Some Economics of Cellular and
Cognitive Radio Networks

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It is becoming increasingly important for wireless network operators to jointly optimize economic and technological decisions for business success. An operator’s decisions may involve the choices and timings of technology adoptions, the amount of resources to invest, and the prices to set for his services. These decisions are coupled with each other and need to be jointly optimized, and such optimization will be challenging when the operator faces limited resources, immature technology, and market competition. This thesis focuses on such issues in two types of networks. We first study the economics of cellular networks, which have the largest market occupancy among all wireless technologies. We then look at the economics of cognitive radios networks, which represent one of the main development trends for wireless technologies in the near future.

In the first part of this thesis, we study a cellular operator’s economic and technological decisions related to network upgrade, service differentiation, and
social applications. First, we develop a game theoretic model for studying competitive operators’ upgrade timing decisions from the existing 3G cellular technology to the next generation (4G) technology. Our analysis shows that operators often select different upgrade times to avoid severe competition. The operator upgrading earlier has advantage in increasing market share, while the one upgrading later benefit from decreased upgrade cost and a more mature 4G market. Second, we study an operator’s economic incentive of deploying femtocell service on top of his existing macrocell service. The femtocell can resolve the issue of poor signal receptions for indoor users in 4G networks, but need to occupy the operator’s limited spectrum resources. Finally, we try to understand how an operator can provide economic incentives for the heterogeneous smartphone users to collaborate in social applications (e.g., data acquisition and distributed computing). Under asymmetric information, we design efficient incentive mechanisms that reward smartphone users according to their different sensitivities to privacy loss, energy and computing efficiencies.

In the second part of this thesis, we study how investment flexibility, sensing uncertainty, and sensing security in cognitive radio networks affect a secondary (unlicensed) operator’s decisions. First, we study a secondary operator, who can flexibly acquire wireless spectrum through both dynamic spectrum leasing and spectrum sensing. We jointly study an operator’s investment choices and pricing strategy to the end users to maximize his profit. Compared to
spectrum leasing, spectrum sensing is unreliable but has a small cost. Second, we consider a competitive market with two operators, and study their competition in both investment and pricing. We show that end users significantly benefit from such market competition, and the operators’ total profit loss due to competition is lower bounded by 25% of the maximum. Finally, an operator may want to deploy collaborative spectrum sensing to improve sensing accuracy, but this approach is vulnerable to data falsification attacks. We design effective attack prevention mechanisms through proper attack detection and punishment.
論文題目：蜂窩和認知無線電網絡中的經濟學

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摘要：

對於無線網絡運營商來說，聯合優化其經濟和技術方面的決策以獲得商業成功正在變得越來越重要。一個運營商的決策可能包括對技術的選取和部署時間的把握、資源投資的數量，以及針對他所提供的服務的定價。考慮到這些決策彼此之間有關聯，我們需要對這些決策進行聯合優化，特別是當運營商面對有限的資源、不成熟的技術和市場競爭時，該優化會變得困難。本論文綜合考慮兩類網絡中的這些因素。注意到在所有無線技術中蜂窩網絡擁有最廣泛的市場佔有率，我們先研究蜂窩網絡中的經濟學。然後我們研究認知無線電中的經濟學，考慮到該技術代表了未來無線技術發展的一個主要趨勢。

在本論文的第一部分，我們研究一個蜂窩網絡運營商在經濟和技術方面的決策，涉及到網路升級、服務分類和社交應用。首先，我們提出了一
套博弈論模型來研究互相競爭的運營商從目前 3G 蜂窩技術升級為未來一代 (4G) 技術的部署時間。我們的分析指出運營商通常會選擇不同的升級時間以避免激烈的競爭。升級早的運營商在市場佔有方面有優勢，而升級晚的運營商只需承擔少量的升級成本並將面對一個更成熟的 4G 市場。其次，我們研究一個運營商是否有經濟動機在他已有的蜂窩基站 (macrocell) 的基礎上再鋪設家庭基站 (femtocell)。家庭基站能解決 4G 網絡中室內用戶信號接收差的問題，但是該服務會佔用運營商原本就有限的頻譜資源。最後，我們嘗試去理解一個運營商該如何為異構的智慧手機用戶提供經濟刺激來鼓勵他們協助社交應用（比如，信息收集和分布式計算）的建立。信息不對稱的情況下，我們設計了有效的激勵機制來根據智慧手機用戶不同的隱私損失，使用能耗和計算效率來提供獎勵。

在本論文的第二部分，我們在認知無線電網絡中研究投資的便利性、頻譜感知的不確定性和安全性將如何影響一個次級（沒有頻譜執照的）運營商的決策。首先，我們研究一個可以通過動態頻譜租賃和頻譜感知兩種靈活方式來獲得無線頻譜的次級運營商。我們聯合研究該運營商的投資選擇和對底層用戶的定價策略來使其利益最優。與動態頻譜租賃相比，頻譜感知不穩定但是能節約投資成本。其次，我們考慮一個包含兩個運營商的競爭市場，並研究他們之間在投資和定價方面的競爭。我們指出該競爭會給底層用戶帶來顯著好處，而給運營商們帶來的收益總損失不會超過 25%。最後，一個運營商可能想利用多用戶合作式頻譜感知技術來提高感知的精確性，但是該技術容易遭受數據偽造攻擊 (data falsification attacks)。我們通過合適的攻擊檢測和懲罰設計了有效的機制以防範攻擊。
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Chapter 1

Introduction

This thesis addresses a problem at the nexus of wireless communications and microeconomics, and exploits the interactions between economic and technological decisions made by wireless network operators. A network operator’s decisions may involve the choices and timings of technology adoptions, the amount of wireless resources to invest, and the prices to set for his services. Though requiring a high initial investment, an advanced technology can improve the resource utilization and users’ qualities of services (QoS). The resource investment decisions should be made depending on the technology, the user population, and the types of wireless applications. The service prices need to reflect the relationship between supply and demand, where users’ heterogeneity needs to be taken into account. An operator needs to make proper decisions while facing one or more of the following challenges:
• **Limited resources:** Although a wireless network operator may adjust his investment level in the long run, the total resource is often fixed and limited in the short term. For example, current static licensing of frequency spectrum lasts for years or even decades, and an operator needs to wait for a long time till the next round of spectrum allocation.

• **Technology development:** The deployment cost of a new technology decreases over time, and thus it is costly to deploy the technology early in order to become a market leader. Furthermore, the introduction of a new technology may have a negative impact on the existing technology offered by the same operator. These factors need to be jointly considered with the positive impact brought by the new technology (e.g., a larger market share).

• **Market competition:** Multiple operators often need to compete with each other in obtaining limited resources, providing high quality services, and offering low prices to attract users.

This thesis focuses on such issues in the following two types of networks. We first study the economics of *cellular networks*, which have the largest market occupancy among all wireless technologies. We then look at the economics of *cognitive radio networks*, which represent one of the main trends for the development of wireless technologies in the near future. Cognitive radios can also apply to cellular networks to improve the wireless spectrum utilization,
and a cellular network can also be a cognitive radio network.

1.1 Economics of Cellular Networks

There are some recent trends for the operation of cellular networks: (a) cellular users have increasing demands for fast wireless data access allocation, and (b) cellular users own more powerful and diversified smartphones. We try to address the new economic issues related to these two trends.

To satisfy users’ ever-increasing data demands, all cellular operators have urgent needs to improve the network capacity. There are several approaches to achieve this, and here we consider the introduction of 4G technology for improving overall data rate and the deployment of femtocells for improving indoor coverage. When users are outdoor, the fourth-generation (4G) networks based on OFDMA scheme provide much higher data rates (up to hundreds of megabits per second) than the current 3G networks [101]. However, even with 4G service, an operator still cannot resolve the poor signal reception problem for his indoor users. 4G technologies operate at wider and higher frequency bands (e.g., 2496MHz-to-2690MHz for TD-LTE in U.S.), and severe signal attenuation arises at these high frequencies especially for indoor users separated by walls from outdoor cellular base stations. To resolve this problem, researchers further proposed the idea of femtocell (e.g., [129–131]), where indoor mobile users communicate with indoor femtocells that connected with
cellular networks through wireline networks.

On the mobile device side, smartphone has become the mainstream in mobile phone market. Today’s smartphones have enough processing power and memory to perform many tasks that are not possible before, but different smartphones still have quite different capabilities such as energy and computing efficicencies [161, 162]. Given millions of powerful smartphones sold annually, some companies (e.g., Apple or Google) have started to encourage smartphone users’ collaboration to build some social applications or services (e.g., location-based services and distributed computing) [163,164].

In the first part of the thesis, we study a cellular operator’s economic and technological decisions related to network upgrade, service differentiation, and social applications. First, we study operators’ 4G network upgrade timing decisions in a competitive market. One might expect competitive operators in the same cellular market to upgrade to the 4G service at about the same time. However, many industry examples show that symmetric 4G upgrades is not common in practice, even when multiple operators have obtained the necessary spectrum and technology for upgrade [103]. Though an early upgrade leads to a larger market share, an operator needs to consider the decreasing upgrade cost and users’ dynamics in switching operators and services when planning the upgrade.

Second, to resolve indoor users’ poor signal receptions in 4G service, we
further study an operator’s economic incentive to deploy femtocell service on top of his existing macrocell service. We need to perform network optimization by considering several practical factors that are critical to the deployment of femtocell service: (a) femtocell service needs to share the limited spectrum bands with the macrocell service, (b) introduction of femtocell service should not decrease users’ payoffs, (c) femtocell service will bring in additional operational cost, and (d) femtocell service can only often provide limited spatial coverage.

Finally, we are interested in understanding how an operator can provide economic incentives for heterogeneous smartphone users to collaborate in social applications. We focus on two specific applications: data acquisition and distributed computing. In data acquisition, a company wants to acquire location data from enough smartphone users to build a database for location-based services. In distributed computing application, a company wants to engage smartphones in distributed computation tasks to solve some complex commercial or engineering problems with minimum cost. We study how to motivate smartphone users’ collaborations in both applications and designs incentive mechanisms to reward users’ efforts under asymmetric information.

1.2 Economic Viability of Cognitive Radio Networks

Wireless spectrum is typically considered as a scarce resource, but most spec-
trum bands are often under-utilized even in densely populated urban areas (\cite{2}). In the current static licensing approach, spectrum licenses are only given to primary network operators for a long term who serve their primary users, while secondary unlicensed operators and their secondary unlicensed users have no usage right of spectrum. To achieve more efficient spectrum utilization, people have proposed several dynamic spectrum access approaches such as hierarchical-access and dynamic exclusive use (\cite{3–9}). The hierarchical-access approach allows a secondary network operator or users to opportunistically access the spectrum without affecting the normal operation of the primary operator. This can be done by a secondary operator or user through sensing in the licensed bands to find and utilize “spectrum holes” tentatively unused by primary users (e.g., \cite{3,6}). As the secondary operator or user does not know primary users’ activities before sensing, the amount of useful spectrum obtained through sensing is uncertain (e.g. \cite{15,47,99}) and malicious users could explore this fact to attack other users during the sensing process \cite{86–88}. The dynamic exclusive use approach, on the other hand, allows a primary operator to dynamically trade the usage right of its licensed spectrum with a secondary operator or user in the spectrum market. This eliminates the need of spectrum sensing and the uncertainty of resource availability, but may increase the system signalling overhead.

In the second part of the thesis, we study how a secondary operator’s in-
investment flexibility, sensing uncertainty, and sensing security in cognitive radio networks affect his economic and technological decisions. We first consider a secondary operator who can obtain spectrum resource via both spectrum sensing and dynamic spectrum leasing. Both investment approaches can be short-term or even real-time (e.g., [3, 17–19]), which are much more flexible than the current static licensing approach. With these flexible resource investment approaches, a secondary operator can quickly respond to the market changes induced by the changes of user population and wireless applications. The operator faces some challenges in making his decisions:

- **Supply uncertainty:** spectrum sensing leads to supply uncertainty, since there is no guarantee on the amount of useful spectrum obtained through sensing. If the operator senses a large spectrum band or the primary users’ activity level is low, he may obtain excessive bandwidth and waste unnecessary energy and time in sensing. If the operator only senses a small spectrum band or the primary users’ activity level is high, he may not have enough spectrum to serve his users.

- **Investment cost:** Despite the uncertainty, spectrum sensing may still be desirable to obtain spectrum due to its low cost. The cost of sensing mainly includes the sensing time and energy, while in dynamic spectrum leasing the secondary operator needs to explicitly interact with the primary operator and pay for the trading.
• **Operator competition**: if multiple operators compete in the same market, they will engage in competition in both spectrum acquisition and service pricing. One operator needs to consider others’ decisions when optimizing his own decisions.

We are interested in understanding an operator’s optimal decision in balancing the low cost of sensing and the reliability of leasing. Furthermore, we will study the interaction between competitive operators in investment and pricing, and examine how such competition affect operators’ and users’ performances.

When an operator’s decisions involves sensing, he wants to further improve the sensing accuracy to reduce miss-detection and false-positive in sensing. One way to improve sensing accuracy is *collaborative spectrum sensing*, which involves multiple users at different locations to improve the poor sensing performance of individual users [83–85]. However, a user might have an incentive to misrepresent his sensing result, in order to increase his own chance of using the spectrum resource, and collaborative sensing is vulnerable to sensing data falsification attacks [86–88]. Thus we need to design efficient attack-prevention mechanisms involving both attack detection and punishment. Different from the prior work, we will consider the worst case where the attackers can cooperate with each other and eavesdrop any other users’ sensing reports.
1.3 Outline and Contributions

In Part I, we focus on the economics of cellular networks. We investigate a cellular operator’s economic incentive to deploy new technologies to increase network capacity, and smartphone users’ economic incentives to engage in collaborations in building social applications.

In Chapter 2, we analyze the cellular operators’ timing of 4G network upgrades using models where users can switch operators and services. Being the first to upgrade 3G to 4G service, an operator increases his market share but takes more risk and pays a higher upgrade cost because 4G technology matures over time. We consider a 4G competition market and develop a game theoretic model for studying operators’ interactions. The analysis shows that operators select different upgrade times to avoid severe competition. One operator takes the lead to upgrade, using the benefit of a larger market share to compensate for the larger cost of an early upgrade. We further show that the availability of 4G upgrade may decrease both operators’ profits due to increased competition. Perhaps surprisingly, the profits can increase with the upgrade cost.

In Chapter 3, we investigate the economic incentive for a cellular operator to add femtocell service on top of his existing macrocell service. We model the interactions between a cellular operator and users as a Stackelberg game and study the operator’s spectrum allocations and pricing in the two services. In the ideal case where the femtocell service has the same full spatial coverage
as the macrocell service, we show that the operator will choose to provide femtocell service only. The availability of spectrum spatial reuse strengthens the operator’s incentive to deploy femtocell service. However, if the operator needs to consider users’ reservation payoffs, femtocell operational cost and limited coverage, he always provides the macrocell service besides the femtocell service.

In Chapter 4, we analyze and compare different incentive mechanisms for a client to motivate the collaboration of smartphone users on both data acquisition and distributed computing applications. Data acquisition from a large number of users is essential to build a rich database and support emerging location-based services. We propose a reward-based collaboration mechanism, where the client announces a total reward to be shared among collaborators, and the collaboration is successful if there are enough users willing to collaborate. Distributed computing aims to solve computational intensive problems in a distributed and inexpensive fashion. We study how the client can design an optimal contract by specifying different task-reward combinations for different user types.

In Part II, we consider economics of cognitive radio networks and study secondary operators’ investment and pricing decisions in the spectrum market. We also investigate the security issue in sensing approach.

In Chapter 5, we study the optimal investment and pricing decisions of a
CHAPTER 1. INTRODUCTION

monopoly secondary operator under spectrum supply uncertainty. By modeling the interactions between the operator and secondary users as a multi-stage Stackelberg game, we show the operator’s investment and pricing decisions follow threshold structures. Furthermore, spectrum sensing is shown to significantly improve the operator’s expected profit and users’ payoffs.

In Chapter 6, we present a comprehensive analytical study of two competitive secondary operators’ investment (i.e., spectrum leasing) and pricing strategies, taking into account operators’ leasing heterogeneity and users’ wireless heterogeneity. We model the interactions between operators and users as a three-stage dynamic game, and completely characterize the dynamic game’s equilibria. We show that both operators’ investment and pricing equilibrium decisions process interesting threshold properties. We further show that the operator competition will not bring in large profit loss to each operator, but will significantly increase users’ payoffs.

In Chapter 7, we study how to prevent sensing data falsification attacks in collaborative spectrum sensing, where malicious users (attackers) submit manipulated sensing reports to mislead the fusion center’s decision on spectrum occupancy and they may not follow the fusion center’s decision regarding spectrum access. To effectively prevent such attacks, we propose two novel attack-prevention mechanisms with direct and indirect punishments. Unlike prior work, the proposed simple mechanisms do not require the fusion center
to identify and exclude attackers.

The conclusion of this thesis, Chapter 8, discusses open issues and further research directions.
Part I

Economics of Cellular Networks
Chapter 2

Economics of 4G Cellular Network Upgrade

2.1 Introduction

The third generation (3G) of cellular wireless networks was launched during the last decade. It has provided users with high-quality voice channels and moderate data rates (up to 2 Mbps). However, 3G service cannot seamlessly integrate the existing wireless technologies (e.g., GSM, wireless LAN, and Bluetooth) [100], and cannot satisfy users’ fast growing needs for high data rates. Thus, most major cellular operators worldwide plan to deploy the fourth-generation (4G) networks to provide much higher data rates (up to hundreds of megabits per second) and integrate heterogeneous wireless technologies [101]. The 4G technology is expected to support new services such as high-quality
CHAPTER 2. ECONOMICS OF 4G CELLULAR NETWORK UPGRADE

video chat, video conferencing, and multi-player online games [103].

One might expect competitive operators in the same cellular market to upgrade to a 4G service at about the same time. However, many industry examples show that symmetric 4G upgrades do not happen in practice, even when multiple operators have obtained the necessary spectrum and technology for upgrade.\(^1\) In South Korea, for example, Korean Telecom took the lead to deploy the world’s first 4G network using WiMAX technology in 2006, whereas SK Telecom started to deploy 4G network using more mature LTE technology in 2011.\(^2\) In the US, Sprint deployed the first 4G WiMAX network at the end of 2008, Verizon waited until the end of 2010 to deploy his 4G LTE network, and AT&T planed to deploy his 4G LTE network at the end of 2011 [103]. In China, China Mobile and China Unicom are the two dominant cellular operators, and China Mobile has decided to first deploy 4G LTE network during 2012-2013. In Hong Kong, CSL Telecom started to deploy his 4G LTE network in 2011, and the other operators are believed to upgrade in the future [104]. Thus, the key question we want to answer in this chapter is the following: How do the cellular operators decide when to upgrade to 4G networks?

In this chapter, we analyze the timing of operators’ 4G upgrades in different

\(^1\)For instance, in the US market, all major operators had acquired the spectrum they needed for 4G upgrade by 2008. Verizon and AT&T acquired 700MHz spectrum in early 2008 [105], and Sprint can use some of Clearwire’s spectrum [106].

\(^2\)In December 2010, the International Telecommunication Union (ITU) formally recognized current WiMAX and LTE technologies as two leading 4G technologies [102].
models, including both a 4G *monopoly* market and a 4G *competition* market. Operators need to pay the cost of 4G upgrade, which decreases over time as 4G technology matures. There are two key factors that affect the operators’ upgrade decisions: namely, *4G upgrade cost* and *user switching cost*. An existing 3G user can switch to the 4G service of the same operator or of a different operator, depending on how large the switching cost is. In a monopoly market where only a dominant operator can choose to upgrade to 4G, this operator can use the 4G service to capture a larger market share from small operators. In a competition market where multiple operators can choose to upgrade, we analyze the operators’ interactions as a non-cooperative game. We study how the users’ inter-network switching cost affects the operators’ upgrade decisions, and our findings are consistent with the asymmetric upgrades observed in the industry.

The key results and contributions of this chapter are summarized as follows.

- A *revenue-sharing model between operators*: Most existing works only study a single network’s revenue by exploring the network effect (*e.g.*, [107, 113]), and the results may not apply in a competitive market. In Section 2.2, we study two interconnected networks, where each operator shares the revenue of the inter-network traffic with the other operator.

- *Monopolist’s optimal timing of 4G upgrade*: By upgrading early, the 4G monopolist in Section 2.3 obtains a large market share and a large revenue
because of 4G’s Quality of Service (QoS) improvement, but it cannot exploit the cost depreciation over time. When the upgrade cost is relatively low, he upgrades at the earliest available time; otherwise he postpones his upgrade to benefit from the upgrade cost reduction.

- **Competitive operators’ symmetric upgrades under no inter-network switching:** In Section 2.4, we develop a game theoretic model for studying competitive operators’ interactions. In Section 2.5, we consider an extreme situation where users do not switch operators. In this situation, the operators upgrade simultaneously, irrespectively of whether the upgrade cost is high or low.

- **Competitive operators’ asymmetric upgrades under inter-network switching:** In Section 2.6, users can switch operators. By upgrading early, an operator captures a large market share and the 4G’s QoS improvement can compensate for the large upgrade cost. The other operator, however, postpones his upgrade to avoid severe competition and benefits from cost reduction. *The availability of 4G upgrade may decrease both operators’ profits because of the increased competition, and paradoxically, their profits may increase with the upgrade cost.* We also show that operators with heterogenous network parameters have greater incentives to upgrade at different times.
2.1.1 Related Work

Network Effect and Network Value

In telecommunications, the network effect is the added value that a user derives from the presence of other users [114]. In a network with $N$ users, each user perceives a value that increases with $N$. If each user attaches the same value to the possibility of connecting to any one of the other $N - 1$ users, it may be considered that he perceives a network value proportional to $N - 1$. Then the total value of the network is proportional to $N(N - 1)$, or roughly $N^2$, which is known as the Metcalfe’s Law [113]. A refined model was suggested by Briscoe et al. [107], where each user perceives a value of order $\log(N)$. In that model, a user ranks the other users in decreasing order of importance and assigns a value $1/k$ to the $k$-th user in that order, for a total value $1 + 1/2 + \cdots + 1/(N - 1) \approx \log(N)$. The resulting total network value is then $N \log(N)$, which is appropriate for cellular networks shown by several quantitative studies [107].

Network Upgrade

Recently, there has been a growing interest in studying the economics of network upgrades [108–110]. Musacchio et al. [108] studied the upgrade timing game between two interconnected Internet Service Providers (ISPs), where one ISP’s architecture upgrade also benefits the other because of the network effect. This free-riding effect may make the second operator postpone his upgrade
or even never decide to upgrade. Jiang et al. [109] studied a network security game, where one user’s investment (upgrade) can reduce the propagation of computer viruses to all users. In our problem, however, one operator may benefit from the other’s upgrade only when he also upgrades, letting his 4G users communicate with existing 4G users in other networks. Moreover, our model characterizes the dynamics of users switching between operators and/or services. These dynamics imply that an operator can obtain a larger market share by upgrading earlier, and this weakens the free-riding effect. Sen et al. [110] studied the users’ adoption and diffusion of a new network technology in the presence of an incumbent technology. Our work is different from that study in that we are not focusing on technology competition to attract users, but on the operators’ competition in upgrade timing to obtain greater profits. Moreover, the switching cost is not considered in [110], whereas it is an important parameter of our model.

2.2 System Model

2.2.1 Value of Cellular Networks

In this chapter, we adopt the $N \log(N)$ Law, where the network value with $N$ users is proportional to $N \log(N)$. The operator of a cellular network prefers a large network value; this is because the revenue he obtains by charging users can be proportional to the network value. Notice that the value of a 4G
network is larger than a 3G network even when two networks have the same number of users. This is because the communication between two 4G users is more efficient and more frequent than between two 3G users. Moreover, 4G provides new services to strengthen users’ interactions and increase the network value, e.g., through introducing high-quality video chat, video conferencing, and multi-player online games [103]. Because the average data rate in the 4G service is 5-10 times faster than the 3G (both downlink and uplink), a 4G network can support many new applications. We denote the efficiency ratio between 3G and 4G services as $\gamma$ with $0 < \gamma < 1$. That is, by serving all his users via 4G rather than 3G services, an operator obtains a larger (normalized) revenue $N \log(N)$ instead of $\gamma N \log(N)$. Note that this result holds for a single operator’s network that is not connected to other networks.

Next we discuss the revenues of multiple operators whose networks (e.g., two 3G networks) are interconnected. For the purpose of illustration, we consider two networks that contain $N_1$ and $N_2$ users, respectively. The whole market covers $N = N_1 + N_2$ users. We assume that two operators’ 3G (and later 4G) services are equally good to users, and the efficiency ratio $\gamma$ is the same for both operators. The traffic between two users can be intra-network (when both users belong to the same operator) or inter-network (when two users belong to different operators), and the revenue calculations in the two cases are different. We assume that the user who originates the communication
session (irrespective of whether the same network or to the other network) pays for the communication. This is motivated by the industry observations in EU and many Asian countries. Before analyzing each operator’s revenue, we first introduce two practical concepts in cellular market: “termination rate” and “user ignorance”.

When two users of the same operator communicate with each other, the calling user only pays operator 1. But when an operator 1’s user calls an operator 2’s user, operator 2 charges a termination rate for the incoming call. We denote the two operators’ revenue-sharing portion per inter-network call as $\eta$, where the value of $\eta \in (0, 1)$ depends on the agreement between the two operators or on governments’ regulation on termination rate.

User ignorance is a unique problem in the wireless cellular network, where users are often not able to identify which specific network they are calling. Mobile number portability further exacerbates this problem. Thus a typical user’s evaluation of two interconnected 3G networks does not depend on which network he belongs to, and equals $\gamma \log(N)$ where $N = N_1 + N_2$. We assume a call from any user terminates at a user in network $i \in \{1, 2\}$ with a probability of $N_i/N$ [115]. The operators’ revenues when they are both

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3Our model can also be extended to the case where both involved users in a communication session pay for their communication. This is what happening in US cellular market.

4In the US, termination rate follows “Bill and Keep” and is low. Then operator 1 can keep most of the calling user’s payment. In EU, however, termination rate follows “Calling Party Pays” and is much higher. Then most of the calling user’s payment to operator 1 is used to compensate for the termination rate charged by operator 2 [111].
Lemma 1. When two operators both provide 3G services, operator 1’s revenue is \( \gamma N_1 \log(N) \) and operator 2’s revenue is \( \gamma N_2 \log(N) \).

Proof. For a user in operator 1’s network, his communication brings \( \frac{N_1}{N} \gamma \log(N) + \eta \frac{N_2}{N} \gamma \log(N) \) amount of revenue to operator 1 and \( (1 - \eta) \frac{N_2}{N} \gamma \log(N) \) amount of revenue to operator 2. Similarly, for a user in operator 2’s network, his communication brings \( (1 - \eta) \frac{N_1}{N} \gamma \log(N) \) amount of revenue to operator 1 and \( \eta \frac{N_1}{N} \gamma \log(N) + \frac{N_2}{N} \gamma \log(N) \) amount of revenue to operator 2. Thus operator 1’s total revenue is

\[
N_1 \left( \frac{N_1}{N} \gamma \log(N) + \eta \frac{N_2}{N} \gamma \log(N) \right) + N_2 \left( (1 - \eta) \frac{N_1}{N} \gamma \log(N) \right) = \gamma N_1 \log(N),
\]

and operator 2’s total revenue is \( \gamma N_2 \log(N) \) due to symmetry. \( \square \)

Both operators’ revenues are linear in their number of users (or market share), and are independent of the sharing portion \( \eta \) of the inter-network revenue. Intuitively, the inter-network traffic between two networks is bidirectional: when a user originates a call from network 1 to another user in network 2, his inter-network traffic generates a fraction \( \eta \) of corresponding revenue to operator 1; when the other user calls back from network 2 to network 1, he generates a fraction \( 1 - \eta \) of the same amount of revenue to operator 1. Thus an operator’s total revenue is independent of \( \eta \). Later, in Section 2.3, we show that such independence on \( \eta \) also applies when the two operators both provide 4G services or provide mixed 3G and 4G services.
2.2.2 User Churn during Upgrade from 3G to 4G Services

When 4G service becomes available in the market (offered by one or both networks), the existing 3G users have an incentive to switch to the new service to experience a better QoS. Such user churn does not happen simultaneously for all users, this is because different users have different sensitivities to quality improvements and switching costs [112]. We use two parameters $\lambda$ and $\alpha$ to model the user churn within and between operators:

- **Intra-network user churn**: If an operator provides 4G in addition to his existing 3G service, his existing 3G users need to update their mobile phones to use the new 4G service. The users also spend time to learn how to use the 4G service on their new phones. We use $\lambda$ to denote the users’ switching rate to the 4G service within the same network.

- **Inter-network user churn**: If a 3G user wants to switch to another network’s 4G service, he either waits till his current 3G contract expires, or pays for the penalty of immediate contract termination. This means that inter-network user churn incurs an additional cost on top of the mobile device update, and thus the switching rate will be smaller than the intra-network user churn. We use $\alpha \lambda$ to denote the users’ inter-network switching rate to 4G service, where $\alpha \in (0, 1)$ reflects the transaction cost of switching operators.
We illustrate the process of user churn through a continuous time model. The starting time $t = 0$ denotes the time when the spectrum resource and the 4G technology are available for at least one operator (see Section 2.3 for monopoly market and Sections 2.5 and 2.6 for competition market). We also assume that the portion of users switching to the 4G service follows the exponential distribution (at rate $\lambda$ for intra-network churn and $\alpha \lambda$ for inter-network churn).\footnote{The assumption of exponential switching process helps us derive closed-form solutions and engineering insights consistent with the industry observations. Using other switching processes (with a similar parameter $\alpha$ to differentiate intra- and inter-network switching) is not likely to change the main conclusions.}

As an example, assume that operator 1 introduces a 4G service at time $t = T_1$ while operator 2 decides not to upgrade. The numbers of operator 1’s 4G users and 3G users at any time $t \geq 0$ are $N_{14G}(t)$ and $N_{13G}(t)$, respectively. The number of operator 2’s 3G users at time $t \geq 0$ is $N_{23G}(t)$. As time $t$ increases (from $T_1$), 3G users in both networks start to churn to 4G service,

$$N_{13G}(t) = N_1 e^{-\lambda \cdot \max(t-T_1,0)}, \quad N_{23G}(t) = N_2 e^{-\alpha \lambda \cdot \max(t-T_1,0)}, \quad \forall t \geq 0,$$

and operator 1’s 4G service gains an increasing market share,

$$N_{14G}(t) = N - N_1 e^{-\lambda \cdot \max(t-T_1,0)} - N_2 e^{-\alpha \lambda \cdot \max(t-T_1,0)}, \quad \forall t \geq 0.$$

We illustrate (2.1) and (2.2) in Fig. 2.1. We can see that operator 1’s early upgrade attracts users from his competitor and increases his market share. Notice that (2.2) increases with $\alpha$, thus operator 1 captures a large market share when $\alpha$ is large (i.e., the switching cost is low).
Figure 2.1: The numbers of users in the operators’ different services as functions of time $t$.

Here, operator 1 chooses his upgrade time at $T_1$ and operator 2 decides not to upgrade.

### 2.2.3 Operators’ Revenues and Upgrade Costs

Because of the time discount, an operator values the current revenue more than the same amount of revenue in the future. We denote the discount rate over time as $S$, and the discount factor is thus $e^{-St}$ at time $t$ according to [117].

We approximate one operator’s 4G upgrade cost as a one-time investment.\(^6\) This can be a good approximation, as an operator’s initial investment of wireless spectrum and infrastructure can be much higher than the maintenance costs in the future. For example, spectrum is a very scarce resource that is allocated (auctioned) infrequently by government agencies. Thus an operator cannot obtain additional spectrum frequently after his 4G upgrade. To ensure a good initial 4G coverage, an operator also needs to update many of the base

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6We plan to study operators’ steady upgrade in our future work, where an operator may be short of financial support to deploy a large-scale 4G network initially. He upgrades his 4G infrastructure from time to time as 4G users’ demand reaches the current network capacity.
stations all at once. Otherwise, 4G users would be unhappy with the service, and this would damage the operator’s reputation. That is why Sprint and Verizon covered many markets in their initial launch of their 4G services [119].

More specifically, we denote the 4G upgrade cost at $t = 0$ as $K$, which discounts over time at a rate $U$. Thus if an operator upgrades at time $t$, he needs to pay an upgrade cost $K e^{-Ut}$. We should point out that the upgrade cost decreases faster than the normal discount rate (i.e., $U > S$). This happens because the upgrade cost decreases because of both technology improvement and time discount. Very often the advance of technology is the dominant factor in determining $U$, and this is discussed further in Section 2.6.

Based on the above discussions on revenue and upgrade cost, an operator’s profit is defined as the difference between his revenue in the long run and the one-time upgrade cost. Without loss of generality, we will normalize an operator’s revenue rate (at any time $t$), total revenue, and upgrade cost by $N \log(N)$, where $N$ is the total number of users in the market.

### 2.3 4G Monopoly Market

We first look at the case where only operator 1 can choose to upgrade from 3G to 4G, while the other operators (one or more) always offer the 3G service because of the lack of financial resources or the necessary technology. This can be a reasonable model, for example, for countries such as Mexico and some
other Latin American ones, where America Movil is the dominant cellular operator in the 3G market. As the world’s fourth-largest cellular network operator, America Movil has the advantage over other small local operators in winning additional spectrum via auctions and obtaining LTE patents, and he is expected to be the 4G monopolist in that area [116].

The key question in this section is how operator 1 should choose his upgrade time $T_1$ from the 3G service to the 4G service. $T_1 = 0$ means that operator 1 upgrades at the earliest time that the spectrum and technology are available, and $T_1 > 0$ means that operator 1 chooses to upgrade later to take advantage of the reduction in the upgrade cost. Because of user churn from the 3G to the 4G service, the operators’ market shares and revenue rates change after time $T_1$. For that reason, we analyze periods $t \leq T_1$ and $t > T_1$ separately.

- **Before 4G upgrade ($t \leq T_1$):** Operator 1’s and other operators’ market shares do not change over time. Operator 1’s revenue rate at time $t$ is
  \[ \pi_{1, t \leq T_1}^{3G-3G}(t) = \gamma \frac{N_1}{N}, \]
  which is independent of time $t$. His revenue during this time period is
  \[ \pi_{1, t \leq T_1}^{3G-3G} = \int_0^{T_1} \pi_{1, t \leq T_1}^{3G-3G}(t)e^{-St}dt = \frac{\gamma N_1}{SN} (1 - e^{-ST_1}). \quad (2.3) \]

- **After 4G upgrade ($t > T_1$):** Operator 1’s market share increases over time, and the other operators’ total market share (denoted by $N_2^{3G}(t)/N$) decreases over time. We denote operator 1’s number of 3G users and
number of 4G users as $N_{13G}(t)$ and $N_{14G}(t)$, respectively, and we have

$$N_{13G}(t) + N_{14G}(t) + N_{23G}(t) = N.$$  

This implies that

$$N_{23G}(t) = (N - N_1)e^{-\alpha(t-T_1)}, N_{13G}(t) = N_1e^{-\lambda(t-T_1)},$$

and

$$N_{14G}(t) = N - N_1e^{-\lambda(t-T_1)} - (N - N_1)e^{-\alpha(t-T_1)}.$$  

Note that a 3G user’s communication with a 3G or a 4G user is still based on the 3G standard, and only the communication between two 4G users can achieve a high QoS. Operator 1’s revenue rate is

$$\pi_{14G-3G}(t) = \frac{\gamma N_{13G}(t)}{N} + \frac{N_{14G}(t)}{N} \left( \frac{N_{14G}(t) + \gamma N_{13G}(t)}{N} + \gamma N_{23G}(t) \right),$$

which is independent of the revenue sharing ratio $\eta$ between the calling party and receiving party. Operator 1’s revenue during this time period is then

$$\pi_{14G-3G}^{t>T_1} = \int_{T_1}^{\infty} \pi_{14G-3G}(t)e^{-St}dt, \tag{2.4}$$

where $t \to \infty$ is an approximation of the long-term 4G service provision (e.g., one decade) before the emergence of the next generation standard. This approximation is reasonable since the revenue in the distant future becomes less important because of discount.

Figure 2.1 illustrates how the numbers of users of operators’ different services change over time. Before operator 1’s upgrade (e.g., $t \leq T_1$ in Fig. 2.1),
the number of total users in each network does not change;\footnote{It is still possible for users to switch between networks within the same 3G service class, but such switching does not on average change the market shares.} after operator 1’s upgrade, operator 1’s and the other operators’ 3G users switch to the new 4G service at rates \(\lambda\) and \(\alpha\lambda\), respectively.

By considering (2.3), (2.4), and the decreasing cost \(Ke^{-ST_1}\), operator 1’s long-term profit when choosing an upgrade time \(T_1\) is

\[
\pi_1(T_1) = \pi_{3G-3G}^{1,t \leq T_1} + \pi_{4G-3G}^{1,t > T_1} - Ke^{-UT_1}
\]

\[
= e^{-ST_1} \left( \frac{1}{S} + (1 - \gamma) \frac{(N_1/N)^2}{2\lambda + S} + (1 - \gamma) \frac{(N-N_1/N)^2}{2\alpha\lambda + S} \right)
\]

\[
- e^{-ST_1} \left( 2(1 - \gamma) \frac{N_1/N}{\lambda + S} + (2 - \gamma) \frac{N-N_1/N}{\alpha\lambda + S} \right)
\]

\[
+ 2e^{-ST_1}(1 - \gamma) \frac{N_1(N-N_1/N)}{(1 + \alpha)\lambda + S} + \frac{N_1\gamma}{NS}(1 - e^{-ST_1}) - Ke^{-UT_1}.
\]

(2.5)

We can show that \(\pi_1(T_1)\) in (2.5) is strictly concave in \(T_1\), thus we can compute the optimal upgrade time \(T_1^*\) by solving the first-order condition. The optimal upgrade time depends on the following upgrade cost threshold in the monopoly 4G market,

\[
K_{th}^{mono} = (1 - \gamma) \frac{U/S}{\lambda + S} + \frac{2N_1^2}{2\alpha\lambda + S} + \frac{2N_1(N-N_1/N)}{(1 + \alpha)\lambda + S}
\]

\[
+ 1 - \gamma \frac{N_1/N}{\alpha\lambda + S} - (2 - \gamma) \frac{N-N_1/N}{\alpha\lambda + S}.
\]

(2.6)

**Theorem 1.** Operator 1’s optimal upgrade time in a monopoly 4G market is:

- Low cost regime (upgrade cost \(K \leq K_{th}^{mono}\)): operator 1 upgrades at \(T_1^* = 0\).
• High cost regime (upgrade cost $K > K_{th}^{mono}$): operator 1 upgrades at

$$T_1^* = \frac{1}{U-S} \log \left( \frac{K}{K_{th}^{mono}} \right) > 0.$$  

(2.7)

Intuitively, an early upgrade gives operator 1 a larger market share and enables him to get a higher revenue via the more efficient 4G service. Such advantage is especially obvious in the low cost regime where the upgrade cost $K$ is small.

Next we focus on the high cost regime, and explore how the network parameters affect operator 1’s upgrade time.

**Observation 1.** Operator 1’s optimal upgrade time $T_1^*$ increases with the upgrade cost $K$, and decreases with $\alpha$ (i.e., increases with the users’ inter-network switching cost).

The proofs of Observation 1 and the following observations are given in our online technical report [121].

**Observation 2** (Figure 2.2). When $K_{th}^{mono} < K < \frac{U}{S} K_{th}^{mono}$, $T_1^*$ first increases and then decreases in $U$. When $K \geq \frac{U}{S} K_{th}^{mono}$, $T_1^*$ monotonically decreases in $U$.

Figure 2.2 shows $T_1^*$ as a function of $U$ and $K$. When $K$ is large or $U$ is large, operator 1 wants to postpone his upgrade until the upgrade cost decreases significantly. A larger $U$ (thus a faster cost-decreasing rate) strengthens his willingness to postpone. When $K$ is small and $U$ is small, the upgrade cost
is small and does not decrease fast enough. In this case, operator 1 chooses to upgrade early if the revenue increase can compensate for the small upgrade cost.

**Observation 3** (Figure 2.3). When operator 1’s original market share $N_1/N$ is large, $T_1^*$ increases in the efficiency ratio $\gamma$ (between 3G and 4G). When $N_1/N$ is small, $T_1^*$ decreases in $\gamma$.

Figure 2.3 shows $T_1^*$ as a function of $\gamma$ and $N_1/N$. When $N_1/N$ is large, operator 1 cannot attract many users from other operators and has smaller incentives to upgrade. As $\gamma$ increases (and thus the QoS gap between 3G and 4G shrinks), he is less interested in 4G service and thus postpones his upgrade. When $N_1/N$ is
Figure 2.3: Operator 1’s optimal upgrade time $T_1^*$ as a function of the efficiency ratio between 3G and 4G services ($\gamma$) and his original market share $N_1/N$. Other parameters are $N = 1000$, $K = 2000$, $S = 1$, $\lambda = 1$, and $U = 2$.

small, operator 1 has limited market share and can attract many more users by upgrading early. As $\gamma$ increases, operator 1 obtains a higher revenue between his existing 3G users and the increasing number of 4G users. Operator 2, however, does not benefit much from this, as he loses his market share due to operator 1’s 4G services. Thus $T_1^*$ decreases in $\gamma$.

2.4 4G Competition Market: Duopoly Model and Game Formulation

In this section, we focus on the competition between multiple operators who can choose to upgrade to 4G services. To make the analysis tractable and
to derive clear engineering insights, we focus on the case of two operators (duopoly) in this chapter. This analysis serves as the first step in understanding the more general oligopoly case. This duopoly model is reasonable in a country like China, where China Mobile and China Unicom are the two dominant cellular operators in the 3G market. A similar situation exists in several other Asian and European countries as well.

The focus of this and the following sections is to understand why in so many existing industry examples (e.g., [103–105]) operators choose to upgrade to 4G services at different times even though they have the resources to upgrade simultaneously. In particular, we examine whether such asymmetric upgrades emerge even when the two operators are similar (e.g., having similar market shares before upgrades (e.g., Verizon has 106.3 million users and AT&T has 98.6 million users in the US). In our online technical report [121], we also examine the case where networks are heterogeneous in nature, in which case we show that operators have greater incentives to upgrade at different times. In the following analysis, we consider two operators that have the same market shares before the 4G upgrades \((N_1 = N_2)\), the same upgrade cost \(K\), and the same cost discount rate \(U\). We will first derive the operators’ profits under any upgrade decisions, and then analyze the duopoly game where each operator chooses the best upgrade time to maximize his profit.
Figure 2.4: Users’ switches over the operators’ services: Phase I with 3G services only, Phase II with operator 1’s 4G service, and Phase III with both operators’ 4G services.

2.4.1 Operators’ Long-term Profits

Let us denote two operators’ upgrade times as $T_1$ and $T_2$, respectively. Because the two operators are symmetric, without loss of generality, we assume in the following example (before Lemma 2) that operator 1 upgrades no later than operator 2 (i.e., $T_1 \leq T_2$). To calculate the operators’ profits, we first need to understand how users churn from 3G to 4G services, and how this affects the operators’ revenue rates over time. Figure 2.4 shows that user churn is different in three phases, depending on how many operators have upgraded.

- **Phase I ($0 \leq t \leq T_1$):** No operator has upgraded and both operators’ market shares do not change. Their revenue rates for both operators are the same, i.e.,

  \[
  \rho_1^{3G-3G}(t) = \rho_2^{3G-3G}(t) = \frac{\gamma}{2},
  \]

- **Phase II ($T_1 < t \leq T_2$):** Operator 1 has upgraded to 4G service but operator 2 has not. The 3G users of two operators switch to operator
1’s 4G service at different rates. The numbers of users in the operators’ different services are

\[ N_{1G}(t) = \frac{N}{2} e^{-\lambda (t-T_1)}, \quad N_{2G}(t) = \frac{N}{2} e^{-\alpha \lambda (t-T_1)}, \]

and

\[ N_{1G}(t) = N - \frac{N}{2} e^{-\lambda (t-T_1)} - \frac{N}{2} e^{-\alpha \lambda (t-T_1)}. \]

Two operators’ revenue rates are

\[ \pi_{1G-3G} = \frac{\gamma}{2} e^{-\lambda (t-T_1)} + \left( 1 - \frac{e^{-\lambda (t-T_1)} + e^{-\alpha \lambda (t-T_1)}}{2} \right) \cdot \left( 1 - \frac{1 - \gamma}{2} \left( e^{-\lambda (t-T_1)} + e^{-\alpha \lambda (t-T_1)} \right) \right), \]

and

\[ \pi_{2G-3G} = \frac{\gamma}{2} e^{-\alpha \lambda (t-T_1)}. \]

- **Phase III** \((t > T_2)\): Both operators have upgraded, and 3G users only switch to the 4G service of their current operator. The numbers of users in operators’ different services are

\[ N_{1G}(t) = \frac{N}{2} e^{-\lambda (t-T_1)}, \quad N_{1G}(t) = N - \frac{N}{2} e^{-\lambda (t-T_1)} - \frac{N}{2} e^{-\alpha \lambda (T_2-T_1)}, \]

and

\[ N_{2G}(t) = \frac{N}{2} e^{\alpha \lambda (T_2-T_1)-\lambda (t-T_2)}, \quad N_{2G}(t) = \frac{N}{2} e^{\alpha \lambda (T_2-T_1)} \left( 1 - e^{-\lambda (T_2-T_2)} \right). \]

Two operators’ revenue rates are

\[ \pi_{1G-4G} = \frac{\gamma}{2} e^{-\lambda (t-T_1)} + \left( 1 - \frac{e^{-\lambda (t-T_1)} + e^{-\alpha \lambda (T_2-T_1)}}{2} \right) \cdot \left( 1 - \frac{1 - \gamma}{2} e^{-\lambda (t-T_1)} - \frac{1 - \gamma}{2} e^{-\alpha \lambda (T_2-T_1)-\lambda (t-T_2)} \right), \]
Figure 2.5: The numbers of users in the operators’ different services as functions of time $t$.

Here, operator 1 chooses his upgrade time at $T_1$ and operator 2 chooses $T_2$ with $T_1 \leq T_2$.

and

$$\pi_2^{4G-4G}(t) = \frac{\gamma e^{-\alpha \lambda (T_2 - T_1) - \lambda (t-T_2)} + 1 - e^{-\lambda (t-T_2)}}{2} \cdot \frac{1 - \frac{\gamma}{2} e^{-\lambda (t-T_1)} - \frac{1}{2} e^{-\alpha \lambda (T_2 - T_1) - \lambda (t-T_2)}}{e^{\alpha \lambda (T_2 - T_1)}}.$$ 

Figure 2.5 summarizes how users churn in the three phases. By integrating each operator’s revenue rate over all three phases, we obtain that operator’s long-term revenue. Recall that an operator’s profit is the difference between his revenue and the one-time upgrade cost. By further considering the symmetric case of $T_1 \leq T_2$, we have the following result.

