\boldsymbol{P}$ from the following general Dirac distribution:

$$\pi^d_{\boldsymbol{p}} = \begin{cases} \frac{1}{|\boldsymbol{\mathcal{P}}^o|}, & \text{if } \boldsymbol{p} \in \boldsymbol{\mathcal{P}}^o, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

However, the above general Dirac distribution is hard to obtain since $\boldsymbol{\mathcal{P}}^o$ is unknown to us. Therefore, we need to sample the space of power configuration $\boldsymbol{\mathcal{P}}$ with a new target distribution, which is more tractable than the general Dirac distribution, and more importantly, satisfies the following two conditions:

- **C1**: it can be obtained without knowing the exact value of $\boldsymbol{\mathcal{P}}^o$.
- **C2**: power configurations in set $\boldsymbol{\mathcal{P}}^o$ have the largest probabilities.

It turns out that the following product-form distribution parameterized by $\sigma > 0$ is a nice choice [16], [17]:

$$\pi^*_{\boldsymbol{p}} = \frac{\exp\left(\frac{1}{\sigma}\sum_{l \in \mathcal{L}} R_{l\boldsymbol{p}}\right)}{\sum_{\boldsymbol{p}' \in \boldsymbol{\mathcal{P}}} \exp\left(\frac{1}{\sigma}\sum_{l \in \mathcal{L}} R_{l\boldsymbol{p}'}\right)}, \forall \boldsymbol{p} \in \boldsymbol{\mathcal{P}}. \quad (7)$$

As we will see later the advantages of this product-form distribution (7) lie in: 1) it can be obtained by designing a time-reversible Markov chain without knowing the value of $\boldsymbol{\mathcal{P}}^o$, 2) Given a positive constant σ, we can see $\boldsymbol{\mathcal{P}}^o = \arg\max_{\boldsymbol{p} \in \boldsymbol{\mathcal{P}}} \pi^*_{\boldsymbol{p}}$. Thus both conditions **C1** and **C2** are satisfied.

In other words, when we sample the power configuration space $\boldsymbol{\mathcal{P}}$ based on the distribution $\pi^*_{\boldsymbol{p}}$ in (7), we actually solve the problem (3) approximately and obtain a close-to-optimal value. In fact, as $\sigma \to 0$, the product form degrades into the general Dirac distribution. Intuitively, starting from any initial power configuration, the system can find a path to optimal power configurations of the set $\boldsymbol{\mathcal{P}}^o$ by "hopping" across the Markov chain. When the designed Markov chain converges, the system will stay in power configuration set $\boldsymbol{\mathcal{P}}^o$ most of the time. In this sense, we approximately solve the original rate maximization problem.

Now we show how to design a time-reversible power-hopping Markov chain, with a state space being the set of all feasible power configurations $\boldsymbol{\mathcal{P}}$ and a stationary distribution being the product-form distribution given by (7). Distributed

MCMC is typically considered under continuous-time (or asynchronous) setting as used in [16], [17], which is also our target scenario. There are two degrees of freedom in designing a time-reversible power-hopping Markov chain in a distributed manner:

- **The state space structure:** the state space of the Markov chain should be connected such that any two states are reachable from each other. To enable distributed implementations, we design the Markov chain state space such that direct transitions between two states correspond to one and only one link (out of L links) adjusting its transmission power levels.
- **Direct transition rates:** for any two states $\boldsymbol{p}, \boldsymbol{p}' \in \boldsymbol{\mathcal{P}}$ with direct transitions, the corresponding transition rates between them should satisfy the detailed balance equation for time-reversibility: $\pi_{\boldsymbol{p}} \cdot q_{\boldsymbol{p},\boldsymbol{p}'} = \pi_{\boldsymbol{p}'} \cdot q_{\boldsymbol{p}',\boldsymbol{p}}$, where $q_{\boldsymbol{p},\boldsymbol{p}'}$ is the transition rate from state $\boldsymbol{p}$ to state $\boldsymbol{p}'$. To enable distributed implementations, transition rates should involve only local network information.