**Lemma 2.** Consider two operators $i, j \in \{1, 2\}$ (with $i \neq j$) upgrading at $T_i$ and $T_j$. Operator $i$’s long-term profit is

$$\pi_i(T_i, T_j) = \begin{cases} 
\pi^{ER}(T_i, T_j), & \text{if } T_i \leq T_j; \\
\pi^{LT}(T_j, T_i), & \text{if } T_i \geq T_j,
\end{cases}$$ 

(2.8)
\[
\pi_{ER}(T_i, T_j) = \frac{\gamma + (2 - \gamma)e^{-ST_i} - e^{-\alpha \lambda(T_j - T_i) - ST_i}}{2S} - \frac{(2 - \gamma)(e^{-ST_i} - e^{-\alpha \lambda(T_j - T_i) - ST_j})}{2(\alpha \lambda + S)}
\]
\[
+ \frac{1 - \gamma}{2} \left( \frac{e^{-ST_i} + e^{-(1+\alpha)\lambda(T_j - T_i) - ST_j}}{2(\alpha \lambda + S)} + \frac{e^{-ST_i} - e^{-(1+\alpha)\lambda(T_j - T_i) - ST_j}}{(1 + \alpha)\lambda + S} \right)
\]
\[
+ \frac{e^{-ST_i} - e^{-2\alpha \lambda(T_j - T_i) - ST_j}}{2(\alpha \lambda + S)} - (1 - \gamma) e^{-ST_i} - \frac{\lambda}{\lambda + S} - Ke^{-UT_i},
\]
\[
- \frac{1 - \gamma}{2(\lambda + S)} e^{-\alpha \lambda(T_j - T_i) - ST_j} \left( 1 - \frac{e^{-\lambda(T_j - T_i)} + e^{-\alpha \lambda(T_j - T_i)}}{2} \right).
\] (2.9)

\[
\pi_{LT}(T_j, T_i) = \frac{e^{-\alpha \lambda(T_j - T_i) - ST_i}}{2} \left( \frac{1}{\lambda + S} - \frac{1 - \gamma}{2} \right) \left( e^{-\lambda(T_j - T_i)} + e^{-\alpha \lambda(T_j - T_i)} \right)
\]
\[
+ \frac{\gamma}{2(\alpha \lambda + S)} \left( e^{-ST_j} - e^{-\alpha \lambda(T_i - T_j) - ST_j} \right) - Ke^{-UT_i}.
\] (2.10)

where \(\pi_{ER}(T_i, T_j)\) and \(\pi_{LT}(T_j, T_i)\) are given in (2.9) and (2.10), respectively.

Note that an operator’s profit \(\pi_i(T_i, T_j)\) is continuous in his upgrade time \(T_i\). When operator \(i\)’s upgrade time \(T_i\) is less than \(T_j\), he increases his market share at rate \(\alpha \lambda\) during the time period from \(T_i\) to \(T_j\); but when \(T_i > T_j\), operator \(i\) loses his market share at rate \(\alpha \lambda\) during the period from \(T_j\) to \(T_i\). This explains why we need two different functions \(\pi_{ER}(T_i, T_j)\) and \(\pi_{LT}(T_j, T_i)\) to completely characterize the long-term profit for each operator.

### 2.4.2 Duopoly Upgrade Game

Next we consider the non-cooperative game theoretical interactions between two operators, where each of them seeks to maximize his long-term profit by choosing the best upgrade time.
Upgrade Game: We model the competition between two operators as follows:

- Players: Operators 1 and 2.
- Strategy spaces: Operator $i \in \{1, 2\}$ can choose upgrade time $T_i$ from the feasible set $\mathcal{T}_i = [0, \infty]$.\(^8\)
- Payoff functions: Operator $i \in \{1, 2\}$ wants to maximize his profit $\pi_i(T_i, T_j)$ defined in (2.8).

Notice that we consider a static game here, where both operators decide when to upgrade at the beginning of time. This is motivated by the fact that we mentioned at the beginning of this section: in many competition markets operators can obtain available resource for 4G upgrade at the same time. After making decision at $t = 0$, one operator does not change his decision later on. This is reasonable when operators can predict the future 4G market adoption.\(^9\)

In Sections 2.5 and 2.6, we analyze the duopoly upgrade game under different switching costs (i.e., the value of $\alpha \in [0, 1]$). Nash equilibrium is a commonly used solution concept for a static game with complete information. At a Nash equilibrium, no player can increase his payoff by deviating unilaterally [67]. We are interested in characterizing the conditions under which an

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\(^8\)Note that $T_i = \infty$ means that operator $i$ never upgrades.

\(^9\)Operators can predict the future market adoption by exploring historical records of the market and some trial of 4G deployment. In the future we will study the incomplete information case, where an operator may learn more information as the time goes and revise his upgrade decision (if he has not upgraded yet).
asymmetric upgrade equilibrium emerges between symmetric operators.

2.5 4G Competition Market: No Inter-network switching

In this section, we consider the case of $\alpha = 0$, i.e., no user switches operators because of a very high switching cost. This corresponds to the case, for example, where the penalty for terminating a 3G contract to the other operator’s 4G service is very high or where a contract lasts for a very long time.\(^{10}\) The analysis here helps us better understand the more general case of $\alpha > 0$ in Section 2.6.

Let us denote the cost threshold in the 4G competition market under $\alpha = 0$ as

$$K_{\text{th1,}\alpha=0}^{\text{comp}} = \frac{3(1 - \gamma)\lambda^2}{4U(\lambda + S)(2\lambda + S)}.$$  \hspace{1cm} (2.11)

**Theorem 2.** With symmetric operators and $\alpha = 0$, there exists a unique symmetric 4G upgrade equilibrium with the following characteristics:

- Low cost regime (upgrade cost $K \leq K_{\text{th1,}\alpha=0}^{\text{comp}}$): Both operators upgrade at $T_1^* = T_2^* = 0$.

\(^{10}\)Note that the penalty of 3G contract termination is not high if the involved user is switching to the 4G service provided by the same operator. Actually, an operator prefers his 3G users switching to his 4G service.
• High cost regime \((\text{upgrade } K > K_{\text{th1,α=0}}^{\text{comp}})\): Both operators upgrade at

\[ T_1^* = T_2^* = \frac{1}{U-S} \log \left( \frac{K}{K_{\text{th1,α=0}}^{\text{comp}}} \right) > 0, \]

which increases with \(K\).

The proof of Theorem 2 is given in Section 2.8.1.

Intuitively, in the low cost regime, both operators are willing to upgrade at \(t = 0\) to maximize the revenue from providing 4G services. In the high cost regime, on the other hand, both operators postpone their upgrades until the upgrade cost is small enough. If only one operator upgrades, his total market share does not change (since \(α = 0\)) and only his own 3G users switch to the 4G service. As the total number of 4G users in the market grows slowly, the revenue from the 4G service does not compensate for the high upgrade cost. This makes the operator reluctant to upgrade alone. When two operators upgrade simultaneously, the total number of 4G users in the market grows faster, and the 4G service brings in a significant revenue increase to both operators.

### 2.6 4G Competition Market: Practical Inter-network Switching Rate

In this section, we consider the case of \(α > 0\), \(i.e.,\) 3G users may switch to the 4G service of a different operator. The equilibrium analysis in this
general case depends on the relationship between \( U \) (upgrade cost discount rate) and \( S \) (money depreciation rate). We assume that \( U \) is much larger than \( S \), \textit{i.e.}, \( U > S + \alpha \lambda \). This represents the practical case where the advance of technology is the dominant factor in determining \( U \), and not many 3G users choose to switch operators when the 4G service is just deployed (\textit{i.e.}, small \( \alpha \)) \cite{118}. For example, Sprint deployed the first 4G network in the US by using WiMAX technology in 2008 when LTE technology was not yet mature. Only two years later, in 2010, LTE could already offer a much lower cost per bit than WiMAX \cite{103}. From 2012, LTE is expected to be the leading technology choice for 4G networks. This example motivates that \( U \) is much larger than \( S \), and we will study whether the operators’ symmetric 4G upgrades can happen in this scenario.\(^{11}\)

Recall that by upgrading at \( T_1 \) and \( T_2 \), the operators receive the profits given in (2.8). In game theory, one operator’s \textit{best response function} is his upgrade time that achieves the largest long-term profit, as a function of a fixed upgrade time of the other operator \cite{67}. A fixed point of the two operators’ best response functions is the Nash equilibrium, and in general there can be more than one such fixed point.

We can show that the operators’ best responses functions \( (T_1^{\text{best}}(T_2) \text{ and } T_2^{\text{best}}(T_1)) \) depend on the upgrade cost \( K \), and in particular, they depend on

\(^{11}\)We also analyze \( U \leq S + \alpha \lambda \) in our online technical report, where we show that, at any possible equilibrium, operators choose different upgrade times \cite{121}.
two cost thresholds \((K_{th1}^{\text{comp}} < K_{th2}^{\text{comp}})\) that lead to three cost regimes: low, medium, and high.

When the upgrade cost \(K\) is less than the first threshold \(K_{th1}^{\text{comp}}\) (i.e., low cost regime), both operators will upgrade at \(t = 0\) to maximize the revenue from the 4G service. By solving

\[
\frac{\partial \pi_{LT}(0, T_i)}{\partial T_i}|_{T_i=0} = 0, \forall i \in \{1, 2\},
\]

we have\(^{12}\)

\[
K_{th1}^{\text{comp}} = \frac{1}{2U} \left( -\frac{1 - \gamma}{2} (\lambda + 3\alpha \lambda + 2S) \left( \frac{1}{\lambda + S} - \frac{1}{2\lambda + S} \right) \right.
\]

\[
+ (1 - \gamma) \left( \frac{(1 - \alpha)\lambda}{\lambda + S} + \frac{\alpha \lambda}{S} \right). \quad (2.13)
\]

When the upgrade cost \(K\) is larger than \(K_{th1}^{\text{comp}}\) (i.e., medium or high cost regimes), at least one operator postpones his upgrade until the upgrade cost decreases sufficiently. In particular, when \(K\) is larger than the second threshold \(K_{th2}^{\text{comp}}\) (i.e., high cost regime), both operators postpone their upgrades. When operator \(i \in \{1, 2\}\) upgrades at \(t = 0\), operator \(j \neq i\) postpones his upgrade to \(T_j^{\text{best}}(0)\), which is the unique solution to

\[
\frac{\partial \pi_{LT}(0, T_j)}{\partial T_j}|_{T_j=T_j^{\text{best}}(0)} = 0. \quad (2.14)
\]

The threshold \(K_{th2}^{\text{comp}}\) can be obtained by solving

\[
\frac{\partial \pi_{ER}(T_i, T_j^{\text{best}}(0))}{\partial T_i}|_{T_i=0} = 0. \quad (2.15)
\]

\(^{12}\)Previous threshold \(K_{th1, \alpha=0}^{\text{comp}}\) in (2.11) is just a special example of (2.13) at \(\alpha = 0\).
Next we illustrate numerically how the two operators’ best response functions \( T_{1}^{\text{best}}(T_2) \) and \( T_{2}^{\text{best}}(T_1) \) change with the upgrade cost \( K \). Figure 2.6 shows that each operator’s best response function is discontinuous in the medium cost regime, and the two best response functions with the same value of \( K \) intersect at two points: \( 0 = T_{1}^{*} < T_{2}^{*} \) and (symmetrically) \( 0 = T_{2}^{*} < T_{1}^{*} \). To illustrate this situation, consider operator 2’s best response \( T_{2}^{\text{best}}(T_1) \) in the case \( K = 0.062 \). If operator 1 upgrades early such that \( T_1 \) is less than 0.05, operator 2 does not upgrade at the same time to avoid a severe competition. If operator 1 upgrades later such that \( T_1 \) is larger than 0.05, operator 2 chooses to upgrade earlier than operator 1 to increase his market.
Figure 2.7: Two operators’ best upgrade responses to each other according to different cost values in the high cost regime. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, $\lambda = 1$ and $\gamma = 0.5$ such that $U > S + \alpha \lambda$.

share. Thus $T_{2}^{\text{best}}(T_{1})$ is discontinuous at $T_{1} = 0.05$.

Figure 2.7 shows that each operator’s best response function is discontinuous in the high cost regime, and the two functions (with the same value of $K$) intersect at two points, equilibria $0 < T_{1}^{*} < T_{2}^{*}$ and (symmetrically) $0 < T_{2}^{*} < T_{1}^{*}$. Unlike Fig. 2.6, the high cost here prevents any operator from choosing the upgrade time $t = 0$.

Figure 2.8 summarizes how operators’ upgrade equilibrium changes as cost $K$ increases: starting with $T_{i}^{*} = T_{j}^{*} = 0$ in low cost regime, then $0 = T_{i}^{*} < T_{j}^{*}$ with increasing $T_{j}^{*}$ in medium cost regime, and finally $0 < T_{i}^{*} < T_{j}^{*}$ with increasing $T_{1}^{*}$ and $T_{2}^{*}$ in high cost regime.
Figure 2.8: Two operators’ equilibrium \((T_1^*, T_2^*)\) changes as cost \(K\) increases. Other parameters are \(\alpha = 0.5\), \(N = 1000\), \(U = 2\), \(S = 1\), \(\lambda = 1\), and \(\gamma = 0.5\) such that \(U > S + \alpha \lambda\).

In the following theorem, we prove that the operators do not choose symmetric upgrades as long as the cost is not low.

**Theorem 3.** The two operators’ 4G upgrade equilibria satisfy the following properties:13

- **Low cost regime \((K \leq K_{th1}^{comp})\): Both operators upgrade at \(T_1^* = T_2^* = 0\).**

- **Medium cost regime \((K_{th1}^{comp} < K \leq K_{th2}^{comp})\): Operators do not upgrade at the same time, and only one operator may upgrade at \(t = 0\). The possible

---

13We have not yet been able to prove the existence of pure equilibria in the medium and high cost regimes. There are two common approaches to prove existence: Theorem 1.2 in [67] with quasi-concave requirement on operators’ profits, Propositions 1.1 and 1.3 in [120] with continuous contraction and pseudo-contraction requirements for operators’ best response functions. None of them apply to our problem with not quasi-concave profits in (2.8) and not continuous best response functions (see Figs 2.6 and 2.7).
equilibria can only be $0 \leq T_1^* < T_2^*$ and (symmetrically) $0 \leq T_2^* < T_1^*$.

- **High cost regime** ($K > K^{comp}_{th2}$): *Operators do not upgrade at the same time, and none of them upgrade at $t = 0$. The possible equilibria can only be $0 < T_1^* < T_2^*$ and (symmetrically) $0 < T_2^* < T_1^*$.*

The proof of Theorem 3 is given in Section 2.8.2.

The asymmetric upgrade structure is different from the symmetric one under no inter-network switching ($\alpha = 0$) in Section 2.5. To understand the intuition behind the asymmetric structure, we summarize the advantages of earlier and later upgrades with $\alpha > 0$ as follows:

- **Earlier upgrade** gives an operator the advantage to attract more users (from the other operator), and enables the operator to collect a higher revenue from the 4G service.

- **Later upgrade** allows an operator to incur a reduced upgrade cost and to take advantage of the network effect in the 4G market (with more existing 4G users) when he upgrades.

In order to fully enjoy the two advantages of earlier or later upgrades, operators will avoid symmetric upgrade. If one operator upgrades much earlier to capture a larger market share that can compensate for a large upgrade cost, the second operator will not upgrade at the same time to avoid severe competition in market share; instead, the second operator will wait until his
Figure 2.9: Two operators’ equilibrium profits ($\pi_1^*, \pi_2^*$) change as cost $K$ increases under large $\gamma = 0.5$. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, and $\lambda = 1$ such that $U > S + \alpha \lambda$.

loss of users and revenue is compensated by the reduction of upgrade cost (with $U > S + \alpha \lambda$).

Next we study how operators’ equilibrium profits change with cost $K$ and the efficiency ratio $\gamma$ between 3G and 4G services in the three cost regimes. Figures 2.9 and 2.10 show operators’ equilibrium profits under large and small $\gamma$ values, respectively.

We first study the large $\gamma$ scenario in Fig. 2.9. Without loss of generality, we focus on the case where operator 1 upgrades no later than operator 2 (i.e., $T_1^* \leq T_2^*$).

- In the low cost regime, by upgrading at $T_1^* = T_2^* = 0$, the two operators’
Figure 2.10: Two operators’ equilibrium profits \((\pi_1^*, \pi_2^*)\) change as cost \(K\) increases under small \(\gamma = 0.1\). Other parameters are \(\alpha = 0.5\), \(N = 1000\), \(U = 2\), \(S = 1\), and \(\lambda = 1\) such that \(U > S + \alpha \lambda\).

- In the medium cost regime, Fig. 2.9 shows that operator 1 receives a larger profit than operator 2 by upgrading at \(T_1^* = 0\). *Perhaps surprisingly, his profit increases with \(K\), whereas operator 2’s profit decreases with \(K\).* Intuitively, the increase of \(K\) encourages operator 2 to further postpone his upgrade and lose more users to operator 1. The change of operator 1’s profit trades off the increases of his market share and upgrade cost. As operator 1’s market share increases, his growing 4G users communicate more with his 3G users via the efficient 3G service under large \(\gamma\). Operator 1’s 3G revenue increases because of a more efficient intra-network traffic,
which helps compensate for the upgrade cost.

- In the high cost regime, Fig. 2.9 shows that both operators have to postpone their upgrades and, surprisingly, both operators’ profits increase with $K$. As $K$ increases, operator 1 further postpones his upgrade and operator 2’s market share decreases more slowly. Thus operators’ competition in the market share is postponed, and under large $\gamma$ operator 2 can obtain more 3G revenue before operator 1’s upgrade. Operator 1, on the other hand, also benefits from his postponement to decrease his upgrade cost. Since operator 2 also postpones his upgrade, operator 1 can still capture a large market share even though he upgrades later. As $K \rightarrow \infty$, no operator upgrades and operators’ profits approach the symmetric 3G profits. Under large $\gamma$, the 4G service is not much better than 3G and the availability of 4G upgrade only intensifies operators’ competition. Compared to traditional 3G scenario, both operators’ profits decrease when the upgrade cost is high. In other words, both operators will be better off if 4G technology is not available in this case.

Figure 2.10 shows how operators’ profits change with $K$ under small $\gamma$. The results are more intuitive; this is because the operators’ profits decrease with $K$ in all three cost regimes. Under small $\gamma$, the availability of the 4G upgrade significantly improves the revenue in each network. A larger $K$ reduces the benefit of upgrades. However, the operators’ profits will not be smaller after
2.7 Summary

This chapter presents the first analytical study of operators’ 4G upgrade decisions. We first analyze a 4G monopoly market, where the monopolist’s optimal upgrade time trades off an increased market share and the decreasing upgrade cost. We then consider a 4G competition market and develop a non-cooperative game to study the operators’ interactions. Our results show that operators select different upgrade times to avoid severe competition in market share. One operator always takes the lead and uses the benefit of a larger market share to compensate for a larger upgrade cost. We further show that the availability of 4G upgrade may decrease both operators’ profits due to their competition, and their profits may increase with the upgrade cost.

2.8 Appendix

2.8.1 Proof Sketch of Theorem 2

We establish the properties of the operators’ equilibria in the two cost regimes, respectively.
Proof of equilibrium in the low cost regime

We want to prove that $T_1^* = T_2^* = 0$ is the unique equilibrium under $K \leq K_{\text{th}, \alpha=0}^{\text{comp}}$.

- We first show $T_1^* = T_2^* = 0$ is an equilibrium by proving that no operator has an incentive to deviate. Given $T_1^* = 0$, operator 2’s profit is $\pi_{LT}(0, T_2)$ when upgrading at $T_2$. We can show that $\pi_{LT}(0, T_2)$ has a unique maximum $T_{2\text{best}}(0) = 0$. Thus operator 2 will not deviate from $T_2^* = 0$. Similarly, we can prove that given $T_2^*$, operator 1 will not deviate from $T_1^* = 0$.

- We then prove the uniqueness of the equilibrium by contradiction. Without the loss of generality, we suppose there is another equilibrium with $0 < T_1^* \leq T_2^*$. Note that $T_1^*$ cannot equal zero, otherwise operator 2 will also upgrade at $t = 0$ and we reach the same equilibrium. Next we show that $0 < T_1^* < T_2^*$ cannot be an equilibrium by proving operator 2 will choose his upgrade at the same time. This can be proved by showing operator 2’s profit $\pi_{LT}(T_1, T_2)$ with any $T_1 < T_2$ decreases with $T_2$. Thus the possible other equilibrium can only be $T_1^* = T_2^* > 0$. However, this is not possible since operator 1’s profit $\pi_{ER}(T_1, T_2)$ with any $T_1 = T_2$ decreases
with $T_1$, i.e.,

$$
\frac{\partial \pi^{ER}(T_1, T_2)}{\partial T_1}|_{T_1=T_2} \leq -\frac{1 - \gamma}{4} e^{-ST_1} \frac{3\lambda^2}{(2\lambda + S)(\lambda + S)} + K_{th1,\alpha=0}^{\text{comp}} U e^{-UT_1}
$$

$$
< -e^{-ST_1} \frac{3(1 - \gamma)\lambda^2}{4(2\lambda + S)(\lambda + S)} + K_{th1,\alpha=0}^{\text{comp}} U e^{-ST_1} = 0.
$$

Proof of equilibrium in high cost regime

By following a similar proof as in the low cost regime, we can prove that $T_1^* = T_2^*$ in (2.12) is the unique equilibrium under $K > K_{th1,\alpha=0}^{\text{comp}}$. The difference is that given $T_i^*$, we need to consider the possibilities of the other operator $j$ upgrading earlier or later than $T_i^*$.

2.8.2 Proof Sketch of Theorem 3

We divide our proof into three parts. First, we prove the equilibrium in the low cost regime. Then we prove the asymmetric upgrade structure in the other two regimes. Finally, we prove that both operators postpone their upgrades in the high cost regime.

Proof of equilibrium in the low cost regime

By following a similar proof as in Section 2.8.1, we can prove that $T_1^* = T_2^* = 0$ is the unique equilibrium in the low cost regime.
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Proof of asymmetric upgrades in medium and high cost regimes

Next we focus on $K > K_{\text{comp}}^{th1}$, and prove by contradiction that symmetric upgrades $T_1^* = T_2^*$ cannot be an equilibrium. Suppose $T_1^* = T_2^* > 0$ is an equilibrium, where no operator upgrades at $t = 0$ because $K > K_{\text{comp}}^{th1}$. Next we check if an operator (e.g., operator 1) will deviate from this equilibrium.

- The supposed equilibrium $T_1^* = T_2^*$ should guarantee that operator 1 will not choose to upgrade earlier than $T_2^*$, i.e.,

$$\left. \frac{\partial \pi^{ER}(T_1, T_2^*)}{\partial T_1} \right|_{T_1 = T_2^*} \geq 0.$$  \hspace{1cm} (2.16)

- The supposed equilibrium $T_1^* = T_2^*$ should guarantee that operator 1 will not choose to upgrade later than $T_2^*$, i.e.,

$$\left. \frac{\partial \pi^{LT}(T_2^*, T_1)}{\partial T_1} \right|_{T_1 = T_2^*} \leq 0.$$  \hspace{1cm} (2.17)

However, we can show that

$$\left. \frac{\partial \pi^{ER}(T_1, T_2^*)}{\partial T_1} \right|_{T_1 = T_2^*} - \left. \frac{\partial \pi^{LT}(T_2^*, T_1)}{\partial T_1} \right|_{T_1 = T_2^*} = \alpha \lambda e^{-ST_2} \left( \frac{1}{2\lambda + S} - \frac{1}{\lambda + S} \right) < 0,$$

which means that (2.16) and (2.17) cannot be satisfied at the same time. Thus $T_1^* = T_2^*$ cannot be an equilibrium.

Proof of postponed upgrades in the high cost regime

Finally, we prove that no operator upgrades at $t = 0$ by using a contradiction argument. Since $K > K_{\text{comp}}^{th1}$, at least one operator postpones his upgrade.
Without loss of generality, we suppose that $0 = T_1^* < T_2^*$ is an equilibrium, where we have proved that only asymmetric upgrades exist at any possible equilibrium. We need to explore whether any operator has an incentive to deviate.

- Given $T_1^* = 0$, operator 2’s profit is $\pi^{LT}(0, T_2)$. He will not deviate only when $T_2^*$ already maximizes $\pi^{LT}(T_1, T_2)$ (i.e., $T_2^* = T_2^{\text{best}}(0)$).

- Given $T_2^* = T_2^{\text{best}}(0)$, operator 1’s profit is $\pi^{ER}(T_1, T_2^{\text{best}}(0))$ by upgrading at $T_1 \leq T_2^{\text{best}}(0)$. We find that operator 1 has an incentive to deviate from $T_1^* = 0$. Indeed, since $K > K_{\text{comp}}^{\text{th2}}$,

$$\frac{\partial \pi^{ER}(T_1, T_2^{\text{best}}(0))}{\partial T_1} \bigg|_{T_1=0} > 0. \quad (2.19)$$

Thus $0 = T_1^* < T_2^*$ cannot be an equilibrium, and both operators postpone their upgrades.
Chapter 3

Economics of Femtocell Service Provision

3.1 Introduction

The next generation 4G cellular systems aim at providing end users with high data rates and reliable services by operating at wider and higher frequency bands (e.g., 2496MHz-to-2690MHz for TD-LTE in U.S.). However, severe signal attenuation at these high frequencies often causes poor signal receptions for indoor users, who are separated by walls from outdoor cellular base stations of macrocells.\textsuperscript{1}

To solve the poor signal reception problem for indoor users, researchers...\textsuperscript{1}A macrocell in a cellular network provides radio coverage served through a high-power cellular base station [124].
have proposed the idea of femtocell (e.g., [129–131]). Compared to macrocells, femtocells are short-range, low deployment cost, and low power user-deployed tiny base stations. A user can deploy a femtocell at home and connect it to the wireline broadband Internet connection, e.g., the digital subscriber line (DSL). Femtocells are often managed by the same operator that controls the macrocells, and they can provide better quality of service (QoS) to indoor users as they are very close to users’ cell phones. Despite the obvious motivation to deploy femtocell service, the operator needs to carefully consider several issues that will affect the economic return of the femtocell service.

First, the femtocell service needs to share the limited licensed bands with the macrocell service. There are two types of sharing schemes. The first scheme is “separate carriers”, where the femtocells and macrocells occupy non-overlapping spectrum bands (e.g., [132, 138, 141]). The second scheme is “shared carriers” (or “partially shared carriers”), where macrocells and femtocells operate on (partially) overlapping bands (e.g., [131, 142, 150]). Each scheme has its certain advantages and disadvantages.

- “Separate carriers” scheme is easy to manage and can avoid interferences between macrocells and femtocells. For example, China Unicom (one of the three main wireless service providers in mainland China and the first one deploying femtocell since 2009) is in strong favor of this scheme [158]. This scheme can support two types of access schemes: the public
access where femtocell network becomes an extension of the macrocell network and can be used by any user of the operator, and the private access scheme where only registered femtocell users can gain access [132]. However, “separate carriers” scheme reduces the available spectrum for both services.

- “Shared carriers” (or “partially shared carriers”) scheme is an efficient frequency allocation scheme where macrocells and femtocells operate on (partially) the same spectrum (i.e., co-channel operation). In this case, the operation of femtocells leads to increased interferences for macrocell users, and affects macrocell capacity and service quality ([132, 141]). Proper interference management is very challenging, as femtocells are deployed by users and thus the interference is often hard to predict by the operator. Also, a key requirement for co-channel operation is to allow public access on all femtocells (and thus may not support private access) [132]. The purpose of this requirement is to mitigate femtocells’ interferences to macrocell users by forcing macrocell users to switch to femtocell service whenever possible. However, such public access leads to significant signaling overhead due to increased handovers [137].

In this chapter, we will focus on the first “separate carriers” scheme.

Second, when an operator introduces the femtocell service and charges a higher price, some users who originally experience good macrocell service qual-
ity may experience a decrease in payoff with the femtocell service due to a higher femtocell payment. It is important to ensure the satisfaction of these users by keeping the original macrocell service available at the original price. This will limit the resource allocation to femtocell service.

Third, although femtocells are low in deployment costs, the femtocell service may incur additional operational cost compared to macrocells. Femtocell users’ traffics need to go through wireline broadband Internet connections. The wireline Internet Service Providers (ISPs) may impose additional charges on the femtocell related traffics [143]. Also, since the femtocell users’ traffics will go through the ISP’s network before reaching the cellular operator’s own network, issues such as synchronization with macrocells become more challenging to resolve [144, 145]. Moreover, femtocell service needs billing system integration with macrocell service, and requires additional customer support.

In this chapter, we will discuss the economic incentive of the operator’s femtocell service provision, by considering three issues discussed above. We want to understand when and how the operator should offer the femtocell service, and the corresponding impacts on the original macrocell service. Our main results and contributions include:

- *A Dynamic Decision Model:* We model and analyze the interactions between a cellular operator and users as a two-stage Stackelberg game. Users experience different channel conditions and spectrum efficiencies with
the macrocell service, but all of them achieve a high spectrum efficiency with the femtocell service. Thus users have different preferences between macrocells and femtocells. The operator makes spectrum allocations and pricing decisions for both macrocell and femtocell services to maximize its total profit.

• **Profit-Maximizing with Femtocell Service Only:** If femtocell service has the same maximum coverage as macrocell service, then a profit-maximizing operator will choose to only offer femtocell service to all its users.

• **Dual Service Provision Considering Users’ Reservation Payoffs:** If we consider users’ reservation payoffs as what they can achieve with the original macrocell service, then offering femtocell service only may force some users to leave and thus may not be optimal to the operator. In this case, we characterize when and how the operator should provide the femtocell service together with the macrocell service (i.e., dual services) so that all users achieve payoffs no worse than their reservation payoffs.

• **Impact of Femtocell Spatial Reuse, Operational Cost, and Limited Coverage:** When multiple femtocells can reuse the same spectrum resource, the operator has additional incentives to allocate spectrum to femtocell, and the femtocell price will be reduced to reflect this “small cell” advantage. Moreover, when femtocell service incurs operational cost to the operator or has a smaller coverage than the macrocell service, then the operator
will always serve users by dual services. Furthermore, as cost decreases or coverage increases, more users are served by the femtocell service.

The rest of the chapter is organized as follows. We introduce the network model of macrocell service in Section 7.2, which serves as a benchmark for later analysis. In Section 3.3, we introduce the network model of femtocell service and analyze how the operator provides dual services in terms of spectrum allocations and pricing. Then we extend the results in Section 3.3 to Sections 3.4, 3.5, 3.6, and 3.7, by examining the impacts of users’ reservation payoffs, femtocell frequency reuse, femtocell operational costs, and limited femtocell coverage. We conclude our work in Section 7.6. Some proofs are shown in our online technical report [160].

3.1.1 Related Work

Most prior work on femtocells focused on various technical issues in service provision (e.g., access control and resource management [142,146–149]). Chandrasekhar et al. [142] analyzed the uplink capacity and proposed an interference avoidance strategy for the coexistence of femtocells and macrocells in a CDMA network. Benmesbah et al. [146] proposed a decentralized spectral resource allocation for OFMDA downlink of coexisting macro and femto network. Rangan [147] proposed to control femto-macro interference by interference cancellation and subband scheduling. Shi et al. [149] proposed an analytical model
to explicitly investigate the uplink capacity and coverage of femtocells which coexist within macrocells. Tan et al. [148] studied the power control problem in Rayleigh-fading heterogeneous networks.

Only few papers have studied the economic issues of femtocell service ([138, 139, 151–153]). Claussen et al. [151] explored the financial impact of femtocells on a macrocell network, with a focus on the network deployment costs. Lin et al. [139] compared three deployment types of femtocells, i.e. joint deployment, operator deployment, and user deployment frameworks. Yun et al. [152] studied whether it is economically beneficial for the cellular operator or femtocell owners to provide femtocell service to guest users. Chen et al. [153] further studied how to motivate femtocell owners to open their resource to guest macrocell users. Shetty et al. [138] analyzed the interplay of interferences and service pricing on users’ adoption of femtocells. The key difference between this chapter and the existing literature is that we study the operator’s provision of dual services in terms of both spectrum allocations and pricing decisions. We also characterize the impacts of users’ reservation payoffs, the femtocell operational cost, and the limited femtocell coverage on the service provision.

Our model on dual services is related to the dual-channel model in supply chain management, where a seller adds a direct channel (e.g., Internet selling) on an existing retailing channel to sell the same products ([154,155]). There was often no constraint on the operator’s availability in obtaining resource in
Stage I: (Operator’s pricing) 
Operator decides macrocell price \( p_M \) and announces to users

Stage II: (Users’ demands) 
Each user decides how much bandwidth \( b \) to request from operator

Figure 3.1: Two-stage Stackelberg game between the operator and users in Section 7.2.

these models. In our model, however, the operator’s spectrum is limited, and
the dual services (i.e., femtocell and macrocell services) are different in both
QoS and coverage, which lead to an interesting new formulation and different
engineering insights.

3.2 Benchmark: Macrocell Service Only

As a benchmark, we first look at how the operator prices the macrocell service
to maximize its profit without the choice of the femtocell service. When we
consider the introduction of femtocell service in Sections 3.3, 3.4, 3.6, and 3.7,
the operator should achieve a profit no worse than this benchmark. Also, what
users get in this benchmark will serve as their reservation payoffs in Section
3.4.

We consider an operator who owns a single macrocell.\(^2\) It owns a total \( B \) Hz
wireless spectrum bandwidth to provide macrocell service, where each macro-

\(^2\)The results of this chapter can be extended to a multiple macrocell scenario, where frequency reuse is
allowed over macrocells.
cell user is allocated part of the bandwidth and transmits over the allocated part accordingly. As shown in Fig. 3.1, we model the interactions between the operator and the users as a two-stage Stackelberg game. In Stage I, the operator determines the macrocell price $p_M$ (per unit bandwidth) to maximize its profit. Here, subscript $M$ denotes macrocells. In Stage II, each user decides how much bandwidth to purchase to maximize its payoff.

This usage-based pricing scheme is widely used in today’s cellular macrocell networks, especially in Europe and Asia [78, 157]. In US, AT&T (since December 2009) and Verizon (since July 2011) have adopted the usage-based pricing for wireless data services. Usage-based pricing for femtocell has just started. For example, AT&T’s femtocell service counts the femtocell data usage as part of the regular cellular usage (together with the macrocell data usage), which is subject to usage based pricing [159]. Due to the exponential growth of wireless data traffic and the scarce spectrum resource, we envision that usage-based pricing for both macrocell and femtocell services will become more common in the near future.

We next solve this two-stage Stackelberg game by backward induction.

### 3.2.1 Users’ Bandwidth Demands in Stage II

Different users experience different channel conditions to the macrocell base stations due to different locations, and thus achieve different data rates when using the same amount of bandwidth. We consider that a user has fixed trans-
mission power $P$ per unit bandwidth (e.g., power spectrum density constraint) and his average channel gain in the macrocell is $h$. Without interfering with other users, the user’s *macrocell spectrum efficiency* is thus

$$\theta = \log_2(1 + \text{SNR}) = \log_2 \left( 1 + \frac{Ph}{n_0} \right),$$

where $n_0$ is the background noise power. By obtaining $b$ Hz of spectrum, its achieved data rate is $\theta b$ bits. As users have different channel gains in macrocell service, they perceive different macrocell spectrum efficiency $\theta$. A larger $\theta$ means a better channel condition and a higher spectrum efficiency *when using the macrocell service*. In Section 3.3, we will show that all users achieve the same high spectrum efficiency with femtocell service as the femtocell is always close an indoor user. Note that $\theta$ can be normalized in the range $[0, 1]$ and here we assume the $\theta$ is uniformly distributed (see Fig. 3.2).\(^3\) We also normalize the total user population to be 1 as in [133].

For a user with a macrocell spectrum efficiency $\theta$, it obtains a utility $u(\theta, b)$ when achieving data rate $\theta b$ [125], i.e.,

$$u(\theta, b) = \ln(1 + \theta b).$$

Such utility is commonly-used in economic literature to denote the diminishing return of getting additional resource [54]. The user needs to pay a linear

\(^3\)The uniform distribution is assumed for analytical tractability. Using the uniform distribution to approximate users’ fluid population regarding their locations is widely used (e.g., [134–136]). In our online technical report [160], we show that other continuous distributions (e.g., Gaussian distribution) will not change the main engineering insights obtained in this chapter.
Figure 3.2: Distribution of users’ macrocell spectrum efficiencies $\theta$

payment $p_M b$ to the operator, where the price $p_M$ is announced by the operator in Stage I. The user’s payoff is the difference between its utility and payment, i.e.,

$$\pi_M(\theta, b, p_M) = \ln(1 + \theta b) - p_M b. \quad (3.1)$$

The optimal value of bandwidth demand that maximizes the user’s payoff with the macrocell service is

$$b^*(\theta, p_M) = \begin{cases} \frac{1}{p_M} - \frac{1}{\theta}, & \text{if } p_M \leq \theta, \\ 0, & \text{otherwise}, \end{cases} \quad (3.2)$$

which is decreasing in $p_M$ and increasing in $\theta$ (if $p_M \leq \theta$). The user’s maximum payoff with macrocell service is

$$\pi_M(\theta, b^*(\theta, p_M), p_M) = \begin{cases} \ln\left(\frac{\theta}{p_M}\right) - 1 + \frac{p_M}{\theta}, & \text{if } p_M \leq \theta, \\ 0, & \text{otherwise}, \end{cases} \quad (3.3)$$

which is always nonnegative and is increasing in $\theta$.

### 3.2.2 Operator’s Pricing in Stage I

Next we consider the operator’s optimal choice of price $p_M$ in Stage I. To achieve a positive profit, the operator needs to set $p_M \leq \max_{\theta \in [0,1]} \theta = 1$, so that at least some user purchases some positive bandwidth in Stage II. The
fraction of users choosing macrocell service is $1 - p_M$ as shown in Fig. 3.2. The total user demand is

$$Q_M(p_M) = \int_{p_M}^{1} \left( \frac{1}{p_M} - \frac{1}{\theta} \right) d\theta = \frac{1}{p_M} - 1 + \ln p_M, \quad (3.4)$$

which is a decreasing function of $p_M$. On the other hand, the operator has a limited bandwidth supply $B$, and thus can only satisfy the demand no larger than $B$.

The operator chooses price $p_M$ to maximize its profit, i.e.,

$$\max_{0 < p_M \leq 1} \pi^{\text{operator}}(p_M) = \min \left( Bp_M, p_M Q_M(p_M) \right). \quad (3.5)$$

Notice that the first term in the min operation of (3.5) is increasing in $p_M$, while the second term is decreasing in $p_M$ since

$$\frac{dp_M Q_M(p_M)}{dp_M} = \ln p_M < 1.$$  

By also checking the two terms’ values at the $p_M$ boundary values, we can conclude that the optimal solution to Problem (3.5) is unique and the two terms are equal at the optimality.

**Theorem 4.** The equilibrium macrocell price $p_M^*$ is the unique solution of the following equation:

$$B = \frac{1}{p_M^*} - 1 + \ln p_M^*. \quad (3.6)$$

To facilitate later discussions, we denote this $p_M^*$ as $p_M^{\text{bench}}$. Furthermore, the total user demand $Q_M(p_M^{\text{bench}}) = B$. Finally, the equilibrium price $p_M^{\text{bench}}$ de-
creases with $B$, and the operator’s equilibrium profit $\pi_{\text{operator}}^* = \pi_{\text{operator}}(p_{M}^{\text{bench}})$ increases with $B$.

Notice that all users with a macrocell spectrum efficiency $\theta$ less than $p_{M}^{\text{bench}}$ will not receive macrocell service. When the total bandwidth $B$ is small, the equilibrium macrocell price $p_{M}^{\text{bench}}$ is close to 1 and thus most users will not get service. This motivates the operator to adopt the femtocell service so that it can serve these users and generate additional profits.

### 3.3 Provision of Femtocell Service

We now consider how femtocell service can improve the operator’s profit. Note that the some users preferring one service (e.g., femtocell service) can be rejected if the bandwidth in that service is not enough. This is motivated by the fact that a femtocell with limited bandwidth can only serve several users \[153\]. Similarly, if a macrocell faces demands from too many users, some users will be out of service.

The analysis in this section is based on several assumptions, each of which will be relaxed in later sections.

- Each user has a zero reservation payoff. This means that if a user’s bandwidth demand in macrocell service is not satisfied, he will switch to femtocell as long as its payoff is positive.\(^5\) This assumption will be relaxed

\(^5\)For example, users may have zero reservation payoff if the operator is the monopolist in the local market
in Section 3.4.

- Different femtocells use different spectrum bands and do not have frequency reuse. This assumption will be relaxed in Section 3.5.

- The femtocell service does not incur any additional operational cost compared to the macrocell service. This assumption will be relaxed in Section 3.6.

- The femtocell service has the same maximum coverage as the macrocell service, and each user can access to both macrocell and femtocell services.\(^6\) This assumption will be relaxed in Section 3.7.

It should be noted that we also jointly relax all four assumptions in our online technical report [160], where most engineering insights in Sections 3.4, 3.5, 3.6, and 3.7 still hold.

We are interested in answering the following two questions:

- Is it economically viable for the operator to introduce the femtocell service?

- If so, how should the operator determine resource allocation and pricing for macrocell and femtocell services?

\(^6\)Femtocell full coverage within our focused macrocell is possible if many people deploy femtocells in a popularized area.
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Under the assumptions in this section, we will show that the operator will choose to only provide femtocell service (i.e., no macrocell service) and charge a femtocell price \( p^*_f \) than the optimal macrocell price \( p^*_M \) derived in the benchmark (Section 7.2).

Note that there are two modes for the operator to provide femtocell service: closed access and open access \([130, 139]\). In both modes, femtocell users need to purchase the femtocells from the operator and deploy the devices at their homes. Femtocell users’ device payment is reflected in the femtocell price which is higher than macrocell price. The unique feature of closed access mode is that it automatically hands over femtocell users’ traffics automatically to high QoS femtocell service whenever they are in the proximity of femtocells, while macrocell users’ traffics are always served by macrocell service. Without purchasing the devices and subscribe to femtocell service, a user (i.e., macrocell user) cannot access femtocell service even if he is near to femtocell devices. The open access mode, however, serves all users (including macrocell users) in femtocell service. Though most current femtocell devices support mode switch such that users can set the mode according to their wishes, most femtocell owners prefer closed access mode according to \([139, 140]\). The reason is that femtocell users are not pleased to share the limited capacity and loss privacy. Thus closed access mode (instead of open access one) is widely used in current industry practice, where a user has the choice to become either femtocell user.
(who can also use macrocell service if femtocell service is out of reach) or macrocell user (who can only use macrocell service). As we consider femtocell service has the same maximum coverage as the macrocell service in this section, then femtocell users would not use macrocell service and users only decide to choose femtocell service or macrocell service at the beginning.

More specifically, we will look at a two-stage Stackelberg game as in Fig. 3.3. In Stage I, the operator determines bandwidth allocated to femtocell service (femtocell band $B_F$) and to macrocell service (macrocell band $B_M$), with $B_F + B_M = B$. The operator also determines the femtocell price $p_F$ and macrocell price $p_M$. In Stage II, each user decides which service to choose and how much bandwidth to purchase. If a user’s demand cannot be satisfied by its preferred service, it will purchase bandwidth from the other service.\(^7\) We will again analyze this two-stage Stackelberg game by using backward induction.

\(^7\)Note that a user chooses either femtocell or macrocell service in our closed access mode.
3.3.1 Users’ Service Choices and Bandwidth Demands in Stage II

If a user has a macrocell spectrum efficiency $\theta$, its maximum payoff by using the macrocell service is given in (3.3). Next we consider users’ payoffs by using the femtocell service.

Since femtocell base stations are deployed indoors and are very close to the users’ cell phones, we assume that all users using the femtocell service have equal good channel conditions and all of them achieve the high spectrum efficiency. This means that independent of the macrocell spectrum efficiency $\theta$, each user achieves the same payoff $\pi_F(b)$ when using a bandwidth of $b$ to reach data rate $b$,

$$\pi_F(b, p_F) = \ln(1 + b) - p_F b. \quad (3.7)$$

The user’s optimal demand in femtocells is

$$b^*(p_F) = \begin{cases} \frac{1}{p_F} - 1, & \text{if } p_F \leq 1, \\ 0, & \text{otherwise.} \end{cases} \quad (3.8)$$

A user’s maximum payoff with the femtocell service is

$$\pi_F(b^*(p_F), p_F) = \begin{cases} \ln\left(\frac{1}{p_F}\right) - 1 + p_F, & \text{if } p_F \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad (3.9)$$

which is always nonnegative.

It is clear that a user with a small macrocell spectrum efficiency $\theta$ can get a better payoff by using the femtocell service instead of the macrocell service. Thus there exists a threshold of $\theta$ that separates the users of two services.
Next we define two different types of thresholds.

**Definition 1** (Users’ preferred partition threshold $\theta_{th}^{pr}$). Users with $\theta \in [0, \theta_{th}^{pr})$ prefer to use the femtocell service, and users with $\theta \in [\theta_{th}^{pr}, 1]$ prefer to use the macrocell service.

**Definition 2** (Users’ partition threshold $\theta_{th}$). Users with $\theta \in [\theta_{th}, 1]$ actually receive the macrocell service finally, and users with $\theta \in [0, \theta_{th})$ receive either the femtocell service or no service.

Notice that some users within the range of $[\theta_{th}, 1]$ may not prefer to be served by the macrocell service. The preferred partition threshold $\theta_{th}^{pr}$ only depends on the prices $p_M$ and $p_F$. If all users’ demands from their preferred services are satisfied by large enough $B_F$ and $B_M$, then users’ preferred partition threshold equals users’ partition threshold (i.e., $\theta_{th}^{pr} = \theta_{th}$). However, in general $\theta_{th}$ may be different from $\theta_{th}^{pr}$, depending on the operator’s choice of $B_F$ and $B_M$ in the first stage. We assume that the operator has a higher priority to serve users with larger values of $\theta$ by macrocell service (as it is more efficient for the macrocell service to do so). Thus it is possible for the operator to reject a user’s choice of macrocell service if its $\theta$ value is low.

By comparing a user’s optimal payoff with macrocell and femtocell services in (3.3) and (3.9), we have the following result.

**Lemma 3.** Users’ preferred partition threshold $\theta_{th}^{pr} = p_M/p_F$. Users with a small macrocell spectrum efficiency $\theta < p_M/p_F$ prefer the femtocell service,
and users with a large macrocell spectrum efficiency $\theta > p_M/p_F$ prefer the macrocell service.

Now we introduce the concept of **finalized demand**.

**Definition 3** (User’s Finalized Demand). *If a user’s demand from its preferred service is satisfied, then its finalized demand is its preferred demand. If a user’s demand from its preferred service is not satisfied, then the user may switch to the alternative service and the new demand becomes the finalized demand.*

### 3.3.2 Operator’s Spectrum Allocations and Pricing in Stage I

Now we are ready to study Stage I, where the operator determines $B_F, B_M, p_F$, and $p_M$ to maximize its profit (see Fig. 3.4). Let us denote the operator’s equilibrium decisions as $B_F^*, B_M^*, p_F^*$, and $p_M^*$, which lead to the users’ equilibrium partition threshold (Definition 2) equal to $\theta_{th}^*$. It is clear that the femtocell price $p_F^*$ is larger than the macrocell price $p_M^*$, otherwise all users will choose the femtocell service.
Lemma 4. At the equilibrium, the operator’s total bandwidth \( B \) equals users’ total finalized demand.

Proof of Lemma 4 is given in [160].

Based on Lemma 4, we can further show that bandwidth allocated to each service equals users’ total finalized demand in that service. That is,

\[
B^*_F = \int_{\theta^*_\text{th}}^{1} \left( \frac{1}{p^*_F} - 1 \right) d\theta = \theta^*_\text{th} \left( \frac{1}{p^*_F} - 1 \right), \tag{3.10}
\]

\[
B^*_M = \int_{\theta^*_\text{th}}^{1} \left( \frac{1}{p^*_M} - \frac{1}{\theta} \right) d\theta = \frac{1 - \theta^*_\text{th}}{p^*_M} + \ln \theta^*_\text{th}, \tag{3.11}
\]

and \( B^*_F + B^*_M = B \). This means that it is enough to compute the equilibrium decisions of \( \theta^*_\text{th} \), \( p^*_M \), and \( p^*_F \). The operator’s profit-maximization problem is

\[
\max_{p_M, p_F, \theta_{\text{th}} \in [0,1]} \pi_{\text{operator}}(p_M, p_F, \theta_{\text{th}}) = p_F \left( \theta_{\text{th}} \left( \frac{1}{p_F} - 1 \right) \right) + p_M \left( \frac{1 - \theta_{\text{th}}}{p_M} + \ln \theta_{\text{th}} \right)
\]

subject to

\[
p_M \leq \theta_{\text{th}} \leq 1,
\]

\[
\theta_{\text{th}} \left( \frac{1}{p_F} - 1 \right) + \frac{1 - \theta_{\text{th}}}{p_M} + \ln \theta_{\text{th}} = B. \tag{3.12}
\]

By examining (3.12), we have the following result.

Theorem 5. At the equilibrium, the operator will only provide femtocell service, i.e., \( B^*_F = B \) and \( B^*_M = 0 \). All users will use femtocell service, i.e., users’ equilibrium partition threshold \( \theta^*_\text{th} = 1 \). The equilibrium femtocell price is

\[
p^*_F = \frac{1}{1 + B}, \tag{3.13}
\]
and the operator’s equilibrium profit is

\[ \pi_{\text{operator}}^* = \frac{B}{1 + B}. \] (3.14)

Proof of Theorem 5 is given in [160].

Theorem 5 is intuitive to understand. As the femtocell service provides a higher QoS to all users,\(^8\) the operator can attract the users with small macrocell spectrum efficiency \(\theta\), and sell out the whole bandwidth \(B\) at a price \(p_F^* = 1/(1 + B)\) higher than the equilibrium macrocell price \(p_{M}^{\text{bench}}\) in Theorem 4. This means the operator obtains a higher profit by only providing femtocell service.

However, a user who has a large \(\theta\) (e.g., \(\theta \rightarrow 1\)) will achieve a smaller payoff \(\pi_F(b^*(p_F^*), p_F^*)\) with femtocell service than the payoff \(\pi_M(\theta, b^*(\theta, p_{M}^{\text{bench}}), p_{M}^{\text{bench}})\) with original macrocell service. If we treat \(\pi_M(\theta, b^*(\theta, p_{M}^{\text{bench}}), p_{M}^{\text{bench}})\) as a user’s reservation payoff below which the user will not accept the femtocell service,\(^9\) then the operator can no longer only provide femtocell service. Next section studies this case in details.

### 3.4 Impact of Users’ Reservation Payoffs

In this section, we will consider the operator’s decisions by assuming that each user with a macrocell spectrum efficiency \(\theta\) receives a payoff no less than

---

\(^8\)The only exception will be users with \(\theta = 1\), who have a zero size support under the uniform distribution assumption of \(\theta\).

\(^9\)For example, the user may switch to a different operator who provides macrocell service.
$\pi_M(\theta, b^*, p_M^\text{bench})$ as calculated in (3.3). This means that the operator always needs to provide macrocell service at the same price as $p_M^\text{bench}$ derived based on (3.6). Also, all users’ preferred demands in macrocell service should be satisfied. Next we consider a two-stage decision process similar to Fig. 3.3. The only difference here is that the operator needs to satisfy users’ reservation payoffs.

In this section only, we assume that the operator has a priority to serve the users with the smallest $\theta$ first in femtocell service. This is reasonable since femtocell service aims at improving QoS of indoor users especially for those with a small macrocell spectrum efficiency. These users cannot use macrocell service and will be happy to pay a high price for the femtocell service. For users with a high macrocell spectrum efficiency, they have the additional choice of macrocell service and will not use femtocell service if $p_F$ is high.

We will again use backward induction to analyze the problem. As Stage II is the same as Section 3.3.1, we will focus on the operator’s decisions on $B_M$, $B_F$, and $p_F$ in Stage I.

**Lemma 5.** At the equilibrium, only one of the following is true:

- Only users with $\theta \in [p_M^\text{bench}, 1]$ are served with the macrocell service, and no users are served with the femtocell service.

- All users with $\theta \in [0, 1]$ are served, by either the macrocell service or the femtocell service, and some user with $\theta \geq p_M^\text{bench}$ is served by the macrocell
Proof. We prove this lemma by contradiction. First, assume that all users with \( \theta \in [p_M^{\text{bench}}, 1] \) receive the macrocell service, and some user with \( \theta < p_M^{\text{bench}} \) receive the femtocell service. However, this is impossible as the macrocell users already use a total bandwidth of \( B \) and there is no resource left for the femtocell service. Second, assume that there exists a partition threshold \( \theta_{th}^* > p_M^{\text{bench}} \), such that some users with \( \theta < \theta_{th}^* \) do not receive femtocell service. Due to the constraint on reservation payoffs, all users with \( \theta \in [p_M^{\text{bench}}, \theta_{th}^*) \) receive the femtocell service. Since the operator has a priority to serve the users with small \( \theta \) first in the femtocells, then all users with \( \theta < p_M^{\text{bench}} \) also receive femtocell service. This contracts with the assumption.

Lemma 5 shows that the equilibrium femtocell band is either \( B_F^* = 0 \) or \( B_F^* \geq \int_0^{p_M^{\text{bench}}} \left( \frac{1}{p_F^*} - 1 \right) d\theta \). This implies that when supply \( B \) is low, the operator needs to allocate all its bandwidth supply \( B \) for macrocell service to reach users’ reservation payoffs. Only when \( B \) is high, the operator can serve all users by dual services (i.e., macrocell and femtocell services). This will be further illustrated in Figures 3.5 and 3.7. Notice that as long as supply \( B \) is high, femtocell band needs to serve users more than those with \( \theta \in [0, p_M^{\text{bench}}] \). Thus there is a corresponding sharp increase of \( B_F^* \) (and a sharp decrease of \( B_M^* \)) when supply \( B \) slightly increases from the low to the high regime.

By following a similar analysis as in Section 3.3, we can show that the
Figure 3.5: The operator’s equilibrium femtocell price $p_F^*$ and macrocell price $p_M^{bench}$ as functions of bandwidth supply $B$ considering users’ reservation payoffs.

total supply $B$ equals users’ total finalized demand at the equilibrium (i.e., Lemma 4). The operator’s profit-maximization problem can be simplified as

$$
\max_{p_F, \theta_{th} \in [0,1]} \pi^{\text{operator}}(p_F, \theta_{th}) = p_F \int_0^{\theta_{th}} \left( \frac{1}{p_F} - 1 \right) d\theta + p_M^{\text{bench}} \int_{\theta_{th}}^1 \left( \frac{1}{p_M^{\text{bench}}} - \frac{1}{\theta} \right) d\theta
$$

subject to $p_M^{\text{bench}} \leq \theta_{th} \leq p_M^{\text{bench}}/p_F,$

$$
\int_0^{\theta_{th}} \left( \frac{1}{p_F} - 1 \right) d\theta + \int_{\theta_{th}}^1 \left( \frac{1}{p_M^{\text{bench}}} - \frac{1}{\theta} \right) d\theta = B, \quad (3.15)
$$

where $p_M^{\text{bench}}$ is computed from (3.6), and the right inequality of the first constraint means that the operator cannot violate users’ preferences in macrocell service. In the second constraint, the first and second terms on the left hand side are the users’ finalized total demands in femtocells and macrocells, respectively.
Problem (3.15) is difficult to solve in closed form, so we use numerical results to illustrate some interesting insights.

Figure 3.5 shows the operator’s equilibrium femtocell price $p_F^*$ and macrocell price $p_M^{bench}$ as functions of total bandwidth $B$. Figure 3.5 is consistent with Lemma 5, where the femtocell price is very high ($p_F^* = 1$) in the low supply regime (i.e., $B < 3.5$), thus no users would choose femtocell service and $B_F^* = 0$. Both services are available in the high supply regime (i.e., $B \geq 3.5$), where the femtocell price is lower and becomes attractive to users. We can observe sharp decrease of $p_F^*$ at the boundary value $B = 3.5$ that distinguishes the low and high supply regimes. This is consistent with Lemma 5 where the operator needs to serve users more than those with $\theta \in [0, p_M^{bench}]$ once in the high
supply regime, and thus the femtocell price needs to be significantly reduced to attract users. We further observe that $p^*_F$ decreases faster than $p^*_{M}^{bench}$ in the high supply regime, which means that the operator wants to attract more users to the more efficient femtocell service.

Figure 3.6 shows the users’ equilibrium partition threshold $\theta^*_{th}$ with dual services, comparing to the partition threshold of the macrocell service only case in Section 7.2. In the low supply regime (i.e., $B < 3.5$), two curves overlap as the dual services degenerate to the macrocell service only in this regime. However, in the high supply regime, the operator will announce similar femtocell and macrocell prices (see Fig. 3.5), and most users will choose to use femtocell service (see Fig. 3.6). Unlike the femtocell service only case in Section 3.3 (without considering users’ reservation payoffs), here users with large $\theta$ values will choose to stay with the macrocell service and are not affected by the introduction of femtocell service.

Figure 3.7 shows the operator’s equilibrium bandwidth allocations to dual services (i.e., $B^*_F$ and $B^*_M$) as functions of the total bandwidth supply $B$. Figure 3.7 is directly related to Lemma 5, where only macrocell service is available in the low supply regime (i.e., $B < 3.5$), and both services are available in the high supply regime (i.e., $B \geq 3.5$). Consistent with Lemma 5 and Fig. 3.5, there is a sharp decrease of $B^*_M$ right at the boundary that distinguishes the low and high supply regimes. In the high supply regime, femtocell band $B^*_F$
Figure 3.7: The operator’s equilibrium femtocell band $B^*_F$ and macrocell band $B^*_M$ as functions of supply $B$ considering users’ reservation payoffs increases faster than the macrocell band $B^*_M$. This is because the operator can obtain a higher profit by selling more bandwidth by the femtocell service, which charges users a higher price than the macrocell service.

Figure 3.8 compares the operator’s profits in three different cases: femtocell service only as in Section 3.3, dual services as in this section, and macrocell service only as in Section 7.2. In the low supply regime, dual services degenerate to the macrocell service case. In the high supply regime, the profit of the dual services becomes closer to the femtocell service only case as $B$ increases. This means that considering users’ reservation payoffs will not lead to significant profit loss when the total resource is abundant. In this case, only users with a $\theta$ very close to 1 will stay with the macrocell service and all other users will choose the femtocell service.
3.5 Impact of Femtocell Frequency Reuse

In Section 3.3, we have assumed that different femtocells use different spectrum bands. However, as a femtocell often has a smaller coverage (e.g., tens of meters within a home) than the macrocell (e.g., hundreds of meters), it is often possible to have multiple non-overlapping femtocells within the same macrocell coverage. These non-overlapping femtocells can use the same spectrum band without interfering with each other. This is also called frequency reuse. We will discuss how frequency reuse affects the operator’s provision of femtocell service.

Our analysis here will be based on the simplified scenario as in Section 3.3. We again analyze the two-stage game by using backward induction. Here
the users’ requests in service and bandwidth in Stage II are the same as Section 3.3.1. If the same spectrum is allocated to two different femtocells, then the operator collects twice of the revenue.

Now we are ready to analyze Stage I to derive the operator’s equilibrium decisions. Let us denote the average number of interfering femtocells that cannot use the same frequency spectrum as \( K \), and the frequency reuse factor is then \( 1/K \). We will assume that the total number of femtocells is \( N > K \). Thus after considering frequency reuse, the available spectrum to each femtocell increases from \( B_F/N \) to \( B_F/K \). Then the total available bandwidth to all femtocells will be \( B_F N/K \) instead of \( B_F \).

By following a similar proof as that of Theorem 5, we have the following result.

**Theorem 6.** At the equilibrium, the operator will only provide femtocell service, i.e., \( B^*_F = B \) and \( B^*_M = 0 \). All users will use femtocell service, i.e., users’ equilibrium partition threshold \( \theta^*_{th} = 1 \). The equilibrium femtocell price is

\[
p^*_F = \frac{1}{1 + B F K}.
\]

(3.16)

and the operator’s equilibrium profit is

\[
\pi^{\text{operator}} = \frac{B F K}{1 + B F K}.
\]

(3.17)

which is increasing in the ratio \( N/K \).

By comparing (3.17) to (3.14), we conclude that the operator obtains a
larger profit by adopting frequency reuse for femtocells. This is consistent with the current engineering practice of reducing cell size to increase frequency reuse and improve network capacity. Apparently, smaller femtocell size means a larger $N$, and thus a larger capacity increasing ratio $N/K$ and a larger profit.

### 3.6 Impact of Femtocell Operational Cost

In Section 3.3, we have assumed that there is no additional operational cost of the femtocell service. The data from the femtocells will be delivered through the wireline Internet connection of an ISP back to the operator’s cellular network, free of charge. However, this is only reasonable when the operator and the ISP belong to the same entity or the ISP is sharing-friendly as in [127,128].

In this section, we consider the case where the ISP will charge the operator usage-based fees for using the wireline Internet connection in downloading femtocell users’ traffics from Internet. We are interested in understanding how this additional operational cost affects the provision of femtocell service.

For simplicity, we assume that the operational cost is linearly proportional to femtocell bandwidth with the coefficient $C$. This assumption is reasonable if the ISP adopts the usage-based charging that is linear in femtocell users’
total traffic volume.\textsuperscript{10} Recall that in Section 3.3 we have shown each femtocell user’s data rate and traffic volume are linear in its bandwidth demand, thus all femtocell users’ total traffic volume is also linear in their total bandwidth demand. We focus on the case of $C \in (0, 1)$. It is easy to show that if $C \geq 1$, then the operator will charge a femtocell price $p_F > 1$, and no user will choose the femtocell service based on (3.8). In other words, an operational cost $C \geq 1$ means no femtocell service.

We consider a two-stage decision process similar as Fig. 3.3. The analysis of Stage II is the same as in Section 3.3.1. Here we will focus on the operator’s decisions on $B_M$, $B_F$, $p_M$, and $p_F$ in Stage I. Following a similar analysis as in Section 3.3, we can show that the total bandwidth $B$ equals users’ total finalized demand at the equilibrium (i.e., Lemma 4). Then we can formulate the operator’s profit-maximization problem as

$$
\max_{p_M, p_F, \theta_{th} \in [0, 1]} \pi_{operator}(p_M, p_F, \theta_{th}) = (p_F - C) \theta_{th} \left( \frac{1}{p_F} - 1 \right) + p_M \left( \frac{1 - \theta_{th}}{p_M} + \ln \theta_{th} \right)
$$

subject to $p_M \leq \theta_{th} \leq 1$,

$$
\theta_{th} \left( \frac{1}{p_F} - 1 \right) + \frac{1 - \theta_{th}}{p_M} + \ln \theta_{th} = B. \quad (3.18)
$$

Then we have the following result.

\textsuperscript{10}Even if the ISP adopts other usage-based charging that is non-linear in femtocell traffics, we can still show that the main insights in this section (e.g., Theorem 7) still holds. Moreover, besides the usage-based charging, even if we also consider some immediate investment cost (e.g., software upgrade and customer support), the results in this section can be extended as that cost would not affect the operator’s decisions.
Figure 3.9: The equilibrium femtocell price $p_F^*$ and macrocell price $p_M^*$ as functions of supply $B$ and cost $C$ in dual services considering femtocell operational cost.

**Theorem 7.** With a femtocell operational cost $C \in (0, 1)$, the operator always provides both femtocell service and macrocell service at the equilibrium, and $p_M^* \leq \theta_{th}^* < 1$.

The proof of Theorem 7 is given in [160]. Note that $p_M^*$ is the equilibrium macrocell price, and $\theta_{th}^*$ is the users’ equilibrium partition threshold with dual services. Intuitively, a positive operational cost $C$ forces the operator to charge a higher femtocell price $p_F^*$ than the value in (3.13). However, the small payment from users with a large value of $\theta$ (who only experience a little QoS improvement) in the femtocell service cannot cover the increased operational cost to the operator. As a result, the operator will serve these users by the macrocell service.

Problem (3.18) is difficult to solve in closed form, so we use numerical results
Figure 3.10: The users’ equilibrium partition threshold $\theta_{th}^*$ as a function of supply $B$ and cost $C$ in dual services considering femtocell operational cost.

to illustrate some interesting insights.

Figure 3.9 shows the operator’s equilibrium femtocell price $p_F^*$ and macrocell price $p_M^*$ as functions of bandwidth supply $B$ and femtocell operational cost $C$. The femtocell price $p_F^*$ is always larger than $C$ in order to be profitable. The macrocell price $p_M^*$ does not need to compensate any cost. When $B$ increases, the operator can set $p_M^*$ as low as needed to maximize its profit, but $p_F^*$ is lower-bounded by $C$. This explains why the $p_F^* - p_M^*$ widens as $B$ increases, and such gap becomes even bigger with a larger $C$.

Figure 3.10 shows users’ equilibrium partition threshold $\theta_{th}^*$ as a function of $B$ and $C$. The threshold $\theta_{th}^*$ decreases in both $B$ and $C$, which means that more users will choose to use the macrocell service due to the increase of $p_F^* - p_M^*$.

Figure 3.11 shows the operator’s equilibrium bandwidth allocations to dual
Figure 3.11: The equilibrium femtocell band $B_F^*$ and macrocell band $B_M^*$ as functions of supply $B$ and cost $C$ in dual services considering femtocell operational cost.

Figure 3.12 shows that the operator’s equilibrium profit increases in $B$ and
CHAPTER 3. ECONOMICS OF FEMTOCELL SERVICE PROVISION

3.7 Impact of Limited Femtocell Coverage

In Section 3.3, we have assumed that femtocell service has the same maximum coverage as the macrocell service. In this section, we relax this assumption and consider that femtocell service only covers a small percentage of the macrocell coverage area as illustrated in Figure 3.13.\footnote{For simplicity, we assume that multiple femtocells cannot reuse the same frequency bands.} We will try to understand how the limited coverage affects the provision of femtocell service. We still consider a two-stage decision process similar to Fig. 3.3.

We assume that the femtocell service covers $\eta \in (0, 1)$ portion of the macro-cell coverage area. If users are uniformly distributed in space, then only $\eta$ por-
Figure 3.13: Femtocells cover a limited area compared with the macrocell coverage.

...tion of all users can potentially access both services (called overlapping users). The rest $1 - \eta$ portion of users can only access the macrocell service (called non-overlapping users).

Our results in Section 3.3 show that the operator wants to serve all overlapping users by the more efficient femtocell service if possible. The operator can try to achieve this via two approaches. The first approach is to announce the same price for both macrocell and femtocell services, which makes the macrocell service less attractive than the femtocell service to all overlapping users. This approach, however, means that the macrocell price is too high, and may not be most profitable when $\eta$ is small. The second approach is to allocate all bandwidth $B$ to the femtocell service. However, this may not be most profitable either, since the non-overlapping users will be out of service.

Recall that the operator serves macrocell users with good signals (i.e., a
larger $\theta$) first, thus some users with a small $\theta$ will not be able to receive the macrocell service (even if they prefer so) if the macrocell band is not enough. Let us denote $\theta_{\text{th}}^{\text{non}}$ as the minimum spectrum efficiency among all non-overlapping users served by macrocell service. If a non-overlapping user with $\theta$ is served by macrocell service, another overlapping user with the same $\theta$ should also be able to request and obtain macrocell service successfully. This is because the operator cannot distinguish whether a user is in the femtocell coverage or not. Moreover, the operator wants to serve as many overlapping users as possible by the more efficient femtocell service. Thus we conclude that the partition threshold of overlapping users is $\theta_{\text{th}} = \max(\theta_{\text{th}}^{\text{non}}, p_M/p_F)$.

The total finalized demand of the macrocell service is

\[
Q_M(p_M, p_F, \theta_{\text{th}}^{\text{non}}) = (1 - \eta) \int_{\theta_{\text{th}}^{\text{non}}}^{1} \left( \frac{1}{p_M} - \frac{1}{\bar{\theta}} \right) d\theta + \eta \int_{\max(\theta_{\text{th}}^{\text{non}}, p_M/p_F)}^{1} \left( \frac{1}{p_M} - \frac{1}{\bar{\theta}} \right) d\theta.
\]  

(3.19)

We can show that the operator will allocate $B_M = Q_M(p_M, p_F, \theta_{\text{th}}^{\text{non}})$ for the macrocell service, and will allocate the remaining bandwidth $B_F = B - Q_M(p_M, p_F, \theta_{\text{th}}^{\text{non}})$ to the femtocell service to serve overlapping users with $\theta \in [0, \theta_{\text{th}}]$. Following a similar analysis as in Section 3.3, we can show that the total bandwidth $B$ equals users’ total finalized demand at the equilibrium. That is,

\[
B = Q_M(p_M, p_F, \theta_{\text{th}}^{\text{non}}) + \eta \int_{0}^{\max(\theta_{\text{th}}^{\text{non}}, p_M/p_F)} \left( \frac{1}{p_F} - 1 \right) d\theta.
\]

(3.20)
Theorem 8. At the equilibrium, the operator will satisfy all users’ preferred demands by their preferred services. Non-overlapping and overlapping users’ service partition thresholds are $\theta_{\text{non}}^* = p_M^*$ and $\theta_{\text{th}}^* = p_M^*/p_F^*$. The proof of Theorem 8 is given in [160]. The operator will not leave any non-overlapping users who prefer to be served out of service. Intuitively, the operator wants to serve as many non-overlapping users as possible, which means that the macrocell users’ total demand is large and thus the operator can charge a higher macrocell price. The increase of macrocell price will encourage more overlapping users to choose the femtocell service, which will improve the operator’s profit.

Based on Theorem 8, the operator’s optimization problem is

$$
\max_{p_M, p_F \in [0,1]} \pi_{\text{operator}}(p_M, p_F) = p_F \eta \int_0^{p_M/p_F} \left( \frac{1}{p_F} - 1 \right) d\theta + p_M Q_M(p_M, p_F, \theta_{\text{non}}^*)|_{\theta_{\text{non}}^* = p_M^*},
$$

subject to,

$$p_M \leq p_F,$$

$$B = Q_M(p_M, p_F, \theta_{\text{non}}^*)|_{\theta_{\text{non}}^* = p_M^*} + \eta \int_0^{p_M/p_F} \left( \frac{1}{p_F} - 1 \right) d\theta. \quad (3.21)$$

Problem (3.21) is difficult to solve in closed form, so we use numerical results to illustrate some interesting insights.

Figure 3.14 shows the operator’s equilibrium femtocell price $p_F^*$ and macrocell price $p_M^*$ as functions of bandwidth supply $B$ and femtocell coverage $\eta$. Both equilibrium prices decrease with $B$ to ensure market clearance (supply
equals demand). For each value of coverage \( \eta \), we can observe two different behaviors depending on the value of \( B \):

- **Small \( B \) regime** (e.g., \( B \leq 0.1 \) under \( \eta = 0.2 \), \( B \leq 0.6 \) under \( \eta = 0.5 \), and \( B \leq 3.1 \) under \( \eta = 0.8 \)): the operator will announce the same price for both services, i.e., \( p_F^* = p_M^* \). This means that all overlapping users will be served by the femtocell service. Intuitively, small \( B \) implies limited total resource, and thus it is more important to serve all overlapping users with the more efficient femtocell service than to serve more non-overlapping users with the macrocell service.

- **Large \( B \) regime** (e.g., \( B > 0.1 \) under \( \eta = 0.2 \), \( B > 0.6 \) under \( \eta = 0.5 \), and \( B > 3.1 \) under \( \eta = 0.8 \)): In this case, we have \( p_M^* < p_F^* \). This means that the overlapping users will be served by both femtocell and macrocell services. As the total resource is abundant in this case, it becomes important to provide enough resource for the macrocell service and get enough revenue from the non-overlapping users as well. As the
operator cannot differentiate users based on their coverage areas, some non-overlapping users will also get served by the macrocell service.

We notice that the threshold between two regimes increases with $\eta$, i.e., from $B = 0.1$ when $\eta = 0.2$ to $B = 3.1$ when $\eta = 0.8$. As a larger $\eta$ means more overlapping users, it becomes more attractive to provide the more efficient femtocell service to all overlapping users by setting $p^*_F = p^*_M$.

Figure 3.15 shows the operator’s bandwidth allocations to two services as functions of $B$ and $\eta$. As femtocell coverage $\eta$ increases, the operator will serve more users in femtocells and will allocate more bandwidth to femtocells. Figure 3.16 shows that the operator’s equilibrium profit increases in both $B$ and $\eta$. The operator benefits from the availability to serve more users by the more efficient femtocell service.
CHAPTER 3. ECONOMICS OF FEMTOCELL SERVICE PROVISION

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8

Bandwidth supply $B$

Operator’s equilibrium profit

$\eta = 0.2$

$\eta = 0.5$

$\eta = 0.8$

Figure 3.16: The operator’s equilibrium profit as functions of femtocell coverage $\eta$ and total bandwidth $B$.

3.8 Summary

This chapter studies the economic incentive for a cellular operator to introduce the femtocell service on top of its existing macrocell service. We analyze the operator’s equilibrium decisions in terms of spectrum allocations and pricing of dual services. Compared to the macrocell service, we show that the femtocell service can attract more users at a higher price and increase the operator’s profit. However, the requirement of satisfying users’ reservation payoffs (i.e., what they can achieve without the femtocell service) prevents the operator from only providing femtocell service. We also study the impacts of femtocell frequency reuse, operational cost, and limited coverage on femtocell service provision, where the operator’s profit increases with frequency reuse, decreases in operational cost, and increases in femtocell coverage.
Chapter 4

Smartphone Collaboration on Social Applications

4.1 Introduction

As the emerging technologies (e.g., 4G LTE and femtocells) in cellular networks become more mature, users prefer data service to traditional voice service and smartphone is becoming the mainstream among mobile phones. According to a survey by ComScore in 2010, over 45.5 million people owned smartphones out of 234 million total mobile phone subscribers in the United States [161]. In March 2010, Berg Insight reported that global smartphone shipments increased 74% from 2009 to 2010 [162].

Given millions of smartphones sold annually, some recent phone applications start to utilize the power of smartphone users’ collaborations in cellular
networks [163, 164]. In such an application, there is a client (e.g., Apple or Google in the following examples) who wants to implement some application or service based on user collaborations. We can roughly categorize these applications in two types as follows.

In the first type of data acquisition application, a client wants to acquire enough data from smartphone users to build up a database. According to [164], Apple’s iPhone and Google’s Android smartphones regularly transmit their owners’ location data (including GPS coordinates) back to Apple and Google, respectively. For example, an Android phone collects its location data every few seconds and transmits the data to Google at least several times an hour. The phone also transmits back the name, location, and signal strength of any nearby Wi-Fi networks. After collecting enough location data from users, Google can successfully build a massive database capable of providing location-based services. One service can be live map of auto traffics, where the dynamics of users’ location data on a highway indicate whether there is a traffic jam. Another service can be constructing a large-scale public Wi-Fi map. According to [165], the global location-based service market is growing strongly, and its revenue is expected to increase from US$2.8 billions to US$10.3 billions between 2010 and 2015. In order to perform the above data acquisition, a lot of efforts need to be spent to get users’ consent and protect users’ privacy (e.g., [166–169]). When a user collaborates in this kind of applications, he will
incur a cost such as loss of privacy.

In the second type of distributed computing application, a client wants to solve complex engineering or commercial problems inexpensively using distributed computation power. Smartphones now have powerful and power-efficient processors (e.g., Dual-core A5 chip of Apple iPhone 4S), outstanding battery life, abundant memory, and open operating systems (e.g., Google Android) [170] that make them suitable for complex processing tasks. Since millions of smartphones remain unused most of the time, a client might want to solicit smartphone collaborations in distributed computing (e.g., [171,172]). In this case, a user’s collaboration cost may be due to loss of energy and reduction of physical storage.

In this chapter, we will design incentive mechanism for smartphone collaborations in both data acquisition and distributed computing applications. Then we can compare the similarity and difference in mechanism design for both applications. For each type of applications, we will consider different information scenarios, depending on what the client and users know. In particular, the client may or may not know each smartphone user’s characteristics such as collaboration costs and collaboration efficiencies.

The two types of applications have different requirements and lead to different models. In data acquisition applications, we consider a threshold-based revenue model, where a client can earn a fixed positive revenue only if he can
involve enough (larger than a threshold) smartphone users as collaborators, such that he can build a large enough database to support the application. Since data acquisition only requires simple periodic data reporting, we can assume that users are homogeneous in contribution and efficiency. In distributed computing applications, however, we consider a model where the client’s revenue increases in users’ efforts. Also, users are heterogeneous in computing efficiencies and should be treated differently. For example, the most efficient users should be highly rewarded to encourage them to undertake large tasks.

Our key results and contributions are as follows:

- *New reward-based collaboration mechanism for data acquisition*: In Section 4.2, we model the interactions between the client and users as a Stackelberg game. The client first announces the total reward to be shared among collaborators and the minimum number of collaborators needed. Each user needs to take other users’ decisions into account in estimating the shared reward and the chance of collaboration success.

- *Performance of reward-based mechanism*: Under complete information, the client will only involve users with the lowest costs by offering a small total reward. The client can achieve a similarly good performance under symmetrically incomplete information, when both the client and users do not know users’ cost information. But if users know their costs while the client does not (asymmetrically incomplete information), the client needs
to offer a large total reward to attract enough collaborators. Overall, users benefit from holding private information. Interestingly, the total reward may increase or decrease in the variance of users’ cost distribution, depending on how many collaborators are needed to successfully build the database.

- **New contract-based collaboration mechanism for distributed computing:**
  In Section 4.3, we use contract theory to study how a client efficiently decides different task-reward combinations for heterogeneous users.

- **Performance of contract-based mechanism:** Under complete information, the client involves a user type as long as the client’s preference of the type is larger than the user cost. All collaborators get a zero payoff. But if users have private information and can hold from the client, the client will conservatively target at a smaller group of efficient users with small costs. He has to give most benefits to the collaborators and a collaborator’s payoff increases in the computing efficiency.

### 4.1.1 Related Work

Our first collaboration model on data acquisition is closely related to the literature on location-based services (LBS) [173]. In LBS, a customer needs to report his current location to the database server in order to receive his desired service. Prior work are focusing on how to manage data and how customers can
safely communicate with the database server (e.g., [168, 169, 174]), especially when the massive database has already been built up. Other work considered the technical issues of data collection from users [174]. Our chapter focuses on the client’s problem of incentive mechanism design for attracting enough users (may or may not be LBS customers later) to provide location data, so that the client can build a LBS later on.

Our second collaboration model is relevant to mobile grid computing, which integrates mobile wireless devices into grid computing (e.g., [172, 175–177]). The main focus of mobile grid computing literature is on the technical issues of resource management or load balancing (e.g., [176, 177]). Only few results have considered (mobile) users’ incentives issues in joining in collaboration [172, 178–180]. Kwok et al. in [178] evaluate the impact of selfish behaviors of individual users in a Grid. Subrata et al. in [179] present a Nash bargaining solution for load balancing among multiple clients. Ghosh et al. in [172] and Sim in [180] use a two-player alternating bargaining model to study collaboration between clients and users. The novelty of our model is that a client interacts with all users simultaneously to distribute computing work, and users are heterogeneous in their computing efficiencies and costs. We propose a new contract-based mechanism that maximizes the client’s profit.¹

¹The design of contract-based mechanism here is similar to that in our previous work [181] in methodology, but that work focuses on a different problem on cooperative spectrum sharing and the derived mechanisms are significantly different.
It should also be noted that our model follows a principal-agent structure in different information scenarios, and is quite different from the P2P structure. Note that in a P2P network, each peer interacts with other peers to obtain local storage and uploading services [182], and the incentive mechanisms in P2P network cannot apply to our principal-agent model.

4.2 Collaboration on Data Acquisition Application

4.2.1 System Model of Data Acquisition

In this application, the client is interested in building up a database by collecting information from enough smartphone users. We consider a set $\mathcal{N} = \{1, \cdots, N\}$ of smartphones, and the total number $N$ is publicly known.\textsuperscript{2} User $i \in \mathcal{N}$ has a collaboration cost $C_i > 0$.\textsuperscript{3} We assume that the collaboration costs are independent and identically distributed, with a mean $\mu$ and a cumulative probability distribution function $F(\cdot)$.

We consider a threshold revenue model for the client. If the client attracts at least $n_0$ users as collaborators, he will successfully build the database and receive a revenue of $V$. Otherwise, the client does not receive any revenue.

\textsuperscript{2}We assume that all $N$ users are active. The client (e.g., Apple) can learn the number of active users (e.g., iPhones) by checking users’ usage history, or regularly send control messages to each user for status confirmation.

\textsuperscript{3}A user’s collaboration cost is determined by his privacy loss. The cost can be property loss due to disclosure of bank account information in data reporting to the client or frequent annoyance from unwanted advertising.
CHAPTER 4. SMARTPHONE COLLABORATION ON SOCIAL APPLICATIONS

The client interacts with the users through a two-stage process. In Stage I, the client announces \((R, n_0)\), where \(R\) is the total reward to all users and \(n_0\) is the threshold number of required collaborators. In Stage II, each user chooses to be a collaborator or not.

Assume that there are \(n\) users willing to serve as collaborators in Stage II. There are two models for a collaborator’s payoff:

- **Reward for collaboration effort**: A collaborator \(i\)’s payoff is
  \[
  \left[ \frac{R}{n} - C_i \right] \mathbf{1}_{\{n \geq n_0\}}.
  \]  
  \(\text{(4.1)}\)

  where \(\mathbf{1}_{\{A\}}\) is the indicator function (equals 1 when event \(A\) is true). That is, if the collaboration is successful, user \(i\) pays his collaboration cost \(C_i\), and gets the reward \(R/n\) (equally and fairly spitted among \(n\) collaborators since they undertake the same task in fixed and periodic data reporting).

  We can also view \(R/n\) as in a *lottery* scenario where each collaborator having equal probability \(1/n\) to win the total reward \(R\). In this case, \(n\) users will only collaborate if the client notifies them that \(n \geq n_0\) and the collaboration will be successful. This means that no users will pay collaboration cost if the collaboration is not successful. Here, we assume that the client will truthfully inform the collaborators about the value of \(n\).\(^4\)

\(^4\)In reality, the client may cheat users by announcing a larger value of \(n\), then he can give less reward to each collaborator. But there are some approaches to prevent this. For example, the users may require the client to open all collaborators’ identities for them to check.
• **Reward only with successful collaboration:** A collaborator $i$ receives a payoff
\[
\frac{R}{n} 1_{\{n \geq n_0\}} - C_i.
\] (4.2)

That is, collaborator $i$ always pays his collaboration cost $C_i$, and will get the reward $R/n$ only if the collaboration is successful. This model considers that collaborators will contribute before they know the value of $n$ (which will be announced to them by the client after data acquisition).

In both model, the client obtains a profit of
\[
(V - R) 1_{\{n \geq n_0\}}.
\]

For illustration purpose, we only focus on model (4.1) in this section. The discussion of model (4.2) can be found in Section 4.5.1. It should be noted that users under model (4.1) are more willing to collaborate than under model (4.2), which is not surprising since they face a lower risk in model (4.1). The client also prefers model (4.1) to model (4.2) since he needs to compensate lower risk and fewer cost to motivate users’ collaboration.

The *collaboration game* is a two-stage Stackelberg game [126]. The way to analyze Stackelberg game is backward induction. We will first analyze Stage II, where the users play a game among themselves based on the value of the reward $R$ and the threshold $n_0$.\(^5\) Users reach a *Nash equilibrium* (NE) in this

\(^5\)We consider that each user will join the collaboration as long as his payoff is nonnegative. Yet our results can still be generalized to the case where users have positive reserve payoffs.
stage, if no user can improve his payoff by changing his strategy (collaborate or not) unilaterally. The equilibrium in Stage II leads to a collaboration success probability $P(n \geq n_0; R)$. As we will see, there may be multiple Nash equilibria in Stage II. Then we study Stage I, where the client chooses the value of $R$ to maximize his expected profit $(V - R)P(n \geq n_0; R)$. These two-step analysis enables us to obtain an equilibrium of the whole collaboration game.

Next we will analyze the Stackelberg game, and study how the client’s and the users’ information about the collaboration costs will affect the outcome.

### 4.2.2 Collaboration under Complete Information

We first consider the complete information scenario, where the client and all users know the cost $C_i$ of every user $i \in \mathcal{N}$. This is possible when the client and users have extensive prior collaboration experiences. The equilibrium of the collaboration game is as follows.

**Theorem 9 (Collaboration under Complete Information).** Let $C_0$ be the $n_0$-th smallest collaboration cost among all $N$ users. The collaboration game admits the following unique pure strategy equilibrium.

- If $V < n_0 C_0$, then the client does not initiate the collaboration in Stage I (i.e., setting $R^* = 0$). No user will become collaborator in Stage II.

- If $V \geq n_0 C_0$, the client offers a reward $R^* = n_0 C_0$ in Stage I. In Stage II,

---

$^6$We assume that no two users have the exactly same cost, as there are infinite possible values for the cost.
every user $i$ with $C_i \leq C_0$ collaborates and obtains a nonnegative payoff $C_0 - C_i$, and the remaining $N - n_0$ users decline to collaborate and get a zero payoff. The profit of the client is $V - n_0C_0$.

The proof of Theorem 9 is given in Section 4.5.2. We can show that users will not benefit from using a mixed-strategy. But this may not be the case with symmetrically incomplete information.

4.2.3 Collaboration under Symmetrically Incomplete Information

Now we consider the symmetrically incomplete information scenario, where both the client and the users only know the cumulative probability distribution function $F(\cdot)$ of the collaboration costs.\(^7\) A user $i$ even does not know the precise value of his own cost $C_i$.\(^8\) In this case, we can view all users as homogeneous.

Analysis of Stage II

It turns out that there are multiple equilibria of the collaboration game in Stage II as follows.

\(^7\)The client can estimate $F(\cdot)$ by learning from his collaboration history or making a customer survey. A user can estimate $F(\cdot)$ by checking his or other users’ collaboration experiences. There are many public sources (e.g., the client’s or some third party’s market or customer surveys) that help a user’s cost estimation [165,174].

\(^8\)It is sometimes difficult for a user to know his precise loss of privacy before an actual security threat happens to him. Users may face many possible security threats by losing sensitive information, e.g., direct property loss or advertising harassment.
Theorem 10. (Stage II under Symmetrically Incomplete Information): Stage II admits the following Nash equilibria:

- (No collaboration): If $R < n_0 \mu$, no user will collaborate at any equilibrium in Stage II.

- (Pure strategy NE): If $n_0 \mu \leq R < N \mu$, $n^* = \lfloor \frac{R}{\mu} \rfloor$ users choose to collaborate and the remaining users decline. If $R \geq N \mu$, all $N$ users will collaborate.

- (Mixed strategy NE): If $n_0 \mu < R < N \mu$, every user collaborates with a probability $p^*$, which is the unique solution to

$$E_m \left( \left[ \frac{R}{m+1} - \mu \right] \mathbf{1}_{(m+1 \geq n_0)} \right) = 0,$$

where the expectation $E$ is taken over the random variable $m$ which follows a binomial distribution $B(N-1,p)$.

The proof of no collaboration and pure strategy NE are given in Section 4.5.3.

We note that the pure and mixed strategy equilibria in Theorem 10 share a common parameter range, $n_0 \mu < R < N \mu$.

In the pure strategy NE, a subset of $n^*$ users is picked up among $\binom{N}{n^*}$ possible subsets. Thus there exist multiple pure NEs in this case.\(^9\)

\(^9\)How to select one NE is out of the scope of this chapter, and can be referred to [126]. It should be noted that the client is not interested in selecting a certain NE since all NEs give him the same performance. Furthermore, Theorem 11 shows that the client will not encourage any mixed NE at the first place.
Next we show how the mixed strategy NE $p^*$ is derived. As all users have the same statistical information, we will focus on the symmetric mixed Nash equilibrium. Assume that all users collaborate with a probability $p$. If user $i$ collaborates, his expected payoff is

$$u(R,p) := \mathbb{E}_m \left( \left[ \frac{R}{m + 1} - \mu \right] 1_{\{m+1 \geq n_0\}} \right),$$

where $m$ is the number of users (other than $i$) who collaborate and the expectation is taken over $m$. Note that $m$ follows a binomial distribution $B(N - 1, p)$, and is independent of user $i$’s decision.

Given all the other $N - 1$ users collaborate with the equilibrium probability $p^*$, user $i$’s payoffs by choosing to collaborate or not are the same. Thus $p^*$ should satisfy

$$u(R, p^*) = 0,$$

and is a function of $R$. Thus we can rewrite $p^*$ as $p^*(R)$. One can show that there exists a mixed strategy Nash equilibrium $p^*(R) \in (0,1)$ as long as $n_0 \mu < R < N \mu$. Note that $R \leq n_0 \mu$ leads to $p^*(R) = 0$, which is not a mixed strategy. Also, $R \geq N \mu$ leads to $p^*(R) = 1$, which is not a mixed strategy either.

**Analysis of Stage I**

First we consider the case where users use the mixed strategy in Theorem 10
and collaborate with probability \( p^*(R) \). The client’s expected profit is then

\[
f(R) := \mathbb{E}_n \left( |V - R| 1_{\{n \geq n_0\}} \right),
\]

where the expectation is taken over \( n \) which follows a binomial distribution \((N, p^*(R))\). One can show that \( f(R) \) has a unique maximum \( f(R^*) \), which is positive when \( V > n_0 \mu \). However, under \( n_0 \mu < R < N \mu \) there is always a chance that there are less than \( n_0 \) users choosing to collaborate under the mixed strategy. Thus the client may want to avoid this. Theorem 10 shows that by choosing \( R = n_0 \mu \), the client can guarantee \( n_0 \) collaborators with a pure strategy Nash equilibrium in Stage II. This leads to the following result.

**Theorem 11.** (Stage I under Symmetrically Incomplete Information:) The collaboration game admits the following unique equilibrium.

- If \( V < n_0 \mu \), the client will not initiate the collaboration and choose \( R^* = 0 \).

- If \( V \geq n_0 \mu \), the client will announce a reward \( R^* = n_0 \mu \). A set of \( n_0 \) users will collaborate in Stage II. The collaborators achieve a zero expected payoff, and the client achieves a profit \( V - n_0 \mu \).

### 4.2.4 Collaboration under Asymmetrically Incomplete Information

In this subsection, we study the case where each user \( i \) knows his own exact cost \( C_i \), but not other users’ costs. The client only knows \( F(\cdot) \).
Analysis of Stage II

We have the following result for Stage II.

**Theorem 12.** (Stage II under Asymmetrically Incomplete Information): A user \( i \) will collaborate if and only if \( C_i \leq \gamma^*(R) \). The common equilibrium decision threshold \( \gamma^*(R) \) is the unique solution of \( \Phi(\gamma) = 0 \), where

\[
\Phi(\gamma) := \mathbb{E}_m \left( \left[ \frac{R}{m + 1} - \gamma \right] 1_{\{m+1 \geq n_0\}} \right),
\]

and the expectation is taken over \( m \) which follows a binomial distribution \( B(N - 1, F(\gamma)) \). The equilibrium \( \gamma^*(R) \) satisfies \( \frac{R}{N} < \gamma^*(R) < \frac{R}{n_0} \).

To see why Stage II has the pure NE in Theorem 12, we consider that all users other than \( i \) collaborate if and only if their costs are less than some threshold \( \gamma > 0 \). If user \( i \) collaborates, his payoff is

\[
\left[ \frac{R}{m + 1} - C_i \right] 1_{\{m+1 \geq n_0\}},
\]

where \( m \) follows a binomial distribution \( B(N - 1, F(\gamma)) \) and represents the number of users (other than \( i \)) who collaborate. (Recall that cdf \( F(\gamma) = P(C_i \leq \gamma) \).) Accordingly, the expected payoff of user \( i \) if he collaborates is

\[
\mathbb{E}_m \left( \left[ \frac{R}{m + 1} - C_i \right] 1_{\{m+1 \geq n_0\}} \right),
\]

and zero otherwise. At the Nash equilibrium, (4.5) should equal to 0 when \( C_i = \gamma \). That is, having the common collaboration threshold \( \gamma \) is a Nash
Figure 4.1: $\Phi(\gamma)$ as a function of $\gamma$ and $R$. Other parameters are $n_0 = 40$ and $N = 100$. We consider a uniform cost distribution with $F(\gamma) = \min(\gamma/4, 1)$.

equilibrium if and only if $\Phi(\gamma) = 0$. We denote the solution to (4.4) as $\gamma^*(R)$.

In Section 4.5.4, we prove that there always exists a unique $\gamma^*(R)$, which satisfies $\frac{R}{N} < \gamma^*(R) < \frac{R}{n_0}$.

Figure 4.1 shows $\Phi(\gamma)$ as a function of both $\gamma$ and $R$. The solution $\gamma^*(R)$ to $\Phi(\gamma) = 0$ is always unique and satisfies $\frac{R}{N} < \gamma^*(R) < \frac{R}{n_0}$.\textsuperscript{11} When $R = 100$, for example, we have $\gamma^*(R) = 2$, which is larger than $R/N = 1$ and is smaller than $R/n_0 = 2.5$. It is also interesting to notice that all users share the same decision threshold $\gamma^*(R)$ although they have different costs.

**Theorem 13.** The equilibrium decision threshold $\gamma^*(R)$ increases in $R$, and

\textsuperscript{11}It should be noted that $\gamma^*(R)$ should always be positive, otherwise no users will collaborate and there exists no decision threshold then.
decreases in \( N \) and \( n_0 \).

The proof of Theorem 13 is given in Section 4.5.5.

Intuitively, as \( N \) or \( n_0 \) increases, more users will participate in the collaboration and thus the shared reward per collaborator decreases. As a result, the decision threshold decreases, and each user is less likely to collaborate.

**Analysis of Stage I**

We are now ready to consider Stage I. Given users’ equilibrium strategies based on threshold \( \gamma^*(R) \) in Stage II in Theorem 12, the client chooses reward \( R \) to maximize his expected profit, i.e.,

\[
\max_R f(R) = \mathbb{E}_n \left( (V - R) \mathbf{1}_{\{n \geq n_0\}} \right),
\]

where the expectation is taken over \( n \) which follows a binomial distribution \( B(N, F(\gamma^*(R))) \). A smaller reward \( R \) leads to a larger value of \( V - R \), but decreases the collaboration success probability \( P(n \geq n_0; R) \).

Let us denote the client’s equilibrium choice of reward in Stage I as \( R^* \), which is derived by solving Problem (4.6).

**Theorem 14.** The equilibrium expected profit \( f(R^*) \) of the client increases in \( V \) and \( N \), and decreases in \( n_0 \).

The proof of Theorem 14 is given in Section 4.5.6.

As the client’s revenue \( V \) increases, he benefits more from the collaboration. As the threshold \( n_0 \) increases, however, each user is less likely to collaborate.
Figure 4.2: Client’s expected profit $f(R)$ as a function of $R$ and $N$. Other parameters are $n_0 = 30$ and $V = 100$. Also, we consider a uniform cost distribution with $F(\gamma) = \min(\gamma/3, 1)$.

Thus the client has to give a larger total reward to attract enough collaborators. This decreases his equilibrium expected profit.

Figure 4.2 shows that the client’s expected profit $f(R)$ as a function of $R$ and $N$. We can see that both $f(R)$ and the equilibrium $f(R^*) = \max_R f(R)$ are increasing in $N$. Intuitively, as $N$ increases, more users have small collaboration costs (as the cdf function $F(\cdot)$ does not change), and more users will collaborate under the same total reward. Thus the client can lower the equilibrium reward $R^*$ and obtain a larger profit.