With the above guideline, we design the Markov chain as follows:

- Each power configuration $\boldsymbol{p} \in \boldsymbol{\mathcal{P}}$ is a state of the Markov chain. There are direct transitions between two states $\boldsymbol{p}, \boldsymbol{p}' \in \boldsymbol{\mathcal{P}}$ if and only if $\boldsymbol{p}$ and $\boldsymbol{p}'$ differ in only one link's power level.
- For any two states $\boldsymbol{p}, \boldsymbol{p}'$ that have direct transitions, the corresponding transition rates between them are set to be

$$q_{\boldsymbol{p},\boldsymbol{p}'} = \frac{C}{\exp\left(\frac{1}{\sigma}\left(\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}} - \sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'}\right)\right) + 1}, \quad (8)$$

$$q_{\boldsymbol{p}',\boldsymbol{p}} = \frac{C}{\exp\left(\frac{1}{\sigma}\left(\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'} - \sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}}\right)\right) + 1}, \quad (9)$$

where C is an arbitrary positive constant.

We choose the above transition rates because of the following consideration. Given any power configuration $\boldsymbol{p}$, the total data rate $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}}$ is very hard to obtain locally since it is a global information. Thus we consider the difference between $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}}$ and $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'}$, which is easy to obtain locally. For each link $l \in \mathcal{L}$, we define the rate change of link l as $\Delta R_l(\boldsymbol{p},\boldsymbol{p}') = R_{l\boldsymbol{p}} - R_{l\boldsymbol{p}'}$. Then we know $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}} - \sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'} = \sum_{l\in\mathcal{L}} \Delta R_l(\boldsymbol{p},\boldsymbol{p}')$. Since direct state transition from $\boldsymbol{p}$ to $\boldsymbol{p}'$ corresponds to one and only one link adjusting its transmission power level. For convenience, we denote this link as $\bar{l}$, which changes its power level from $p_{\bar{l}}(\boldsymbol{p})$ to $p_{\bar{l}}(\boldsymbol{p}')$. Such a change affects SINR for *all* links. However, it will only change rates for *some* links due to the stepwise mapping between SINR and transmission rates in (2). We denote $\tilde{L}$ as the set of links with the rate changes because of link $\bar{l}$'s transmission power change, *i.e.*, $\Delta R_l(\boldsymbol{p},\boldsymbol{p}') \neq 0, \forall l \in \tilde{L}$. Therefore, $\Delta R_l(\boldsymbol{p},\boldsymbol{p}') = 0, \forall l \in \mathcal{L} \setminus \tilde{L}$, and $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}} - \sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'} = \sum_{l\in\tilde{L}} \Delta R_l(\boldsymbol{p},\boldsymbol{p}')$. According to (1) and assuming the channel gain is dominated by the distance-based path loss component, $\bar{l}$'s neighboring links are more likely to be affected than links far away from link $\bar{l}$. Thus links in the set $\tilde{L}$ are more likely to be the near neighbors of link $\bar{l}$, hence we regard set $\tilde{L}$ as a "semi-local" set of link $\bar{l}$. In this sense, the difference between $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}}$ and $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'}$, *i.e.*, $\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}} - \sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'}$, is a "semi-local" information and can be obtained through distributed implementations.

We design the following ARM algorithm to implement the designed power-hopping Markov chain. We emphasize that the executions of ARM in different links are asynchronous. In ARM, for each link $l \in \mathcal{L}$, there are three phases after the initialization of power level setting: transmitting phase, rating phase and switching phase.

- **Transmitting Phase:** At the beginning of this phase, each link $l \in \mathcal{L}$ starts a random counter-down timer with a countdown time $T(n)$. The value of $T(n)$ is generated by an exponential distribution with a mean $\frac{1}{nC}$, where $n+1$ is the number of discrete power levels and C is a system parameter chosen according to different system requirements. Within the countdown time $T(n)$, link l transmits at its current power level p_l. In the meantime, link l also observes whether there is a SINR change that causes a non-zero change of link l's transmission rate. If link l observes the need for rate change, it immediately terminates the transmitting phase, and begins the rating phase. Otherwise, link l does not observe any rate change during the time $T(n)$, and it will begin the switching phase when the count-down timer reaches to zero.
- **Rating Phase:** At the beginning of this phase, link l will stop its current count-down timer and broadcast its rate change information. A small number of bits is enough to represent the rating information, because there are only a limited choices of available data rates. Thus we have a low communication complexity. Then link l will restart the transmitting phase again.
- **Switching Phase:** At the beginning of this phase, link l transmits at a randomly chosen new power level $p_l(\boldsymbol{p}')$. Then it will collect the rate change information from all other links (as those links will be triggered to enter the rating phase). Next, it will set its current transmission power level to be $p_l(\boldsymbol{p}')$ with probability $s_{\boldsymbol{p},\boldsymbol{p}'} = \frac{1}{\exp\left(\frac{1}{\sigma}\left(\sum_{l'\neq l} \Delta R_{l'}(\boldsymbol{p},\boldsymbol{p}')\right)\right)+1}$, or it will switch its current transmission power level to be back to p_l with probability $1 - s_{\boldsymbol{p},\boldsymbol{p}'}$. Then link l will restart transmitting phase again.