Next we study how the the client’s equilibrium total reward and profit change with the cdf function $F(\cdot)$ of a user’s collaboration cost. To deliver
Figure 4.3: Client’s equilibrium reward $R^*$ as a function of variance $\delta$ and $n_0$. Other parameters are $N = 80, V = 250, \mu = 3$.

By clean engineering insights, we pick uniform distribution for example, which can be explicitly characterized by mean $\mu$ and variance $\delta$ only.

Observation 4. The client’s equilibrium total reward $R^*$ increases in mean $\mu$ (and his profit decreases in $\mu$). Moreover, as variance $\delta$ increases, the client who strictly requires a large collaborator requirement $n_0$ (compared to $N$) has to announce larger reward $R^*$, whereas the client requiring a small requirement $n_0$ only needs to announce smaller reward $R^*$.

The relationship between $R^*$ and $\mu$ is quite intuitive, as the client needs to decide a larger $R^*$ to compensate each collaborator’s increased cost in expected sense. We next elaborate the impact of $\delta$ on $R^*$.

Figure 4.3 shows $R^*$ as a function of variance $\delta$ and $n_0$, where the client with
smaller $n_0$ requirement can efficiently build the database with smaller reward $R^*$. When the client requires a large $n_0$, he needs to incentivize most users to join the collaboration. As $\delta$ increases, some users are more likely to realize much larger costs than $\mu$ and the conservative client still needs to incentivize them. Thus $R^*$ increases in $\delta$ in this case. When the client only requires a small $n_0$, he can target at those users with smallest costs. As $\delta$ increases, these users are more likely to have much smaller costs than $\mu$ and the client only needs to decide smaller $R^*$ to incentivize them.

We can similarly show in Fig. 4.4 that as $\delta$ increases, the client with large $n_0$ requirement obtains smaller profit $f(R^*)$, whereas the client with small $n_0$ requirement obtains larger $f(R^*)$. Notice that $f(R^*)$ decreases in requirement $n_0$, which is consistent with Theorem 14.

By comparing the performances of the client and users under complete information, symmetrically incomplete information, and asymmetrically incomplete information, we have the following result.

**Theorem 15.** At the equilibrium of the collaboration game, the client obtains the smallest expected profit under asymmetrically incomplete information, whereas the users obtain the smallest (zero) expected payoffs under symmetrically incomplete information.

Theorem 15 shows that the users benefit from knowing their own costs, while the client incurs profit loss when the users know their costs and can hide
Figure 4.4: Client’s equilibrium reward $R^*$ as a function of variance $\delta$ and $n_0$. Other parameters are $N = 80$, $V = 250$, $\mu = 3$.

Recall that the client obtains an expected profit $V - n_0\mu$ under symmetrically incomplete information, and obtains a profit $V - n_0C_0$ under complete information. The relation between these two values depends on $N$, $n_0$, and $F(\cdot)$. Take the uniform distribution $F(\cdot)$ as an example. If $n_0$ is much smaller than $N/2$, the expected value of $C_0$ will be smaller than $\mu$ and the client is better off under complete information.
4.3 Collaboration on Distributed Computing Application

4.3.1 System Model on Distributed Computation

In this type of applications, the client solicits the collaboration of users to perform distributed computing. Different from requiring fixed and periodic data reporting as in data acquisition applications, the client here can assign different amounts of work to different user types. Smartphones are generally different in terms of CPU performance, memory and storage, battery life, and connectivity [175]. Even with the same smartphones, two users may have different phone usage behaviors and different sensitivities (e.g., to power consumption).

We consider a total of $N$ users belonging to a set $\mathcal{I} = \{1, \cdots, I\}$ of $I$ types. Each type $i \in \mathcal{I}$ has $N_i \geq 1$ users, with $\sum_{i \in \mathcal{I}} N_i = N$. A type-$i$ user can perform at most $\bar{t}_i$ units of work, and faces a cost $K_i$ per unit of work he performs. The upper bound of $\bar{t}_i$ reflects the limited battery capacity, time constraint, or other physical constraints. Users know their unit costs before the collaboration, since (i) many factors of these costs (e.g., power consumption) are explicitly reflected by smartphones’ technical specifications, and (ii) users explicitly know their own sensitivities (e.g., to power consumption) in costs. This is different from data acquisition, where costs mainly come from implicit insecurity.
The payoff of a type-\(i\) user who accomplishes \(t\) units of work and receives a reward \(r\) from the client is

\[
    u_i(r, t) = r - K_i t, \text{ for } 0 \leq t \leq \bar{t}_i. \tag{4.7}
\]

Note that the user can always choose not to collaborate with the client and thus receive zero payoff with \(t = r = 0\). Without loss of generality, we order user types in the descending order of the unit cost, i.e., \(K_1 > K_2 > \ldots > K_I\), i.e., a higher type of user has a smaller cost.

By asking each type-\(i\) user to accomplish the amount \(t_i\) of work and rewarding him with \(r_i\), the client’s profit is

\[
    \pi(\{(r_i, t_i)\}_{i \in I}) = \sum_{i \in I} (\theta_i \log(1 + N_i t_i) - N_i r_i). \tag{4.8}
\]

The term \(\theta_i \log(1 + N_i t_i)\) is increasing in users’ efforts and well characterizes the client’s diminishing return (or utility) from the total work \(N_i t_i\) finished by type-\(i\) (as in [183, 184]).\(^\dagger\) The parameter \(\theta_i > 0\) characterizes the client’s preference for work performed by type-\(i\) users, and does not depend on \(K_i\). In particular, \(\theta_i\)’s may or may not be decreasing in \(i\). The term \(N_i r_i\) in (4.8) is the total reward that the client offers to type-\(i\) users. The summation operation in (4.8) is motivated by the fact that many complex engineering or commercial problems can be separated into multiple subproblems and solved in a distributed manner [172].

\(^\dagger\)The assumed logarithmic utility term helps us derive closed-form solutions and engineering insights. Using other concave terms are not likely to change the main conclusions.
By examining (4.7) and (4.8), we can see that the client and users have conflicting objectives. The client wants users to accomplish a larger task, which increases the client’s utility as well as users’ collaboration costs. Users want to obtain a larger reward, which decreases the client’s profit. Next we study how client and users interact through a contract.

4.3.2 Contractual Interactions between Client and Users

Contract theory studies how an economic decision-maker constructs contractual arrangements, especially in the presence of asymmetric (private) information [185]. In our case, the user types are private information.

The client proposes a contract that specifies the relationship between a user’s amount of task $t$ and reward $r$. Specifically, a contract is a set $C = \{(t_1, r_1), \ldots, (t_M, r_M)\}$ of $M \geq 1$ (amount of task, reward)-pairs that are called contract items. The client proposes $C$. Each user selects a contract item $(t_m, r_m)$ and performs the amount of work $t_m$ for the reward $r_m$. According to [185], it is optimal for the client to design a contract item for each type, i.e., $M = I$. Note that a user can always choose not to work for the client, which implies an implicit contract item $(r, t) = (0, 0)$ (often not counted in the total number of contract items). Once a user accepts some contract item, he needs to accomplish the task and the client needs to reward him according to that item.

Each type of users selects the contract item that maximizes his payoff in
(4.7). The client wants to optimize the contract items and maximize his profit in (4.8). We will again focus on a two-stage Stackelberg game, where the client proposes the contract first and users choose the contract items afterwards.

Next, we study how the client determines the contract that maximizes his profit, depending on what information he has about the users’ types. As explained in the beginning of Section 4.3.1, we assume that a user knows his unit cost. This means that we only need to consider two information scenarios, complete information and asymmetrically incomplete information, depending on what the client knows.

### 4.3.3 Contract Design under Complete Information

In this subsection, we study the case where the client knows the type of each user. This makes it possible for the client to monitor and make sure that each type of users accepts only the contract item designed for that type. The client needs to ensure that each user has a non-negative payoff so that the user will accept the contract. In other words, the contract should satisfy the following individual rationality constraints.

**Definition 4 (IR: Individual Rationality).** A contract satisfies the individual rationality constraints if each type-i user receives a non-negative payoff by accepting the contract item for type-i, i.e.,

\[ r_i - K_i t_i \geq 0, \ \forall i \in \mathcal{I}. \]  

(4.9)
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Under complete information, the optimal contract \( C = \{(r_i^*, t_i^*)\}_{i \in \mathcal{I}} \) solves the following problem:

\[
\max_{\{(r_i, t_i)\}_{i \in \mathcal{I}}} \pi(\{(r_i, t_i)\}_{i \in \mathcal{I}}) = \sum_{i \in \mathcal{I}} (\theta_i \log(1 + N_i t_i) - N_i r_i),
\]

subject to: IR constraints (4.9) and \( 0 \leq t_i \leq \bar{t}_i, \forall i \in \mathcal{I} \). (4.10)

It is easy to check that the IR constraints are tight at the optimal solution to Problem (4.10), and the client will leave a zero payoff to each type-\( i \) user with \( r_i^* = K_i t_i^* \). Also, due to the independence of each type in Problem (4.10), we can decompose Problem (4.10) into \( I \) subproblems. For each type \( i \in \mathcal{I} \), the client needs to solve the following subproblem

\[
\max_{t_i} \pi_i(t_i) = \theta_i \log(1 + N_i t_i) - N_i K_i t_i,
\]

subject to: \( 0 \leq t_i \leq \bar{t}_i \). (4.11)

By solving all \( I \) subproblems, we have the following result.

**Theorem 16 (Optimal Contract under Complete Information).** At the equilibrium, the client will hire the type-\( i \) users if \( \theta_i > K_i \). The total involved user type set is

\[
\mathcal{I}_C = \{ i \in \mathcal{I} : \theta_i > K_i \}.
\] (4.12)

The subscript \( C \) in \( \mathcal{I}_C \) refers to the complete information assumption. For a user with type \( i \in \mathcal{I}_C \), the equilibrium contract item is

\[
(r_i^*, t_i^*) = (K_i t_i^*, t_i^*) = \left( \min \left( \frac{\theta_i - K_i}{N_i}, K_i \bar{t}_i \right), \min \left( \frac{\theta_i - K_i}{K_i N_i}, \bar{t}_i \right) \right). \] (4.13)
For a user with type \( i \notin I_C \), the equilibrium contract item is \((r^*_i, t^*_i) = (0, 0)\).

All users (no matter joining collaboration or not) receive a zero payoff. The client’s equilibrium profit is

\[
\pi^* = \sum_{i \in I_C} \min \left( \theta_i \log \left( \frac{\theta_i}{K_i} \right) - \theta_i + K_i, \theta_i \log(1 + N_i \bar{t}_i) - N_i K_i \bar{t}_i \right). \tag{4.14}
\]

**Proof.** By observing Problem (4.11), the client will only hire type-\( i \) users when his marginal utility is larger than marginal cost (i.e., reward to users) at \( t_i = 0 \). That is,

\[
\left. \frac{d\pi_i(t_i)}{dt_i} \right|_{t_i=0} = \left( \frac{N_i \theta_i}{1 + N_i \bar{t}_i} - N_i K_i \right) \bigg|_{t_i=0} = N_i (\theta_i - K_i) > 0,
\]

Thus the client will hire type-\( i \) users only when \( \theta_i > K_i \). Since \( \pi_i(t_i) \) is concave in \( 0 \leq t_i \leq \bar{t}_i \), we can directly examine the first-order condition of \( \pi_i(t_i) \) over \( t_i \) for each type. Then we can derive the equilibrium contract item for type-\( i \) in (4.13).

By substituting all contract items into the objective function in Problem (4.10), we can further derive the client’s equilibrium profit in (4.14).

Intuitively, the client needs to compensate a collaborator’s cost, thus he will hire type-\( i \) users only when his preference characteristic \( \theta_i \) is larger than the unit cost of that type \( K_i \). Users will receive a zero payoff since their private information about unit costs are known to the client.

By looking into all parameters in the equilibrium contract in (4.13) and payoff \( \pi^* \) in (4.14), we have the following observation.
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Observation 5. For $i \in I_C$, the equilibrium task $t^*_i$ to a type-$i$ user increases in $\theta_i$, and decreases in $N_i$ and $K_i$. Also, the client’s equilibrium profit $\pi^*$ increases in $\theta_i$, $N_i$, and $\bar{t}_i$, and decreases in $K_i$.

By looking into (4.13), we also have the following result.

Observation 6. The client may or may not offer a larger task or reward to a higher type-$i$ collaborator, depending on the number of collaborators $N_i$ and the client’s preference characteristic $\theta_i$ for that type.

Notice that a higher type-$i$ collaborator has less unit cost where the client needs to compensate, but the client may not give him a larger task or reward. This can happen when there are too many collaborators of that type, or the client evaluates this type with a small value of $\theta_i$.

4.3.4 Client’s Contract Design under Asymmetrically Incomplete Information

In this subsection, we study the case where the client only has asymmetrically incomplete information about each user’s type. A user’s actual type is only known to himself, and the client and the other users only have a rough estimation on this. We consider that others believe a user belonging to type-$i$ with a probability $q_i$. Everyone knows the total number of users $N$.\footnote{Users can know $N$ by checking the client’s or some third party’s market survey, or the news on recent penetration or shipment of smartphones.}
Feasibility of contract under asymmetrically incomplete information

According to [185], the client’s contract should first be feasible in this scenario. A feasible contract must satisfy both individual rationality (IR) constraints (Definition 4 in Section 4.3.3) and incentive compatibility constraints defined as follows.

**Definition 5 (IC: Incentive Compatibility).** A contract satisfies the incentive compatibility constraints if each type-i user prefers to choose the contract item for his own type, i.e.,

\[ r_i - K_i t_i \geq r_j - K_i t_j, \quad \forall i, j \in I. \] (4.15)

Under asymmetrically incomplete information, the client does not know the number of users \( N_i \) of type-\( i \). Let us denote the users’ numbers of all types as \( \{n_i\}_{i \in I} \), which are random variables following certain distributions and satisfying \( \sum_{i \in I} n_i = N \). Note that the realizations of \( \{n_i\}_{i \in I} \) depend on \( N \) and probabilities \( \{q_i\}_{i \in I} \) of all types that a user may belong to. The client’s profit for a particular realization of \( \{n_i\}_{i \in I} \) is

\[ \pi(\{(r_i, t_i)\}_{i \in I}, \{n_i\}_{i \in I}) = \sum_{i \in I} (\theta_i \log(1 + n_i t_i) - n_i r_i). \] (4.16)
Thus the client’s expected profit is

$$E_{\{n_i\}_{i \in I}}[\pi(\{(r_i, t_i)\}_{i \in I}, \{n_i\}_{i \in I})]$$

\[
= \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \cdots \sum_{n_{I-1}=0}^{N-\sum_{j=2}^{I} n_j} \frac{N!q_1^{n_1} \cdots q_{I-1}^{n_{I-1}} q_I^{N-\sum_{j=1}^{I-1} n_j} n_1! \cdots n_{I-1}! (N - \sum_{j=1}^{I} n_j)!}{\pi(\{(r_i, t_i)\}_{i \in I}, \{n_i\}_{i \in I})}.
\]

\[(4.17)\]

The client’s profit optimization problem as

$$\max_{\{(r_i, t_i)\}_{i \in I}} \mathbb{E}_{\{n_i\}_{i \in I}}[\pi(\{(r_i, t_i)\}_{i \in I}, \{n_i\}_{i \in I})]$$

subject to: IR constraints in (4.9),

IC constraints in (4.15),

$$0 \leq t_i \leq \bar{t}_i, \forall i \in I.$$  \[(4.18)\]

The total number of IR and IC constraints is $I^2$. Next, we show that it is possible to represent these $I^2$ constraints with a set of much fewer equivalent constraints.

**Proposition 1.** (Sufficient and Necessary Conditions for feasibility): For a contract $C = \{(r_i, t_i), \forall i \in I\}$ with user costs $K_1 > \ldots > K_I$, it is feasible if and only if all the following conditions are satisfied:

1. **Condition(+)**: $r_1 - K_1 t_1 \geq 0$;

2. **Condition(↑)**: $0 \leq r_1 \leq \ldots \leq r_I$ and $0 \leq t_1 \leq \ldots \leq t_K$;

3. **Condition(≤)**: For any $i = 2, \ldots, I$,

$$r_{i-1} + K_i(t_i - t_{i-1}) \leq r_i \leq r_{i-1} + K_{i-1}(t_i - t_{i-1}).$$ \[(4.19)\]
The proof of Proposition 1 is given in Section 4.5.7.

Intuitively, \textbf{Condition}(+) ensures that all types of users can get a nonnegative payoff by accepting the contract item \((r_1, t_1)\), as it implies \(r_1 - K_j t_1 \geq 0\) for all \(j \geq 2\). Thus this can replace the IR constraints in (4.9). \textbf{Condition}(↑) and \textbf{Condition}(≤) are related to IC constraints in (4.15). \textbf{Condition}(↑) shows that a user with a higher type should be assigned a larger task, because his unit cost is lower (and more efficient) and the client needs to compensate this user less per unit work. Also, a larger reward should be given to this user for the larger task undertaken by him, otherwise this user will choose another contract item in order to work less. \textbf{Condition}(≤) shows the relation between any two neighboring contract items.

Based on Proposition 1, we can simplify the client’s problem in (4.18) as

\[
\max_{\{(r_i, t_i)\}_{i \in \mathcal{I}}} \mathbb{E}_{\{n_i\}_{i \in \mathcal{I}}} \left[ \pi(\{(r_i, t_i)\}_{i \in \mathcal{I}}, \{n_i\}_{i \in \mathcal{I}}) \right]
\]

subject to, \textbf{Condition}(+), \textbf{Condition}(↑), \textbf{Condition}(≤),

\[
0 \leq t_i \leq \bar{t}_i, \forall i \in \mathcal{I},
\]

where the previous \(I^2\) IR and IC constraints have been reduced to \(I + 2\) constraints.

\textbf{Analysis by sequential optimization}

Now we want to solve the client’s optimal contract. However, (4.20) is not easy to solve as it has coupled variables and many constraints. The way we
solve is a sequential optimization approach: we first derive the optimal rewards \( \{r^*_i(\{t_i\}_i \in \mathcal{I})\}_i \in \mathcal{I} \) given any feasible tasks \( \{t_i\}_i \in \mathcal{I} \), then further derive the optimal tasks \( \{t^*_i\}_i \in \mathcal{I} \) for the optimal contract.

**Proposition 2.** Let \( \mathcal{C} = \{(r_i, t_i)\}_i \in \mathcal{I} \) be a feasible contract with any feasible tasks \( 0 \leq t_1 \leq ... \leq t_I \). The unique optimal rewards \( \{r^*_i(\{t_i\}_i \in \mathcal{I})\}_i \in \mathcal{I} \) satisfy

\[
\begin{align*}
r^*_1(\{t_i\}_i \in \mathcal{I}) &= K_1 t_1, \\
r^*_i(\{t_i\}_i \in \mathcal{I}) &= r^*_{i-1} + K_1(t_i - t_{i-1}) \\
&= K_1 t_1 + \sum_{j=2}^{i} K_j(t_j - t_{j-1}), \, \forall i = 2, ..., I.
\end{align*}
\]  

\((4.21)\)

\((4.22)\)

**Proof (Sketch).** First, we can prove \((4.21)\) by showing that Condition\((+\)) binds at the optimality. This guarantees the IR constraints of the contract. Second, we can prove \((4.22)\) by showing that the left-hand side inequality in Condition\((\leq\)) binds at the optimality. This guarantees the IC constraints of the contract.

Based on Proposition 2, we can greatly simplify the client’s optimization Problem in \((4.20)\) as

\[
\begin{align*}
\max & \quad \mathbb{E}_{\{n_i\}_i \in \mathcal{I}}[\pi(\{(r^*_i(\{t_i\}_i \in \mathcal{I}), t_i\}, \{n_i\}_i \in \mathcal{I})] \\
\text{subject to,} & \quad 0 \leq t_1 \leq ... \leq t_I, \\
& \quad t_i \leq \bar{t}_i, \forall i \in \mathcal{I}.
\end{align*}
\]

\((4.23)\)

Problem \((4.23)\) can be solved by various methods in nonlinear programming \cite{186}. In the following, to avoid a loss of optimality, we use exhaustive search
to solve Problem (4.23). This helps us explicitly compare the client’s performances in different information scenarios. In Section 4.5.8, we also propose an efficient approximated algorithm, which can decompose Problem (4.23) involving $I$ coupled variables into $I$ single-variable subproblems with only small performance loss.

Without solving Problem (4.23), we can already derive some interesting results as follows.

**Theorem 17.** The total involved user type set under asymmetrically incomplete information is

$$\mathcal{I}_A = \{i \in \mathcal{I} : \mathbb{E}_{(n_i)_{i \in I}}[n_i(\theta_i - K_i) - (K_i - K_{i+1}) \sum_{\forall j > i, j \in I} n_j > 0]\}, \quad (4.24)$$

where the subscript $A$ in $\mathcal{I}_A$ refers to the asymmetrically incomplete information assumption.\(^{14}\) Compared with the collaborator set $\mathcal{I}_C$ under complete information case, here the client involves less collaborators, i.e., $|\mathcal{I}_A| \leq |\mathcal{I}_C|$. Moreover, the client assigns a larger task and gives a larger reward to a higher type of collaborator, which may not be the case under complete information (see Observation 6). Only the lowest type of collaborator(s) in set $\mathcal{I}_A$ obtains a zero payoff, and higher types of collaborators in set $\mathcal{I}_A$ obtain positive payoffs that are increasing in their types.

**Proof.** All involved users in set $\mathcal{I}_A$ will receive positive rewards and tasks.

\(^{14}\)Note that the client will design $(r^*, t^*) = (0, 0)$ for the types not in set $\mathcal{I}_A$. Thus the users of these types are not involved as collaborators.
According to Condition\((\uparrow)\), the rewards and tasks are non-decreasing in the types. Let us denote the lowest type of involved users in set \(I_A\) as type-\(\hat{j}\). If \(\hat{j} = 1\), then relation (4.21) shows that a type-1 collaborator receives a zero payoff. If \(\hat{j} > 1\), then any lower type \(k < \hat{j}\) is not in set \(I_A\), and receives zero task and zero reward. By using relation (4.22), we can further derive that \(r^*_j = K_j t^*_j\), which means the lowest type collaborator still obtains a zero payoff.

According to (4.22), the type-\(i\) collaborator’s equilibrium payoff is \(r^*_i = K_i t^*_i = r^*_{i-1} - K_i t^*_{i-1}\), which is strictly larger than type-(\(i-1\)) collaborator’s payoff \(r^*_{i-1} - K_{i-1} t^*_{i-1}\) as \(K_i < K_{i-1}\). Thus a higher type collaborators receive a larger positive payoff.

Next we show which types of users are involved as collaborators. The first derivative of the client’s expected profit in (4.17) over \(t_i\) is

\[
\frac{\partial E_{\{n_i\}_{i \in I}}}{\partial t_i} = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \cdots \sum_{n_{i-1}=0}^{N-\sum_{j=1}^{i-2} n_j} \frac{N! q_1^{n_1} \cdots q_{I-1}^{n_{I-1}} q_I^{N-\sum_{j=1}^{I-1} n_j}}{n_1! \cdots n_{I-1}!(N-\sum_{j=1}^{I-1} n_j)!} 
\left( \frac{n_i \theta_i}{1 + n_i t_i} - n_i K_i - (K_i - K_{i+1}) \sum_{\forall j > i, \forall j \in \mathcal{I}} n_j \right), \forall i \in \mathcal{I}, \tag{4.25}
\]

where \(t_i\) only appears in the last bracket. The client will involve type-\(i\) users only when the last bracket of (4.25) is positive at \(t_i = 0\). This leads to the collaborator set in (4.24). By comparing \(\mathcal{I}_C\) in (4.12) and \(\mathcal{I}_A\) in (4.24), we conclude that \(|\mathcal{I}_A| \leq |\mathcal{I}_C|\).

Intuitively, as the client does not know each user’s type, he needs to provide
incentives (in terms of positive payoffs) to the users to attract them revealing their own types truthfully. If he involves a low type user, he needs to give increasingly higher payoffs to all higher types. Thus he should target at users with high enough types. We have $|I_A|$ smaller than $|I_C|$, which means that some low types belong to set $I_C$ may not be included in set $I_A$. By comparing (4.24) and (4.12) for the highest type-$I$, we know that that this type is involved in both information scenarios.

Recall that under complete information, Observation 6 shows that the client may not give a larger task and reward to a higher type-$i$ collaborator. This can happen when $\theta_i$ is small or the number of users of that type is large. Under asymmetrically incomplete information, however, the IC constraints require the reward and task to be nondecreasing in the collaborator types, independent of $\theta_i$ and the number of users in each type (which is a random variable). Otherwise, some collaborators will have incentives to choose contract items not designed for their own types, and thus violate IC constraints. This is not optimal for the client based on the Revelation Principle [185].

Figure 4.5 shows the client’s optimal contract $\{(r^*_i, t^*_i)\}_{i=1}^3$ for three collaborator types. A higher type-$i$ user obtains a larger task $t^*_i$, a larger reward $r^*_i$, and a larger payoff (not shown in this figure). The slope of the dashed line between two points $(r^*_i, t^*_i)$ and $(r^*_{i+1}, t^*_{i+1})$ equals to cost $K_{i+1}$ (as shown in Proposition 2). In the contract, the ratio between the reward and task (i.e.,
Figure 4.5: The client’s optimal contract items for three types ($I=3$). Other parameters are $N = 120$, $K_1 = 1.5$, $K_2 = 1$, $K_3 = 0.5$, $\theta_i = 5$, and $q_i = 1/3$ for any $i \in \mathcal{I}$.

$r^*_i/t^*_i$) for type-$i$ decreases with the type. Thus a lower type $j < i$ collaborator will not choose the higher contract item $(r^*_i, t^*_i)$, since it is too costly and not be efficient for him to undertake the task. A user will not choose a lower type contract item either, otherwise his payoff (though still positive) will decrease with a smaller reward.

By looking into (4.25), we have the following observation.

**Observation 7.** The client’s optimal task allocation $t^*_i$ to a type-$i$ collaborator increases in the client’s preference characteristic $\theta_i$ and decreases in the collaborator’s cost $K_i$. The client’s equilibrium expected profit increases in $\theta_i$ for all $i \in \mathcal{I}_A$.

Next, we compare the client’s profits under complete and asymmetrically
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Figure 4.6: The ratio of the client’s realized payoffs under asymmetrically incomplete and complete information as a function of users’ realized numbers $\{n_i\}_{i=1}^3$ in three types ($I=3$).

Here we only show $n_1$ and $n_2$, and $n_3$ can be computed as $N - n_1 - n_2$. Other parameters are $N = 120$, $K_1 = 1.1$, $K_2 = 1$, $K_3 = 0.9$, $\theta_i = 5$, $q_i = 1/3$ for any $i \in I$.

Incomplete information.

**Observation 8.** Compared with complete information, the client obtains a smaller equilibrium expected profit under asymmetrically incomplete information. The gap between his realized profit under two information scenarios is minimized when the realization (users’ numbers in different types) is the closest to the expected value.

Figure 4.6 shows the ratio of the client’s realized payoffs under asymmetrically incomplete and complete information, which is a function of users’ realizations $\{n_i\}_{i=1}^3$ in all three types. This ratio is always no larger than 1, as the
client obtains the maximum profit under complete information. This profit ratio reaches its maximum 92% when users’ type realization matches the expected value, i.e., \( n_i = Nq_i = 40 \) for \( i = 1, 2 \) (and thus \( n_3 = N - n_1 - n_2 = 40 \) as well). This is consistent with the fact that the client maximize its expected profit under asymmetrically incomplete information.

4.4 Summary

This chapter analyzes and compares different mechanisms that a client can use to motivate the collaboration of smartphone users on both data acquisition and distributed computing. Our proposed incentive mechanisms cover several possible information scenarios that the client may face in reality.

For data acquisition applications, we propose a reward-based collaboration scheme for the client to attract enough users by giving out the minimum reward. We show that when the client knows the users’ collaboration costs, he only involves users with the lowest costs to build up the database. However, if users can hold their private information from the client, the client needs to offer a larger reward to get enough collaborators.

For distributed computing applications, we use contract theory to study how a client decides different task-reward combinations for many different types of users. Under complete information, the client involves a type of users as long as his preference of that type outweighs that the user’s unit cost.
All collaborators receive a zero payoff in this case. Under asymmetrically incomplete information, however, the client has to offer a larger reward to a higher user type. Most collaborators then receive a positive payoff, and a collaborator’s payoff increases in his type.

4.5 Appendix

4.5.1 Discussion of Model (4.2) in Different Information Scenarios

Here we focus on model (4.2) where the client will reward only with successful collaboration. The analysis of this model is very similar to model (4.1), and in the following we briefly discuss the difference between the two models.

- Under complete information, we can derive the same results as in Theorem 9 for model (4.2), by using a similar analysis.

- Under symmetrically incomplete information, for the equilibrium of Stage II, we can similarly derive the same results as in Theorem 10 for model (4.2) except mixed strategy NE. The mixed strategy NE exists only when $R$ is sufficiently large, and the equilibrium probability $p^*$ in (4.3) is the unique solution to

$$\mathbb{E}_m \left( \frac{R}{m + 1} 1_{\{m+1 \geq n_0\}} - \mu \right) = 0,$$

where the expectation $\mathbb{E}$ is taken over the random variable $m$ that follows a binomial distribution $B(N - 1, p)$. For the equilibrium of the whole
collaboration game, we can still derive the same results as in Theorem 11.

- Under asymmetrically incomplete information, for the equilibrium of Stage II, we can derive a similar equilibrium decision threshold $\gamma^*(R)$ as the solution to

$$E_m \left( \frac{R}{m+1} 1_{\{m+1 \geq n_0\}} - \gamma \right) = 0, \quad (4.26)$$

where the expectation is taken over $m$ that follows a binomial distribution $B(N-1, F(\gamma))$.\(^{15}\) Then we can similarly analyze the client’s maximization problem in (4.6). The difference from model (4.1) is that here the client needs to determine a larger reward $R$ to attract enough users who face a higher risk.

### 4.5.2 Proof of Theorem 9

If $V < n_0C_0$, then the client’s announced total reward $R$ is also smaller than $n_0C_0$ to make a profit. This reward is not enough to compensate even $n_0$ users with smallest costs, thus no users will join. Regarding this, the client will not seek users’ collaboration in Stage I by announcing zero reward $R^* = 0$. Next we focus on $V \geq n_0C_0$.

We first prove the existence of the equilibrium in Theorem 9. In the strategies shown in Theorem 9, involved users will not leave the collaboration since

\(^{15}\)Note that the solution to (4.26) will exist only when $R$ is sufficiently large, and the solution may not be unique. If there exist two solutions (denoted by $\gamma_1^*$ and $\gamma_2^*$ with $\gamma_1^* < \gamma_2^*$), each user $i$ will pick up $\gamma_2^*$ instead of $\gamma_1^*$ since it gives him a larger payoff $\gamma_2^* - C_i$ (i.e., pareto-optimal for all users).
they have non-negative payoffs. Also, those users not in the collaboration will not decide to collaborate, otherwise they receive negative payoffs. The client will not deviate by decreasing or increasing the $R^*$, otherwise he will involve less than $n_0$ users or loss profit, respectively.

We then prove the uniqueness of the equilibrium by contradiction. Note that $R^* = n_0 C_0$ corresponds to a unique state of users’ equilibrium decisions in Theorem 9. Suppose there exists another equilibrium with a different $\hat{R}^* \neq R^*$. If $\hat{R}^* < R^*$, the client cannot attract enough collaborators and the collaboration is not successful; if $\hat{R}^* > R^*$, the client has incentive to decrease $\hat{R}^*$ to $R^*$. Thus there does exist such an equilibrium with $\hat{R}^* \neq R^*$.

4.5.3 Proof of No Collaboration and Pure Strategy NE in Theorem 10

We focus on users’ pure strategies where $R$ is already given. If $R < n_0 \mu$, this reward cannot attract $n_0$ collaborators where each user’s collaboration cost is believed to be $\mu$. Thus the collaboration is not successful and no user will collaborate in Stage II. Next we focus on $R \geq n_0 \mu$.

- If $n_0 \mu \leq R < N \mu$, we prove $n^* = \lfloor \frac{R}{\mu} \rfloor$ by contradiction. Suppose there are $n^* \neq \lfloor \frac{R}{\mu} \rfloor$ collaborators at the equilibrium.
  - If $n^* < \lfloor \frac{R}{\mu} \rfloor$, then another user will join the collaboration and receive nonnegative expected payoff (nonnegative payoff $\frac{R}{n^*+1} - \mu$ when
collaboration is successful and zero payoff otherwise).

- If \( n^* > \lfloor \frac{R}{\mu} \rfloor \), then some involved user will leave the collaboration since he receives negative expected payoff (negative payoff \( \frac{R}{n^*} - \mu \) if the collaboration is successful and zero payoff otherwise).

Thus there are \( n^* = \lfloor \frac{R}{\mu} \rfloor \) collaborators at the equilibrium.

- If \( R \geq N\mu \), each user can join the collaboration and receive non-negative expected payoff and thus \( n^* = N \).

### 4.5.4 Proof of Existence And Uniqueness of Equilibrium Threshold in Theorem 12

Recall that \( \Phi(\gamma) \) is given in (4.4). Here we want to prove that there exists a unique solution \( \gamma^*(R) \) (or simply \( \gamma^* \)) to \( \Phi(\gamma) = 0 \), which satisfies \( \frac{R}{N} < \gamma^* < \frac{R}{n_0} \).

We divide the proof into the following three parts, depending on relation between \( R \) and \( \gamma^* \). For simplicity, we represent \( F(\gamma^*) \) as \( F^* \).

- Suppose that there exists a solution \( \gamma^* \) to \( \Phi(\gamma) = 0 \) in (4.4) which satisfies \( R \leq n_0\gamma^* \). Since \( \Phi(\gamma^*) \) is increasing in \( R \), we have \( \Phi(\gamma^*) \leq \Phi(\gamma^*) \lvert_{R=n_0\gamma^*} \).

That is,

\[
\Phi(\gamma^*) \leq \sum_{m=n_0-1}^{N-1} \left( \frac{n_0\gamma^*}{m+1} - \gamma^* \right) \binom{N-1}{m} (F^*)^m (1-F^*)^{N-1-m},
\]

which is negative due to our consideration of \( n_0 < N \) and \( F^* > 0 \). Thus there does not exist any solution \( \gamma^* \) to \( \Phi(\gamma) = 0 \) satisfying \( R \leq n_0\gamma^* \) in
Stage II.

- Suppose that there exists a solution $\gamma^*$ to $\Phi(\gamma) = 0$ which satisfies $R \geq N\gamma^*$. We have $\Phi(\gamma^*) \geq \Phi(\gamma^*)|_{R=N\gamma^*}$. That is,

$$\Phi(\gamma^*) \geq \sum_{m=n_0-1}^{N-1} \left( \frac{N\gamma^*}{m+1} - \gamma^* \right) \begin{pmatrix} N-1 \\ m \end{pmatrix} (F^*)^m (1 - F^*)^{N-1-m},$$

which is positive due to our consideration of $n_0 < N$ and $F^* > 0$. Thus there does not exist any solution $\gamma^*$ to $\Phi(\gamma) = 0$ satisfying $R \geq N\gamma^*$ in Stage II.

- When $n_0\gamma < R < N\gamma$, we first show that there exists a solution $\gamma^*$ to $\Phi(\gamma) = 0$ and then prove its uniqueness. We can check that $\lim_{\gamma \to (R/N)^+} \Phi(\gamma) > 0$ and $\lim_{\gamma \to (R/n_0)^-} \Phi(\gamma) < 0$. Due to the continuity of $\Phi(\gamma)$ on $\gamma$, there exists a solution $\gamma^*$ to $\Phi(\gamma) = 0$. Next we prove the uniqueness of the solution by contradiction.

Suppose there exist at least two different solutions to $\Phi(\gamma) = 0$. The first derivative $\Phi(\gamma)$ over $\gamma$ at one solution (denoted as $\gamma^*$ with corresponding $F^*$) is nonnegative. But we have

$$\frac{\partial \Phi(\gamma^*)}{\partial \gamma} = \sum_{m=n_0-1}^{N-1} \begin{pmatrix} N-1 \\ m \end{pmatrix} (F^*)^{m-1} (1 - F^*)^{N-m-2}$$

$$\cdot \left[ -F^*(1-F^*) + \frac{dF^*}{d\gamma} \left( \frac{R}{m+1} - \gamma^*(m + (1-N)F^*) \right) \right],$$
which is smaller than

\[
\sum_{m=n_0-1}^{N-1} \binom{N-1}{m} (F^*)^{m-1}(1-F^*)^{N-m-2} \\
\cdot \left[ \frac{dF^*}{d\gamma} (\frac{R}{m+1} - \gamma^*)(m + (1-N)F^*) \right].
\] (4.27)

By substituting \( \Phi(\gamma^*) = 0 \) with \( F = F^* \) into (4.27), we can show

\[
\frac{\partial \Phi(\gamma^*)}{\partial \gamma} < \sum_{m=n_0-1}^{N-1} \binom{N-1}{m} (F^*)^{m-1}(1-F^*)^{N-m-2} \left( \frac{R}{m+1} - \gamma^* \right) m \frac{dF^*}{d\gamma} < 0,
\] (4.28)

where \( F(\cdot) \) is an increasing function. This contradicts with our supposition that the first derivative of \( \partial \Phi(\gamma^*)/\partial \gamma \) is nonnegative. This ends our proof of the existence of unique solution \( \gamma^* \) to \( \Phi(\gamma) = 0 \).

4.5.5 Proof of Theorem 13

We first prove the relation between \( \gamma^* \) and \( R \). Recall that (4.28) has shown that \( \Phi(\gamma^*) \) is decreasing in \( \gamma^* \), while (4.4) shows that \( \Phi(\gamma^*) \) is linearly increasing in \( R \). By applying implicit function theorem, we can derive

\[
\frac{d\gamma^*}{dR} = -\frac{\partial \Phi(\gamma^*)}{\partial \gamma} / \frac{\partial \Phi(\gamma^*)}{\partial R} > 0.
\]

Thus \( \gamma^* \) is increasing in \( R \).

Next we prove the relation between \( \gamma^* \) and \( n_0 \). Let us denote \( F(\gamma^*) \) as \( F^* \)
and define

\[ \phi(m) := \left( \frac{R}{m+1} - \gamma^* \right) \binom{N-1}{m} (F^*)^m (1 - F^*)^{N-1-m}, \]

then we can rewrite \( \Phi(\gamma^*) \) in (4.4) as \( \sum_{n_0-1}^{N-1} \phi(m) \). Since Section 4.5.4 shows that \( n_0 \gamma^* < R < N \gamma^* \), \( \phi(m) \) is positive when \( m \) is small and is negative when \( m \) is large. As \( n_0 \) increases to \( n_0+1 \), previous positive term \( \phi(n_0 - 1) \) in \( \Phi(\gamma^*) \) disappears while all negative terms still remain. Hence, \( \Phi(\gamma^*) \) decreases with current \( n_0 \). Recall that we have shown in (4.28) that \( \Phi(\gamma^*) \) is decreasing in \( \gamma^* \), thus \( \gamma^* \) is decreasing in \( n_0 \) due to \( \Phi(\gamma^*) = 0 \).

Next we prove the relation between \( \gamma^* \) and \( N \). As \( N \) increases to \( N+1 \), we have an additional negative term \( \phi(N) \) appeared in the (4.4) (denoted by \( \tilde{\Phi}(\gamma^*) \)). For a previous term \( \phi(m) \) with \( n_0 - 1 \leq m \leq N - 1 \), it changes to

\[ \tilde{\phi}(m) = \left( \frac{R}{m+1} - \gamma^* \right) \binom{N}{m} (F^*)^m (1 - F^*)^{N-m}. \]

Thus we can rewritten \( \tilde{\phi}(m) = (1 - F^*)\phi(m) \frac{N}{N-m} \), where the fraction term is increasing in \( m \). Then the absolute value of a previously negative term \( \phi(m) \) (with large \( m \)) is relatively enlarged compared to a positive term (with small \( m \)). Hence, the summation of the first \( N \) terms in \( \tilde{\Phi}(\gamma^*) \) is negative, and \( \tilde{\Phi}(\gamma^*) \) with an additional negative term \( \phi(N) \) is further decreased to be negative. Recall that we have shown in (4.28) that \( \Phi(\gamma^*) \) is decreasing in \( \gamma^* \), thus \( \gamma^* \) is decreasing in \( N \) due to \( \Phi(\gamma^*) = 0 \).
4.5.6 Proof of Theorem 14

Recall that (4.6) shows that $f(R)$ is linearly increasing in $V$ for any $R$ values, thus the client’s equilibrium expected payoff $f(R^*)$ is increasing in $V$.

Next we prove that $f(R)$ and $f(R^*)$ are decreasing in $n_0$. Notice that the increase of $n_0$ decreases the number of (positive) summation terms in $f(R)$, and affects $F^*$ (i.e., $F(\gamma^*)$) in each term. Recall that Theorem 13 has shown that $\gamma^*$ and thus $F^*$ are decreasing in $n_0$. Thus if we can show that $f(R)$ is also increasing in $F^*$, then $f(R)$ is decreasing in $n_0$.

The partial derivative of $f(R)$ over $F^*$ is

$$\frac{\partial f(R)}{\partial F^*} = (V - R) \cdot \sum_{n=n_0}^{N} (n - NF^*) \binom{N}{n} (F^*)^{n-1} (1 - F^*)^{N-n-1}. \quad (4.29)$$

According to Theorem 12, the equilibrium collaborator number is

$$n^* = \sum_{n=0}^{N} n \binom{N}{n} (F^*)^n (1 - F^*)^{N-n},$$

which leads to $n^* = Np^*$. Thus we have

$$\left( V - R \right) \sum_{n=0}^{N} (n - NF^*) \binom{N}{n} (F^*)^{n-1} (1 - F^*)^{N-n-1}$$

$$= \frac{V - R}{F^* (1 - F^*)} (n^* - NF^*) = 0. \quad (4.30)$$

Notice that the sign of each term in the summation operation of (4.30) is decided by the relation between $n$ and $NF^*$, thus a term with small $n$ is negative and a term with large $n$ is positive. Compared to (4.30), $\partial f(R)/\partial F^*$ in (4.29)
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has less negative terms in the summation operation and is thus positive. Thus we conclude that \( f(R) \) and equilibrium \( f(R^*) \) are decreasing in \( n_0 \).

4.5.7 The proof of Proposition 1

Proof of sufficient conditions

We use mathematical induction to prove the three conditions in Proposition 1 are sufficient conditions for contract feasibility. Let us denote \( C(l) \) as a subset which contains the first \( l \) task-reward combinations in the contract \( C \). That is, \( C(l) = \{(r_i, t_i)\}_{i=1}^l \).

We first show that \( C(1) \) is feasible. Since there is only one user type, the contract is feasible as long as it satisfies IR constraint for type-1. This is true due to Condition(+) in Proposition 1.

Next we show that if contract \( C(l) \) is feasible, then the new contract \( C(l+1) \) by adding new item \((r_{l+1}, t_{l+1})\) is also feasible. To achieve this, we need to show the following results.

- **Result I:** the IC and IR constraints for type-(\( l+1 \)) users:
  \[
  \begin{align*}
  r_{l+1} - K_{l+1}t_{l+1} &\geq r_i - K_{i+1}t_i, \forall i = 1, \ldots, l \\
  r_{l+1} - K_{l+1}t_{l+1} &\geq 0,
  \end{align*}
  \] (4.31)

- **Result II:** for the original \( l \) types already contained in the contract \( C(l) \), the IC constraints are still satisfied after adding the new type-(\( l+1 \)):
  \[
  r_i - K_i t_i \geq r_{l+1} - K_{l+1} t_{l+1}, \forall i = 1, \ldots, l.
  \] (4.32)
Note that the new contract $C(l + 1)$ will satisfy the IR constraints for all original $l$ types of users, since the original contract $C(l)$ is feasible.

Proof of Result I in (4.31): First, we prove the IC constraint for type-$(l+1)$. Since contract $C(l)$ is feasible, the IC constraint for a type-$i$ user must hold, i.e.,

$$r_j - K_l t_j \leq r_l - K_t t_l, \forall j = 1, ..., l.$$  

Also, the left inequality of (4.19) in Condition($\leq$) can be transformed to

$$r_l + K_{l+1}(t_{l+1} - t_l) \leq r_{l+1}.$$  

By combining the above two inequalities, we have

$$r_j - K_l t_j + K_{l+1}(t_{l+1} - t_l) \leq r_{l+1} - K_l t_l, \forall j = 1, ..., l. \quad (4.33)$$

Notice that $K_{l+1} < K_l$ and $t_j \leq t_l$ in Condition($\uparrow$), we also have

$$K_{l+1}(t_l - t_j) \leq K_l(t_l - t_j).$$

By substituting this inequality into (4.33), we have

$$r_{l+1} - K_{l+1} t_{l+1} \geq r_j - K_{l+1} t_j, \quad (4.34)$$

which is actually the IC constraint for type-$(l + 1)$.

Next, we show that the IR constraint for type-$(l + 1)$. Since $K_{l+1} < K_j$ for any $j \leq l$, then

$$r_j - K_{l+1} t_j \geq r_j - K_j t_j.$$
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By combining this inequality and (4.34), we have

\[ r_{l+1} - K_{l+1}t_{l+1} \geq r_j - K_j t_j \geq 0, \]
due to the IR constraint for type-\(j\). Thus we prove the IR constraint for type-(\(l + 1\)) in (4.31).

**Proof of Result II in (4.32):** Since contract \(C(l)\) is feasible, the IC constraint for type-\(j\) holds, i.e.,

\[ r_l - K_j t_l \leq r_j - K_j t_j, \forall j = 1, ..., l. \]

Also, we can transform the right inequality of (4.19) in Condition(\(\leq\)) to

\[ r_{l+1} \leq r_l + K_l(t_{l+1} - t_l). \]

By combining the above two inequalities, we conclude

\[ r_{l+1} - K_j t_l \leq K_l(t_{l+1} - t_l) + r_j - K_j t_j. \]

Notice that \(K_l < K_j\) and \(t_{l+1} \geq t_l\) in Condition(\(\uparrow\)), we also have

\[ K_l(t_{l+1} - t_l) \leq K_j(t_{l+1} - t_l). \]

By combining the above two inequalities, we conclude

\[ r_j - K_j t_j \geq r_{l+1} - K_j t_{l+1}, \forall j = 1, ..., l, \]

which is actually the IC constraint for type-\(j\) in (4.32).

**Proof of necessary conditions**

Now we prove the three conditions in Proposition 1 are necessary conditions for contract feasibility. It is easy to see that Condition(\(\uparrow\)) is just the IR condition.
for type-1 in a feasible contract. Also, the right inequality of Condition(≤) can be derived from the IC constraint for type-(i − 1), and the left inequality can be derived from the IC constraint for type-i.

Next we prove Condition(↑) is also the necessary condition. We divide the proof into two parts.

- We first prove that if $K_i > K_j$ then $t_i \leq t_j$ by contradiction. Suppose $t_i > t_j$, then we have

$$K_i(t_i - t_j) > K_j(t_i - t_j), \quad (4.35)$$

due to $K_i > K_j$. Notice that the feasible contract satisfies the IC constraints for type-$i$ and type-$j$ users, we have

$$r_i - K_i t_i \geq r_j - K_i t_j,$$

and

$$r_j - K_j t_j \geq r_i - K_j t_i.$$

By combining the above two inequalities, we conclude

$$K_i t_i + K_j t_j \leq K_i t_j + K_j t_i,$$

which contradicts with (4.35).

- We then prove that $t_i \geq t_j$ if and only if $r_i \geq r_j$.