We summarize the algorithm by a form of flowchart in Figure 1.

Remarks: ARM does not require synchronization or explicit coordination, and the only communication overhead is the simple rate change information, which makes it suitable for parallel and distributed implementation in wireless ad hoc networks. To implement ARM in LTE system, we can adopt methods similar to those proposed in [18]. One is the over-the-air method, using the existing CQI (channel quality indicator) channel in LTE with extra rate change information. The other is the backhaul method, using the existing low-bit-rate X2 channel in LTE to exchange rate change information among transmitters and basestations. Since the changes of rates have very limited values due to limited power levels and MCSs, the overall signaling overhead is low.

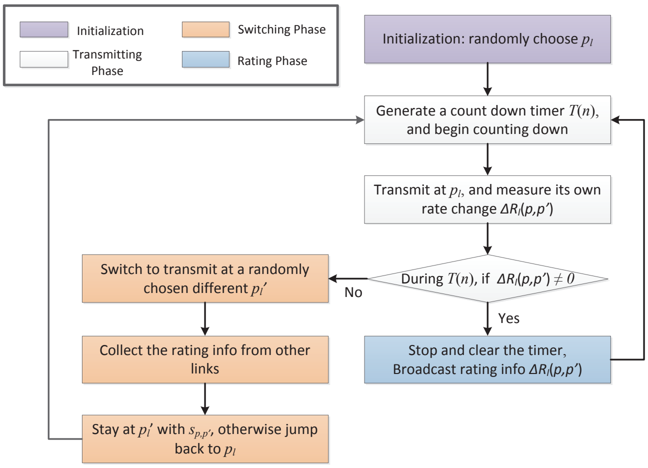

Fig. 1. Flowchart of ARM

We can show the following result:

Proposition 1. *The ARM algorithm realizes a time-reversible Markov chain with the desired stationary distribution in* (7).

We can understand ARM in an intuitive way. In this algorithm, "good states" (*i.e.,* having a high total data rate) or "bad states" (*i.e.,* having a low total data rate) are voted through links' rating information. With the carefully designed transition rates, ARM makes the system prefer to jumping into "good states" more often, but also risk to try "bad states" so as to avoid being trapped in a local optimal state (where the risk is parameterized by the approximation factor σ). As a result, the system stays in "good states" most of the time, and eventually stays in the "best states" most of the time.

Next we study two issues related to the practical implementation of ARM: Robustness and Convergence Time.

A. Robustness of ARM

In practice, one link may obtain inaccurate values of rating information (*i.e.,* $\Delta R_l(\boldsymbol{p}, \boldsymbol{p}')$) from other links. The inaccuracy comes from two sources of perturbation:

- **Transmission errors or losses**: erroneous wireless transmissions of rating information between links lead to inaccurate rating messages.
- **Local estimation**: to further reduce the complexity of ARM and facilitate the distributed implementation, each link may collect the rating information only from neighboring links. Such a local estimation leads to inaccurate total rating information.

Consequently, with perturbations, the new perturbed Markov chain may **not** converge to the desired stationary distribution $\pi^*_{\boldsymbol{p}}$ (7), resulting in another stationary distribution and a performance gap. To characterize such gap, we adopt the quantization error model proposed in [19]. In this model, for each state $\boldsymbol{p} \in \boldsymbol{\mathcal{P}}$ with total data rate $\sum_{l\in L} R_{l\boldsymbol{p}}$, we assume that its corresponding perturbation error belongs to the bounded set $[-\Delta_{\boldsymbol{p}}, \Delta_{\boldsymbol{p}}]$, where $\Delta_{\boldsymbol{p}}$ is the error bound. We also assume that $\Delta_{\boldsymbol{p}}$ is upper bounded, *i.e.,* $\Delta_{\boldsymbol{p}} \leq \Delta_{\max}$ for any $\boldsymbol{p} \in \boldsymbol{\mathcal{P}}$, and the perturbed $\sum_{l\in L} R_{l\boldsymbol{p}}$ takes only one of the following $2n_{\boldsymbol{p}} + 1$ discrete values:

$$\left\{ \sum_{l\in L} R_{l\boldsymbol{p}} + \frac{j}{n_p}\Delta_{\boldsymbol{p}}, j \in \{-n_{\boldsymbol{p}}, \ldots, n_{\boldsymbol{p}}\} \right\},$$

where $n_{\boldsymbol{p}}$ is a positive constant. Furthermore, with probability $\gamma_{j,\boldsymbol{p}}$, the perturbed $\sum_{l\in L} R_{l\boldsymbol{p}}$ takes the value $\sum_{l\in L} R_{l\boldsymbol{p}} + \frac{j}{n_{\boldsymbol{p}}}\Delta_{\boldsymbol{p}}$, $\forall j \in \{-n_{\boldsymbol{p}}, \ldots, n_{\boldsymbol{p}}\}$. Here $\gamma_{j,\boldsymbol{p}}$ is symmetric, *i.e.,* $\gamma_{j,\boldsymbol{p}} = \gamma_{-j,\boldsymbol{p}}$ and $\sum_{j=-n_{\boldsymbol{p}}}^{n_{\boldsymbol{p}}} \gamma_{j,\boldsymbol{p}} = 1$.

Let $\phi_{\max}$ denote the aggregate rates of all links under the optimal power configuration, i.e., $\phi_{\max} = \sum_{l\in L} R_{l\boldsymbol{p}}, \forall \boldsymbol{p} \in \boldsymbol{\mathcal{P}}^o$; let ϕ_a denote the expected aggregate rates of all links with the power-hopping Markov chain, i.e., $\phi_a = \sum_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} \pi^*_{\boldsymbol{p}} \cdot (\sum_{l\in L} R_{l\boldsymbol{p}})$; and let ϕ_e denote the expected aggregate rates of all links with the perturbed power-hopping Markov chain, *i.e.,* $\phi_e = \sum_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} \pi^e_{\boldsymbol{p}} \cdot (\sum_{l\in L} R_{l\boldsymbol{p}})$, where $\pi^e_{\boldsymbol{p}}$ is the stationary distribution of the perturbed Markov chain. By perturbation analysis developed in [19], we have the following result:

Theorem 1. *(a) The stationary distribution of the perturbed power-hopping Markov chain is*

$$\pi^e_{\boldsymbol{p}} = \frac{\beta_{\boldsymbol{p}} \cdot \exp\left(\frac{1}{\sigma}\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}}\right)}{\sum_{\boldsymbol{p}'\in\boldsymbol{\mathcal{P}}} \beta_{\boldsymbol{p}'} \cdot \exp\left(\frac{1}{\sigma}\sum_{l\in\mathcal{L}} R_{l\boldsymbol{p}'}\right)}, \forall \boldsymbol{p} \in \boldsymbol{\mathcal{P}} \tag{10}$$

where $\beta_{\boldsymbol{p}} = \sum_{j=-n_{\boldsymbol{p}}}^{n_{\boldsymbol{p}}} \gamma_{j,\boldsymbol{p}} \cdot \exp\left(\frac{1}{\sigma} \cdot \frac{j\Delta_{\boldsymbol{p}}}{n_{\boldsymbol{p}}}\right), \forall \boldsymbol{p} \in \boldsymbol{\mathcal{P}}.$
(b) Bounds on optimality gap for both power-hopping Markov chain and its perturbed counterpart are:

$$0 \leq \phi_{\max} - \phi_a \leq L \cdot \sigma \cdot \log(n+1) - \sigma \cdot \log|\boldsymbol{\mathcal{P}}^o|, \tag{11}$$

$$0 \leq \phi_{\max} - \phi_e \leq L \cdot \sigma \cdot \log(n+1) - \sigma \cdot \log|\boldsymbol{\mathcal{P}}^o| + \Delta_{\max}. \tag{12}$$

Remarks:

- The upper bounds on the optimality gap for both the power-hopping Markov chain in (11) and its perturbed counterpart in (12) increase linearly with the number of links L and approximation factor σ, increase log-linearly with the number of discrete power levels $n+1$, and decrease log-linearly with the number of optimal power configurations $|\boldsymbol{\mathcal{P}}^o|$.
- The upper bound on the optimality gap of perturbed Markov chain in (12) is quite general, as it only increases linearly with the maximum perturbation error $\Delta_{\max}$ and it is independent of the number of quantized error levels $\{n_{\boldsymbol{p}}\}$ and the distribution of quantized errors $\{\gamma_{j,\boldsymbol{p}}\}$.