- If $t_i > t_j$, we want to prove $r_i > r_j$. Due to the IC constraint for type-$i$, we have

$$r_i - K_i t_i \geq r_j - K_i t_j,$$
which can be transformed to

\[ r_i - r_j \geq K_i(t_i - t_j). \]

Since \( t_i > t_j \), we can derive \( r_i > r_j \) from the above inequality.

- If \( r_i > r_j \), we want to prove that \( t_i > t_j \). Due to the IC constraint for type-\( j \), we have

\[ r_j - K_j t_j \geq r_i - K_j t_i, \]

which can be transformed to

\[ K_j(t_i - t_j) \geq r_i - r_j. \]

Since \( r_i > r_j \), we can derive \( t_i > t_j \) from the above inequality.

- Using a similar analysis, we can prove that \( r_i = r_j \) if and only if \( t_i = t_j \).

### 4.5.8 Approximated Algorithm to Solve Problem (4.23)

Notice that the optimization problem in (4.23) is difficult to solve, where \( I \) variables are coupled. Here we propose an efficient approximated algorithm to solve Problem (4.23).

We can find that each variable (\( t_i \) for users in type-\( i \)) is independent of each other in the objective function of Problem (4.23), and the objective function is strictly concave in each variable. Thus we can directly examine the first-order condition for each variable if we do not consider the constraint \( 0 \leq t_1 \leq ... \leq t_I \).
where variables are coupled. By checking the first-order conditions (making (4.25) equal to zero for each \( i \in \mathcal{I} \)), we can derive the optimal solutions as \( \tilde{t}_i \) for each \( i \in \mathcal{I} \), which also need to be upper-bounded by \( \bar{t}_i \) for each \( i \in \mathcal{I} \). Thus the optimal solutions are

\[
\hat{t}_i = \min(\tilde{t}_i, \bar{t}_i), \forall i \in \mathcal{I}.
\]

Since \( 0 \leq t_1 \leq ... \leq t_I \) should always be satisfied, we present \( I \) options to adjust the relation between different tasks in finalizing tasks \( \{t_i^*\}_{i \in \mathcal{I}} \):

1) \( t_1^* = \hat{t}_1 \) and \( t_{j+1}^* = \max(\hat{t}_{j+1}, t_j^*) \) for any \( j \geq 1 \);

... 

i) \( t_i^* = \hat{t}_i \), \( t_{j-1}^* = \min(\hat{t}_{j-1}, t_j^*) \) for any \( j \leq i \), and \( t_{i+1}^* = \max(\hat{t}_{i+1}, t_i^*) \) for any \( l \geq i \);

... 

I) \( t_I^* = \hat{t}_I \), and \( t_{i-1}^* = \min(\hat{t}_{i-1}, t_i^*) \) for any \( i \leq I \).

Then we pick one out of the above \( I \) options depending on which option gives the highest value for the objective in Problem (4.23). This algorithm greatly simplifies the computation complexity by decomposing Problem (4.23) into \( I \) single-variable subproblems. Let us denote the upperbound of all feasible task allocations as \( T \). Then the proposed approximated algorithm significantly simplifies the computational complexity from roughly \( O(T^I) \) to current \( O(I \cdot T) \) through proper decomposition.
Compared to the optimal exhaustive search, we can show our approximated algorithm only introduces small or zero performance loss for the client. The gap is zero if the constraint $0 \leq t_1 \leq ... \leq t_I$ does not bind at the optimality. This can happen when the client does not have much smaller preference characteristics on higher user types, or higher types do not involve many more users than lower types.
Part II

Economics of Cognitive Radio Networks
Chapter 5

Monopoly Spectrum Market

Using Cognitive Radios

5.1 Introduction

Wireless spectrum is typically considered as a scarce resource, and is traditionally allocated through static licensing. Field measurements show that, however, most spectrum bands are often under-utilized even in densely populated urban areas ([2]). To achieve more efficient spectrum utilization, people have proposed various dynamic spectrum access approaches including hierarchical-access and dynamic exclusive use ([3–9]). Hierarchical-access allows a secondary (unlicensed) network operator or users to opportunistically access the spectrum without affecting the normal operation of the primary operator who serves the primary (licensed) users. Dynamic exclusive use allows a prima-
ry operator to dynamically transfer and trade the usage right of its licensed spectrum to a third party (e.g., a secondary network operator or a secondary end-user) in the spectrum market. This chapter considers a secondary operator who obtains spectrum resource via both spectrum sensing as in the hierarchical-access approach and dynamic spectrum leasing as in the dynamic exclusive use approach.

Spectrum sensing obtains awareness of the spectrum usage and existence of primary users, by using geolocation and database, beacons, or cognitive radios (e.g., [10–13]). The primary users are oblivious to the presence of secondary cognitive network operators or users. The secondary network operator or users can sense and utilize the unused “spectrum holes” in the licensed spectrum without violating the usage rights of the primary users (e.g., [3,6]). Since the secondary operator or users does not know the primary users’ activities before sensing, the amount of useful spectrum obtained through sensing is uncertain (e.g. [15,47,99]).

With dynamic spectrum leasing, a primary operator allows secondary users to operate in their temporarily unused part of spectrum in exchange of economic return (e.g., [4,6,16]). The dynamic spectrum leasing can be short-term or even real-time (e.g., [17–19]), and can be at a similar time scale of the spectrum sensing operation.

In this chapter, we study the operation of a cognitive radio network that
CHAPTER 5. MONOPOLY SPECTRUM MARKET USING COGNITIVE RADIOS

consists of a cognitive mobile virtual network operator (C-MVNO) and a group of secondary unlicensed users. The word “virtual” refers to the fact that the operator does not own the wireless spectrum bands or even the physical network infrastructure [20]. The C-MVNO serves as the interface between the primary operator and the secondary users which is similar to MVNO.\(^1\) The word “cognitive” refers to the fact that the operator can obtain spectrum resource through both spectrum sensing using the cognitive radio technology (\([3, 6]\)) and dynamic spectrum leasing from the primary operator (\([4, 6, 16]\)). The operator then resells the obtained spectrum (bandwidth) to secondary users to maximize its profit. The proposed model is a hybrid of the hierarchical-access and dynamic exclusive use models. It is applicable in various network scenarios, such as achieving efficient utilization of the TV spectrum in IEEE 802.22 standard [26]. This standard suggests that the secondary system should operate on a point-to-multipoint basis, i.e., the communications will happen between secondary base stations and secondary customer-premises equipment. The base stations can be operated by one or several C-MVNOs introduced in this chapter.

\(^1\)References [24,25] show that it can be more efficient for the primary operator to hire an MVNO as intermediary to retail its spectrum resource, as MVNO can have a better understanding of local user population and users’ demand. MVNOs can partially share the network investment cost and introduce new services as supplement to existing services provided by the primary operators [21]. Some regulators also wants primary operators to open their networks or resources to MVNOs such that more competition is introduced into the market [22].
Compared with a traditional MVNO who only leases spectrum through long-term contracts, a C-MVNO can dynamically adjust its sensing and leasing decisions to match the changes of users’ demand at a short time scale. Moreover, sensing often offers a cheaper way to obtain spectrum compared with leasing. The cost of sensing mainly includes the sensing time and energy, and does not include explicit cost paid to the primary operator. With a mature spectrum sensing technology, sensing cost should be reasonable low (otherwise there is no point of using cognitive radio). Spectrum leasing, however, involves direct negotiation with the primary operator. When the primary operator determines the cost of leasing, it needs to calculate its opportunity cost, i.e., how much revenue the spectrum can provide if the primary operator provides services directly over it. It is reasonable to believe that the leasing cost is more expensive than the sensing cost in most cases.\(^2\) Although sensing is cheaper, the amount of spectrum obtained through sensing is often uncertain due to the stochastic nature of primary users’ traffic. It is thus critical for a C-MVNO to find the right balance between cost and uncertainty.

Our key results and contributions are summarized as follows. For simplicity, we refer to the C-MVNO as “operator”, secondary users as “users”, and “dynamic leasing” as “leasing”.

- A Stackelberg game model: We model and analyze the interactions be-

\(^2\)The analysis of this chapter also covers the case where sensing is more expensive than leasing, which is a trivial case to study.
between the operator and the users in the spectrum market as a Stackelberg game. As the leader, the operator makes the sensing, leasing, and pricing decisions sequentially. As the followers, users then purchase bandwidth from the operator to maximize their payoffs. By using backward induction, we prove the existence and uniqueness of the equilibrium, and show how various system parameters (i.e., sensing and leasing costs, users’ transmission power and channel conditions) affect the equilibrium behavior. Despite the complexity of the model, we are able to fully characterize the unique equilibrium behaviors of the operator and users.

- **Threshold structures of the optimal investment and pricing decisions**: At the equilibrium, the operator will sense the spectrum only if the sensing cost is cheaper than a threshold. Furthermore, it will lease some spectrum only if the resource obtained through sensing is below a threshold. Finally, the operator will charge a constant price to the users if the total bandwidth obtained through sensing and leasing does not exceed a threshold. The thresholds are easy to compute and the corresponding decisions rules are easy to implement in practice.

- **Fair and predictable QoS**: The operator’s optimal pricing decision is independent of the users’ wireless characteristics. Each user receives a payoff that is proportional to its channel gain and transmission power, which leads to the same signal-to-noise (SNR) for all users.
• **Impact of spectrum sensing:** We show that the availability of sensing always increases the operator’s profit in the *expected* sense. The actual realization of the profit at a particular time heavily depends on the spectrum sensing results. Users always get better payoffs when the operator performs spectrum sensing.

Section 7.2 introduces the network model and problem formulation. In Section 5.3, we analyze the game model through backward induction. We discuss various insights obtained from the equilibrium analysis and present some numerical results in Section 5.4. In Section 5.5, we show the impact of spectrum sensing on both the operator and users. In Section 5.6, we extend our work to the incomplete information case, where the operator does not know about the distribution of sensing realization factor and needs to learn over time. We conclude in Section 7.6 and outline some future research directions.

### 5.1.1 Related Work

There is a growing interest in studying the investment and pricing decisions of cognitive network operators recently. Several auction mechanisms have been proposed to study the investment problems of cognitive network operators (e.g., [27–29]). Other recent results studied the pricing decisions of the cognitive network operators who interact with a group of secondary users (e.g., [30–38]). [27] considered users’ queueing delays and obtained most results
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through simulations. [31] presented a recent survey on the spectrum sharing games of network operators and cognitive radio networks. [32] studied the competition among multiple service providers without modeling users’ wireless details. [33] considered a pricing competition game of two operators and adopted a simplified wireless model for the users. [34] derived users’ demand functions based on the acceptance probability model for the users. [35] explored demand functions based on both quality-sensitive and price-sensitive buyer population models. [36] formulated the interaction between one primary user (monopolist) and multiple secondary users as a Stackelberg game. The primary user uses some secondary users as relays and leases its bandwidth to those relays to collect revenue. [37] studied a multiple-level spectrum market among primary, secondary, and tertiary services where global information is not available. [38] considered the short-term spectrum trading between multiple primary users and multiple secondary users. The spectrum buying behaviors of secondary users are modeled as an evolutionary game, while selling behaviors of primary users are modeled as a noncooperative game. [34–38] obtained most interesting results through simulations. There are only few papers (e.g., [19,37,39]) that jointly considered the spectrum investment and service pricing problem as this chapter. None of the above work considered the impact of supply uncertainty due to spectrum sensing.

Our model of spectrum uncertainty is related to the random-yield model in
supply chain management (e.g., [40–42]). The unique wireless aspects of the system model lead to new solutions and insights in our problem.

This chapter represents a first attempt of understanding how spectrum uncertainty impacts the economic decisions of a cognitive radio operator. To obtain interesting insights, we focus on a stylized model where a monopolist operator faces a group of secondary users. There are many more interesting research issues in this area. Some are further discussed in Section 7.6.

5.2 Network Model

5.2.1 Background on Spectrum Sensing and Leasing

To illustrate the opportunity and trade-off of spectrum sensing and leasing, we consider a primary operator who divides its licensed spectrum into two types:

\[
\text{Spectrum Owner's Service Band} \quad \text{Spectrum Owner's Transference Band}
\]

\[
t = 1
\]

\[
t = 2
\]

Channels, PUs' Activity Band, Operator's Sensed Band, Operator's Leased Band

Figure 5.1: Operator's Investment in Spectrum Sensing and Leasing

Our model of dynamic spectrum leasing in transference band falls into the “exclusive-use” model, and spectrum sensing with opportunistic access falls into the “shared-use” model in [5]. Our model is a general combination of these well known models in literature.
• **Service Band**: This band is reserved for serving the spectrum owner’s primary users (PUs). Since the PUs’ traffic is stochastic, there will be some unused spectrum which changes dynamically. The operator can sense and utilize the unused portions. There are no explicit communications between the primary operator and the operator.

• **Transference Band**: The primary operator temporarily does not use this band. The operator can lease the bandwidth through explicit communications with the primary operator. Since the transference band is not used for serving primary users, there are no “spectrum holes” and there is no need for sensing in this band.

Due to the short-term property of both sensing and leasing, the operator needs to make both the sensing and leasing decisions in each time slot.

The example in Fig. 1 demonstrates the dynamic opportunities for spectrum sensing, the uncertainty of sensing outcome, and the impact of sensing or leasing decisions. The primary operator’s entire band is divided into small 34 channels.

- **Time slot 1**: PUs use channels 1–4 and 11–15. The operator is unaware of this and senses channels 3–8. As a result, it obtains 4 unused channels (5–8). It leases additional 9 channels (20–28) from the transference band.

---

4Channel 16 is the guard band between the service and transference bands.
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- Time slot 2: PUs change their behavior and use channels 1 – 6. The operator senses channels 5 – 14 and obtains 8 unused channels (7 – 14). It leases additional 5 channels (23 – 27) from the transference band.

The choice of time slot length depends on characteristics of the primary traffic. The optimization of time slot length has been extensively studied in [43,44,99], where secondary users maximize their overall access time under the constraint that primary users should be sufficiently protected (e.g., the primary users’ outage probability is below some threshold). In our simulations, we choose the length of time slot such that the probability that primary users’ activities change within a time slot is very small. This ensures that the outage probability due to secondary users’ access is tolerable to primary users.

5.2.2 Notations and Assumptions

We consider a cognitive network with one operator and a set $\mathcal{I} = \{1, \ldots, I\}$ of users. The operator has the cognitive capability and can sense the unused spectrum. One way to realize this is to let the operator construct a sensor network that is dedicated to sensing the radio environment in space and time [45]. The operator will collect the sensing information from the sensor network and provide it to the unlicensed users, or providing “sensing as service”. If the operator owns several base stations, then each base station is responsible for collecting sensing information in a certain geographical area. As mentioned
## Table 5.1: Key Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s$</td>
<td>Sensing bandwidth</td>
</tr>
<tr>
<td>$B_l$</td>
<td>Leasing bandwidth</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Unit sensing cost</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Unit leasing cost</td>
</tr>
<tr>
<td>$\alpha \in [0,1]$</td>
<td>Sensing realization factor</td>
</tr>
<tr>
<td>$\mathcal{I} = {1, \ldots , I}$</td>
<td>Set of secondary users</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Unit price</td>
</tr>
<tr>
<td>$w_i$</td>
<td>User $i$’s bandwidth allocation</td>
</tr>
<tr>
<td>$r_i$</td>
<td>User $i$’s data rate</td>
</tr>
<tr>
<td>$P_{\text{max}}^i$</td>
<td>User $i$’s maximum transmission power</td>
</tr>
<tr>
<td>$h_i$</td>
<td>User $i$’s channel gain</td>
</tr>
<tr>
<td>$n_0$</td>
<td>Noise power density</td>
</tr>
<tr>
<td>$g_i = P_{\text{max}}^i h_i / n_0$</td>
<td>User $i$’s wireless characteristic</td>
</tr>
<tr>
<td>$\text{SNR}_i = g_i / w_i$</td>
<td>User $i$’s SNR</td>
</tr>
<tr>
<td>$G = \sum_{i \in \mathcal{I}} g_i$</td>
<td>Users’ aggregate wireless characteristics</td>
</tr>
<tr>
<td>$R$</td>
<td>Operator’s profit</td>
</tr>
</tbody>
</table>
in [45], there has been significant current research efforts in the context of an European project SENDORA [46], which aims at developing techniques based on sensor networks for supporting coexistence of licensed and unlicensed wireless users in a same area. The users are equipped with software defined radios and can tune to transmit in a wide range of frequencies as instructed by the operator, but do not necessarily have the cognitive sensing capacity. Since the secondary users do not worry about sensing, they can spend most of their time and energy on actual data transmissions. Such a network structure puts most of the implementation complexity at the operator side and reduces the user equipment complexity, and thus might be easier to implement in practice than a “full” cognitive network.

The key notations of this chapter are listed in Table 5.1 with some explanations as follows.

- **Investment decisions** $B_s$ and $B_l$: the operator’s sensing and leasing band-widths, respectively.

- **Sensing realization factor** $\alpha$: when the operator senses a total bandwidth of $B_s$ in a time slot, only a proportion of $\alpha \in [0, 1]$ is unused and can be used by the operator. $\alpha$ is a random variable and depends on the primary users’ activities. With perfect sensing results, users can use bandwidth

---

5Even with the cognitive sensing capability, a secondary user may suffer from poor detection performance such a high missed detection probability. The sensor network infrastructure established by the operator can realize space diversity and reach good detection performance [12].
up to $B_\alpha$ without generating interferences to the primary users. We first consider the case where the operator already knows the distribution of $\alpha$.\(^6\) We will relax this assumption in Section 5.6 and discuss how the operator learn $\alpha$ distribution over time.

- **Cost parameters $C_s$ and $C_l$:** the operator’s fixed sensing and leasing costs per unit bandwidth, respectively. Sensing cost $C_s$ depends on the operator’s sensing technologies. We focus on the commonly used energy detection for sensing technology [3]. To track and measure the energy of received signal, the operator needs to use a bandpass filter to square the output signal and then integrate over a proper observation interval. Thus the sensing cost only involves time and energy spent on channel sampling and signal processing ( [23,43]). Sensing over different channels often needs to be done sequentially due to the potentially large number of channels open to opportunistic spectrum access and the limited power/hardware capacity of cognitive radios ( [51]). The larger sensing bandwidth and the more channels, the longer time and higher energy it requires ( [52]). For simplicity, we assume that total sensing cost is linear in the sensing bandwidth $B_s$. Leasing cost $C_l$ is determined through the negotiation between the operator and the primary operator and is assumed to be larger than $C_s$.\(^7\)

\(^6\)This is reasonable if the operator can extensively measure PUs’ activity patterns beforehand [47, 48], and then approximate the $\alpha$ distribution accurately as in [49,50].

\(^7\)If $C_l$ is smaller than $C_s$, then the case becomes trivial as the operator will only lease spectrum. In a
Stage I: Operator determines sensing amount $B_s$  
realize available bandwidth $B_s\alpha$

Stage II: Operator determines leasing amount $B_l$

Stage III: Operator announces price $\pi$ to market

Stage IV: End-users determine the demands for bandwidth from the operator

Figure 5.2: A Stackelberg Game

- **Pricing decision $\pi$:** the operator’s choice of price per unit bandwidth to the users.

### 5.2.3 A Stackelberg Game

We consider a Stackelberg Game between the operator and the users as shown in Fig. 5.2. The operator is the Stackelberg leader: it first decides the sensing amount $B_s$ in Stage I, then decides the leasing amount $B_l$ in Stage II (based on the sensing result $B_s\alpha$), and then announces the price $\pi$ to the users in Stage III (based on the total supply $B_s\alpha + B_l$). Finally, the users choose their bandwidth demands to maximize their individual payoffs in Stage IV.

For a more general model, the primary operator can choose the value of $C_l$ to maximize its own profit. We will study this model in our future work.
We note that “sensing followed by leasing and pricing” is optimal for the operator to maximize its profit. Assuming sensing (though unreliable) is cheaper than leasing, the operator should observe sensing result first and then lease only if sensing does not provide enough resource. If the operator determines sensing, leasing and pricing simultaneously, then it is likely to “over-lease” expensive resource (compared with “sensing followed by leasing”) to avoid having too little resource when $\alpha$ is small. Also, simultaneously determination of price makes it harder to reach the market equilibrium where supply equals demand.

We can also show that optimizing the leasing and pricing decisions sequentially (as in our chapter) leads to the same profit if we optimize them simultaneously.

### 5.3 Backward Induction of the Four-stage Game

The Stackelberg game falls into the class of dynamic game, and the common solution concept is the Subgame Perfect Equilibrium (SPE, or simply as \textit{equilibrium} in this chapter). Note that the traditional Nash equilibrium investigates players’ simultaneous actions in static game, thus is not applicable to our dynamic model [53]. A general technique for determining the SPE is the backward induction ([54]). We will start with Stage IV and analyze the users’ behaviors given the operator’s investment and pricing decisions. Then we will look at Stage III and analyze how the operator makes the pricing decision
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given investment decisions and the possible reactions of the users in Stage IV. Finally we proceed to derive the operator’s optimal leasing decision in Stage II and then the optimal sensing decision in Stage I. The backward induction captures the sequential dependence of the decisions in four stages.

5.3.1 Spectrum Allocation in Stage IV

In Stage IV, end-users determine their bandwidth demands given the unit price $\pi$ announced by the operator in stage III.

Each user can represent a transmitter-receiver node pair in an ad hoc network, or a node that transmits to the operator’s base station in an uplink scenario. We assume that users access the spectrum provided by the operator through OFDM (Orthogonal frequency-division multiplexing) to avoid mutual interferences.\(^8\) User $i$’s achievable rate (in nats) is:\(^9\)

$$r_i(w_i) = w_i \ln \left(1 + \frac{P_{\text{max}}^i h_i}{n_0 w_i}\right), \quad (5.1)$$

where $w_i$ is the allocated bandwidth from the operator, $P_{\text{max}}^i$ is user $i$’s maximum transmission power, $n_0$ is the noise power per unit bandwidth, $h_i$ is user $i$’s channel gain between user $i$’s transmitter to the operator’s secondary base station in an uplink scenario. To obtain rate in (5.1), user $i$ spreads its maximum transmission power $P_k^\text{max}$ across the entire allocated bandwidth $w_i$. To

---

\(^8\)We focus on a single OFDMA cell case, where users transmit over orthogonal bands. The interference management across multiple cells is beyond the scope of this chapter.

\(^9\)We assume that the operator only provides bandwidth without restricting the application types. This assumption has been commonly used in dynamic spectrum sharing literature, e.g., [16, 19, 32, 38].
simplify the notation, we let \( g_i = \frac{P_i^{\max} h_i}{n_0} \), thus \( g_i/w_i \) is the user \( i \)'s signal-to-noise ratio (SNR). Here we focus on best-effort users who are interested in maximizing their data rates. Each user only knows its local information (i.e., \( P_i^{\max}, h_i, \) and \( n_0 \)) and does not know anything about other users.

From a user’s point of view, it does not matter whether the bandwidth has been obtained by the operator through spectrum sensing or dynamic leasing. Each unit of allocated bandwidth is perfectly reliable for the user.

To obtain closed-form solutions, we first focus on the high SNR regime where \( \text{SNR} \gg 1 \). This is motivated by the fact that users often have limited choices of modulation and coding schemes, and thus may not be able to decode a transmission if the SNR is below a threshold. In the high SNR regime, the rate in (5.1) can be approximated as

\[
    r_i(w_i) = w_i \ln \left( \frac{g_i}{w_i} \right). \tag{5.2}
\]

Although the analytical solutions in Section 5.3 are derived based on (5.2), we emphasize that all the major engineering insights remain true in the general SNR regime. A formal proof is in Section 5.4.

A user \( i \)'s payoff is a function of the allocated bandwidth \( w_i \) and the price \( \pi \),

\[
    u_i(\pi, w_i) = w_i \ln \left( \frac{g_i}{w_i} \right) - \pi w_i, \tag{5.3}
\]

i.e., the difference between the data rate and the linear payment \( (\pi w_i) \). Payoff \( u_i(\pi, w_i) \) is concave in \( w_i \), and the unique bandwidth demand that maximizes
the payoff is
\[ w_i^*(\pi) = \arg \max_{w_i \geq 0} u_i(\pi, w_i) = g_i e^{-(1+\pi)}, \] (5.4)
which is always positive, linear in \( g_i \), and decreasing in price \( \pi \). Since \( g_i \) is linear in channel gain \( h_i \) and transmission power \( P_{\text{max}}^i \), then a user with a better channel condition or a larger transmission power has a larger demand.

Equation (5.4) shows that each user \( i \) achieves the same SNR:
\[ \text{SNR}_i = \frac{g_i}{w_i^*(\pi)} = e^{(1+\pi)}. \]
but a different payoff that is linear in \( g_i \),
\[ u_i(\pi, w_i^*(\pi)) = g_i e^{-(1+\pi)}. \]
We denote users’ aggregate wireless characteristics as \( G = \sum_{i \in I} g_i \). The users’ total demand is
\[ \sum_{i \in I} w_i^*(\pi) = Ge^{-(1+\pi)}. \] (5.5)
Next, we consider how the operator makes the investment (sensing and leasing) and pricing decisions in Stages I-III based on the total demand in eq. (5.5).\(^{10}\) In particular, we will show that the operator will always choose a price in Stage III such that the total demand (as a function of price) does not exceed the total supply.

5.3.2 Optimal Pricing Strategy in Stage III

We focus on the uplink transmissions in an infrastructure based secondary
\(^{10}\)We assume that the operator knows the value of \( G \) through proper feedback mechanism from the users.
network (like the one proposed in IEEE 802.22 standard), where the secondary
users need to communicate directly with the secondary base station (i.e., the
operator). Similar as today’s cellular network, a user needs to register with
the operator when it enters and leaves the network. Thus at any given time,
the operator knows precisely how many users are using the service. Equation
(5.4) shows that each user’s demand depends on the received power (i.e., the
product of its transmission power and the channel gain) at the secondary base
station in the uplink cellular network. This can be measured at the secondary
base station when the user first registers with the operator. Thus the operator
knows the exact demand from the users as well as user population in our model.

In Stage III, the operator determines the optimal pricing considering users’
total demand (5.5), given the bandwidth supply $B_s \alpha + B_l$ obtained in Stage
II. The operator profit is

$$R(B_s, \alpha, B_l, \pi) = \min \left( \pi \sum_{i \in I} w_i^*(\pi), \pi (B_l + B_s \alpha) \right)$$

$$- (B_s C_s + B_l C_l), \quad (5.6)$$

which is the difference between the revenue and total cost. The min oper-
ation denotes the fact that the operator can only satisfy the demand up to
its available supply. The objective of Stage III is to find the optimal price
$\pi^*(B_s, \alpha, B_l)$ that maximizes the profit, that is,

$$R_{III}(B_s, \alpha, B_l) = \max_{\pi \geq 0} R(B_s, \alpha, B_l, \pi). \quad (5.7)$$
Figure 5.3: Different intersection cases of $D(\pi)$ and $S(\pi)$

The subscript “III” denotes the best profit in Stage III.

Since the bandwidths $B_s$ and $B_l$ are given in this stage, the total cost $B_sC_s + B_lC_l$ is already fixed. The only optimization is to choose the optimal price $\pi$ to maximize the revenue, i.e.,

$$\max_{\pi \geq 0} \min \left( \pi \sum_{i \in I} w_i^*(\pi), \pi (B_l + B_s\alpha) \right).$$

The solution of problem (5.8) depends on the bandwidth investment in Stages I and II. Let us define $D(\pi) = \pi \sum_{i \in I} w_i^*(\pi)$ and $S(\pi) = \pi (B_l + B_s\alpha)$.

Figure 5.3 shows three possible relationships between these two terms, where $S_j(\pi)$ (for $j = 1, 2, 3$) represents each of the three possible choices of $S(\pi)$ depending on the bandwidth $B_l + B_s\alpha$:

- $S_1(\pi)$ (excessive supply): No intersection with $D(\pi)$;
Table 5.2: Optimal Pricing Decision and Profit in Stage III

<table>
<thead>
<tr>
<th>Total Bandwidth Obtained in Stages I and II</th>
<th>Optimal Price $\pi^* (B_s, \alpha, B_l)$</th>
<th>Optimal Profit $R_{III}(B_s, \alpha, B_l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excessive Supply Regime: $B_l + B_s \alpha \geq Ge^{-2}$</td>
<td>$\pi^{ES} = 1$</td>
<td>$R_{III}^{ES}(B_s, \alpha, B_l) = Ge^{-2} - B_s C_s - B_l C_l$</td>
</tr>
<tr>
<td>Conservative Supply Regime: $B_l + B_s \alpha &lt; Ge^{-2}$</td>
<td>$\pi^{CS} = \ln \left( \frac{G}{B_l + B_s \alpha} \right) - 1$</td>
<td>$R_{III}^{CS}(B_s, \alpha, B_l) = (B_l + B_s \alpha) \ln \left( \frac{G}{B_l + B_s \alpha} \right) - B_s (\alpha + C_s) - B_l (1 + C_l)$</td>
</tr>
</tbody>
</table>

- $S_2(\pi)$ (excessive supply): intersect once with $D(\pi)$ where $D(\pi)$ has a non-negative slope;

- $S_3(\pi)$ (conservative supply): intersect once with $D(\pi)$ where $D(\pi)$ has a negative slope.

In the excessive supply regime, $\max_{\pi \geq 0} \min (S(\pi), D(\pi)) = \max_{\pi \geq 0} D(\pi)$, i.e., the max-min solution occurs at the maximum value of $D(\pi)$ with $\pi^* = 1$. In this regime, the total supply is larger than the total demand at the best price choice. In the conservative supply regime, the max-min solution occurs at the unique intersection point of $D(\pi)$ and $S(\pi)$. The above observations lead to the following result.

**Theorem 18.** The optimal pricing decision and the corresponding optimal profit at Stage III can be characterized by Table 5.2.
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The proof of Theorem 18 is given in Section 5.8.1. Note that in the excessive supply regime, some bandwidth is left unsold (i.e., $S(\pi^*) > D(\pi^*)$). This is because the acquired bandwidth is too large, and selling all the bandwidth will lead to a very low price that decreases the revenue (the product of price and sold bandwidth). The profit can be apparently improved if the operator acquires less bandwidth in Stages I and II. Later analysis in Stages II and I will show that the equilibrium of the game must lie in the conservative supply regime if the sensing cost is non-negligible. In the conservative supply regime, the optimal price still guarantees supply equal to demand (i.e., market equilibrium).

5.3.3 Optimal Leasing Strategy in Stage II

In Stage II, the operator decides the optimal leasing amount $B_l$ given the sensing result $B_s, \alpha$:

$$R_{II}(B_s, \alpha) = \max_{B_l \geq 0} R_{III}(B_s, \alpha, B_l). \quad (5.9)$$

We decompose problem (5.9) into two subproblems based on the two supply regimes in Table 5.2,

1. Choose $B_l$ to reach the excessive supply regime in Stage III:

$$R_{II}^{ES}(B_s, \alpha) = \max_{B_l \geq \max\{G - B_s, \alpha, 0\}} R_{III}^{ES}(B_s, \alpha, B_l). \quad (5.10)$$
Table 5.3: Optimal Leasing Decision and Profit in Stage II

<table>
<thead>
<tr>
<th>Given Sensing Result</th>
<th>Optimal Leasing Amount $B_l^*$</th>
<th>Optimal Profit $R_{II}(B_s, \alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CS1) $B_s\alpha \leq Ge^{-(2+C_l)}$</td>
<td>$B_l^{CS1} = Ge^{-(2+C_l)} - B_s\alpha$</td>
<td>$R_{II}^{CS1}(B_s, \alpha) = Ge^{-(2+C_l)} + B_s(\alpha C_l - C_s)$</td>
</tr>
<tr>
<td>(CS2) $B_s\alpha \in (Ge^{-(2+C_l)}, Ge^{-2}]$</td>
<td>$B_l^{CS2} = 0$</td>
<td>$R_{II}^{CS2}(B_s, \alpha) = B_s\alpha \ln \left(\frac{Ge}{B_s \alpha}\right) - B_s(\alpha + C_s)$</td>
</tr>
<tr>
<td>(ES3) $B_s\alpha &gt; Ge^{-2}$</td>
<td>$B_l^{ES3} = 0$</td>
<td>$R_{II}^{ES3}(B_s, \alpha) = Ge^{-2} - B_s C_s$</td>
</tr>
</tbody>
</table>

2. Choose $B_l$ to reach the conservative supply regime in Stage III:

$$R_{II}^{CS}(B_s, \alpha) = \max_{0 \leq B_l \leq Ge^{-2} - B_s\alpha} R_{II}^{CS}(B_s, \alpha, B_l), \quad (5.11)$$

To solve subproblems (5.10) and (5.11), we need to consider the bandwidth obtained from sensing.

- **Excessive Supply** ($B_s\alpha > Ge^{-2}$): in this case, the feasible sets of both subproblems (5.10) and (5.11) are empty. In fact, the bandwidth supply is already in the excessive supply regime as defined in Table II, and it is optimal not to lease in Stage II.

- **Conservative Supply** ($B_s\alpha \leq Ge^{-2}$): first, we can show that the unique optimal solution of subproblem (5.10) is $B_l^* = Ge^{-2} - B_s\alpha$. This means that the optimal objective value of subproblem (5.10) is no larger than that of subproblem (5.11), and thus it is enough to consider subproblem (5.11) in the conservative supply regime only.
Base on the above observations and some further analysis, we can show the following:

**Theorem 19.** *In Stage II, the optimal leasing decision and the corresponding optimal profit are summarized in Table 5.3.*

The proof of Theorem 19 is given in Section 5.8.2. Table 5.3 contains three cases based on the value of $B_s \alpha$: (CS1), (CS2), and (ES3). The first two cases involve solving the subproblem (5.11) in the conservative supply regime, and the last one corresponds to the excessive supply regime. Although the decisions in cases (CS2) and (ES3) are the same (i.e., zero leasing amount), we still treat them separately since the profit expressions are different.

It is clear that we have an optimal *threshold* leasing policy here: the operator wants to achieve a total bandwidth equal to $G e^{-(2+C_l)}$ whenever possible. When the bandwidth obtained through sensing is not enough, the operator will lease additional bandwidth to reach the threshold; otherwise the operator will not lease.

### 5.3.4 Optimal Sensing Strategy in Stage I

In Stage I, the operator will decide the optimal sensing amount to maximize its expected profit by taking the uncertainty of the sensing realization factor $\alpha$ into account. The operator needs to solve the following problem

\[ R_I = \max_{B_s \geq 0} R_{II} (B_s), \]
where \( R_{II}(B_s) \) is obtained by taking the expectation of \( \alpha \) over the profit functions in Stage II (i.e., \( R_{II}^{CS1}(B_s, \alpha) \), \( R_{II}^{CS2}(B_s, \alpha) \), and \( R_{II}^{ES3}(B_s, \alpha) \) in Table 5.3).

To obtain closed-form solutions, we assume that the sensing realization factor \( \alpha \) follows a uniform distribution in \([0, 1] \). In Section 5.4.1, we prove that the major engineering insights also hold under any general distribution.

To avoid the trivial case where sensing is so cheap that it is optimal to sense a huge amount of bandwidth, we further assume that the sensing cost is non-negligible and is lower bounded by \( C_s \geq (1 - e^{-2C_l})/4 \).

To derive function \( R_{II}(B_s) \), we will consider the following three intervals:

1. Case I: \( B_s \in [0, Ge^{-(2+C_l)}] \). In this case, we always have \( B_s \alpha \leq Ge^{-(2+C_l)} \) for any value \( \alpha \in [0, 1] \), which corresponds to case (CS1) in Table 5.3.

   The expected profit is

   \[
   R_{II}^{1}(B_s) = E_{\alpha \in [0,1]} \left[ R_{II}^{CS1}(B_s, \alpha) \right] = Ge^{-(2+C_l)} + B_s \left( \frac{C_l}{2} - C_s \right),
   \]

   which is a linear function of \( B_s \). If \( C_s > C_l/2 \), \( R_{II}^{1}(B_s) \) is linearly decreasing in \( B_s \); if \( C_s < C_l/2 \), \( R_{II}^{1}(B_s) \) is linearly increasing in \( B_s \).

2. Case II: \( B_s \in (Ge^{-(2+C_l)}, Ge^{-2}] \). Depending on the value of \( \alpha \), \( B_s \alpha \) can
be in either case (CS1) or case (CS2) in Table 5.3. The expected profit is

\[
R_{II}^2(B_s) = E_{\alpha \in \left[0, \frac{G e^{-(2+C_l)}}{B_s}\right]} \left[ R_{II}^{CS1}(B_s, \alpha) \right]
\]

\[
+ E_{\alpha \in \left[\frac{G e^{-(2+C_l)}}{B_s}, 1\right]} \left[ R_{II}^{CS2}(B_s, \alpha) \right]
\]

\[
= \frac{B_s}{2} \ln \left( \frac{G}{B_s} \right) - \frac{B_s}{4} + \frac{B_s}{4} \left( \frac{G e^{-(2+C_l)}}{B_s} \right)^2 - B_s C_s.
\]

\(R_{II}^2(B_s)\) is a strictly concave function of \(B_s\) since its second-order derivative

\[
\frac{\partial^2 R_{II}^2(B_s)}{\partial B_s^2} = - \frac{1}{2B_s} \left[ \left( \frac{G e^{-(2+C_l)}}{B_s} \right)^2 - 1 \right] < 0
\]

as \(B_s > G e^{-(2+C_l)}\) in this case.

3. Case III: \(B_s \in (G e^{-2}, \infty)\). Depending on the value of \(\alpha\), \(B_s\) can be any of the three cases in Table 5.3. The expected profit is

\[
R_{II}^3(B_s) = E_{\alpha \in \left[0, \frac{G e^{-(2+C_l)}}{B_s}\right]} \left[ R_{II}^{CS1}(B_s, \alpha) \right]
\]

\[
+ E_{\alpha \in \left[\frac{G e^{-(2+C_l)}}{B_s}, \frac{G e^{-2}}{B_s}\right]} \left[ R_{II}^{CS2}(B_s, \alpha) \right]
\]

\[
+ E_{\alpha \in \left[\frac{G e^{-2}}{B_s}, 1\right]} \left[ R_{II}^{ES3}(B_s, \alpha) \right]
\]

\[
= \left( \frac{G}{e^2} \right)^2 \frac{e^{-2C_l} - 1}{4B_s} - B_s C_s + \frac{G}{e^2}.
\]

Because its first-order derivative

\[
\frac{\partial R_{II}^3(B_s)}{\partial B_s} = \left( \frac{G e^{-2}}{B_s} \right)^2 \frac{1 - e^{-2C_l}}{4} - C_s < 0,
\]

as \(B_s > G e^{-2}\) in this case, \(R_{II}^3(B_s)\) is decreasing in \(B_s\) and achieves its maximum at \(B_s = G e^{-2}\).
CHAPTER 5. MONOPOLY SPECTRUM MARKET USING COGNITIVE RADIOS

Table 5.4: Choice of Optimal Sensing Amount in Stage I

<table>
<thead>
<tr>
<th>Optimal Sensing Decision $B_s^*$</th>
<th>Expected Profit $R_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Sensing Cost Regime: $C_s \geq C_l/2$</td>
<td>$B_s^* = 0$</td>
</tr>
<tr>
<td>Low Sensing Cost Regime: $C_s \in [(1-e^{-2C_l})/4, C_l/2]$</td>
<td>$B_s^* = B_s^{L*}$, solution to eq. (5.13)</td>
</tr>
</tbody>
</table>

To summarize, the operator needs to maximize

$$R_{II}(B_s) = \begin{cases} 
R_{II}^1(B_s), & \text{if } 0 \leq B_s \leq Ge^{-(2+C_l)}; \\
R_{II}^2(B_s), & \text{if } Ge^{-(2+C_l)} < B_s \leq Ge^{-2}; \\
R_{II}^3(B_s), & \text{if } B_s > Ge^{-2}. 
\end{cases} \quad (5.12)$$

We can verify that Case II always achieves a higher optimal profit than Case III. This means that the optimal sensing will only lead to either case (CS1) or case (CS2) in Stage II, which corresponds to the conservative supply regime in Stage III. This confirms our previous intuition that equilibrium is always in the conservative supply regime under a non-negligible sensing cost, since some resource is wasted in the excessive supply regime (see discussions in Section 5.3.2).

Table 5.4 shows that the sensing decision is made in the following two cost regimes:

- **High sensing cost regime** ($C_s > C_l/2$): it is optimal not to sense. Intu-
Figure 5.4: Expected profit in Stage II under different sensing and leasing costs

itively, the coefficient $1/2$ is due to the uniform distribution assumption of $\alpha$, i.e., on average obtaining one unit of available bandwidth through sensing costs $2C_s$.

- **Low sensing cost regime** ($C_s \in \left[\frac{1-e^{-2C_l}}{4}, \frac{C_t}{2}\right]$): the optimal sensing amount $B_s^{L*}$ is the unique solution to the following equation:

$$
\frac{\partial R^2_{II}(B_s)}{\partial B_s} = \frac{1}{2} \ln \left( \frac{1}{B_s/G} \right) - \frac{3}{4} \ln \left( \frac{e^{-2+C_l}}{2B_s/G} \right)^2 = 0. \quad (5.13)
$$

The uniqueness of the solution is due to the strict concavity of $R^2_{II}(B_s)$ over $B_s$. We can further show that $B_s^{L*}$ lies in the interval of $[Ge^{-(2+C_l)}, Ge^{-2}]$ and is linear in $G$. Finally, the operator’s optimal expected profit is

$$
R^L_I = \frac{B_s^{L*}}{2} \ln \left( \frac{G}{B_s^{L*}} \right) - \frac{B_s^{L*}}{4} + \frac{1}{4B_s^{L*}} \left( \frac{G}{e^{2+C_l}} \right)^2 - B_s^{L*} C_s. \quad (5.14)
$$

Based on these observations, we can show the following:
Table 5.5: The Operator’s and Users’ Equilibrium Behaviors

<table>
<thead>
<tr>
<th>Sensing Cost Regimes</th>
<th>Sensing Cost: High: $C_s \geq \frac{C_l}{2}$</th>
<th>Low Sensing Cost: $\frac{1-e^{-2C_l}}{4} \leq C_s \leq \frac{C_l}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Sensing Amount $B_s^*$</td>
<td>0</td>
<td>$B_s^{L_s} \in [Ge^{-(2+C_l)}, Ge^{-2}]$, solution to eq. (5.13)</td>
</tr>
<tr>
<td>Sensing Realization Factor $\alpha$</td>
<td>$0 \leq \alpha \leq 1$</td>
<td>$0 \leq \alpha \leq \tilde{\alpha}$</td>
</tr>
<tr>
<td>Optimal Leasing Amount $B_l^*$</td>
<td>$Ge^{-(2+C_l)}$</td>
<td>$Ge^{-(2+C_l)} - B_s^{L_s} \tilde{\alpha}$</td>
</tr>
<tr>
<td>Optimal Price $\pi^*$</td>
<td>$1 + C_l$</td>
<td>$1 + C_l$</td>
</tr>
<tr>
<td>Expected Profit $R_I$</td>
<td>$R_I^H = Ge^{-(2+C_l)}$</td>
<td>$R_I^L$ in eq. (5.14)</td>
</tr>
<tr>
<td>User i’s SNR</td>
<td>$e^{(2+C_l)}$</td>
<td>$e^{(2+C_l)}$</td>
</tr>
<tr>
<td>User i’s Payoff</td>
<td>$g_i e^{-(2+C_l)}$</td>
<td>$g_i e^{-(2+C_l)}$</td>
</tr>
</tbody>
</table>

**Theorem 20.** In Stage I, the optimal sensing decision and the corresponding optimal profit are summarized in Table 5.4. The optimal sensing amount $B_l^*$ is linear in $G$.

Figure 5.4 shows two possible cases for the function $R_{II}(B_s)$. The vertical dashed line represents $B_s = e^{-(2+C_l)}$. For illustration purpose, we assume $G = 1$, $C_l = 2$, and $C_s = \{0.8, 1.2\}$. When the sensing cost is large (i.e., $C_s = 1.2 > C_l/2$), $R_{II}(B_s)$ achieves its optimum at $B_s = 0$ and thus it is optimal not to sense. When the sensing cost is small (i.e., $C_s = 0.8 < C_l/2$), $R_{II}(B_s)$ achieves its optimum at $B_s > e^{-(2+C_l)}$ and it is optimal to sense a positive amount of spectrum.
5.4 Equilibrium Summary and Numerical Results

Based on the discussions in Section 5.3, we summarize the operator’s equilibrium sensing/leasing/pricing decisions and the equilibrium resource allocations to the users in Table 5.5. These decisions can be directly computed by the operator in each time slot without using any iterative algorithm. Several interesting observations are as follows.

**Observation 9.** Both the optimal sensing amount $B_s^*$ (either 0 or $B_s^{L^*}$) and leasing amount $B_l^*$ are linear in the users’ aggregate wireless characteristics $G = \sum_{i \in I} P_{i}^{\text{max}} h_i / n_0$.

The linearity enables us to normalize optimal sensing and leasing decisions by users’ aggregate wireless characteristics, and study the relationships be-
between the normalized optimal decisions and other system parameters as in Figs. 5.5 and 5.6.

Figure 5.5 shows how the normalized optimal sensing decision $B_s^*/G$ changes with the costs. For a given leasing cost $C_l$, the optimal sensing decision $B_s^*$ decreases as the sensing cost $C_s$ becomes more expensive, and drops to zero when $C_s \geq C_l/2$. For a given sensing cost $C_s$, the optimal sensing decision $B_s^*$ increases as the leasing cost $C_l$ becomes more expensive, in which case sensing becomes more attractive. Note that the sensing decision $B_s^*$ is the same in each time slot if the users’ population and channel conditions do not change.

Figure 5.6 shows how the normalized optimal leasing decision $B_l^*/G$ depends on the costs $C_l$ and $C_s$ as well as the sensing realization factor $\alpha$ in the low sensing cost regime (denoted by “L”). In all cases, a higher value $\alpha$ means
more bandwidth is obtained from sensing and there is a less need to lease. Figure 5.6 confirms the threshold structure of the optimal leasing decisions in Section 5.3.3, i.e., no leasing is needed whenever the bandwidth obtained from sensing reaches a threshold. Comparing different curves, we can see that the operator chooses to lease more as leasing becomes cheaper or sensing becomes more expensive. For high sensing cost regime, the optimal leasing amount only depends on $C_l$ and is independent of $C_s$ and $\alpha$, and thus is not shown here. Note that the leasing decision $B_l^*$ may change with the sensing realization factor $\alpha$, which depends on the burstiness of the primary user’s stochastic traffic.

**Observation 10.** The optimal pricing decision $\pi^*$ in Stage III is independent of users’ aggregate wireless characteristics $G$. 
Observation 10 is closely related to Observation 9. Since the total bandwidth is linear in $G$, the “average” resource allocation per user is “constant” at the equilibrium. This implies that the price must be independent of the user population change, otherwise the resource allocation to each individual user will change with the price accordingly.

**Observation 11.** The optimal pricing decision $\pi^*$ in Stage III is non-increasing in $\alpha$ in the low sensing cost regime.

First, in the low sensing cost regime where the sensing result is poor (i.e., $\alpha$ is small as the third column in Table 5.5), the operator will lease additional resource such that the total bandwidth reaches the threshold $Ge^{-2(2+C_l)}$. In this case, the price is a constant and is independent of the value of $\alpha$. Second, when the sensing result is good (i.e., $\alpha$ is large as in the last column in Table 5.5), the total bandwidth is large enough. In this case, as $\alpha$ increases, the amount of total bandwidth increases, and the optimal price decreases to maximize the profit.