B. Convergence Time of ARM

Now we study the mixing time (convergence time) of the designed Markov chain. First, we introduce the definition of total variation distance [20] between any two probability distributions $\boldsymbol{\pi}$ and $\boldsymbol{\pi}'$ over state space $\mathcal{P}$ as follows:

$$||\pi - \pi'||_{TV} \triangleq \frac{1}{2}\sum_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} |\pi_{\boldsymbol{p}} - \pi'_{\boldsymbol{p}}|. \tag{13}$$

Now let $\pi_t(\boldsymbol{p})$ denote the probability distribution of all states in $\boldsymbol{\mathcal{P}}$ at time t, given that the initial state is $\boldsymbol{p}$ and the system evolves according to ARM. Then the mixing time of the designed Markov chain [20] is defined as follows:

$$t_{\text{mix}}(\epsilon) \triangleq \inf \left\{ t \geq 0 : \max_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} ||\pi_t(\boldsymbol{p}) - \pi^*||_{TV} \leq \epsilon \right\}, \quad (14)$$

where π^* is the stationary distribution shown in (7).

We denote a threshold value $\sigma_{th} = \frac{L\cdot r_m}{\ln(1+\frac{1+1/n}{L-1})}$. We also use $\sim$ to represent the relationship of the same order of magnitude. Then we have the following results on the upper bound of the convergence time (mixing time):

Theorem 2. *If $\sigma > \sigma_{th}$, the upper bound of mixing time is $t_{mix}(\epsilon) \sim O\left(\exp(\frac{1}{\sigma}) \cdot \ln\frac{L}{\epsilon}\right)$. Otherwise when $0 < \sigma \leq \sigma_{th}$, the upper bound of mixing time is $t_{mix}(\epsilon) \sim \Omega\left(\exp\left(\frac{L}{\sigma} + \ln\ln\frac{1}{\epsilon}\right)\right)$.*

The proof adopts spectral analysis method [20] to obtain both a lower bound and an upper bound of t_{mix} for general values of σ. It also adopts path coupling method [20] to obtain a tight upper bound of t_{mix} for some values of σ.

When σ is a given parameter, we consider the scaling of the optimality gap and mixing time with the network size L. We observe the following trade-off between the optimality gap (Theorem 1) and its mixing time (Theorem 2):

- As $\sigma \to 0$, the optimality gap approaches zero while the upper bound of its mixing time scales with $\Omega(\exp(L))$ and approaches infinity (slow-mixing).
- As $\sigma \to \infty$, the optimality gap approaches infinity while the bound of its mixing time scales with $O(\ln(L))$ and remains limited (fast-mixing).

This resembles the phase transition phenomenon in statistics physics. Since $\sigma_{th} = \frac{L\cdot r_m}{\ln(1+\frac{1+1/n}{L-1})}$, with small values of L and n, σ_{th} can be very large, which means that we can only have the slow mixing results even when σ is large (but still smaller than the threshold σ_{th}). This is the theoretical result from the worst-case analysis. However, in practice, even small values of σ can lead to fast mixing. In fact, simulations in next section show that ARM converges very fast for small values of σ.

IV. Performance Evaluation

1) Simulation Setting: We focus on a downlink transmission scenario in a wireless network of seven hexagon cells, shown in Figure 2, where small cells (*i.e.,* the gray cells in Figure 2) are densely deployed to cover an area. The border effect is taken care by the wrap-around technique[1] [14]

The main system parameters of simulations are summarized in Table II based on the current standardizations of practical cellular system [6]. There are 11 popular MCSs available for each cell in our simulation, and the spectrum efficiency of each MCS and its required SINR level have been listed in previously Table I. We also consider both the exponential power level set and the uniform power level set, where the maximum number of power level sets is five and a full power is 20 dBm.

[1]The wrap-around is commonly used in practical system simulation, where the simulated network is visualized as a torus with edges warping around to the opposite edges, so that each cell has a complete set of interfering cells. The white cells in Figure 2 denote the wrap-around mappings of the considered network.

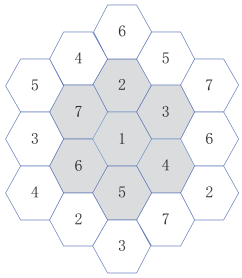

Fig. 2. Seven-cell Network with Corresponding Wrap Around: the gray cells are the network we considered, and the white cells denote the wrap-around mappings.