Figure 5.7 shows how the optimal price changes with various costs and $\alpha$ in the low sensing cost regime. It is clear that price is first a constant and then starts to decrease when $\alpha$ is larger than a threshold. The threshold decreases in the optimal sensing decision of $B_s^{L^*}$: a smaller sensing cost or a higher leasing cost will lead to a higher $B_s^{L^*}$ and thus a smaller threshold.

It is interesting to notice that the equilibrium price only changes in a time
Figure 5.8: Optimal price $\pi^*$ over time with different sensing costs and $\alpha$ realizations

slot where the sensing realization factor $\alpha$ is large. This means that although operator has the freedom to change the price in every time slot, the actual variation of price is much less frequent. This makes it easier to implement in practice. Figure 5.8 illustrates this with different sensing costs and $\alpha$ realizations. In each time slot, a realization of $\alpha$ distribution is drawn and we can derive equilibrium price from Table 5.5. The left two subfigures correspond to the realizations of $\alpha$ and the corresponding prices with $C_s = 0.48$ and $C_l = 1$. As the sensing cost $C_s$ is quite high in this case, the operator does not rely
heavily on sensing. As a result, the variability of $\alpha$ (in the upper subfigure) has very small impact on the equilibrium price (in the lower subfigure). In fact, the price only changes in 11 out 50 time slots, and the maximum amplitude variation is around 10%. The right two figures correspond to the case where $C_s = 0.35$ and $C_l = 1$. As sensing cost is cheaper in this case, the operator senses more and the impact of $\alpha$ on price is larger. The price changes in 30 out of 50 time slots, and the variation in amplitude can be as large as 30%.

**Observation 12.** The operator will sense the spectrum only if the sensing cost is lower than a threshold. Also, it will lease additional spectrum only if the spectrum obtained through sensing is below a threshold. Furthermore, it will charge a constant price to the users if the total bandwidth obtained through sensing and leasing does not exceed a threshold.

**Observation 13.** Each user $i$ obtains the same SNR independent of $g_i$ and a payoff linear in $g_i$.

Observation 13 shows that users obtains fair and predictable resource allocation at the equilibrium. In fact, a user does not need to know anything about the total number and payoffs of other users in the system. It can simply predict its QoS if it knows the cost structure of the network ($C_s$ and $C_l$).\footnote{The analysis of the game, however, does not require the users to know $C_s$ or $C_l$.} Such property is highly desirable in practice.

Finally, users achieve the same high SNR at the equilibrium. The SNR value
is either \(e^{2+C_i}\) or \(G/(B^*_\alpha)\), both of which are larger than \(e^2\). This means that the approximation ratio \(\ln(\text{SNR}_i)/\ln(1+\text{SNR}_i) > \ln(e^2)/\ln(1+e^2) \approx 94\%\). The ratio can even be close to one if the price \(\pi\) is high.

In Sections 5.3.1 and 5.3.4, we made the high SNR regime approximation and the uniform distribution assumption of \(\alpha\) to obtain closed-form expressions. Next we show that relaxing both assumptions will not change any of the major insights.

### 5.4.1 Robustness of the Observations

**Theorem 21.** Observations 9-13 still hold under the general SNR regime (as in (5.1)) and any general distribution of \(\alpha\).

**Proof.** We represent a user \(i\)'s payoff function in the general SNR regime,

\[
u_i(\pi, w_i) = w_i \ln \left(1 + \frac{g_i}{w_i}\right) - \pi w_i.
\]  

(5.15)

The optimal demand \(w^*_i(\pi)\) that maximizes (5.15) is \(w^*_i(\pi) = g_i/Q(\pi)\), where \(Q(\pi)\) is the unique positive solution to \(F(\pi, Q) := \ln(1+Q) - \frac{Q}{1+Q} - \pi = 0\). We find the inverse function of \(Q(\pi)\) to be \(\pi(Q) = \ln(1+Q) - \frac{Q}{1+Q}\). By applying the implicit function theorem, we can obtain the first-order derivative of function \(Q(\pi)\) over \(\pi\) as

\[
Q'(\pi) = -\frac{\partial F(\pi, Q)/\partial \pi}{\partial F(\pi, Q)/\partial Q} = \frac{(1 + Q(\pi))^2}{Q(\pi)},
\]

(5.16)

which is always positive. Hence, \(Q(\pi)\) is increasing in \(\pi\).
User $i$’s optimal payoff is

$$u_i(\pi, w_i^*(\pi)) = \frac{g_i}{Q(\pi)}[\ln(1 + Q(\pi)) - \pi].$$

(5.17)

As a result, a user’s optimal SNR equals $g_i/w_i^*(\pi) = Q(\pi)$ and is user-independent. The total demand from all users equals $G/Q(\pi)$, and the operator’s investment and pricing problem is

$$R^* = \max_{B_s \geq 0} \mathbb{E}_{\alpha \in [0,1]} \left[ \max_{B_l \geq 0} \min_{\pi \geq 0} \left( \frac{\pi G}{Q(\pi)}, \pi(B_l + B_s\alpha) \right) - B_s C_s - B_l C_l \right].$$

(5.18)

Define $\tilde{R}^* = \frac{R^*}{G}$, $\tilde{B}_l = \frac{B_l}{G}$, and $\tilde{B}_s = \frac{B_s}{G}$. Then solving (5.18) is equivalent to solving

$$\tilde{R}^* = \max_{\tilde{B}_s \geq 0} \mathbb{E}_{\alpha \in [0,1]} \left[ \max_{\tilde{B}_l \geq 0} \min_{\pi \geq 0} \left( \frac{\pi}{Q(\pi)}, \pi(\tilde{B}_l + \tilde{B}_s\alpha) \right) - \tilde{B}_s C_s - \tilde{B}_l C_l \right].$$

(5.19)

In Problem (5.19), it is clear that the operator’s optimal decisions on leasing, sensing and pricing do not depend on users’ aggregate wireless characteristics. This is true for any continuous distribution of $\alpha$. And a user’s optimal payoff in eq. (5.17) is linear in $g_i$ since $Q(\pi)$ is independent of users’ wireless characteristics. This shows that Observations 9, 10, and 13 hold for the general SNR regime and any general distribution of $\alpha$. We can also show that Observations 11 and 12 hold in the general case, with a detailed proof in Section 5.8.3. \qed
5.5 The Impact of Spectrum Sensing Uncertainty

The key difference between our model and most existing literature (e.g., [19, 27, 28, 32, 34, 35]) is the possibility of obtaining resource through the cheaper but uncertain approach of spectrum sensing. Here we will elaborate the impact of sensing on the performances of operator and users by comparing with the baseline case where sensing is not possible. Note that in the high sensing cost regime it is optimal not to sense, as a result, the performance of the operator and users will be the same as the baseline case. Hence we will focus on the low sensing cost regime in Table 5.5.

Observation 14. The operator’s optimal expected profit always benefits from the availability of spectrum sensing in the low sensing cost regime.
Figure 5.10: Operator’s normalized optimal realized profit as a function of $\alpha$.

Figure 5.9 illustrates the normalized optimal expected profit as a function of the sensing cost. We assume leasing cost $C_l = 2$, and thus the low sensing cost regime corresponds to the case where $C_s \in [0, 2]$ in the figure. It is clear that sensing achieves a better optimal expected profit in this regime. In fact, sensing leads to 250% increase in profit when $C_s = 0.2$. The benefit decreases as the sensing cost becomes higher. When sensing becomes too expensive, the operator will choose not to sense and thus achieve the same profit as in the baseline case.

**Theorem 22.** The operator’s realized profit (i.e., the profit for a given $\alpha$) is a strictly increasing function in $\alpha$ in the low sensing cost regime. Furthermore, there exists a threshold $\alpha_{th} \in (0, 1)$ such that the operator’s realized profit is larger than the baseline approach if $\alpha > \alpha_{th}$.
Proof. As in Table 5.5, we have two cases in the low sensing cost regime:

• If \( \alpha \leq Ge^{-(2+C_l)}/B_s^{L_s} \), then substituting \( B_s^{L_s} \) into \( R_{l_II}^{CS1}(B_s, \alpha) \) in Table 5.3 leads to the realized profit

\[
R_{l_II}^{CS1}(\alpha) = Ge^{-(2+C_l)} - B_s^{L_s}C_s + B_s^{L_s}\alpha C_l,
\]

which is strictly and linearly increasing in \( \alpha \).

• If \( \alpha \geq Ge^{-(2+C_l)}/B_s^{L_s} \), then substituting \( B_s^{L_s} \) into \( R_{l_II}^{CS2}(B_s, \alpha) \) in Table 5.3 leads to the realized profit

\[
R_{l_II}^{CS2}(\alpha) = B_s^{L_s}\alpha \left( \ln \left( \frac{G}{B_s^{L_s}\alpha} \right) - 1 \right) - B_s^{L_s}C_s.
\]

Because the first-order derivative

\[
\frac{\partial R_{l_II}^{CS2}(\alpha)}{\partial \alpha} = B_s^{L_s}\left( \ln \left( \frac{G}{B_s^{L_s}\alpha} \right) - 2 \right) > 0,
\]

as \( B_s^{L_s} \leq Ge^{-2} \), \( R_{l_II}^{CS2}(\alpha) \) is strictly increasing in \( \alpha \).

We can also verify that \( R_{l_II}^{CS1}(\alpha) = R_{l_II}^{CS2}(\alpha) \) when \( \alpha = Ge^{-(2+C_l)}/B_s^{L_s} \). Therefore, the realized profit is a continuous and strictly increasing function of \( \alpha \).

Next we prove the existence of threshold \( \alpha_{th} \). First consider the extreme case \( \alpha = 0 \). Since the operator obtains no bandwidth through sensing but still incurs some cost, the profit in this case is lower than the baseline case. Furthermore, we can verify that \( R_{l_II}^{CS2}(1) > R_{l}^{H} \) in Table 5.5, thus the realized profit at \( \alpha = 1 \) is always larger than the baseline case. Together with the continuity and strictly increasing nature of the realized profit function, we have proven the existence of threshold of \( \alpha_{th} \). \( \square \)
Figure 5.10 shows the realized profit as a function of $\alpha$ for different costs. The realized profit is increasing in $\alpha$ in both cases. The “crossing” feature of the two increasing curves is because the optimal sensing $B_s^*$ is larger under a cheaper sensing cost ($C_s = 0.5$), which leads to larger realized profit loss (gain, respectively) when $\alpha \to 0$ ($\alpha \to 1$, respectively). This shows the tradeoff between improvement of expected profit and the large variability of the realized profit.

**Theorem 23.** *Users always benefit from the availability of spectrum sensing in the low sensing cost regime.*

*Proof.* In the baseline approach without sensing, the operator always charges the price $1 + C_l$. As shown in Table 5.5, the equilibrium price $\pi^*$ with sensing is always no larger than $1 + C_l$ for any value of $\alpha$. Since a user’s payoff is
strictly decreasing in price, the users always benefit from sensing.

Figure 5.11 shows how a user $i$'s normalized realized payoff $u_i^*/g_i$ changes with $\alpha$. The payoff linearly increases in $\alpha$ when $\alpha$ becomes larger than a threshold, in which case the equilibrium price becomes lower than $1 + C_l$. A smaller sensing cost $C_s$ leads to more aggressive sensing and thus more benefits to the users.

5.6 Learning the Distribution of Sensing Realization Factor $\alpha$

Our previous analysis assumes that the operator knows the distribution of sensing realization factor $\alpha$ beforehand. When such information is not available, the operator can learn the distribution through machine learning [55]. Next we propose a machine learning algorithm, where the operator uses the sensing realizations of previous time slots to update the distribution of $\alpha$.

Let us denote the probability density function (pdf) of $\alpha$ as $f(\alpha)$ over the support of $[0, 1]$. Although $f(\alpha)$ is in general continuous, we can approximate it through a proper discretization, i.e., representing the pdf by a probability mass function (pmf) over $N + 1$ equally spaced values (with the first and last values equal to 0 and 1, respectively).

The overall learning process is divided into several learning rounds. Each learning round consists of $M$ time slots. In the $k$th learning round, the operator
builds an empirical distribution of the $\alpha$ distribution, an $N + 1$ long vector $Record_k$, based on the observation of sensing results over the $M$ time slots in this round. At the end of the learning round, the operator updates the $\alpha$ distribution estimation $Distr_{k+1}$ based on the current value of $Distr_k$ and $Record_k$. The machine learning algorithm for updating the distribution of $\alpha$ is shown in Algorithm 1.

Algorithm 1 Machine Learning Algorithm

1: Initiates $Distr_1$ by an arbitrary distribution at $k = 1$.

2: while $k = 1$ or $Distr_k \neq Distr_{k-1}$ do

3: Compute $B^*_s$ according to $Distr_k$

4: Initialize the empirical distribution $Record_k = (0, ..., 0)$ (with $N + 1$ entries)

5: for time slot $m = 1$ to $M$ do

6: Senses the spectrum and records the $\alpha$ realization

7: Updates $Record_k$ with the current $\alpha$ realization by adding one to the corresponding entry. For example, if $\alpha = 0.36$ and $N = 100$, then the operator increases the 37th entry of $Record_k$ by one.

8: Compute $B^*_l$ and $\pi^*$ according to Tables 5.3 and 5.2 in the $m$th time slot

9: end for

10: Update $Distr_{k+1} = \beta Distr_k + (1 - \beta) \frac{Record_k}{M}$, where $\beta \in [0, 1]$ is the discount factor

11: $k := k + 1$

12: end while

A proper choice of the learning round length $M$ is important. If $M$ is too large, then the distribution update takes much time. If $M$ is too small, then the operator needs to frequently recompute its sensing decision according
Figure 5.12: Operator’s learning of the distribution of $\alpha$ over learning rounds with $\beta = 0.8$ to the updated distribution in each round. This increases the computation overhead.

5.6.1 Performance Evaluation of Machine Learning

We evaluate the performance of the proposed machine learning algorithm for updating the $\alpha$ distribution. For the illustration purpose, we assume that $\alpha$ follows a normal distribution with mean $m = 0.5$ and standard deviation $\delta = 0.15$\(^{12}\). Notice that the proposed algorithm works for any distribution of $\alpha$. The operator starts with an initial “guess” $Distr_1$ of uniform distribution. We assume $N = 100$ and $M = 5000$ in the simulation.

Figure 5.12 shows the operator’s estimation of the distribution of $\alpha$ over

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\(^{12}\)This choice of $m$ and $\delta$ ensures that almost all $\alpha$ realizations fall into the feasible range $[0, 1]$.\)
Figure 5.13: Operator’s adaptation of its sensing decision $B_s^*$ over learning rounds with different $\beta$ values multiple learning rounds with $\beta = 0.8$. It is generated by following Algorithm 1, where at each round the operator’s belief of $\alpha$ distribution is updated. The round 1 line corresponds to the uniform distribution of $Distr_1$. After 40 rounds, the operator obtains a pmf that approximations the real normal distribution very well.

Figure 5.13 illustrates the impact of discount factor $\beta$ on the convergence of the sensing decision $B_s^*$. Both curves converge as the number of learning rounds increases. When $\beta$ is small (e.g., $\beta = 0.5$), the operator’s sensing decision converges fast with many fluctuations. When $\beta$ is large (e.g., $\beta = 0.8$), the operator’s sensing decision converges slowly with less fluctuations. The operator needs to trade off convergence speed and fluctuations by choosing
the proper $\beta$.

In summary, using the proposed machine learning algorithm, the operator can quickly learn the distribution of $\alpha$, and can make good use of spectrum resource by adapting its sensing decisions dynamically.

5.7 Summary

This chapter represents some initial results towards understanding the new business models, opportunities, and challenges of the emerging cognitive virtual mobile network operators (C-MVNOs) under supply uncertainty. Here we focus on studying the trade-off between the cost and uncertainty of spectrum investment through sensing and leasing. We model the interactions between the operator and the users by a Stackelberg game, which captures the wireless heterogeneity of users in terms of maximum transmission power levels and channel gains.

We have discovered several interesting features of the game equilibrium. We show that the operator’s optimal sensing, leasing, and pricing decisions follow nice threshold structures. The availability of sensing always increases the operator’s expected profit, despite that the realized profit in each time slot will have some variations depending on the sensing result. Moreover, users always benefit in terms of payoffs when sensing is performed by the operator.

Through the analytical and simulation study of an idealized model in this
chapter, we have obtained various interesting engineering and economical insights into the operations of C-MVNOs. We hope that this chapter can contribute to the further understanding of proper network architecture decisions and business models of future cognitive radio systems.

5.8 Appendix

5.8.1 Proof of Theorem 18

Given the total bandwidth $B_l + B_s\alpha$, the objective of Stage III is to solve the optimization problem (5.8), i.e., $\max_{\pi \geq 0} \min(D(\pi), S(\pi))$. First, by examining the derivative of $D(\pi)$, i.e., $\partial D(\pi) / \partial \pi = (1 - \pi)Ge^{-(1+\pi)}$, we can see that the continuous function $D(\pi)$ is increasing in $\pi \in [0, 1]$ and decreasing in $\pi \in [1, +\infty]$, and $D(\pi)$ is maximized when $\pi = 1$. Since $S(\pi)$ always increases in $\pi$ and $D(\pi)$ is concave over $\pi \in [0, 1]$, $S(\pi)$ intersects with $D(\pi)$ if and only if $\frac{\partial D(\pi)}{\partial \pi} > \frac{\partial S(\pi)}{\partial \pi}$ at $\pi = 0$, i.e., $B_l + B_s\alpha < Ge^{-1}$.

Next we divide our discussion into the intersection case and the non-intersection case:

1. Given $B_l + B_s\alpha \leq Ge^{-1}$, $S(\pi)$ intersects with $D(\pi)$. By solving equation $S(\pi) = D(\pi)$ the intersection point is $\pi = \ln\left(\frac{G}{B_l + B_s\alpha}\right) - 1$. There are two subcases:

   - when $B_l + B_s\alpha \leq Ge^{-2}$, $S(\pi)$ intersects with $D(\pi)$, and $\min(D(\pi), S(\pi))$
is maximized at the intersection point, i.e., $\pi^* = \ln\left(\frac{G}{B_l + B_s\alpha}\right) - 1$. (See $S_3(\pi)$ in Fig. 5.3.)

- when $B_l + B_s\alpha \geq Ge^{-2}$, $S(\pi)$ intersects with $D(\pi)$, and $\min(D(\pi), S(\pi))$ is maximized at the maximum value of $D(\pi)$, i.e., $\pi^* = 1$. (See $S_2(\pi)$ in Fig. 5.3.)

2. Given $B_l + B_s\alpha \geq Ge^{-1}$, $S(\pi)$ doesn’t intersect with $D(\pi)$. Then $\min(D(\pi), S(\pi))$ is maximized at the maximum value of $D(\pi)$, i.e., $\pi^* = 1$. (See $S_1(\pi)$ in Fig. 5.3.)

5.8.2 Proof of Theorem 19

Given the sensing result $B_s\alpha$, the objective of Stage II is to solve the decomposed two subproblems (5.10) and (5.11), and select the best one with better optimal performance. Since $R_{III}^{ES}(B_s, \alpha, B_l)$ in subproblem (5.10) is linearly decreasing in $B_l$, its optimal solution always lies at the lower boundary of the feasible set (i.e., $B_l^* = \max\{Ge^{-2} - B_s\alpha, 0\}$). We compare the optimal profits of two subproblems (i.e., $R_{II}^{ES}(B_s, \alpha)$ and $R_{II}^{CS}(B_s, \alpha)$) for different sensing results:

1. Given $B_s\alpha > Ge^{-2}$, the obtained bandwidth after Stage I is already in excessive supply regime. Thus it is optimal not to lease for subproblem (5.10) (i.e., $B_l^{ES3} = 0$ of case (ES3) in Table 5.3).

2. Given $0 \leq B_s\alpha \leq Ge^{-2}$, the optimal leasing decision for subproblem
(5.11) is $B_l^* = Ge^{-2} - B_s\alpha$ and we have $R^{ES}_{III}(B_s, \alpha, B_l) = R^{CS}_{III}(B_s, \alpha, B_l)$ when $B_l = Ge^{-2} - B_s\alpha$, thus the optimal objective value of (5.10) is always no larger than that of (5.11) and it is enough to consider the conservative supply regime only. Since

$$\frac{\partial^2 R^{CS}_{III}(B_s, \alpha, B_l)}{\partial B_l^2} = -\frac{1}{B_l + B_s\alpha} < 0,$$

$R^{CS}_{III}(B_s, \alpha, B_l)$ is concave in $0 \leq B_l \leq Ge^{-2} - B_s\alpha$. Thus it is enough to examine the first-order condition

$$\frac{\partial R^{CS}_{III}(B_s, \alpha, B_l)}{\partial B_l} = \ln \left( \frac{G}{B_l + B_s\alpha} \right) - 2 - C_l = 0,$$

and the boundary condition $0 \leq B_l \leq Ge^{-2} - B_s\alpha$. This results in optimal leasing decision $B_l^* = \max(Ge^{-(2+C_l)} - B_s\alpha, 0)$ and leads to $B_l^{CS1} = Ge^{-(2+C_l)} - B_s\alpha$ and $B_l^{CS2} = 0$ of cases (CS1) and (CS2) in Table 5.3.

By substituting $B_l^{CS1}$ and $B_l^{CS2}$ into $R^{CS}_{III}(B_s, \alpha, B_l)$ in Table 5.2, we derive the corresponding optimal profits $R^{CS1}_{III}(B_s, \alpha)$ and $R^{CS2}_{III}(B_s, \alpha)$ in Table 5.3. $R^{ES3}_{III}(B_s, \alpha)$ can also be obtained by substituting $B_l^{ES3}$ into $R^{ES}_{III}(B_s, \alpha, B_l)$.

### 5.8.3 Supplementary Proof of Theorem 21

In this section, we prove that Observations 11 and 12 hold for the genera case (i.e., the general SNR regime and a general distributions of $\alpha$). We first show that Observation 12 holds for the general case.
Threshold structure of sensing

It is not difficult to show that if the sensing cost is much larger than the leasing cost, the operator has no incentive to sense but will directly lease. Thus the threshold structure on the sensing decision in Stage I still holds for the general case. We ignore the details due to space limitations.

Threshold structure of leasing

Next we show the threshold structure on leasing in Stage II also holds. Similar as in the proof of Theorem 18, we define \( D(\pi) = \pi \frac{G}{Q(\pi)} \) and \( S(\pi) = \pi (B_s \alpha + B_l) \).

- We first show that \( D(\pi) \) is increasing when \( \pi \in [0, 0.468] \) and decreasing when \( \pi \in [0.468, +\infty) \). To see this, we take the first-order derivative of \( D(\pi) \) over \( \pi \),

\[
D'(\pi) = \frac{2Q(\pi)^2 + Q(\pi) - (1 + Q(\pi))^2 \ln(1 + Q(\pi))}{Q(\pi)^3},
\]

which is positive when \( Q(\pi) \in [0, 2.163) \) and negative when \( Q(\pi) \in [2.163, +\infty) \). Since eq. (5.16) shows that \( Q(\pi) \) is increasing in \( \pi \) and \( \pi(Q) \mid_{Q=2.163} = 0.468 \), as a result \( D(\pi) \) is increasing in \( \pi \in [0, 0.468] \) and decreasing in \( \pi \in [0.468, +\infty) \). In other words, \( D(\pi) \) is maximized at \( \pi = 0.468 \).

- Next we derive the operator’s optimal pricing decision in Stage III. Figure 5.14 shows two possible intersection cases of \( S(\pi) \) and \( D(\pi) \). \( B_{th1} \) is defined as the total bandwidth obtained in Stages I and II (i.e., \( B_s \alpha + B_l \))
such that $S(\pi)$ intersects with $D(\pi)$ at $\pi = 0.468$. Here is how the optimal pricing is determined:

- If $B_s \alpha + B_l \geq B_{th1}$ (e.g., $S_1(\pi)$ in Fig. 5.14), the optimal price is $\pi^* = 0.468$. The total supply is no smaller (and often exceeds) the total demand.

- If $B_s \alpha + B_l < B_{th1}$ (e.g., $S_2(\pi)$ in Fig. 5.14), the optimal price occurs at the unique intersection point of $S(\pi)$ and $D(\pi)$ (where $D(\pi)$ has a negative first-order derivative). The total supply equals total demand.

- Now we are ready to show the threshold structure of the leasing decision.

- If the sensing result from Stage I satisfies $B_s \alpha \geq B_{th1}$, then the
an operator will not lease. This is because leasing will only increase the total cost without increasing the revenue, since the optimal price is fixed at \( \pi^* = 0.468 \) and thus revenue is also fixed at \( D(\pi^*) \).

Let us focus on the case where the sensing result from Stage I satisfies \( B_s(\alpha) < B_{th1} \). Let us define \( B = B_s(\alpha) + B_t \), then we have \( B = G/Q(\pi) \) and \( \pi = \ln(1 + G/B) - G/(G + B) \). This enables us to rewrite \( D(\pi) \) as a function of total resource \( B \) only,

\[
D(B) = B \left[ \ln \left( 1 + \frac{G}{B} \right) - \frac{G}{G+B} \right].
\]

The first-order derivative of \( D(B) \) is

\[
D'(B) = \ln \left( 1 + \frac{1}{B/G} \right) - \frac{1}{1+B/G} - \frac{1}{(1+B/G)^2}, \tag{5.20}
\]
which denotes the increase of revenue $D(B)$ due to unit increase in bandwidth $B$. Since obtaining each unit bandwidth has a cost of $C_l$ in Stage II, the operator will only lease positive amount of bandwidth if and only if $D'(B_s\alpha) > C_l$. To facilitate the discussions, we will plot the function of $D'(B/G)$ in Fig. 5.15, with the understanding that $D'(B/G) = D'(B)G$. The intersection point of $B/G = 0.462$ in Fig. 5.15 corresponds to the point of $\pi = 0.468$ in Fig. 5.14. The positive part of $D'(B)$ on the left side of $B/G = 0.462$ in Fig. 5.15 corresponds to the part of $D(\pi)$ with a negative first-order derivative in Fig. 5.14. For any value $C_l$, Fig. 5.15 shows that there exists a unique threshold $B_{th2}(C_l)$ such that $D'(B_{th2}(C_l))/G = C_lG$, i.e., $D'(B_{th2}(C_l)) = C_l$. Then the optimal leasing amount will be $B_{th2}(C_l) - B_s\alpha$ if the bandwidth obtained from sensing $B_s\alpha$ is less than $B_{th2}(C_l)$, otherwise it will be zero.

**Threshold structure of pricing and Observation 11**

Based on the proofs above, we show that Observation 11 also holds for the general case as follows. Let us denote the optimal sensing decision as $B_s^*$, and consider two sensing realizations $\alpha_1$ and $\alpha_2$ in time slots 1 and 2, respectively. Without loss of generality, we assume that $\alpha_1 < \alpha_2$.

- If $B_s^*\alpha_2 \geq B_{th1}$, then the optimal price in time slot 2 is $\pi^* = 0.468$ (see Fig. 5.14). The optimal price in time slot 1 is always no smaller than
• If $B_s^*\alpha_1 < B_s^*\alpha_2 < B_{th1}$, then we need to consider three subcases:

- If $B_s^*\alpha_1 < B_s^*\alpha_2 \leq B_{th2}(C_l)$, then the operator will lease up to the threshold in both time slots, i.e., $B_l^* = B_{th2}(C_l) - B_s^*\alpha_1$ in time slot 1 and $B_l^* = B_{th2}(C_l) - B_s^*\alpha_2$ in time slot 2. Then optimal prices in both time slots are the same.

- If $B_s^*\alpha_1 \leq B_{th2}(C_l) < B_s^*\alpha_2$, then the operator will lease $B_l^* = B_{th2}(C_l) - B_s^*\alpha_1$ in time slot 1 and will not lease in time slot 2. Thus the total bandwidth in time slot 1 is smaller than that of time slot 2, and the optimal price in time slot 1 is larger.

- If $B_{th2}(C_l) \leq B_s^*\alpha_1 < B_s^*\alpha_2$, then the operator in both time slots will not lease and total bandwidth in time slot 1 is smaller, and the optimal price in time slot 1 is larger.

To summarize, the optimal price $\pi^*$ in Stage III is non-increasing in $\alpha$. And the operator will charge a constant price ($\pi^* = 0.468$) to the users as long as the total bandwidth obtained through sensing and leasing does not exceed the threshold $B_{th2}(C_l)$.  

$\blacksquare$
Chapter 6

Competitive Spectrum Market

Using Cognitive Radios

6.1 Introduction

Recall that in Chapter 2, we only study one secondary operator’s economic decisions and our focus is how the supply uncertainty in spectrum sensing affect the operator’s investment and pricing. Yet as there are more and more operators in the global spectrum market,¹ some operators are competing for the same local market in terms of spectrum acquisition and service pricing (e.g., Virgin Mobile USA and Simple Mobile compete for California market in US).

¹Started from late 1990s, there are over 400 mobile virtual network operators owned by over 360 companies worldwide as of February 2009 [57].
In this chapter, we study the competition between secondary operators in spectrum acquisition and pricing to serve a common pool of secondary users. To abstract the interactions among operators, we focus on two operator case (i.e., duopoly) and will study multiple operator case (i.e., oligopoly) in our future work. As the operator competition in the current spectrum market only focuses on spectrum leasing and has not introduce spectrum sensing yet, we only consider the operators’ spectrum leasing approach to acquire resource. The operators will dynamically lease spectrum from primary operators, and then compete to sell the resource to the secondary users to maximize their individual profits. We would like to understand how the operators make the equilibrium investment (leasing) and pricing (selling) decisions, considering operators’ heterogeneity in leasing costs and wireless users’ heterogeneity in transmission power and channel conditions.

We adopt a three-stage dynamic game model to study the (secondary) operators’ investment and pricing decisions as well as the interactions between the operators and the (secondary) users. In Stage I, the two operators simultaneously lease spectrum (bandwidth) from the primary operators with different leasing costs. In Stage II, the two operators simultaneously announce their spectrum retail prices to the users. In Stage III, each user determines how much resource to purchase from which operator. Each operator wants to maximize its profit, which is the difference between the revenue collected from its
users and the cost paid to the primary operator.

Key results and contributions of this chapter include:

- *An appropriate wireless spectrum sharing model:* We assume that heterogeneous users share the spectrum using orthogonal frequency division multiplexing (OFDM) technology. Then a user’s achievable rate and thus its spectrum demand depend on its allocated bandwidth, maximum transmission power, and channel condition. This model is more suitable to our problem than the generic economic models used in related literature ([19,32,34,35]). It can also provide more engineering insights on how different wireless network parameters in the spectrum sharing model (e.g., users’ various wireless characteristics) impact the operators’ leasing and pricing decisions.

- *Symmetric pricing structure:* We show the two operators always choose the same equilibrium price, even when they have different leasing costs and make different investment decisions. Moreover, this price is independent of users’ transmission powers and channel conditions.\(^2\)

- *Threshold structures of investment and pricing equilibrium:* We show that both operators’ investment and pricing equilibrium decisions process interesting threshold properties. For example, when the two operators’

\(^2\)Such independency is good for the development of spectrum market, since a user does not need to worry about how variations of user population and wireless characteristics change its performance in spectrum trading.
leasing costs are close, both operators will lease positive spectrum. Otherwise, one operator will choose not to lease and the other operator becomes the monopolist. For pricing, a positive pure strategy equilibrium exists only when the total spectrum investment of both operators is less than a threshold.

- **Fair service quality achieved by users**: We show that each user achieves the same signal-to-noise (SNR) that is independent of the users’ population and wireless characteristics.

- **Impact of competition**: We show that the operators’ competition leads to a maximum 25% loss of their total profit compared with a coordinated case. The users, however, always benefit from the operators’ competition by achieving better payoffs.

Next we briefly discuss the related literature. In Section 6.2, we describe the network model and game formulation. In Section 6.3, we analyze the dynamic game through backward induction and calculate the duopoly leasing/pricing equilibrium. We discuss various insights obtained from the equilibrium analysis in Section 6.4. In Section 6.6, we show the impact of duopoly competition on the total operators’ profit and the users’ payoffs. We conclude in Section 6.7 together with some future research directions.
Table 6.1: Key Notations

<table>
<thead>
<tr>
<th>Notations</th>
<th>Physical Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_i, B_j$</td>
<td>Leasing bandwidths of operators $i$ and $j$</td>
</tr>
<tr>
<td>$C_i, C_j$</td>
<td>Costs per unit bandwidth paid by operators $i$ and $j$</td>
</tr>
<tr>
<td>$p_i, p_j$</td>
<td>Prices per unit bandwidth announced by operators $i$ and $j$</td>
</tr>
<tr>
<td>$\mathcal{K} = {1, \ldots, K}$</td>
<td>Set of the users in the network</td>
</tr>
<tr>
<td>$P_k^{\text{max}}$</td>
<td>User $k$’s maximum transmission power</td>
</tr>
<tr>
<td>$h_k$</td>
<td>User $k$’s channel gain between its transceiver</td>
</tr>
<tr>
<td>$n_0$</td>
<td>Noise power per unit bandwidth</td>
</tr>
<tr>
<td>$g_k = P_k^{\text{max}} h_k / n_0$</td>
<td>User $k$’s wireless characteristic</td>
</tr>
<tr>
<td>$G = \sum_{k \in \mathcal{K}} g_k$</td>
<td>The users’ aggregate wireless characteristics</td>
</tr>
<tr>
<td>$w_{ki}, w_{kj}$</td>
<td>User $k$’s bandwidth demand from operator $i$ or $j$</td>
</tr>
<tr>
<td>$r_k$</td>
<td>User $k$’s data rate</td>
</tr>
<tr>
<td>$\mathcal{K}_i^P, \mathcal{K}_j^P$</td>
<td>Preferred user sets of operators $i$ and $j$</td>
</tr>
<tr>
<td>$D_i, D_j$</td>
<td>Preferred demands of operators $i$ and $j$</td>
</tr>
<tr>
<td>$\mathcal{K}_i^R, \mathcal{K}_j^R$</td>
<td>Realized user sets of operators $i$ and $j$</td>
</tr>
<tr>
<td>$Q_i, Q_j$</td>
<td>Realized demands of operators $i$ and $j$</td>
</tr>
<tr>
<td>$R_i, R_j$</td>
<td>Revenues of operators $i$ and $j$</td>
</tr>
<tr>
<td>$\pi_i, \pi_j$</td>
<td>Profits of operators $i$ and $j$</td>
</tr>
<tr>
<td>$T_\pi$</td>
<td>Total profit of both operators</td>
</tr>
</tbody>
</table>
6.1.1 Related Work

Recently, researchers have started to study the economic aspect of dynamic spectrum access, such as the secondary operators’ strategies of spectrum acquisition from primary operators and service provision to the users. For example, several auction mechanisms have been proposed for the primary operator to allocate spectrum ([27, 58–60]). Auction is a good choice when a primary operator does not have a good estimation of how much the spectrum is worth to the users, and thus relies on the bids of the users to determine the pricing and resource allocation. When a primary operator has the complete network information, however, he can simply price the spectrum accordingly and lease to the users.

For operators’ service provision, most related results looked at the pricing interactions between network operators and the secondary users ([19, 27, 32, 34, 35, 62, 63]). There are few papers that jointly studied resource investment and service pricing decisions for intermediary secondary operator(s) ([19, 64, 65]) as this chapter. Such joint optimization is very important since different investment amounts at the beginning weigh heavily on later pricing and service accommodation capabilities of operators. Jia and Zhang [19] only used a generic economic model for total users’ spectrum demand without much wireless details. Niyato et al. [64] did not explicitly analyze the joint optimization over leasing and pricing resource and extensive simulations are mainly used.
instead. Also, Duan et al. [65] only considered a single operator case without discussing operator competition. In multiple competitive operator case, the operators’ strategy spaces are often coupled and is hard to analyze over multiple stages.

The key difference of this work here is that we present a comprehensive analytical study that characterizes both the competitive operators’ equilibrium investment and pricing decisions, with heterogeneous leasing costs for the operators and an appropriate wireless spectrum sharing model for the users.
6.2 Network and Game Model

We consider two operators \((i, j \in \{1, 2\} \text{ and } i \neq j)\) and a set \(\mathcal{K} = \{1, \ldots, K\}\) of users in an ad hoc network as shown in Fig. 6.1. The operators obtain wireless spectrum from different primary operators with different leasing costs, and compete to serve the same set \(\mathcal{K}\) of users. Each user has a transmitter-receiver pair. We assume that users are equipped with software defined radios and can transmit in a wide range of frequencies as instructed by the operators, but do not have the capability of spectrum sensing in cognitive radios.\(^3\) Such a network structure puts most of the implementation complexity for dynamic spectrum leasing and allocation on the operators, and thus is easier to implement than a “full” cognitive radio network especially for a large number of users. A user may switch among different operators’ services (e.g. WiMAX, 3G) depending on operators’ prices. It is important to study the competition among multiple operators as operators are normally not cooperative.

The interactions between the two operators and the users can be modeled as a *three-stage dynamic game*, as shown in Fig. 6.2. Operators \(i\) and \(j\) first simultaneously determine their leasing bandwidths in Stage I, and then simultaneously announce the prices to the users in Stage II. Finally, each user

\(^3\)Spectrum sensing is the most important functionality of cognitive radios, which enables users to actively monitor the external radio environments to communicate efficiently without interfering primary users. The capability of spectrum sensing includes comprehensive monitoring of frequency spectrum, user behavior, and network state over time.
Figure 6.2: Three-stage dynamic game: the duopoly’s leasing and pricing, and the users’ resource allocation

chooses to purchase bandwidth from only one operator to maximize its payoff in Stage III.

The key notations of the chapter are listed in Table 6.1. Some are explained as follows.

- **Leasing decisions $B_i$ and $B_j$:** leasing bandwidths of operators $i$ and $j$ in Stage I, respectively.

- **Costs $C_i$ and $C_j$:** the fixed positive leasing costs per unit bandwidth for operators $i$ and $j$, respectively. These costs are determined by the negotiation between the operators and their own spectrum suppliers.

- **Pricing decisions $p_i$ and $p_j$:** prices per unit bandwidth charged by operators $i$ and $j$ to the users in Stage II, respectively.

- **The User $k$’s demand $w_{ki}$ or $w_{kj}$:** the bandwidth demand of a user $k \in K$ from operator $i$ or $j$. A user can only purchase bandwidth from one operator.
6.2.1 Users’ and Operators’ Models

OFDM has been proposed as a promising physical layer choice for dynamic spectrum sharing ([66, 68]). We assume that the users share the spectrum using OFDM to avoid mutual interferences. The main analysis in this chapter assumes that users are located close-by, and thus no two users will transmit over the same channel (also called subcarriers in the OFDM literatures [69,70]). We also relax this assumption in our online technical report [75] and show that our results can be extended to the case with spectrum spatial reuse.

If a user \( k \in \mathcal{K} \) obtains bandwidth \( w_{ki} \) from operator \( i \), then it achieves a data rate (in nats) of ([71])

\[
r_k(w_{ki}) = w_{ki} \ln \left( 1 + \frac{P_{\text{max}} h_k}{n_0 w_{ki}} \right), \tag{6.1}
\]

where \( P_{\text{max}} \) is user \( k \)'s maximum transmission power, \( n_0 \) is the noise power density, \( h_k \) is the channel gain between user \( k \)'s transmitter and receiver. The channel gain \( h_k \) is independent of the operator, as the operator only sells bandwidth and does not provide a physical infrastructure.\(^4\) Here we assume that user \( k \) spreads its power \( P_{\text{max}}^k \) across the entire allocated bandwidth \( w_{ki} \).

To simplify later discussions, we let

\[
g_k = \frac{P_{\text{max}}^k h_k}{n_0},
\]

\(^4\)We also assume that the channel condition is independent of transmission frequencies, such as in the current 802.11d/e standard [72] where the channels are formed by interleaving over the tones. In other words, each user experiences a flat fading over the entire spectrum.
thus $\frac{g_k}{w_{ki}}$ is the user $k$’s SNR. The rate in (6.1) is calculated based on the Shannon capacity.

To obtain closed-form solutions, we first focus on the high SNR regime where SNR $\gg 1$. This will be the case where a user has limited choices of modulation and coding schemes, and thus can not decode a transmission if the SNR is below some threshold. In the high SNR regime, the rate in (6.1) can be approximated as

$$r_k(w_{ki}) = w_{ki} \ln \left( \frac{g_k}{w_{ki}} \right).$$

(6.2)

Although the analytical solutions in Section 6.3 are derived based on (6.2), we will show later in Section 6.5 that all major engineering insights remain unchanged in the general SNR regime.

If a user $k$ purchases bandwidth $w_{ki}$ from operator $i$, it receives a payoff of

$$u_k(p_i, w_{ki}) = w_{ki} \ln \left( \frac{g_k}{w_{ki}} \right) - p_i w_{ki},$$

(6.3)

which is the difference between the data rate and the payment. The payment is proportional to price $p_i$ announced by operator $i$. This linear pricing scheme has been widely used in the literature ([73, 74]).

For an operator $i$, its profit is the difference between the revenue and the total cost, i.e.,

$$\pi_i(B_i, B_j, p_i, p_j) = p_i Q_i(B_i, B_j, p_i, p_j) - B_i C_i,$$

(6.4)

where $Q_i(B_i, B_j, p_i, p_j)$ and $Q_j(B_i, B_j, p_i, p_j)$ are realized demands of operators $i$ and $j$. The concept of realized demand will be defined later in Definition 9.
6.3 Backward Induction of the Three-Stage Game

A common approach of analyzing dynamic game is backward induction to find the subgame perfect equilibrium (SPE) ([67]). Subgame perfect equilibrium (or simply, equilibrium) represents a Nash equilibrium of every subgame of the original game. In this chapter, we start with Stage III and analyze the users’ behaviors given the operators’ investment and pricing decisions. Then we look at Stage II and analyze how operators make the pricing decisions taking the users’ demands in Stage III into consideration. Finally, we look at the operators’ leasing decisions in Stage I knowing the results in Stages II and III. Throughout the chapter, we will use “bandwidth”, “spectrum”, and “resource” interchangeably.

In the following analysis, we only focus on pure strategy SPE and rule out mixed SPE in the multi-stage game.\(^5\) Such a methodology has been widely used in the literature [81, 82]. Following the definition in [81], we use conditionally SPE to denote an SPE with pure strategies only, where the network’s pure strategies constitute a Nash equilibrium in every subgame. The concept of conditionally SPE is motivated by the concept of SPE but rules out mixed strategies. In Section 6.3.2, we will show that a conditionally SPE will not include any investment decisions \((B_i, B_j)\) in the medium investment regime in Stage I. Otherwise there is no pure strategy Nash equilibrium for pricing in

\(^5\)For interested readers, we have provided some preliminary analysis of mixed strategy SPE in [75].
Stage II, and it will not be a conditionally SPE.\textsuperscript{6}

Following very similar statements in [81], we list several reasons to focus on conditionally SPE in this chapter without considering mixed strategies.

- First, we want to emphasize the result that a pure strategy pricing equilibrium may not exist in Stage II, as this result highlights the very important Edgeworth paradox for the medium investment regime (which will be introduced in Section 6.3.2). Such result reveals the special structure of our problem and leads to important engineering insights for practical network design.

- Second, a standard criticism of mixed strategy equilibrium is that they impose very large informational burdens on users [67]. If operators choose prices according to mixed strategies, users need to consider price distributions (from which the final prices will be drawn by operators) when they choose which operator to purchase from. When the operators’ leasing costs change over time, the leasing amounts and the corresponding mixed pricing strategies can also be time-varying. Given all these complexities, it is unlikely that end users will have the computational capacities and willingness to calculate the “equilibrium choices” in real spectrum market. In other words, the analysis results when allowing mixed strategies

\textsuperscript{6}If we do not focus on the concept of conditionally SPE, there may be an SPE with mixed strategies. For example, in the pricing subgame in Stage II, mixed pricing strategy Nash equilibrium can exist in the medium investment regime, which is supported by our analysis in [75] and [79].
may not be very relevant for engineering practice.

- Third, two operators need to run the randomization procedure in the pricing stage of each time slot if they adopt mixed pricing strategies. However, such randomization over time may be too complicated to implement in practice in a short time scale [78].

In the following analysis, we derive the conditionally SPE, which is also referred to as equilibrium for simplicity.

### 6.3.1 Spectrum Allocation in Stage III

In Stage III, each user needs to make the following two decisions based on the prices $p_i$ and $p_j$ announced by the operators in Stage II:

1. Which operator to choose?

2. How much to purchase?

If a user $k \in \mathcal{K}$ obtains bandwidth $w_{ki}$ from operator $i$, then its payoff $u_k(p_i, w_{ki})$ is given in (6.3). Since this payoff is concave in $w_{ki}$, the unique demand that maximizes the payoff is

$$
\begin{equation}
    w^*_k(p_i) = \arg\max_{w_{ki} \geq 0} u_k(p_i, w_{ki}) = g_k \exp(-(1 + p_i)).
\end{equation}
$$

Demand $w^*_k(p_i)$ is always positive, linear in $g_k$, and decreasing in price $p_i$. Since $g_k$ is linear in channel gain $h_k$ and transmission power $P_k^{\text{max}}$, then a user with a better channel condition or a larger transmission power has a larger
demand. It is clear that \( w^*_k(p_i) \) is upper-bounded by \( g_k \exp(-1) \) for any choice of price \( p_i \geq 0 \). In other words, even if operator \( i \) announces a zero price, user \( k \) will not purchase infinite amount of resource since it can not decode the transmission if \( \text{SNR}_k = g_k/w_{ki} \) is low.

Eqn (6.5) shows that every user purchasing bandwidth from operator \( i \) obtains the same SNR

\[
\text{SNR}_k = \frac{g_k}{w^*_k(p_i)} = \exp(1 + p_i),
\]
and obtains a payoff linear in \( g_k \)

\[
u_k(p_i, w^*_k(p_i)) = g_k \exp(-(1 + p_i)).
\]

**Which Operator to Choose?**

Next we explain how each user decides which operator to purchase from. The following definitions help the discussions.

**Definition 6.** The Preferred User Set \( \mathcal{K}_i^P \) includes the users who prefer to purchase from operator \( i \).

**Definition 7.** The Preferred Demand \( D_i \) is the total demand from users in the preferred user set \( \mathcal{K}_i^P \), i.e.,

\[
D_i(p_i, p_j) = \sum_{k \in \mathcal{K}_i^P(p_i, p_j)} g_k \exp(-(1 + p_i)).
\] (6.6)

The notations in (6.6) imply that both set \( \mathcal{K}_i^P \) and demand \( D_i \) only depend on prices \( (p_i, p_j) \) in Stage II and are independent of operators’ leasing decisions \( (B_i, B_j) \) in Stage I. Such dependance can be discussed in two cases:
1. **Different Prices** ($p_i < p_j$): every user $k \in K$ prefers to purchase from operator $i$ since

$$u_k(p_i, w_{ki}(p_i)) > u_k(p_j, w_{kj}(p_j)).$$

We have $K^P_i = K$ and $K^P_j = \emptyset$, and

$$D_i(p_i, p_j) = G \exp(-(1 + p_i)) \text{ and } D_j(p_i, p_j) = 0,$$

where $G = \sum_{k \in K} g_k$ represents the aggregate wireless characteristics of the users. This notation will be used heavily later in the chapter.

2. **Same Prices** ($p_i = p_j = p$): every user $k \in K$ is indifferent between the operators and randomly chooses one with equal probability. In this case,

$$D_i(p, p) = D_j(p, p) = G \exp(-(1 + p))/2.$$

Now let us discuss how much demand an operator can actually satisfy, which depends on the bandwidth investment decisions $(B_i, B_j)$ in Stage I. It is useful to define the following terms.