TABLE II
System Parameters

Number of small cells	9
Maximum transmission power of each cell	20dBm
Radius of each cell	20m
Noise power spectrum density	-174dB/Hz
Bandwidth	10M
Pathloss	$20\log_{10}(d) + 38.46$ dB

In the current system, a cell usually has multiple channels to support multiple users, and the cell local scheduler (*e.g.,* the proportional fairness scheduler used in most cellular system) will schedule at most one user for each channel to avoid the interference within the same cell. To simplify the simulation, we only simulate the one channel case here. The multiple channel case can be viewed as simple duplicates of single channel case (under an ideal channel allocation algorithm). Thus in each experiment of the simulation, we randomly put one user (*i.e.,* mobile devices) in each cell, which represents the link chosen by each cell's local scheduler. Each cell runs ARM independently to determine the transmission power and MCS of each base station. We run ARM with different values of approximation factor σ and different power level sets, and compare its performances with the optimal solutions generated by exhaustive search.

2) One Typical Experiment: We show results of one typical experiment as an example. In this experiment, the power level set is $[0, \frac{1}{2}, 1]$ and approximate factor $\sigma = 0.05$. Figure 3 shows the user positions in this experiment. Figure 4 shows the average spectrum efficiency of each cell as time elapses. For the time setting in all simulations, the mean time $1/C$ is set as 1 unit time. We purposely set the number of transitions very large for each experiment, *e.g.,* 10^5 to guarantee the convergence. However, in most of experiments, we find that ARM actually converges very fast as the one shown in Figure 4, where the network performance (*i.e.,* 10.9871 bit/s/Hz) by 5000 iterations is close to optimal (*i.e.,* 11 bit/s/Hz by Exhaustive Search). After 10^5 units of time, ARM's performance reaches 10.9982 bit/s/Hz, with a performance gap less than 0.02%. Note that this performance gap is also closely related to the approximation factor σ, which we will discuss in next subsection.

3) Impacts of Approximation Factor σ: For the experiment described in previous Section IV-2, Table III records the

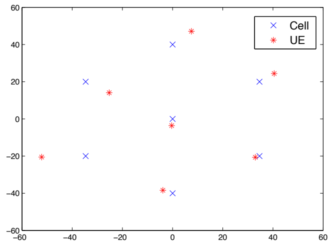

Fig. 3. User Position

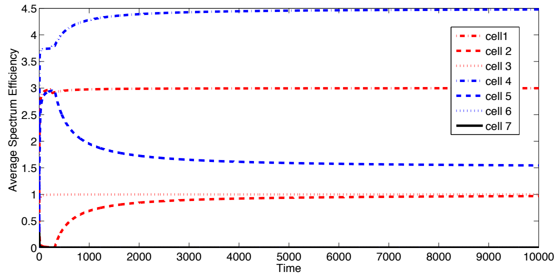

Fig. 4. Average Spectrum Efficiency

sum spectrum efficiency and performance gap of ARM under different values of σ. Here ES denotes performances obtained by exhaustive search, and ARM's performances are obtained after 10^5 transitions. This result is consistent with Theorem 1,

TABLE III
PERFORMANCE IN ONE TYPICAL EXPERIMENT

σ	1	0.5	0.2	0.1	0.05	ES
Spectrum Efficiency	8.9155	9.8764	10.7623	10.9929	10.9982	11
Performance Gap	19%	10%	2.1%	0.06%	0.02%	0

in the sense that smaller values of σ usually lead to higher spectrum efficiencies and lower performance gap. Then we run 1000 experiments and records the average sum spectrum efficiencies of exhaustive search (ES) and ARM (after 10^5 transitions), respectively. Results are shown in Table IV.

TABLE IV
AVERAGE PERFORMANCE OF 1000 EXPERIMENTS

σ	1	0.5	0.2	0.1	0.05
ES	10.3755				
ARM	8.2202	9.2148	10.1230	10.3118	10.3567
Performance Gap	23%	12%	2.7%	0.7%	0.2%

4) Impacts of Power Levels: Continuous power levels can be regarded as the extreme case of discrete power level, where the number of power levels is infinite. Given a fixed power range (*e.g.,* minimum and maximum), more power levels will provide more flexibility to power control and thus may improve the performance. On the other hand, more power levels will lead to a higher computation complexity.

To study the impact of number of power levels, we run 1000 experiments with both exponential power level sets and uniform power level sets, where the maximum number of power levels is 5. For each experiment, we run ARM with $\sigma = 0.05$, and 10^5 transitions.