**Definition 8.** The Realized User Set $K^R_i$ includes the users whose demands are satisfied by operator $i$.

**Definition 9.** The Realized Demand $Q_i$ is the total demand of users in the Realized User Set $K^R_i$, i.e.,

$$Q_i(B_i, B_j, p_i, p_j) = \sum_{k \in K^R_i(B_i, B_j, p_i, p_j)} g_k \exp(-(1 + p_i)).$$
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Notice that both $K_i^R$ and $Q_i$ depend on prices $(p_i, p_j)$ in Stage II and leasing decisions $(B_i, B_j)$ in Stage I. Calculating the Realized Demands also requires considering two different pricing cases.

1. Different prices $(p_i < p_j)$: The Preferred Demands are $D_i(p_i, p_j) = G \exp(-(1 + p_i))$ and $D_j(p_i, p_j) = 0$.

   - If Operator $i$ has enough resource (i.e., $B_i \geq D_i(p_i, p_j)$): all Preferred Demand will be satisfied by operator $i$. The Realized Demands are
     \[
     Q_i = \min(B_i, D_i(p_i, p_j)) = G \exp(-(1 + p_i)),
     \]
     \[
     Q_j = 0.
     \]

   - If Operator $i$ has limited resource (i.e., $B_i < D_i(p_i, p_j)$): since operator $i$ cannot satisfy the Preferred Demand, some demand will be satisfied by operator $j$ if it has enough resource. Since the realized demand
     \[
     Q_i(B_i, B_j, p_i, p_j) = B_i = \sum_{k \in K_i^R} g_k \exp(-(1 + p_i)),
     \]
     then $\sum_{k \in K_i^R} g_k = B_i \exp(1 + p_i)^7$. The remaining users want to purchase bandwidth from operator $j$ with a total demand of $\frac{G - B_i \exp(1 + p_i)}{\exp(1 + p_j)}$.

---

7 In this chapter, we consider a large number of users and each user is non-atomic (infinitesimal). Thus an individual user’s demand is infinitesimal to an operator’s supply and we can claim equality holds for $Q_i = B_i$. 
Thus the Realized Demands are

\[ Q_i = \min(B_i, D_i(p_i, p_j)) = B_i, \]
\[ Q_j = \min \left( B_j, \frac{G - B_i \exp(1 + p_i)}{\exp(1 + p_j)} \right). \]

2. *Same prices* \((p_i = p_j = p)\): both operators will attract the same Preferred Demand \(G \exp(-(1 + p))/2\). The Realized Demands are

\[ Q_i = \min(B_i, D_i(p, p) + \max(D_j(p, p) - B_j, 0)) \]
\[ = \min(B_i, \frac{G}{2\exp(1+p)} + \max\left(\frac{G}{2\exp(1+p)} - B_j, 0\right)), \]
\[ Q_j = \min(B_j, D_j(p, p) + \max(D_i(p, p) - B_i, 0)) \]
\[ = \min(B_j, \frac{G}{2\exp(1+p)} + \max\left(\frac{G}{2\exp(1+p)} - B_i, 0\right)). \]

### 6.3.2 Operators’ Pricing Competition in Stage II

In Stage II, the two operators simultaneously determine their prices \((p_i, p_j)\) considering the users’ preferred demands in Stage III, given the investment decisions \((B_i, B_j)\) in Stage I.

An operator \(i\)’s profit is defined earlier in (6.4). Since the payment \(B_iC_i\) is fixed at this stage, operator \(i\)’s profit maximization problem is equivalent of maximization of its revenue \(p_iQ_i\). Note that users’ total demand \(Q_i\) to operator \(i\) depends on the received power of each user (product of its transmission power and channel gain). We assume that an operator \(i\) knows users’ transmission powers and channel conditions. This can be achieved similarly as in today’s cellular networks, where users need to register with the operator when they
enter the network and frequently feedback the channel conditions. Thus we assume that an operator knows the user population and user demand.

**Game 1** (Pricing Game). *The competition between the two operators in Stage II can be modeled as the following game:*

- **Players:** two operators $i$ and $j$.
- **Strategy space:** operator $i$ can choose price $p_i$ from the feasible set $\mathcal{P}_i = [0, \infty)$. Similarly for operator $j$.
- **Payoff function:** operator $i$ wants to maximize the revenue $p_i Q_i(B_i, B_j, p_i, p_j)$. Similarly for operator $j$.

At an equilibrium of the pricing game, $(p^*_i, p^*_j)$, each operator maximizes its payoff assuming that the other operator chooses the equilibrium price, i.e.,

$$p^*_i = \arg \max_{p_i \in \mathcal{P}_i} p_i Q_i(B_i, B_j, p_i, p^*_j), \quad i = 1, 2, i \neq j.$$  

In other words, no operator wants to unilaterally change its pricing decision at an equilibrium.

Next we will investigate the existence and uniqueness of the pricing equilibrium. First, we show that it is sufficient to only consider symmetric pricing equilibrium for Game 1.

**Proposition 3.** *Assume both operators lease positive bandwidth in Stage I, i.e., $\min (B_i, B_j) > 0$. If pricing equilibrium exists, it must be symmetric $p^*_i = p^*_j$.***
Figure 6.3: Pricing equilibrium types in different \((B_i, B_j)\)

The proof of Proposition 3 is given in our online technical report [75]. The intuition is that no operator will announce a price higher than its competitor to avoid losing its Preferred Demand. This property significantly simplifies the search for all possible equilibria.

Next we show that the symmetric pricing equilibrium is a function of \((B_i, B_j)\) as shown in Fig. 6.3.

**Theorem 24.** The equilibria of the pricing game are as follows.

- **Low Investment Regime:** \((B_i + B_j \leq G \exp(-2))\) as in region \((L)\) of Fig. 6.3): there exists a unique nonzero pricing equilibrium

\[
p_i^*(B_i, B_j) = p_j^*(B_i, B_j) = \ln \left( \frac{G}{B_i + B_j} \right) - 1.
\]  

(6.7)

The operators’ profits in Stage II are

\[
\pi_{II,i}(B_i, B_j) = B_i \left( \ln \left( \frac{G}{B_i + B_j} \right) - 1 - C_i \right).
\]  

(6.8)
\[ \pi_{II,j}(B_i, B_j) = B_j \left( \ln \left( \frac{G}{B_i + B_j} \right) - 1 - C_j \right). \tag{6.9} \]

- Medium Investment Regime \((B_i + B_j > G \exp(-2) \text{ and } \min(B_i, B_j) < G \exp(-1)\) as in regions (M1)-(M3) of Fig. 6.3): there is no pricing equilibrium.

- High Investment Regime \((\min(B_i, B_j) \geq G \exp(-1)\) as in region (H) of Fig. 6.3): there exists a unique zero pricing equilibrium

\[ p_i^*(B_i, B_j) = p_j^*(B_i, B_j) = 0, \]

and the operators’ profits are negative for any positive values of \(B_i\) and \(B_j\).

Proof of Theorem 24 is given in Section 6.8.1. Intuitively, higher investments in Stage I will lead to lower equilibrium prices in Stage II. Theorem 24 shows that the only interesting case is the low investment regime where both operators’ total investment is no larger than \(G \exp(-2)\), in which case there exists a unique positive symmetric pricing equilibrium. Notice that same prices at equilibrium do not imply same profits, as the operators can have different costs \((C_i \text{ and } C_j)\) and thus can make different investment decisions \((B_i \text{ and } B_j)\) as shown next.

Note that our equilibrium results in medium investment regime are consistent with the well-known Edgeworth paradox (\([53]\)) in economics. Edgeworth paradox describes a situation where two players cannot reach a state of equi-
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librium with pure strategies. Each operator faces capacity constraints when determining pricing decisions in Stage II. The choice of both operators charging zero prices is not an equilibrium in the medium investment regime, since at least one operator can raise its price and obtain non-zero revenue. Nor is the case where one operator charges less the other an equilibrium, since the lower price operator can profitably raise its price towards the other. Nor is the case where both operators charge the same positive price, since at least one operator can lower its price slightly and increase its profit.

The above non-equilibrium cases will not happen in the low investment regime where operators have very limited resources. This is because that in the low investment regime no operator can satisfy the whole demand alone, and thus it is possible for the two operators to share the market at the equilibrium.

Also, these non-equilibrium cases will not happen in the high investment regime, where both operators have more resources than users’ total demand even the price is zero. In this regime, we can ignore the resource constraints (similar to the Bertrand competition) and the zero price equilibrium is the same as the Bertrand paradox ([54]). In the Bertrand paradox, either operator deviating from zero price cannot attract any demand from its competitor who can already serve all users.
6.3.3 Operators’ Leasing Strategies in Stage I

In Stage I, the operators need to decide the leasing amounts \((B_i, B_j)\) to maximize their profits. Based on Theorem 24, we only need to consider the case where the total bandwidth of both the operators is no larger than \(G \exp(-2)\).

We emphasize that the analysis of Stage I is not limited to the case of low investment regime; we actually also consider the medium investment regime and the high investment regime. The key observation is that an SPE will not include any investment decisions \((B_i, B_j)\) in the medium investment regime, as it will not lead to a pricing equilibrium in Stage II. Moreover, any investment decisions in the high investment regime lead to zero operator revenues and are strictly dominated by any decisions in low investment regime. After the above analysis, the operators only need to consider possible equilibria in the low investment regime in Stage I.

**Game 2 (Investment Game).** *The competition between the two operators in Stage I can be modeled as the following game:*

- **Players:** two operators \(i\) and \(j\).

- **Strategy space:** the operators will choose \((B_i, B_j)\) from the set \(\mathcal{B} = \{(B_i, B_j) : B_i + B_j \leq G \exp(-2)\}\). Notice that the strategy space is coupled across the operators, but the operators do not cooperate with each other.

- **Payoff function:** the operators want to maximize their profits in (6.8) and
At an equilibrium of the investment game, \((B_i^*, B_j^*)\), each operator has maximized its payoff assuming that the other operator chooses the equilibrium investment, i.e.,

\[
B_i^* = \arg \max_{0 \leq B_i \leq G \exp(-2)} \pi_{II,i}(B_i, B_j^*), \quad i = 1, 2, i \neq j.
\]

To calculate the investment equilibria of Game 2, we can first calculate operator \(i\)'s best response given operator \(j\)'s (not necessarily equilibrium) investment decision, i.e.,

\[
B_i^*(B_j) = \arg \max_{0 \leq B_i \leq G \exp(-2)} \pi_{II,i}(B_i, B_j), \quad i = 1, 2, i \neq j.
\]

By looking at operator \(i\)'s profit in (6.8), we can see that a larger investment decision \(B_i\) will lead to a smaller price. The best choice of \(B_i\) will achieve the best tradeoff between a large bandwidth and a small price.

After obtaining best investment responses of duopoly, we can then calculate the investment equilibria, given different costs \(C_i\) and \(C_j\).

**Theorem 25.** The duopoly investment (leasing) equilibria in Stage I are summarized as follows.

- Low Costs Regime \((0 < C_i + C_j \leq 1\), as region \((L)\) in Fig. 6.4): there exists infinitely many investment equilibria characterized by

\[
B_i^* = \rho G \exp(-2), \quad B_j^* = (1 - \rho) G \exp(-2),
\]

\[(6.10)\]
Table 6.2: Operators’ and Users’ Behaviors at Equilibria (assuming $C_i \leq C_j$)

<table>
<thead>
<tr>
<th>Costs regimes</th>
<th>Low costs: $C_i + C_j \leq 1$</th>
<th>High comparable costs: $C_i + C_j &gt; 1$ and $C_j - C_i \leq 1$</th>
<th>High incomparable costs: $C_j &gt; 1 + C_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of equilibria</td>
<td>Infinite</td>
<td>Unique</td>
<td>Unique</td>
</tr>
<tr>
<td>Investment equilibria</td>
<td>$(\rho Ge^{-2}, (1 - \rho) Ge^{-2})$, with $C_j \leq \rho \leq (1 - C_i)$</td>
<td>$(\frac{(1+C_i-C_j)G}{2e^{\frac{C_i+C_j+3}{2}}}, \frac{(1+C_i-C_j)G}{2e^{\frac{C_i+C_j+3}{2}}})$</td>
<td>$(Ge^{-(2+C_i)}, 0)$</td>
</tr>
<tr>
<td>Pricing equilibrium $(p_i^<em>, p_j^</em>)$</td>
<td>$(1, 1)$</td>
<td>$(\frac{C_i+C_j+1}{2}, \frac{C_i+C_j+1}{2})$</td>
<td>$(1 + C_i, N/A)$</td>
</tr>
<tr>
<td>Profits $(\pi_{I,i}, \pi_{I,j})$</td>
<td>$\pi_{I,i}^L = \rho(1 - C_i)Ge^{-2}$, $\pi_{I,j}^L = (1 - \rho)(1 - C_j)Ge^{-2}$</td>
<td>$\pi_{I,i}^{HC} = \frac{(1+C_i-C_j)^2}{2} Ge^{-\frac{(C_i+C_j+3)}{2}}$, $\pi_{I,j}^{HC} = \frac{(1+C_i-C_j)^2}{2} Ge^{-\frac{(C_i+C_j+3)}{2}}$</td>
<td>$\pi_{I,i}^{HI} = Ge^{-(2+C_i)}$, $\pi_{I,j}^{HI} = 0$</td>
</tr>
<tr>
<td>User $k$’s bandwidth demand</td>
<td>$g_k e^{-2}$</td>
<td>$g_k e^{-\frac{(C_i+C_j+3)}{2}}$</td>
<td>$g_k e^{-(2+C_i)}$</td>
</tr>
<tr>
<td>User $k$’s SNR</td>
<td>$e^2$</td>
<td>$e^{\frac{C_i+C_j+3}{2}}$</td>
<td>$e^2 + C_i$</td>
</tr>
<tr>
<td>User $k$’s payoff</td>
<td>$g_k e^{-2}$</td>
<td>$g_k e^{-\frac{(C_i+C_j+3)}{2}}$</td>
<td>$g_k e^{-(2+C_i)}$</td>
</tr>
</tbody>
</table>
where $\rho$ can be any value that satisfies

$$C_j \leq \rho \leq 1 - C_i. \quad (6.11)$$

The operators’ profits are

$$\pi^L_{i,i} = B^*_i (1 - C_i),$$

$$\pi^L_{i,j} = B^*_j (1 - C_j),$$

where “L” denotes the low costs regime.

- High Comparable Costs Regime ($C_i + C_j > 1$ and $|C_j - C_i| \leq 1$, as region (HC) in Fig. 6.4): there exists a unique investment equilibrium

$$B^*_i = \frac{(1 + C_j - C_i)G}{2} \exp\left(-\frac{C_i + C_j + 3}{2}\right), \quad (6.12)$$

$$B^*_j = \frac{(1 + C_i - C_j)G}{2} \exp\left(-\frac{C_i + C_j + 3}{2}\right). \quad (6.13)$$
The operators’ profits are

\[
\pi_{I,i}^{HC} = \left( \frac{1 + C_j - C_i}{2} \right)^2 G \exp\left( -\left( \frac{C_i + C_j + 3}{2} \right) \right), \\
\pi_{I,j}^{HC} = \left( \frac{1 + C_i - C_j}{2} \right)^2 G \exp\left( -\left( \frac{C_i + C_j + 3}{2} \right) \right),
\]

where “HC” denotes the high comparable costs regime.

- **High Incomparable Costs Regime** \((C_j > 1 + C_i \text{ or } C_i > 1 + C_j, \text{ as regions } (HI) \text{ and } (HI') \text{ in Fig. 6.4})\): For the case of \(C_j > 1 + C_i\), there exists a unique investment equilibrium with

\[
B^*_i = G \exp(-(2 + C_i)), \quad B^*_j = 0,
\]

i.e., operator \(i\) acts as the monopolist and operator \(j\) is out of the market.

The operators’ profits are

\[
\pi_{I,i}^{HI} = G \exp(-(2 + C_i)), \quad \pi_{I,j}^{HI} = 0,
\]

where “HI” denotes the high incomparable costs. The case of \(C_i > 1 + C_j\) can be analyzed similarly.

The proof of Theorem 25 is given in Section 6.8.2. Let us further discuss the properties of the investment equilibrium in three different costs regimes.

**Low Costs Regime** \((0 < C_i + C_j \leq 1)\)

In this case, both the operators have very low costs. It is the best response for each operator to lease as much as possible. However, since the strategy set
in the Investment Game is coupled across the operators (i.e., $\mathcal{B} = \{(B_i, B_j) : B_i + B_j \leq G \exp(-2)\}$), there exist infinitely many ways for the operators to achieve the maximum total leasing amount $G \exp(-2)$. We can further identify the focal point, i.e., the equilibrium that the operators will agree on without prior communications ([67]). The details can be found in our online technical report [75].

**High Comparable Costs Regime** ($C_i + C_j > 1$ and $|C_j - C_i| \leq 1$)

First, the high costs discourage the operators from leasing aggressively, thus the total investment is less than $G \exp(-2)$. Second, the operators’ costs are comparable, and thus the operator with the slightly lower cost does not have sufficient power to drive the other operator out of the market.

**High Incomparable Costs Regime** ($C_j > 1 + C_i$ or $C_i > 1 + C_j$)

First, the costs are high and thus the total investment of two operators is less than $G \exp(-2)$. Second, the costs of the two operators are so different that the operator with the much higher cost is driven out of the market. As a result, the remaining operator thus acts as a monopolist.

### 6.4 Equilibrium Summary

Based on the discussions in Section 6.3, we summarize the equilibria of the three-stage game in Table 6.2, which includes the operators’ investment deci-
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sions, pricing decisions, and the resource allocation to the users. Without loss of generality, we assume \( C_i \leq C_j \) in Table 6.2. The equilibrium for \( C_i > C_j \) can be described similarly.

Several interesting observations are as follows.

Observation 15. The operators’ equilibrium investment decisions \( B_i^* \) and \( B_j^* \) are linear in the users’ aggregate wireless characteristics \( G = \sum_{k \in K} g_k = \sum_{k \in K} P_{\text{max}} h_k / n_0 \).

This shows that the operators’ total investment increases with the user population, users’ channel gains, and users’ transmission powers.

Observation 16. The symmetric equilibrium price \( p_i^* = p_j^* \) does not depend on users’ wireless characteristics.

Observations 15 and 16 are closely related. Since the total investment is linearly proportional to the users’ aggregate characteristics, the “average” equilibrium resource allocation per user is “constant” and does not depend on the user population. Since resource allocation is determined by the price, this means that the price is also independent of the user population and wireless characteristics.

Observation 17. The operators can determine different equilibrium leasing and pricing decisions by observing some linear thresholds in Figures 6.3 and 6.4.

For equilibrium investment decisions in Stage I, the feasible set of investment costs can be divided into three regions by simple linear thresholds as in
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Fig. 6.4. As leasing costs increase, operators invest less aggressively; as the leasing cost difference increases, the operator with a lower cost gradually dominates the spectrum market. For the equilibrium pricing decisions, the feasible set of leasing bandwidths is also divided into three regions by simple linear thresholds as well. A meaningful pricing equilibrium exists only when the total available bandwidth from the two operators is no larger than a threshold (see Fig. 6.3).

Observation 18. Each user $k$’s equilibrium demand is positive, linear in its wireless characteristic $g_k$, and decreasing in the price. Each user $k$ achieves the same SNR independent of $g_k$, and obtains a payoff linear in $g_k$.

Observation 18 shows that the users receive fair resource allocation and service quality. Such allocation does not depend on the wireless characteristics of the other users.

Observation 19. In the High Incomparable Costs Regime, users’ equilibrium SNR increases with the costs $C_i$ and $C_j$, and the equilibrium payoffs decrease with the costs.

As the costs $C_i$ and $C_j$ increase, the pricing equilibrium ($p_i^* = p_j^*$) increases to compensate the loss of the operators’ profits due to increased costs. As a result, each user will purchase less bandwidth from the operators. Since a user spreads its total power across the entire allocated bandwidth, a smaller bandwidth means a higher SNR but a smaller payoff.
Finally, we observe that the users achieve a high SNR at the equilibrium. The minimum equilibrium SNR that users achieve among the three costs regime is $\exp(2)$. In this case, the ratio between the high SNR approximation and Shannon capacity, $\ln(\text{SNR})/\ln(1 + \text{SNR})$, is larger than 94%. This validates our assumption on the high SNR regime. The next section, on the other hand, shows that most of the insights remain valid in the general SNR regime.

### 6.4.1 How Network Dynamics Affect Equilibrium Decisions

Our analysis so far has not considered network dynamics, as we have focused on a single time slot where an operator knows users’ channel conditions through proper feedback mechanisms. In this subsection, we will look at how the equilibrium results in Table 6.2 change over multiple time slots with the network dynamics. Note that operators still have the complete network information in each time slot. Users are myopic in the sense that they do not take into account the effect of time-varying network parameters on future prices when they determine bandwidth demand in the current time slot.

First, we consider the case where the spectrum available for leasing changes over time. Intuitively, when a primary operator faces a strong demand from its own primary users, it will have less spectrum resource for the virtual operator and will set a higher leasing cost. Here, we look at the case where operators’ leasing costs $C_i$ and $C_j$ change over time according to some Markov decision processes. We write two costs as $C_i(t)$ and $C_j(t)$ to emphasize their dependan-
cies in time. For the illustration purpose, we consider three possible values for both $C_i(t)$ and $C_j(t)$: 0.4, 0.8, and 2, and the transition probabilities (same for two operators) are shown in Fig. 6.5.

Figure 6.6 shows how costs $C_i(t)$ and $C_j(t)$, equilibrium leasing decisions $B_i^*$ and $B_j^*$, and pricing decisions $p_i^*$ and $p_j^*$ change over time. Here we represent a price $N/A$ in Table 6.2 as a zero price. This means that whenever we see a zero price in the figure, the corresponding operator does not participate in the game and the other operator becomes the monopoly in the market. We observe that as an operator’s leasing cost increases, its leasing amount decreases. The operator with a lower cost will lease more and will become the monopolist if its cost is much lower than its competitor (i.e., with $|C_j(t) - C_i(t)| > 1$ in the high incomparable costs regime). In this case, its competitor decides not to lease. As costs increase, operators’ symmetric prices tend to increase to compensate the costs. When two costs are low (with $C_i(t) + C_j(t) \leq 1$), both operators...
Figure 6.6: Costs, equilibrium bandwidth and pricing decisions as functions of time slots. Here we fix $\rho$ to be 0.5.

Second, we can consider the dynamics of users’ channel gains and their population over time. Users’ channel gains may follow, for example, different Rayleigh distributions. Also, there can be users departing or entering the network over different time slots. As a result, users’ aggregate wireless characteristics $G$ will change over time. Table 6.2 has clearly shown that operators’ leasing amounts and profits will change proportionally to $G$. But the equilibrium prices will not be affected, since operators will balance their leasing amounts with users’ demands. For the sake of space, we will not show additional plots for this case.
6.5 Equilibrium Analysis under General SNR Regime

In Sections 6.3 and 6.4, we computed the equilibria of the three-stage game based on the high SNR assumption in (6.2), and obtained five important observations (Observations 1-5). The high SNR assumption enables us to obtain closed-form solutions of the equilibria analysis and clear engineering insights.

In this section, we further consider the more general SNR regime where a user’s rate is computed according to (6.1) instead of (6.2). We will follow a similar backward induction analysis, and extend Observations 15, 16, 18, 19, and pricing threshold structure of Observation 17 to the general SNR regime.

We first examine the pricing equilibrium in Stage II.

**Theorem 26.** Define $B_{th} := 0.462G$. The pricing equilibria in the general SNR regime are as follows.

- Low Investment Regime ($B_i + B_j \leq B_{th}$ as in region (L) of Fig. 6.7):
there exists a unique pricing equilibrium

\[ p_i^*(B_i, B_j) = p_j^*(B_i, B_j) = \ln \left( 1 + \frac{G}{B_i + B_j} \right) - \frac{G}{B_i + B_j + G}. \] (6.14)

The operators’ profits in Stage II are

\[ \pi_i(B_i, B_j) = B_i \left[ \ln \left( 1 + \frac{G}{B_i + B_j} \right) - \frac{G}{B_i + B_j + G} - C_i \right], \] (6.15)

\[ \pi_j(B_i, B_j) = B_j \left[ \ln \left( 1 + \frac{G}{B_i + B_j} \right) - \frac{G}{B_i + B_j + G} - C_j \right]. \] (6.16)

- High Investment Regime \((B_i + B_j > B_{th}\) as in region \((H)\) of Fig. 6.7): there is no pricing equilibrium.

Proof of Theorem 26 is given in [75]. This result is similar to Theorem 24 in the high SNR regime, and shows that the pricing equilibrium in the general SNR regime still has a threshold structure in Observation 17.

Unlike Theorem 24, here we only have two investment regimes. The “high investment regime” in Theorem 24 is gone, and the “medium investment regime” in Theorem 24 corresponds to the high investment regime here. Intuitively, the high SNR assumption in Section 6.3 requires each user to demand relatively small amount of bandwidth to spread its transmission power efficiently, thus the users’ total demand is not elastic to prices and is always upper-bounded by \(G \exp(-1)\) in Fig. 6.3. But in the general SNR case, users’ demands are elastic to prices and is no longer upper-bounded. Hence, we only have two regimes here. For more details, please refer to [75].
Based on Theorem 26, we are ready to prove Observations 15, 16, 18, and 19 in the general SNR regime.

**Theorem 27.** Observations 15, 16, 18, and 19 in Section 6.4 still hold for the general SNR regime.

Proof of Theorem 27 is given in [75].

### 6.6 Impact of Operator Competition

We are interested in understanding the impact of operator competition on the operators’ profits and the users’ payoffs. As a benchmark, we will consider the *coordinated* case where both operators jointly make the investment and pricing decisions to maximize their total profit. In this case, there does not exist competition between the two operators. However, it is still a Stackelberg game between a single decision maker (representing both operators) and the users. Then we will compare the equilibrium of this Stackelberg game with that of the duopoly game as in Section 6.4.

#### 6.6.1 Maximum Profit in the Coordinated Case

We analyze the coordinated case following a three stage model as shown in Fig. 6.8. Compared with Fig. 6.2, the key difference here is that a single decision maker makes the decisions for two operators in both Stages I and II. In other words, the two operators coordinate with each other.
Stage I: A decision maker determines leasing amounts $B_i$ and $B_j$.

Stage II: A decision maker announces prices $p_i$ and $p_j$ to the market.

Stage III: Each end-user determines its bandwidth demand from one operator.

Figure 6.8: The three-stage Stackelberg game for the coordinated operators

Again we use backward induction to analyze the three-stage game. The analysis of Stage III in terms of the spectrum allocation among the users is the same as in Section 6.3.1 (still assuming the high SNR regime), and we focus on Stages II and I. Without loss of generality, we assume that $C_i \leq C_j$.

In Stage II, the decision maker maximizes the following total profit $T_{\pi}$ by determining prices $p_i$ and $p_j$:

$$T_{\pi}(B_i, B_j, p_i, p_j) = \pi_i(B_i, B_j, p_i, p_j) + \pi_j(B_i, B_j, p_i, p_j),$$

where $\pi_i(B_i, B_j, p_i, p_j)$ is given in (6.4) and $\pi_j(B_i, B_j, p_i, p_j)$ can be obtained similarly.

**Theorem 28.** In Stage II, the optimal pricing decisions for the coordinated operators are as follows:

- If $B_i > 0$ and $B_j = 0$, then operator $i$ is the monopolist and announces a price

$$p_i^{co}(B_i, 0) = \ln \left( \frac{G}{B_i} \right) - 1. \quad (6.17)$$
Similar result can be obtained if $B_i = 0$ and $B_j > 0$.

- If $\min(B_i, B_j) > 0$, then both operator $i$ and $j$ announce the same price

$$p_i^{co}(B_i, B_j) = p_j^{co}(B_i, B_j) = \ln\left(\frac{G}{B_i + B_j}\right) - 1.$$ 

Proof of Theorem 28 can be found in our online technical report [75]. Theorem 28 shows that both operators will act together as a monopolist in the pricing stage.

Now let us consider Stage I, where the decision maker determines the leasing amounts $B_i$ and $B_j$ to maximize the total profit:

$$\max_{B_i, B_j \geq 0} T_\pi(B_i, B_j)$$

$$= \max_{B_i, B_j \geq 0} B_i(p_i^{co}(B_i, 0) - C_i) + B_j(p_j^{co}(0, B_j) - C_j), \quad (6.18)$$

where $p_i^{co}(B_i, B_j)$ and $p_j^{co}(B_i, B_j)$ are given in Theorem 28. In this case, operator $j$ will not lease (i.e., $B_j^{co} = 0$) as operator $i$ can lease with a lower cost. Thus the optimization problem in (6.18) degenerates to

$$\max_{B_i \geq 0} T_\pi(B_i) = \max_{B_i \geq 0} B_i(p_i^{co}(B_i, 0) - C_i).$$

This leads to the following result.

**Theorem 29.** In Stage I, the optimal investment decisions for the coordinated operators are

$$B_i^{co}(C_i, C_j) = G \exp(-(2 + C_i)), \quad B_j^{co}(C_i, C_j) = 0, \quad (6.19)$$
and the total profit is

\[ T_{\pi}^{co}(C_i, C_j) = G \exp(- (2 + C_i)) \].

### 6.6.2 Impact of Competition on Operators’ Profits

Let us compare the total profit obtained in the competitive duopoly case (Theorem 25) and the coordinated case (Theorem 29).

**Low Costs Regime** (0 \( < \) \( C_i + C_j \leq \) 1)

First, the total equilibrium leasing amount in the duopoly case is \( B_i^* + B_j^* = G \exp(-2) \), which is larger than the total leasing amount \( G \exp(-(2 + C_i)) \) in the coordinated case. In other words, operator competition leads to a more aggressive overall investment. Second, the total profit at the duopoly equilibria is

\[ T_{\pi}^{L}(C_i, C_j, \rho) = [\rho(1 - C_i) + (1 - \rho)(1 - C_j)]G \exp(-2), \]

where \( \rho \) can be any real value in the set of \([C_j, 1 - C_i]\). Each choice of \( \rho \) corresponds to an investment equilibrium and there are infinitely many equilibria in this case as shown in Theorem 25. The minimum profit ratio between the duopoly case and the coordinated case optimized over \( \rho \) is

\[ T_{\pi}^{R}(C_i, C_j) \triangleq \min_{\rho \in [C_j, 1 - C_i]} \frac{T_{\pi}^{L}(C_i, C_j, \rho)}{T_{\pi}^{co}(C_i, C_j)}. \]

Since \( T_{\pi}^{L}(C_i, C_j, \rho) \) is increasing in \( \rho \), the minimum profit ratio is achieved at

\[ \rho^* = C_j. \]
CHAPTER 6. COMPETITIVE SPECTRUM MARKET USING COGNITIVE RADIOS

This means

$$T_{\pi R^L}(C_i, C_j) = [C_j(1 - C_i) + (1 - C_j)^2] \exp(C_i). \quad (6.20)$$

Although (6.20) is a non-convex function of $C_i$ and $C_j$, we can numerically compute the minimum value over all possible values of costs in this regime

$$\min_{(C_i, C_j) : 0 < C_i + C_j \leq 1} T_{\pi R^L}(C_i, C_j) = \lim_{\epsilon \to 0} T_{\pi R^L}(\epsilon, 0.5 + \epsilon) = 0.75.$$ 

This means that the total profit achieved at the duopoly equilibrium is at least 75% of the total profit achieved in the coordinated case under any choice of cost parameters in the Low Costs Regime.

**High Comparable Costs Regime** ($C_i + C_j > 1$ and $C_j - C_i \leq 1$)

First, the total duopoly equilibrium leasing amount is $B_i^* + B_j^* = G \exp(-\left(\frac{C_i + C_j + 3}{2}\right))$ which is greater than $G \exp(-(2 + C_i))$ of the coordinated case. Again, competition leads to a more aggressive overall investment. Second, the total profit of duopoly is

$$T_{\pi HC}^H(C_i, C_j) = \frac{1 + (C_j - C_i)^2}{2} G \exp(-\left(\frac{C_i + C_j + 3}{2}\right)).$$

And the profit ratio is

$$T_{\pi R^{HC}}(C_i, C_j) \triangleq \frac{T_{\pi HC}^H(C_i, C_j)}{T_{\pi}^{\pi co}(C_i, C_j)} = \frac{1 + (C_j - C_i)^2}{2} \exp(\frac{1 - (C_j - C_i)}{2}),$$
which is a function of the cost difference $C_j - C_i$. Let us write it as $T_\pi R^{HC}(C_j - C_i)$. We can show that it is a convex function and achieves its minimum at 

$$
\min_{(C_i, C_j): C_i + C_j > 1, 0 \leq C_j - C_i \leq 1} T_\pi R^{HC}(C_j - C_i) = T_\pi R^{HC}(2 - \sqrt{3}) = 0.773.
$$

**High Incomparable Costs Regime** ($C_j - C_i > 1$)

In this case, only one operator leases a positive amount at the duopoly equilibrium and achieves the same profit as in the coordinated case. The profit ratio is 1.

We summarize the above results as follows.

**Theorem 30 (Operators’ Profit Loss).** *Comparing with the coordinated case, the operator competition leads to a maximum total profit loss of 25% in the low costs regime.*

Since low leasing costs lead to aggressive leasing decisions and thus intensive competitions among operators, it is not surprising to see that the maximum profit loss happens in the low cost regime. For detailed discussions on the relationship between the profit loss and the costs, see our online technical report [75].

### 6.6.3 Impact of Competition on Users’ Payoffs

**Theorem 31.** *Comparing with the coordinated case, users obtain same or higher payoffs under the operators’ competition.*
By substituting (6.19) into (6.17), we obtain the optimal price in the coordinated case as $1+C_i$. This means that user $k$’s payoff equals to $g_k \exp(-2 - C_i)$ in all three costs regimes. According to Table 6.2, users in the duopoly competition case have the same payoffs as in coordinated case in the high incomparable costs regime. The payoffs are larger in the other two costs regimes with the competitor competition. The intuition is that operator competition in those two regimes leads to aggressive investments, which results in lower prices and higher user payoffs.

6.7 Summary

Dynamic spectrum leasing enables the secondary network operators to quickly obtain the unused resources from the primary operator and provide flexible services to secondary end-users. This chapter studies the competition between two secondary operators and examines the operators’ equilibrium investment and pricing decisions as well as the users’ corresponding achieved service quality and payoffs.

We model the economic interactions between the operators and the users as a three-stage dynamic game. Our appropriate OFDM-based spectrum sharing model captures the wireless heterogeneity of the users in terms of maximum transmission power levels and channel gains. The two operators engage in investment and pricing competitions with asymmetric costs. We have discovered
several interesting features of the game’s equilibria. For example, the operators can determine different equilibrium leasing and pricing decisions by observing some linear thresholds. We also study the impact of operator competition on operators’ total profit loss and the users’ payoff increases. Compared with the coordinated case where the two operators cooperate to maximize their total profit, we show that at the maximum profit loss due to competition is no larger than 25%. We also show that the users always benefit from competition by achieving the same or better payoffs. Although we have focused on the high SNR regime when obtaining closed-form solutions, we show that most engineering insights summarized in Section 6.4 still hold in the general SNR regime. Due to the page limit, more detailed discussions and all proofs can be found in the online technical report [75].

6.8 Appendix

6.8.1 Proof of Theorem 24

Assume, without loss of generality, that $B_i \leq B_j$. Based on Proposition 3, in the following analysis we examine all possible $(B_i, B_j)$ regions labeled (a)-(f) in Fig. 6.9, and check if there exists a symmetric pricing equilibrium (i.e., $p^*_i = p^*_j$) in each region.

(a) If $B_j \geq B_i \geq G \exp(-1)$, both the operators have adequate bandwidths to cover the total preferred demand which reaches its maximum $G \exp(-1)$
at zero price.

- if $p_i^* = p_j^* > 0$, then operator $i$ attracts and realizes half of the total preferred demand. But when operator $i$ slightly decreases its price, it attracts and realizes the total preferred demand, and thus doubles its revenue.

- if $p_i^* = p_j^* = 0$, any operator can not attract or realize any preferred demand by unilaterally deviating from (increasing) its price.

Hence, $p_i^* = p_j^* = 0$ is the unique equilibrium in region (a).

(b-c) If $B_i \leq G \exp(-2) < G \exp(-1) \leq B_j$ or $G \exp(-2) < B_i < G \exp(-1) \leq B_j$, operator $j$ has adequate bandwidth while operator $i$ only has limited bandwidth.
– if \( p^*_i = p^*_j > 0 \), then operator \( j \) will slightly reduce its price to attract and realize the total preferred demand.

– if \( p^*_i = p^*_j = 0 \), then operator \( j \) will increase its price and still have positive realized demand. This is because operator \( i \) does not have enough supply to satisfy the total preferred demand.

Hence, there doesn’t exist an equilibrium in regions \((b-c)\).

(d-e) If \( G \exp(-2) \leq B_i \leq B_j < G \exp(-1) \) or \( B_i \leq G \exp(-2) \leq B_j < G \exp(-1) \), we have shown in the proof of Proposition 3 that possible pricing equilibrium will not exceed 1. We find possible pricing equilibrium given operator \( j \)'s leasing amount.

– if \( p^*_i = p^*_j > \ln \left( \frac{G}{B_j} \right) - 1 \), then operator \( j \) has enough bandwidth to cover the total preferred demand and it will slightly decrease its price to attract a larger preferred demand.

– if \( p^*_i = p^*_j \leq \ln \left( \frac{G}{B_j} \right) - 1 \), then the operator \( j \) has limited bandwidth and it will make decision depending on operator \( i \)'s supply.

* if \( B_i \leq G \exp(-(1 + p^*_j))/2 \), then the operator \( j \) will slightly decrease its price if \( B_i + B_j > G \exp(-(1 + p^*_j)) \), or increase its price to 1 if \( B_i + B_j \leq G \exp(-(1 + p^*_j)) \).

* if \( B_i > G \exp(-(1 + p^*_j))/2 \), then the operator \( j \) will slightly reduce its price.
Hence, there doesn’t exist a pricing equilibrium in regions \((d-e)\).

(f) If \(B_i \leq B_j \leq G \exp(-2)\), we will first show that total supply equals total preferred demand at any possible equilibrium (i.e., \(p_i^* = p_j^* = \ln\left(\frac{G}{B_i+B_j}\right) - 1\)).

- Suppose that at an equilibrium \(p_i^* = p_j^* < \ln\left(\frac{G}{B_i+B_j}\right) - 1\) and thus the total supply is less than the total preferred demand. Then operator \(j\) will slightly increase its price without changing much its realized demand, and thus receive a greater revenue.

- Suppose that at an equilibrium \(p_i^* = p_j^* \geq \ln\left(\frac{G}{B_i+B_j}\right) - 1\) and thus the total supply is greater than the total preferred demand. Thus we have \(B_j > G \exp(-(1 + p_j^*))/2\). Operator \(j\) will slightly reduce its price to attract much more preferred demand and receive a greater revenue.

Thus we have \(p_i^* = p_j^* = \ln\left(\frac{G}{B_i+B_j}\right) - 1\) at any possible equilibrium. Then we check if such \((p_i^*, p_j^*)\) is an equilibrium for the following two cases.

- If \(B_i + B_j > G \exp(-2)\), then we have \(p_i^* = p_j^* < 1\). Since operator \(j\) already has its individual supply equal to its realized demand, then operator \(i\) acts as a monopolist serving its own users in the monopolist’s high investment regime in the proof of Proposition 3. Then operator \(i\) will increase its price to 1.
Table 6.3: Best Investment Response $B^*_i(B_j)$ of operator $i$ in Stage I

<table>
<thead>
<tr>
<th>Response $B^*_i(B_j)$</th>
<th>Low individual cost $0 &lt; C_i \leq 1$</th>
<th>High individual cost $C_i &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small competitor’s decision $B_j &lt; C_i \cdot G \exp(-2)$</td>
<td>the solution to $\partial \pi_{II,i}(B_i, B_j)/\partial B_i = 0$</td>
<td>N/A</td>
</tr>
<tr>
<td>Large competitor’s decision $B_j \geq C_i \cdot G \exp(-2)$</td>
<td>$G \exp(-2) - B_j$</td>
<td>N/A</td>
</tr>
<tr>
<td>Small competitor’s decision $B_j &lt; G \exp(-(1 + C_i))$</td>
<td>N/A</td>
<td>the solution to $\partial \pi_{II,i}(B_i, B_j)/\partial B_i = 0$</td>
</tr>
<tr>
<td>Large competitor’s decision $B_j \geq G \exp(-(1 + C_i))$</td>
<td>N/A</td>
<td>0</td>
</tr>
</tbody>
</table>

- If $B_i + B_j \leq G \exp(-2)$, then we have $p^*_i = p^*_j \geq 1$. Each operator acts as a monopolist serving its own users in the monopolist’s low investment regime in the proof of Proposition 3. And it’s optimal for each operator to stick with its current price.

Thus there exists a unique pricing equilibrium $p^*_i = p^*_j = \ln \left( \frac{G}{B_i + B_j} \right) - 1$ for the low investment regime $B_i + B_j \leq G \exp(-2)$ in region (f).

The same results can be extended to symmetric regions $(a')-(f')$ in Fig. 6.9.

6.8.2 Proof of Theorem 25

The best investment response of operator $i$ is summarized in Table 6.3 with detailed proof in [75]. An investment equilibrium $(B^*_i, B^*_j)$ corresponds to a
fixed iteration point of two functions $B_i^*(B_j)$ and $B_j^*(B_i)$. In the following analysis, we examine all possible costs $(C_i, C_j)$ regions labeled (I)-(III) in Fig. 6.10, and check if there exists any equilibrium in each region.

(I) If $C_i \leq 1$ and $C_j \leq 1$, both the operators are in low individual cost regime.

- If $B_i^* \geq C_j G \exp(-2)$ and $B_j^* \geq C_i G \exp(-2)$, there exist infinitely many investment equilibria characterized by (6.10) and (6.11). Since $B_i^* \geq C_j G \exp(-2)$ and $B_j^* \geq C_i G \exp(-2)$, $C_i + C_j \leq 1$ is further required for existence of equilibria.

- If $B_i^* < C_j G \exp(-2)$ and $B_j^* \geq C_i G \exp(-2)$, then by solving equations $B_i^*(B_j^*) = G \exp(-2) - B_j^*$, and

$$\frac{\partial \pi_{H,j}(B_i, B_j)}{\partial B_j} \bigg|_{B_i = B_i^*, B_j = B_j^*} = 0,$$

we have $B_i^* = C_j G \exp(-2)$ and $B_j^* = (1 - C_j) G \exp(-2)$. But the
value of $B_i^*$ is not smaller than $C_j G \exp(-2)$.

- If $B_i^* \geq C_j G \exp(-2)$ and $B_j^* < C_i G \exp(-2)$, we can also show that there does not exist any equilibrium in this case by a similar argument as above.

- If $B_i^* < C_j G \exp(-2)$ and $B_j^* < C_i G \exp(-2)$, then by solving equations

\[
\frac{\partial \pi_{II,i}(B_i, B_j)}{\partial B_i} \bigg|_{B_i = B_i^*, B_j = B_j^*} = 0,
\quad \frac{\partial \pi_{II,j}(B_i, B_j)}{\partial B_j} \bigg|_{B_i = B_i^*, B_j = B_j^*} = 0,
\]

we have $B_i^*$ in (6.12) and $B_j^*$ in (6.13). And $C_i + C_j > 1$ is further required for existence of this equilibrium.

Hence, in region $(I)$, there exist infinitely many equilibria satisfying (6.10) and (6.11) when $C_i + C_j \leq 1$, and there exists a unique equilibrium satisfying (6.12) and (6.13) when $C_i + C_j > 1$.

(II) If $C_i > 1$ and $0 < C_j \leq 1$, operator $i$ is in high individual cost regime and operator $j$ is in low individual cost regime.

- If $B_i^* \geq C_j G \exp(-2)$ and $B_j^* \geq G \exp(-(1 + C_i))$, then we have $B_i^* = 0$ and $B_j^* = G \exp(-2)$. But the value of $B_i^*$ is not greater than $C_j G \exp(-2)$.

- If $B_i^* \geq C_j G \exp(-2)$ and $B_j^* < G \exp(-(1 + C_i))$, then by solving
equations $B^*_j(B^*_i) = G \exp(-2) - B^*_i$, and

$$\frac{\partial \pi_{II,i}(B_i, B_j)}{\partial B_i} |_{B_i=B^*_i, B_j=B^*_j} = 0,$$

we have $B^*_i = (1 - C_i)G \exp(-2)$ and $B^*_j = C_iG \exp(-2)$. But the value of $B^*_j$ is not less than $G \exp(-(1 + C_i))$.

- If $B^*_i < C_jG \exp(-2)$ and $B^*_j \geq G \exp(-(1 + C_i))$, then by solving equations $B^*_i(B^*_j) = 0$, and

$$\frac{\partial \pi_{II,i}(B_i, B_j)}{\partial B_j} |_{B_i=B^*_i, B_j=B^*_j} = 0,$$

we have $B^*_i = 0$ and $B^*_j = G \exp(-(2 + C_j))$. And $C_i > 1 + C_j$ is further required for existence of this equilibrium.

- $B^*_i < C_jG \exp(-2)$ and $B^*_j < G \exp(-(1 + C_i))$, then by solving equations

$$\frac{\partial \pi_{II,i}(B_i, B_j)}{\partial B_i} |_{B_i=B^*_i, B_j=B^*_j} = 0,$$

$$\frac{\partial \pi_{II,j}(B_i, B_j)}{\partial B_j} |_{B_i=B^*_i, B_j=B^*_j} = 0,$$

we have $B^*_i$ in (6.12) and $B^*_j$ in (6.13). And $C_i \leq 1 + C_j$ is further required for existence of this equilibrium.

Hence, in region (II), there exists a unique investment equilibrium $(B^*_i, B^*_j)$ satisfying (6.12) and (6.13) when $C_i \leq 1 + C_j$, and there exists a unique equilibrium satisfying $B^*_i = 0$ and $B^*_j = G \exp(-(2 + C_j))$ when $C_i > 1 + C_j$. 
(III) If \( C_i > 1 \) and \( C_j > 1 \), then both the operators are in high individual cost regime.

- If \( B_i^* < G \exp(-(1 + C_j)) \) and \( B_j^* < G \exp(-(1 + C_i)) \), then by solving equations

\[
\frac{\partial \pi_{II,i}(B_i, B_j)}{\partial B_i} \bigg|_{B_i = B_i^*, B_j = B_j^*} = 0,
\]
\[
\frac{\partial \pi_{II,j}(B_i, B_j)}{\partial B_j} \bigg|_{B_i = B_i^*, B_j = B_j^*} = 0,
\]

we have \( B_i^* \) in (6.12) and \( B_j^* \) in (6.13). And \( C_i - 1 < C_j < C_i + 1 \) is further required for existence of this equilibrium.

- If \( B_i^* < G \exp(-(1 + C_j)) \) and \( B_j^* \geq G \exp(-(1 + C_i)) \), then by solving equations \( B_i^*(B_j^*) = 0 \), and

\[
\frac{\partial \pi_{j}(B_i, B_j)}{\partial B_j} \bigg|_{B_i = B_i^*, B_j = B_j^*} = 0,
\]

we have \( B_i^* = 0 \) and \( B_j^* = G \exp(-(2 + C_j)) \). And \( C_j \leq C_i - 1 \) is further required for existence of this equilibrium.

- If \( B_i^* \geq G \exp(-(1 + C_j)) \) and \( B_j^* < G \exp(-(1 + C_i)) \), then we can similarly show that there exists a unique equilibrium \( B_i^* = G \exp(-(2 + C_i)) \) and \( B_j^* = 0 \) only when \( C_j \geq C_i + 1 \).