First in Table V, we show performances with the exponential power level sets. The sum spectrum efficiency by exhaustive search with enumeration and by ARM are compared. The "ES Improvement" in the third row computes the ES performance difference between the current n and the previous $n-1$ power levels, where $n = 3, 4, 5$. Similarly, we also calculate "ARM Improvement" in the fifth row. The last row records the absolute performance gap between ES and ARM. Performances for the uniform power level sets are shown in Table VI. Note that for two and three power levels, uniform power level sets coincide with the exponential power level sets.

TABLE V
PERFORMANCE COMPARISON FOR EXPONENTIAL POWER LEVEL SETS

Number of Power Levels	2	3	4	5
ES (bit/s/Hz)	9.9337	10.3755	10.4713	10.4808
ES Improvement (bit/s/Hz)	–	0.4418	0.0958	0.0095
ARM (bit/s/Hz)	9.9241	10.3567	10.4555	10.4593
ARM Improvement (bit/s/Hz)	–	0.4326	0.0988	0.0038
Performance Gap	0.0096	0.0188	0.0158	0.0215

TABLE VI
PERFORMANCE COMPARISON FOR UNIFORM POWER LEVEL SETS

Number of Power Levels	2	3	4	5
ES (bit/s/Hz)	9.9337	10.3755	10.5683	10.6263
ES Improvement (bit/s/Hz)	–	0.4418	0.1928	0.0580
ARM (bit/s/Hz)	9.9241	10.3567	10.5418	10.5897
ARM Improvement (bit/s/Hz)	–	0.4326	0.1851	0.0479
Performance Gap	0.0096	0.0188	0.0265	0.0366

In both Table V and VI, We find that the more power levels, the better performance. However the marginal improvement of performance due to an additional power level quickly diminishes. Moreover, we observe that the performance gap between ES and ARM is slightly increasing with the increasing number of power levels. It is because that we use the same stopping time for all experiments, but more power levels actually need more time to converge to the stationary result of ARM. Intuitively, more power levels leads to larger state spaces and longer time to converge, and thus higher complexity.

The above results imply that a small number of fixed power levels can achieve close-to-optimal performance with a low complexity. For example, four power levels should be good enough. Even having two power levels can already achieve over 93% of the performance that can be achieved by five power levels.

In addition, we also observe that for the same number of power levels, the uniform power level set has better performances than the exponential power level set.

5) Robustness of ARM: As we discussed in SectionIII-A, ARM is robust to the inaccuracy of rating information. To validate this result, we consider a more complex cellular network (*i.e.,* 61 cells in total) shown in Figure 5. We run ARM with $\sigma = 0.05$ and different uniform power level sets.

As shown in Figure 5, we consider two scenarios. In one scenario, each cell only receives the rating information from its immediate neighboring cells, *i.e.,* the first-hop cells. In the other scenario, each cell only receives the rating information from both the first-hop and the second-hop cells. Any rating information from other cells will be ignored. The performance is shown in Table VII.

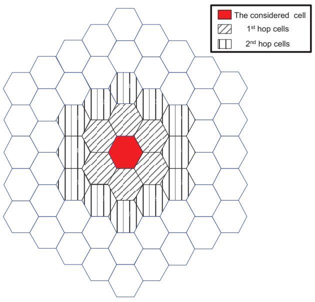

Fig. 5. 61-cell network

TABLE VII
IMPACT OF INFORMATION LOSS: PERFORMANCE OF 1000 EXPERIMENTS

	ARM	ARM w./ 1-hop LR	ARM w./ 2-hop LR
2 power levels	64.9571	61.7144	64.8478
Performance Ratio	100 %	95.01%	99.83%
3 power levels	68.0084	60.7702	65.6973
Performance Ratio	100 %	89.13%	96.52%
4 power levels	69.1290	60.2273	65.5371
Performance Ratio	100%	87.12%	94.80%
5 power levels	70.7057	59.7104	65.3180
Performance Ratio	100 %	85.21%	93.21%

We can see that in the worst-case scenario, ARM with only one-hop local rating information can still maintain more than 85% of the original performance without information loss. When two-hop local rating information is considered, the worse-case performance increases to over 93% of the original performance. The intuition is that the interference decreases with distance, hence the immediate neighboring cells' feedbacks are the most important ones, while the feedbacks from remote cells are less important. This property guarantees ARM's robustness, and will be useful for reducing communication complexity of ARM when implementing it in practical systems.

We also find that the less number of power levels, the better the performance of ARM with local rating. Intuitively, the larger number of power levels, the more sensitive the mapping from SINR to rate , and consequently the more perturbations due to the loss of the rating information. Comparatively, a smaller number of power levels is less insensitive to the information loss, *e.g.,* for two power levels, one-hop local rating already maintains 95% of the original performance.

V. CONCLUSIONS

In this paper we propose ARM, a novel distributed algorithm to solve the discrete power control problem under the physical interference model and the practical setting of limited numbers of power levels and modulation-coding schemes. By both mathematical analysis and extensive simulations, we show that ARM can achieve close-to-optimal performances. Moreover, this algorithm is simple and robust, and thus shows great potentials for interference management in practical wireless systems. There are several interesting directions for the future work, including the design of optimal power level set, the characterization of the impact of information loss for general network topology, and the extension of ARM for time-variant networking environment.

ACKNOWLEDGMENT

This work was partially supported by a NSFC grant (No.61302114), and the General Research Funds (Project Number CUHK 412713 and 14202814) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

REFERENCES

[1] M. Chiang, P. Hande, T. Lan, and C. W. Tan, "Power control in wireless cellular networks," *Foundations and Trends in Networking*, vol. 2, no. 4, pp. 381–533, 2008.
[2] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 641–646, 1993.
[3] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 7, pp. 1341–1347, 1995.
[4] L. Qian, Y. J. Zhang, and J. Huang, "Mapel: Achieving global optimality for a non-convex wireless power control problem," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1553–1563, 2009.
[5] J. Huang, R. A. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 5, pp. 1074–1084, 2006.
[6] S. Sesia, I. Toufik, and M. Baker, *LTE: the UMTS long term evolution*. Wiley Online Library, 2009.
[7] S. Zhou, X. Wu, and L. Ying, "Distributed power control and coding-modulation adaptation in wireless networks using annealed gibbs sampling," in *Proceedings of IEEE INFOCOM*, March 2012, pp. 3016–3020.
[8] C. Wu and D. P. Bertsekas, "Distributed power control algorithms for wireless networks," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 2, pp. 504–514, 2001.
[9] C. W. Sung and W. S. Wong, "A distributed fixed-step power control algorithm with quantization and active link quality protection," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 2, pp. 553–562, 1999.
[10] A. Gjendemsjø, D. Gesbert, G. E. Oien, and S. G. Kiani, "Binary power control for sum rate maximization over multiple interfering links," *IEEE Transactions on Wireless Communications*, vol. 7, no. 8, pp. 3164–3173, 2008.
[11] H. Zhang, L. Venturino, N. Prasad, P. Li, S. Rangarajan, and X. Wang, "Weighted sum-rate maximization in multi-cell networks via coordinated scheduling and discrete power control," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 6, pp. 1214–1224, 2011.
[12] D. Wang, X. Xu, X. Chen, X. Tao, Y. Yin, and H. Haas, "Discrete power allocation via ant colony optimization for multi-cell ofdm systems," in *IEEE VTC Fall*, September,2012.
[13] S. Li, Z. Shao, and J. Huang, "Arm: Anonymous rating mechanism for discrete power control," 2014, https://www.dropbox.com/s/t3capss9jdw2ww2/ARMreport.pdf.
[14] "3gpp tr 36.814," *The Mobile Broadband Standard*, available online: http://www.3gpp.org/DynaReport/36814.htm.
[15] Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 57–73, 2008.
[16] M. Chen, S. Liew, Z. Shao, and C. Kai, "Markov approximation for combinatorial network optimization," *IEEE Transactions on Information Theory*, vol. 59, no. 10, pp. 6301 – 6327, 2013.
[17] L. Jiang and J. Walrand, "A distributed csma algorithm for throughput and utility maximization in wireless networks," *IEEE/ACM Transactions on Networking (TON)*, vol. 18, no. 3, pp. 960–972, 2010.
[18] M. Andrews and L. Zhang, "Utility optimization in heterogeneous networks via csma-based algorithms," in *11th International Symposium on Modeling & Optimization in Mobile, Ad Hoc & Wireless Networks (WiOpt)*, May 2013, pp. 372–379.
[19] S. Zhang, Z. Shao, and M. Chen, "Optimal Distributed P2P Streaming under Node Degree Bounds," in *Proceedings of IEEE ICNP*, October,2010.
[20] D. A. Levin, Y. Peres, and E. L. Wilmer, *Markov chains and mixing times*. American Mathematical Soc., 2009.