- If \( B_i^* \geq G \exp(-(1 + C_j)) \) and \( B_j^* \geq G \exp(-(1 + C_i)) \), then we have \( B_i^* = 0 \) and \( B_j^* = 0 \). However, the value of \( B_i^* \) is not greater than \( G \exp(-(1 + C_j)) \).
Hence, in region (III), there exists a unique equilibrium satisfying (6.12) and (6.13) when $C_{i} - 1 < C_{j} < C_{i} + 1$; there exists a unique equilibrium satisfying $B_{i}^{*} = 0$ and $B_{j}^{*} = G \exp(-(2 + C_{j}))$ when $C_{j} \leq C_{i} - 1$; and there exists a unique equilibrium with $B_{i}^{*} = G \exp(-(2 + C_{i}))$ and $B_{j}^{*} = 0$ when $C_{j} \geq C_{i} + 1$.

The same results can be extended to symmetric region ($II'$) in Fig. 6.10.
Chapter 7

Security Protection in Collaborative Spectrum Sensing

7.1 Introduction

In previous two chapters, we focus on the economic return of a secondary operator regardless of potential security problems in cognitive radio networks. In fact, if a secondary party (e.g., secondary operator or user) engages in spectrum sensing, security problems may arise and we need to design an effective attack prevention mechanism beforehand. As different sensing technologies require different prevention mechanisms, in this chapter we focus on collaborative spectrum sensing which is a commonly used technology with good performance. Next we further introduce the background of collaborative spectrum sensing and point out why it is vulnerable to attacks.
As a key technology for realizing opportunistic spectrum access while protecting PU communications, spectrum sensing aims to detect the presence or absence of a primary signal with high accuracy. To provide sufficient protection, researchers have proposed collaborative spectrum sensing to improve detection performance by exploiting sensor location diversity [83–85].

Collaborative sensing, however, is vulnerable to critical attacks, such as sensing data falsification attacks, while its detection is difficult. In CRNs, sensors can be deployed in unattended and hostile environments, and thus can be compromised by attackers. Thus, compromised or malicious sensors can intentionally send distorted sensing results to the fusion center in order to disrupt the incumbent detection process [86–88]. Such attacks can be easily launched due to the openness of the low-layer protocols stacks of cognitive radio devices [89]. However, it is challenging for the fusion center to accurately validate the integrity of sensing reports because of the two unique features in spectrum sensing—unpredictability in wireless channel signal propagations and lack of coordination between PUs and SUs. The sensing data falsification attack will ultimately result in a waste of spectrum opportunities (in the form of false alarms), and/or excessive interference to the PU communications (in the form of missed detections). Therefore, this poses a significant threat to the implementation of cognitive radio technology, and thus calls for efficient attack detection and prevention mechanisms.
Table 7.1: Key Results for different attack scenarios

<table>
<thead>
<tr>
<th>Attack Scenarios</th>
<th>Attack-and-run</th>
<th>Stay-with-attacks</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Punishment</td>
<td>Attacks happen and honest SUs always lose transmission opportunities</td>
<td></td>
</tr>
<tr>
<td>(Sec.7.3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct Punishment</td>
<td>Completely prevent attacks</td>
<td></td>
</tr>
<tr>
<td>(Sec.7.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indirect Punishment</td>
<td>Cannot prevent attacks</td>
<td>If attackers focus on long-term reward, we can completely prevent attacks; otherwise can partially prevent attacks.</td>
</tr>
<tr>
<td>(Sec.7.5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this chapter, we consider an attack scenario in which multiple attackers (i.e., compromised SUs/sensors) cooperate to maximize their aggregate spectrum utilization in cognitive radio networks (CRNs). Despite the serious threat posed by collaborated attacks, attacker collaboration have not been fully considered in CRNs. We focus on the particularly challenging attack scenario in which attackers can overhear all honest SUs’ sensing reports, whereas the honest SUs are unaware of the existence of attackers. This information asymmetry gives the attackers maximum capability to launch attacks and achieve their goals. We design attack-prevention mechanisms that safeguard collaborative sensing in such a challenging scenario, which constitutes the main contribution of this chapter.

We consider two different attack scenarios: the “attack-and-run” scenario
in which attackers only care about an immediate reward, and the “stay-with-attacks” scenario in which attackers care about the long-term reward. We first analyze the impact of attacks on honest SUs in the absence of attack-prevention mechanisms. Then, we propose two attack-prevention mechanisms: a direct punishment scheme that can effectively prevent attacks in both scenarios mentioned above, and an indirect punishment scheme that is easier to implement and effectively prevents attacks in the “stay-with-attacks” scenario. The key idea of both mechanisms is to discourage attackers from launching attacks by designing efficient attack detection and punishment strategies.

The key results and organization of this chapter are summarized as follows.

- **A spectrum-sharing model with collision penalty:** In Section 7.2, we introduce the concept of collision penalty, which requires the SUs to compensate a PU for collision in utilizing the spectrum. It is designed to protect the PU’s exclusive spectrum usage and encourage the PU’s opening of its licensed spectrum to SUs.

- **Understanding cooperative attackers’ optimal behaviors:** In Section 7.3, we theoretically show that in the absence of attack-prevention mechanisms, attackers will utilize all spectrum opportunities exclusively, whereas honest SUs cannot transmit and may even suffer from the collision penalty caused by attackers (see Table 7.1).

- **Effective direct punishment:** In Section 7.4, we design a direct punish-
ment mechanism that can detect attacks and punish the attackers. This requires an efficient way for the fusion center to directly punish SUs. The proposed mechanism can prevent all attacks in both “attack-and-run” and “stay-with-attacks” scenarios (see Table 7.1). We further show that a single attacker makes the network most vulnerable under this mechanism.

- **Effective indirect punishment:** In Section 7.5, we propose an indirect attack-prevention mechanism that is easy to implement when direct punishment is infeasible. The key idea is to terminate collaborative sensing when an attack is detected. The proposed mechanism can prevent all attacks if the attackers care enough about their long-term reward (see Table 7.1). Unlike the direct punishment, the presence of a larger number of attackers may make the network more vulnerable.

### 7.1.1 Related Work

There has been a growing interest in attack-resilient collaborative spectrum sensing in CRNs (e.g., [86–88, 90]). Such prior work mainly focus on mechanisms for detecting and filtering out abnormal sensing reports.

Our work is different from existing approaches in three aspects. First, we consider cooperation among attackers, so the attacks are much more challenging to prevent. Second, unlike the previous work which focused on sensing data falsification attacks, we also consider the case where the attackers violate
the fusion center’s decision regarding spectrum access. Finally, our proposed attack-prevention mechanisms can easily prevent attacks without differentiating attackers from honest SUs.

7.2 Preliminary

7.2.1 CRN Model and Assumptions

We consider an infrastructure-based secondary CRN, which consists of a single and a set of SUs (or sensors). The fusion center coordinates SUs’ collaborative spectrum sensing and their access to a licensed PU channel. We assume that the fusion center is maintained by a trusted network administrator and has high computation power. For collaborative spectrum sensing, all SUs (i) measure the primary signal strength on the same target channel, (ii) make local binary decisions on the presence or absence of the primary signal, and (iii) report the binary decisions to the fusion center [84, 93]. Based on the reported sensing results, the fusion center makes a global decision and broadcasts this result to the SUs.

There is a set of $\mathcal{N} = \{1, \ldots, N\}$ SUs in the network, $M$ of which are attackers as shown in Fig. 7.1. We assume that there is at least one honest SU in the network, i.e., $N - M \geq 1$; otherwise, it would be infeasible to defeat attacks. The honest SUs fairly share the licensed channel among themselves when the channel is available to them (i.e., it is not being used by the PU). The
**Figure 7.1:** An illustration of cooperative spectrum sensing in cognitive radio networks: The figure shows a secondary network with $N = 6$ SUs including $M = 2$ malicious SUs (i.e., attackers). The SUs periodically perform spectrum sensing and report the local (binary) decisions to the fusion center (the solid arrows). The fusion center makes a final decision and announces it to the SUs (the dotted arrows).

Attackers (i.e., malicious or compromised SUs), on the other hand, behave to maximize their own aggregate reward (e.g., achievable throughput) by manipulating their sensing reports so that the fusion center makes a wrong decision. In particular, we focus on the case that attackers can overhear all honest SUs’ sensing reports to the fusion center before they collaboratively manipulating their sensing results. We assume that attackers can communicate with each other (and thus know the number of attackers), while the honest SUs only communicate with the fusion center. The honest SUs do not have to be strategic, and they do not need to make decisions by considering other honest SUs and attackers’ decisions.
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To make the analysis tractable and obtain useful engineering insights, we make three assumptions throughout the chapter:

A1. All SUs have the same detection performance in terms of primary false alarm ($P_f$) and missed detection ($P_m$) probabilities.$^1$

A2. The PU’s spectrum occupancy is the same for all SUs and is independent across different time slots.$^2$

A3. All SUs have the same transmission rate in utilizing the licensed channel.

In Section 7.7.1, we relax both assumptions A1 and A3 by studying SUs’ heterogeneous detection performances and heterogeneous transmission rates. We show that our proposed attack-prevention mechanisms still apply.

Regarding the PU’s temporal channel usage statistics, we denote $P_I$ as the probability that the channel is actually idle. Thus, the channel is busy with the probability $1 - P_I$. We assume that SUs (including attackers and fusion center) know the probability $P_I$ before collaborative spectrum sensing as in [88, 94]. This is reasonable if SUs and fusion center can collect PU’s

---

1A false alarm occurs when an SU detects an idle channel as busy, and a missed detection occurs when an SU detects a busy channel as idle. The detection performance depends on the SU’s physical location (relative to the primary transmitter) and fading environment.

2This assumption is frequently used in the literature (e.g., [88,94]), and is reasonable when we approximate the case where PU’s traffic changes fast (e.g., wireless microphones) and the time slot is relatively long. We need to study the correlation between spectrum occupancies when PU’s traffic changes slowly over time (e.g., TV transmitters). Analyzing the correlated case requires a much more complicated Markov decision process (MDP) model than the one we used in Section 7.5, and we consider this as a future direction.
activity information from PU side and calculate $P_I$ using various methods as in [95]. Such information collection is possible for SUs by examining PU’s published historical activity report or purchasing the history report from PU directly.

### 7.2.2 Spectrum Sensing and Opportunistic Access Model

We assume a time-slotted model for opportunistic spectrum access. Such time-slotted channel access model has been widely assumed in the literature [77, 96, 97], including the IEEE 802.22 standard draft [98]. Each time slot includes two phases:

- **Phase I (Collaborative Spectrum Sensing):** As shown in Fig. 7.1, each SU performs sensing individually and makes a local binary decision (i.e., 0/1) on channel occupancy: 1 if it detects the PU’s signal (i.e., busy), and 0 otherwise (i.e., idle). All honest SUs truthfully report their sensing decisions to the fusion center. The attackers, on the other hand, overhear the sensing reports from the honest SUs before sending their own reports (which may be different from their actual local sensing decisions) to the fusion center. Based on the reports from all SUs (including the attackers), the fusion center makes a global decision and broadcasts it to all SUs in the network. We assume that the sensing reports and announcements are communicated via a dedicated and reliable control channel with no
communication errors.$^3$

- Phase II (Spectrum Sharing): If the fusion center announces the channel to be idle, then honest SUs will transmit in Phase II. If it announces the channel to be busy, then honest SUs will wait. The attackers may transmit or wait in both cases. We assume that SUs who transmit in Phase II equally share the transmission time. More advanced link scheduling and power control may improve the overall network performance in Phase II, but is not the focus of this chapter. Let us normalize the total transmission rate of the channel to 1.$^4$ More specifically, $X$ SUs transmitting together leads to $1/X$ rate for each involved SU by using TDMA mode.

### 7.2.3 Collision Penalty

In order to increase social welfare, the government regulatory bodies (e.g., FCC in the U.S. and Ofcom in the U.K.) are pushing new spectrum-sharing schemes to allow the coexistence of PUs and SUs. There are two main obstacles in persuading PUs to share their licensed spectrum bands: (i) PUs’ fear of interference or service disruption caused by SUs, and (ii) lack of economic incentives to PUs for spectrum sharing. To achieve these goals while efficiently preventing attacks, we adopt the notion of “collision penalty”, similar in [99],

---

$^3$Under this one-hop network configuration, the attackers can overhear the control channel and easily decode honest SUs’ reports like the fusion center.

$^4$If the total transmission rate of the channel is $r$ ($\neq 1$), we can change $C_p$ and $C_b$ (defined later) to $C_p/r$ and $C_b/r$ and all results will go through.
as an incentive mechanism to allow for an efficient PU-SU coexistence. When a collision happens, we assume that the PU will charge a collision penalty $C_p$ to all SUs in the network. This collision penalty will compensate PUs for potential performance loss due to collisions.\footnote{The penalty $C_p$ can be in the form of monetary payments from SUs, or reduced transmission opportunities of SUs, or cooperative transmission by SUs to improve the PU’s performance [78,99].} The reasons why PU charges all SUs can be found in [91].

We now define the PU’s expected utility in one time slot as the sum of the PU’s successful transmission rate and collision penalty collected from $N$ SUs, i.e.,

$$U_{PU}(C_p) = (1 - \gamma(C_p))V(r_{PU}) + \gamma(C_p)NC_p,$$

(7.1)

where $\gamma(C_p)$ is the collision probability of the PU’s transmission due to SUs’ aggressive access and is decreasing in $C_p$, $r_{PU}$ is the PU’s transmission rate, and $V(r_{PU})$ is PU’s utility of achieving rate $r_{PU}$. A larger $C_p$ makes SUs more conservative in spectrum access and leads to a lower $\gamma(C_p)$. Hence, a larger $C_p$ achieves a high successful transmission rate (in the first term in Eq. (7.1)), but may also lead to a low compensation from SUs (the second term in Eq. (7.1)).

### 7.2.4 Decision Fusion Rule

At the end of Phase I in each time slot, the fusion center collects a binary sensing report $D_i \in \{0 \text{ (idle)}, 1 \text{ (busy)}\}$ from each SU $i \in \mathcal{N}$, and makes a decision using the following $n$-out-of-$N$ rule [84]:
According to Eq. (7.2), the fusion center infers the channel to be busy $H_1$ when at least $n$-out-of-$N$ SUs report 1 (busy); otherwise, it infers the channel to be idle $H_0$. The optimal selection of the threshold $n$ depends on the system parameters and the reward functions of the SUs [93]. When $n = 1$, we have the OR-rule.

Of the general $n$-out-of-$N$ rules, We adopt OR-rule throughout this chapter. Ghasemi and Sousa [92] showed that the OR-rule performs better than other rules in many cases of practical interest. We further show the following theoretical result.

**Theorem 32.** At the fusion center, the OR-rule outperforms the other $n$-out-of-$N$ rules ($n > 1$) when the collision penalty $C_p$ satisfies the following condition.

\[
\text{Condition I: } C_p < \frac{P_I}{1 - P_I} \left(\frac{1 - P_f}{P_m}\right)^N \frac{1}{N (1 - P_m)(1 - P_f)} \frac{P_m P_f}{N},
\]

(7.3)

The lower-bound of $C_p$ in **Condition I** discourages the SUs from transmitting when at least one SU reports 1 (busy). The upper-bound of $C_p$ in **Condition I** encourages SUs to transmit when all $N$ SUs report 0 (idle). In the rest of the chapter, we assume that $C_p$ always satisfies **Condition I**. More
Table 7.2: Attackers’ optimal behaviors and honest SUs’ behaviors

<table>
<thead>
<tr>
<th>Sensing Decisions</th>
<th>Attackers’ optimal behaviors</th>
<th>Honest SUs’ behaviors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sum_{i \in N} D_i = 0 )</td>
<td>Attack by reporting falsely and transmitting exclusively</td>
<td>Wait</td>
</tr>
<tr>
<td>( \sum_{i \in N} D_i = k \geq 1 )</td>
<td>If ( P_{N,k}^I &lt; MP_{N,K}^B C_p ), do not attack; otherwise attack by reporting truthfully and transmitting exclusively</td>
<td>Wait</td>
</tr>
</tbody>
</table>

Discussion of Condition I can be found in [91].

For the ease of reading, we denote the following two conditional probabilities depending on SUs’ sensing reports:

\[
P_{N,k}^I := Pr \left( \text{idle} \mid \sum_{i \in N} D_i = k \right) = \frac{P_I (1 - P_f)^{N-k} P_f^k}{P_I (1 - P_f)^{N-k} P_f^k + (1 - P_I) (P_m)^{N-k} (1 - P_m)^k}, \quad (7.4)
\]

\[
P_{N,k}^B := Pr \left( \text{busy} \mid \sum_{i \in N} D_i = k \right) = 1 - P_{N,k}^I. \quad (7.5)
\]

7.3 Attackers’ Behaviors Without Punishment

In this section, we analyze the behavior of cooperative attackers when the system lacks attack-prevention mechanisms. The results in this section will serve as a benchmark for the proposed attack-prevention mechanisms in Sections 7.4.
and 7.5.

We first define some useful notations.

- **State set $S$:** A state $s \in S$ describes the local sensing decisions of the honest SUs and attackers: $(\sum_{i \in N \setminus M} D_i, \sum_{i \in M} D_i)$. The size of set $S$ is $(N - M + 1)(M + 1)$. The attackers know the exact state in a particular time slot by overhearing the honest SUs’ reports to the fusion center.

- **Attackers’ action set $A$:** The action $a_m$ of an attacker $m \in M$ is a tuple, (report to the fusion center in Phase I, spectrum access decision in Phase II), which has 4 possibilities: (idle, wait), (busy, wait), (idle, transmit), and (busy, transmit). Define $a = \{a_m, \forall m \in M\}$ as all attackers’ action vector, and $A$ includes all possible $a$.

- **Attackers’ expected aggregate reward $R(a, s)$:** This reward depends on the state $s$ and the attackers’ actions $a$ in one time slot. It denotes the difference between the attackers’ aggregate transmission rate and their expected payment to PU due to usage collision in one time slot.

For each state $s$, the attackers choose $a$ to maximize the expected aggregate reward in a single time slot, i.e.,

$$\max_{a \in A} R(a, s). \quad (7.6)$$

We discuss the solution to Eq. (7.6) in the three following cases. Due to the

---

6The value of $D_i$ can be either 0 or 1, thus $\sum_{i \in N \setminus M} D_i$ ranges from 0 to $N - M$ and $\sum_{i \in M} D_i$ ranges from 0 to $M$.\n
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page limit, we skip all proofs here which can be found in our online technical report [91].

7.3.1 All SUs sense the channel idle

**Proposition 4.** Given the state $s = \left(\sum_{i \in \mathcal{N} \setminus \mathcal{M}} D_i = 0, \sum_{i \in \mathcal{M}} D_i = 0\right)$, the cooperative attackers’ optimal actions are: at least one attacker adopts the action (busy, transmit) and the other attackers (if any) adopt the action (idle, transmit). That is, at least one attacker will report the channel busy in Phase I and all attackers will transmit exclusively over the channel in Phase II. The fusion center will announce a wrong decision $\mathcal{H}_1$ in this case. The attackers’ expected aggregate reward is:

$$R(a, s) = P_{N,0}^I - MP_{N,0}^B C_p > 0,$$  \hspace{1cm} (7.7)

where the definitions of $P_{N,0}^I$ and $P_{N,0}^B$ are given in Eqs. (7.4) and (7.5), respectively. An honest SU does not transmit, but may suffer from the collision penalty caused by attackers and receives a negative expected reward

$$R_{\text{honestSU}}(s) = -P_{N,0}^B C_p < 0.$$ \hspace{1cm} (7.8)

Proposition 4 shows that an attack always happens when all SUs sense the channel idle.

7.3.2 All honest SUs sense the channel idle, but some attacker(s) senses the channel busy

Here we define the attackers’ aggregate sensing result $\sum_{i \in \mathcal{M}} D_i$ as $\bar{M}$. 
Proposition 5. Given the state

$$s = \left( \sum_{i \in \mathcal{N} \setminus \mathcal{M}} D_i = 0, \sum_{i \in \mathcal{M}} D_i = \bar{M} \geq 1 \right),$$

the cooperative attackers’ optimal actions are as follows.

- If $P^I_{N,M} < MP^B_{N,M}C_p$, then at least one attacker adopts the action (busy, wait) and the other attackers (if any) adopt the action (idle, wait). This leads to a correct announcement $\mathcal{H}_1$ (busy) at the fusion center. Since no one transmits, the attackers and the honest SUs all get zero reward,

$$R(a, s) = R_{\text{honestSU}}(s) = 0. \quad (7.9)$$

- If $P^I_{N,M} \geq MP^B_{N,M}C_p$, then at least one attacker adopts the action (busy, transmit) and the other attackers (if any) adopt the action (idle, transmit). This leads to a correct announcement $\mathcal{H}_1$ (busy) at the fusion center. Only attackers will transmit exclusively in Phase II, their expected aggregate reward is:

$$R(a, s) = P^I_{N,M} - MP^B_{N,M}C_p > 0. \quad (7.10)$$

An honest SU does not transmit in Phase II, but may suffer from the collision penalty caused by attackers’ transmissions and receives a negative expected reward

$$R_{\text{honestSU}}(s) = -P^B_{N,M}C_p < 0. \quad (7.11)$$

Proposition 5 indicates that an attack only happens when the benefit of
exclusive transmission is large enough to compensate the potential collision penalty for the attackers.

7.3.3 Some honest SUs sense the channel busy

Proposition 6. Given the state

$$s = \left( \sum_{i \in \mathcal{N} \setminus \mathcal{M}} D_i = K \geq 1, \sum_{i \in \mathcal{M}} D_i = \bar{M} \geq 0 \right),$$

the announcement at the fusion center is always correct with $\mathcal{H}_1$ (busy), and the attackers’ optimal actions are as follows.

- If $P_{\mathcal{N},K+\bar{M}}^I < M P_{\mathcal{N},K+\bar{M}}^B C_p$, then each attacker can either take the action (busy, wait) or (idle, wait). Since no one transmits, the attackers and the honest SUs all get zero reward,

$$R(a, s) = R_{\text{honest SU}}(s) = 0. \tag{7.12}$$

- If $P_{\mathcal{N},K+\bar{M}}^I \geq M P_{\mathcal{N},K+\bar{M}}^B C_p$, then each attacker can either take the action (busy, transmit) or (idle, transmit). As only attackers will transmit in Phase II, their expected aggregate reward is:

$$R(a, s) = P_{\mathcal{N},K+\bar{M}}^I - M P_{\mathcal{N},K+\bar{M}}^B C_p. \tag{7.13}$$

An honest SU does not transmit in Phase II, but may suffer from the collision penalty caused by attackers’ transmissions and receives a negative reward.

Note that this state includes the case that all honest SUs sense the channel busy and (some) attackers sense idle.
expected reward

\[ R_{honestSU}(s) = - \frac{P^B_{N,K}}{M} C_p < 0. \]  \hspace{1cm} (7.14)

Propositions 4-6 indicate that, without any attack-prevention mechanism, the attackers will utilize the spectrum opportunities exclusively, whereas the honest SUs will never transmit regardless of their sensing decisions. What is worse, the honest SUs may suffer from the collision penalty caused by the attackers. These results are summarized in Table 7.2.

Note that our current analytical results focus on one time slot, where the attackers want to maximize their expected aggregate reward in the current time slot (i.e., the “attack-and-run” scenario). Since attackers’ behaviors are independent over time slots, the above analytical results also hold for the “stay-with-attacks” scenario.

7.4 Attack-Prevention Mechanism: A Direct Punishment

In this section, we consider the case in which the fusion center can directly charge a punishment to the SUs when attacks are identified. We focus on the “attack-and-run” scenario in a single time slot. The analysis also applies to the “stay-with-attacks” scenario as in Section 7.3. With the proper choice of punishment, the proposed mechanism ensures that no attack will happen and no one will be punished.
Let us denote the direct punishment as $C_b$, which is different from the collision penalty $C_p$ introduced in Section 7.2.3. The fusion center will only charge the punishment to all SUs when the PU detects an attack. Let us consider the following scenario:

- When the announcement at the fusion center is $H_1$ (busy) in Phase I and a collision happens in Phase II, the fusion center knows that an attack happens (as honest SUs will not transmit in Phase II). In this case, all SUs are charged a direct punishment $C_b$ by the fusion center (in addition to the collision penalty $C_p$ charged by the PU).\(^8\)

Note that when the announcement at the fusion center is $H_0$ (idle) in Phase I, no direct punishment will be triggered even if there is a collision in Phase II. This is because attackers will not share the spectrum access opportunity with honest SUs as in Proposition 4, and such collision can only the result of the missed detections of spectrum sensing.

The effectiveness of the attack-prevention mechanism depends on the choice of the punishment $C_b$. Theorem 33 shows that a large enough $C_b$ can prevent all possible attacks.

**Theorem 33.** For $M$ attackers in the network, there exists a threshold $C_b^{th}(M)$,\(^8\)
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Figure 7.2: Direct punishment threshold $C_b^{th}(M)$ for different $M$ and $N$ cases with $(P_f, P_f, P_m, C_p)=(0.6, 0.08, 0.08, 6e+10)$.

\[ C_b^{th}(M) = \max \left( \frac{P_f P_m}{(1-P_f)(1-P_m)} \frac{1}{M} - C_p \left( \frac{1}{M} - \frac{1}{N} \right) \right) \cdot \frac{P_f}{1-P_f} \left( \frac{1-P_f}{P_m} \right)^N, \quad \forall M \geq 1, \quad (7.15) \]

such that any value $C_b > C_b^{th}(M)$ can prevent all attack scenarios described in Section 7.3.

The proof of Theorem 33 is given in [91]. Next, we examine how the numbers of honest SUs and attackers affect the threshold $C_b^{th}(M)$.

**Observation 20.** $C_b^{th}(M)$ is decreasing in the number of attackers $M$ and increasing in the number of honest SUs $N - M$. If the fusion center does not know the number of attackers, it should set the threshold to be $C_b^{th}(1) =$
max_{M \geq 1} C_b^{th}(M) to prevent all attacks.

Figure 7.2 shows the value of threshold $C_b^{th}(M)$ as a function of $M$ for different values of $N$.\textsuperscript{9} When the number of attackers increases, the total penalty to the group of attackers also increases when an attack is confirmed (while the total transmission rate does not change), which discourages the attacks to happen.

Figure 7.2 also shows that $C_b^{th}(M)$ increases with the number of honest SUs $N - M$ for any fixed $M$. This is because the more honest SUs’ sensing reports are overheard by the attackers, the more accurately the attackers can estimate the actual channel state, and thus more likely the attackers will launch an attack. As a result, a higher $C_b$ is required to prevent attackers from manipulating their sensing reports. Thus, the single attacker scenario (i.e., $M = 1$) is the most challenging case for this attack-prevention mechanism.

\textbf{Observation 21.} The threshold $C_b^{th}(1)$ is increasing in the idle probability $P_I$ and non-increasing in collision penalty $C_p$.\footnote{Since $P_f$ and $P_m$ must be less than 10\% in 802.22 WRAN standard draft, thus the probability to trigger direct punishment is very small under this choice of $P_f$ and $P_m$. As a result, high $C_b^{th}(M) = C_b^{th}(M)/r$ value is determined in Fig. 7.2 to eliminate the attack benefit.}
7.5 Attack-Prevention Mechanism: An Indirect Punishment

The direct punishment scheme may be difficult to enforce for certain types of networks due to practical constraints, such as implementation overhead and complexity. For example, if the direct punishment is in the form of monetary payments from SUs to the fusion center, the fusion center needs to have reliable channels to collect and monitor such payments [78]. In this section we propose an indirect punishment scheme that can effectively prevent attacks in the “stay-with-attacks” scenario as long as the attackers care enough about future rewards. The key idea is to terminate collaborative sensing once the fusion center detects an attack, which forces the attackers to rely on their own sensing results in the future. This prevents attackers from overhearing honest SU sensing reports, and results in an increase in missed detection probability for attackers. Therefore, such indirect punishment will reduce the attackers’ incentives to attack.

The indirect punishment works as follows:

- When the fusion center announces $H_1$ (busy) in Phase I and a collision happens in Phase II, the indirect punishment is triggered and there is no collaborative sensing in future time slots.\(^{10}\)

\(^{10}\)The fusion center can achieve this by broadcasting to all SUs that there is no need to report local sensing decisions in the future.
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Note that when the fusion center announces $H_0$ (idle) in Phase I, no indirect punishment will be triggered even if there is a collision in Phase II.

Similar to the direct punishment mechanism in Section 7.4, no indirect punishment will be triggered if all SUs behave honestly. The effectiveness of the indirect punishment depends on the attackers’ performance when they are isolated from the honest SUs.

In the rest of the section, we make the following assumption:

$$A4: \quad C_p > \frac{P_I}{1-P_f} \frac{1-P_f}{P_m}.$$  \hspace{1cm} (7.16)

$A4$ is derived from $P_{1,0}^I - P_{1,0}^B C_p < 0$, which implies that a single SU will not transmit based on its own sensing decision (since it can be quite unreliable after the collaborative sensing breaks down) even without interference from the other SUs. $A4$ is quite mild. When the number of SUs is reasonable (i.e., $N > 7$), Condition I in Eq. (7.3) directly guarantees the satisfaction of $A4$ in Eq. (7.16). Note that $A4$ only applies to this section.

To analyze the attackers’ dynamic decisions in the long-term “stay-with-attacks” scenario, we formulate the problem as a Markov decision process (MDP). More specifically, we consider an infinite horizon Markov decision process $(S', A', P, R)$, where the group of cooperative attackers is the only decision-maker (collectively) over time.

- **State set $S'$**: A state $s \in S'$ describes the attackers’ knowledge of honest SUs’ sensing decisions, their own sensing decisions, and whether the
indirect punishment is triggered: \((\sum_{i \in \mathcal{N} \setminus \mathcal{M}} \bar{D}_i, \sum_{i \in \mathcal{M}} D_i, \text{Punishment})\).

When \(\text{Punishment} = \text{off}\), \(\sum_{i \in \mathcal{N} \setminus \mathcal{M}} \bar{D}_i = \sum_{i \in \mathcal{M}} D_i\). When \(\text{Punishment} = \text{on}\), \(\sum_{i \in \mathcal{N} \setminus \mathcal{M}} \bar{D}_i = \text{Unknown}\) as the attackers do not know the honest SUs’ sensing decisions. The size of set \(\mathcal{S}'\) is \([ (N - M + 1)(M + 1) + (M + 1) ]\).

The attackers know the state during each time slot.

- **Attackers’ action set \(\mathcal{A}'\):** The action \(a_m\) of an attacker \(m \in \mathcal{M}\) is a tuple: (report to the fusion center, spectrum access decision). When the indirect punishment is not triggered, there are four possible actions: (idle, wait), (busy, wait), (idle, transmit), and (busy, transmit). When the indirect punishment is triggered, an attacker’s action can be \((N/A, \text{transmit})\) or \((N/A, \text{wait})\), where \(N/A\) means that the attackers do not report. We define \(a = \{a_m, \forall m \in \mathcal{M}\}\) as the action vector of all attackers and \(\mathcal{A}'\) contains all feasible values of \(a\).

- **Transition probability \(P(a, s, s')\):** The transition probability that actions \(a\) in a state \(s\) at time slot \(t\) will lead to state \(s'\) in time slot \(t+1\) is \(P(a, s, s') = Pr(s_{t+1} = s'| s_t = s, a_t = a)\). This depends on both state \(s\) and actions \(a\), and is independent of time \(t\).

- **Attackers’ expected aggregated reward \(R(a, s)\):** The attackers’ received reward after taking actions \(a\) in state \(s\) of a time slot.

Compared to the reward in the current time slot, the attackers may value future rewards less. This can be captured by a discount factor \(\delta \in (0, 1)\). We
further define a stationary policy $u$ as a mapping between the set of states $S'$ to the action set $A'$. In other words, a policy defines what action to take in each possible state. The attackers’ objective is to choose a policy $u$ from policy set $U$ to maximize the long-term expected aggregate reward:

$$\max_{u \in U} \sum_{t=0}^{\infty} \delta^t R(u(s), s),$$  \hspace{1cm} (7.17)

Let us denote the attackers’ optimal long-term expected aggregate rewards by $LR^H$ and $LR^{DH}$ if they behave honestly and dishonestly, respectively.

Since attackers’ behaviors and rewards before and after the indirect punishment are quite different, we need to study them separately. Here we first consider the attackers’ behaviors before the punishment. Let us consider the case where at least one SU senses the channel busy, i.e., $\sum_{i \in N} D_i = K \geq 1$. The attackers’ optimal behaviors can be classified into two cases:

- **Non-aggressive Transmission**: The attackers will not attack for any $K \geq 1$, which is true if

$$\text{Case.NT} : P_{N,1}^I - M P_{N,1}^B C_p < 0,$$  \hspace{1cm} (7.18)

where the attackers’ exclusive transmission opportunity does not compensate their collision penalty.

- **Aggressive Transmission**: The attackers may attack even if $K \geq 1$, which is true if

$$\text{Case.AT} : P_{N,1}^I - M P_{N,1}^B C_p \geq 0.$$  \hspace{1cm} (7.19)
In the rest of this section, we focus on Case NT with \( M \geq 1 \) attackers. The discussion for Case AT with \( M \geq 1 \) is given in Section 7.7.2.

We analyze the conditions under which attacks can be completely prevented via an indirect punishment. We first need to understand the attackers’ performance degradation once the indirect punishment is triggered. Since the attackers are cooperative, they can always exchange sensing information among themselves. Depending on whether the attackers will transmit after the indirect punishment, we have two cases:

- **Weak Cooperation**: The attackers will not transmit even when all attackers sense the channel idle,

  \[
  \text{Case.WC} : \quad P_{M,0}^I - MP_{M,0}^B C_p \leq 0. \tag{7.20}
  \]

  This means that the attackers feel that their own sensing results are not reliable enough (with a high missed detection probability). Case.WC also implies that the attackers will definitely not transmit if one or more attackers sense the channel busy. Due to assumption A4, the reward in Eq. (7.20) is an increasing function of the number of attackers \( M \). Then we can also write Eq. (7.20) as an upper bound of \( M \), i.e., Case.WC corresponds to a small number of attackers \( M \).

- **Strong Cooperation**: The attackers will transmit when all attackers sense the channel idle,

  \[
  \text{Case.SC} : \quad P_{M,0}^I - MP_{M,0}^B C_p > 0. \tag{7.21}
  \]
This means that the attackers feel that their own sensing results (collectively) are accurate enough (with a low missed detection probability) even taking the collision penalty $C_p$ into consideration. We can also write Eq. (7.21) as a lower bound of $M$, i.e., Case.SC corresponds to a large number of attackers $M$.

Obviously, it is more challenging to prevent attacks in Case.SC than Case.WC. However, we can show that in Case.SC the attackers’ expected aggregate reward in one time slot with punishment triggered is always less than their reward when they always behave honestly. In other words, as long as the attackers care enough about future reward (i.e., the discount factor $\delta$ is high enough), we can still prevent attacks even in Case.SC (and thus in Case.WC as well).

**Lemma 6.** The attackers’ optimal long-term expected aggregate rewards in Case.WC and Case.SC are

\[
LR^H_{WC} = LR^H_{SC} = \Pr \left( \sum_{i \in N} D_i = 0 \right) \left( P_{N,0}^I \frac{1}{N} - P_{N,0}^B C_p \right) \frac{M}{1 - \delta}, \tag{7.22}
\]

\[
LR^{DH}_{WC} = \frac{\Pr \left( \sum_{i \in N} D_i = 0 \right) (P_{N,0}^I - MP_{N,0}^B C_p)}{1 - \delta (1 - \Pr \left( \sum_{i \in N} D_i = 0 \right) P_{B,0}^N)}, \tag{7.23}
\]

and $LR^{DH}_{SC}$ in

\[
LR^{DH}_{SC} = LR^{DH}_{WC} + \frac{\delta}{1 - \delta} \Pr \left( \sum_{i \in N} D_i = 0 \right) \cdot \frac{P_{N,0}^B \Pr \left( \sum_{i \in M} D_i = 0 \right) (P_{M,0}^I - MP_{M,0}^B C_p)}{1 - \delta (\Pr \left( \sum_{i \in N} D_i > 0 \right) + \Pr \left( \sum_{i \in N} D_i = 0 \right) P_{N,0}^I)} \tag{7.24}.
\]

Here the superscripts “H” and “DH” indicates honest and dishonest behaviors of attackers, respectively.
\[ \delta_{\text{SC}}^\text{th}(M) = \left( 1 + \frac{1 - P_I}{P_I} \left( \frac{P_m}{1 - P_f} \right)^N \right) \left( \frac{P_I(1 - P_f)^N}{N} - \frac{(1 - P_I)(P_m)^N C_p}{N} - \frac{(1 - P_I)(P_f)^M}{M} - \frac{(1 - P_f)(P_m)^M C_p}{N} \right)^{-1}. \] (7.26)

The proof of Lemma 6 is given in Section 7.7.3, where we can show that \( LR_{\text{WC}}^{\text{DH}} < LR_{\text{WC}}^{\text{H}} \) and \( LR_{\text{SC}}^{\text{DH}} < LR_{\text{SC}}^{\text{H}} \) when \( \delta \) goes close to 1. This leads to the following result.

**Theorem 34.** The indirect punishment can prevent all attack in “stay-withattacks” scenario if the discount factor \( \delta \) satisfies the following condition:

- Weak cooperation (Case.WC): for any \( 1 \leq M < N \), we need \( \delta > \delta_{\text{WC}}^\text{th}(M) \) where

\[ \delta_{\text{WC}}^\text{th}(M) = \frac{1}{1 + \frac{P_I(1 - P_f)^N}{N} - \frac{(1 - P_I)(P_m)^N C_p}{N} - \frac{(1 - P_f)(P_m)^M}{M} - \frac{(1 - P_f)(P_m)^M C_p}{N}}. \] (7.25)

- Strong cooperation (Case.SC): for any \( 1 \leq M < N \), we need \( \delta > \delta_{\text{SC}}^\text{th}(M) \) where \( \delta_{\text{SC}}^\text{th} \) is given in Eq. (7.26).

If the fusion center does not know the number of attackers, \( M \), it can choose \( \delta > \max_{0 < M < N} \delta_{\text{WC}}^\text{th}(M) \) and \( \delta > \max_{0 < M < N} \delta_{\text{SC}}^\text{th}(M) \) for the two cases, respectively.

Although it is not shown in Theorem 34, we want to mention that the indirect punishment can still partially prevent attacks even \( \delta \) is less than the
discount factor threshold. Intuitively, attackers do not want to trigger indirect punishment and lose the opportunity to overhear honest SUs’ sensing results. Thus they will behave more conservatively compared to the case with no indirect punishment. For example, if some SUs’ sensing results indicate a busy channel state, the attackers will not attack to trigger the long-term punishment.

We have the following interesting observations.

**Observation 22.** (Impact of network size:) Both $\delta_{WC}^{th}(M)$ in Case.WC and $\delta_{SC}^{th}(M)$ in Case.SC are increasing in the number of the honest SUs $N - M$. Threshold $\delta_{WC}^{th}(M)$ is decreasing in the number of the attackers $M$, while $\delta_{SC}^{th}(M)$ is increasing in the number of the attackers $M$. 

Figure 7.3: Discount factor threshold $\delta_{SC}^{th}(M)$ with $(P_I, P_f, P_m, C_p) = (0.6, 0.08, 0.08, 3e + 18)$. 

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Figure 7.3: Discount factor threshold $\delta_{SC}^{th}(M)$ with $(P_I, P_f, P_m, C_p) = (0.6, 0.08, 0.08, 3e + 18)$.
Figure 7.3 plots $\delta_{SC}^{th}(M)$ as a function of $N$ and $M$ in Case.SC. The corresponding result in Case.WC can also be obtained based on Eq. (7.25).

With more honest SUs $N - M$, attackers have a less incentive to share the spectrum with honest SUs in the long-term and a higher incentive to attack and transmit exclusively. Thus a higher $\delta$ is needed to prevent attacks.

A larger number of attackers $M$ has two effects: (a) a higher total collision penalty (whenever a collision happens), and (b) attackers’ better estimation of channel condition (once the punishment is triggered). It turns out that effect (a) dominates in Case.WC and effect (b) dominates in Case.SC, which explains why the $\delta$ threshold decreases in $M$ in Case.WC and increases in $M$ in Case.SC. In Fig. 7.3, the most attack-vulnerable case happens when almost all SUs are attackers ($M \to N$), in which case $LR_{SC}^{DH} \to LR_{SC}^{H}$ and $\delta_{SC}^{th}(M)$ in (7.26) is close to 1.

**Observation 23.** (Impact of collision penalty $C_p$:) $\delta_{WC}^{th}(M)$ in Case.WC is increasing in the collision penalty $C_p$, while $\delta_{SC}^{th}(M)$ in Case.SC is decreasing in $C_p$.

In Case.WC, the collision penalty $C_p$ only affects the time slots before the punishment is triggered. A higher $C_p$ means a smaller long-term expected reward as a conservative honest SU (by comparing $LR_{WC}^{H}$ in Eq. (7.22) to $LR_{WC}^{DH}$ in Eq. (7.23)), and thus more incentives to attack. In Case.SC, a larger $C_p$ hurts the reward of attackers more after punishment than before punishment. This
is because the transmission probability before punishment is \( Pr(\sum_{i \in N} D_i = 0) \) (i.e., all SUs sense idle), which is smaller than the transmission probability after punishment \( Pr(\sum_{i \in M} D_i = 0) \) (i.e., all attackers sense idle). Thus a larger \( C_p \) discourages the attacks in Case SC.

### 7.6 Summary

Collaborative spectrum sensing is vulnerable to sensing data falsification attacks. In this chapter, we focus on a challenging attack scenario in which multiple cooperative attackers can overhear the honest SU sensing reports, but the honest SUs are unaware of the existence of attackers. We proposed two attack-prevention mechanisms with direct and indirect punishments. Both mechanisms do not require identification of the attackers. The direct punishment can effectively prevent all attacks in both “attack-and-run” and “stay-with-attacks,” and the indirect punishment can prevent all attacks in the long-run if the attackers care enough about their future rewards.

### 7.7 Appendix

#### 7.7.1 Relaxation of Assumptions A1 and A3

Here we will relax Assumption A1 and consider the general case where SUs have heterogeneous detection performances (i.e., different false alarm probabilities \( P_f \) and missed detection probabilities \( P_m \)) and transmission rates. We
are interested to know whether our attack-prevention mechanisms (with some minor modification of system parameters) can still apply, and how to change punishments to attackers. Due to the page limit, we only examine the direct punishment here. The effectiveness of the indirect punishment can be shown similarly.

Observation 20 in Section 7.4 showed that the single attacker scenario ($M = 1$) is the most challenging attack scenario for the direct punishment mechanism. Thus we will focus on the single attacker scenario to check the effectiveness of this mechanism. We label the attacker as the $N$th SU, and we denote its false alarm probability and missed detection probability as $P_{f,A}$ and $P_{m,A}$, respectively. For the ease of analysis, we still consider all honest SUs having the same $P_f$ and $P_m$.\footnote{If we consider different detection performances for honest SUs, the analysis becomes more complicated without adding more meaningful insights. Intuitively, as honest SUs’ overall detection performance becomes more precise, the attacker can predict the channel state more precisely by overhearing honest SUs’ reports.}

We denote the attacker’s transmission rate as $r_A$, and an honest SU $i < N$ has a different transmission rate $r_i$. When all SUs share the same transmission opportunity using TDMA mode, the attacker obtains a data rate of $r_A/N$.

First of all, we need to change the two notations in (7.4) and (7.5) as follows. When $0 \leq k \leq N - 1$, honest SUs sense the channel busy ($\sum_{i=1}^{N-1} D_i = k$) and the attacker senses idle ($D_N = 0$). The condition probability that the channel
is actually idle is

\[ P_{N,(k)+(0)}^I := \Pr(\text{idle} | \sum_{i=1}^{N-1} D_i = k, D_N = 0) = \frac{P_I(1 - P_f)^{N-1-k} P_f^k (1 - P_{f,A})}{P_I(1 - P_f)^{N-1-k} P_f^k (1 - P_{f,A}) + (1 - P_I)(1 - P_m)^k P_m^{N-1-k}(1 - P_{m,A})}. \]  \hspace{1cm} (7.27)

The conditional probability that the channel is actually busy is

\[ P_{N,(k)+(0)}^B = 1 - P_{N,(k)+(0)}^I. \]  \hspace{1cm} (7.28)

When \(0 \leq k \leq N - 1\), honest SUs sense the channel busy \(\left(\sum_{i=1}^{N-1} D_i = k\right)\) and the attacker also senses busy \((D_N = 1)\). The conditional idle and busy probabilities are respectively

\[ P_{N,(k)+(1)}^I := \Pr(\text{idle} | \sum_{i=1}^{N-1} D_i = k, D_N = 1) = \frac{P_I(1 - P_f)^{N-1-k} P_f^k P_{f,A}}{P_I(1 - P_f)^{N-1-k} P_f^k P_{f,A} + (1 - P_I)(1 - P_m)^k P_m^{N-1-k}(1 - P_{m,A})}, \]  \hspace{1cm} (7.29)

and

\[ P_{N,(k)+(1)}^B = 1 - P_{N,(k)+(1)}^I. \]  \hspace{1cm} (7.30)

With the help of above notations, we can similarly analyze how the direct punishment works and how to determine the punishment as in Section 7.4.

**Theorem 35.** For the single attacker in the network, there exists a threshold

\[ C_b^{th} = \frac{P_I}{1 - P_I} \left(\frac{1 - P_f}{P_m}\right)^{N-1} \frac{1 - P_{f,A}}{P_{m,A}} \frac{N - 1}{N} r_A, \]  \hspace{1cm} (7.31)

such that any value \(C_b > C_b^{th}\) can prevent all attack scenarios described in Section 7.3.
Proof. By examining different possible states as in Section 7.3, we can derive the attacker’s optimal behavior. Then in response to the attacker’s optimal behavior, we find the proper value of direct punishment $C_b$ to prevent all attacks.

State $s = \left( \sum_{i=1}^{N-1} D_i = 0, D_N = 0 \right)$

When the sensing results are all 0, then the attackers may report truthfully or falsely in the Phase I:

- If the attacker reports 0 in Phase I, then the announcement at the fusion center is $H_0$ (idle), and all honest SUs will transmit. It is easy to check that the attacker will also transmit and receive a positive expected reward

$$R_A(s) = P^I_{N,(0),(0)} \frac{r_A}{N} - P^B_{N,(0),(0)} C_p > 0.$$ (7.32)

- If the attacker reports 1 in Phase I, then the announcement at the fusion center is $H_1$ (busy) and all honest SUs will not transmit.

- If the attacker chooses to transmit, its expected reward is

$$R_A(s) = P^I_{N,(0),(0)} r_A - P^B_{N,(0),(0)} (C_p + C_b),$$ (7.33)

which may or may not be larger than (7.32).

- If the attacker waits, its expected reward equals 0 and is less than (7.32).
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To prevent attacks in this state, a high value of $C_b$ should be set to make (7.32) larger than (7.33). In other words,

$$C_b > \frac{P_f(1 - P_{f,A})}{(1 - P_f)P_{m,A}} \left( \frac{1 - P_f}{P_m} \right)^{N-1} \frac{N - 1}{N} r_A,$$

(7.34)

where we denote the term in the right-hand side as threshold $C_{b}^{th1}$. It is easy to check that $C_{b}^{th1}$ is decreasing in both $P_{f,A}$ and $P_{m,A}$, but is increasing in $P_f$, $N$, and $r_A$.

State $s = \left( \sum_{i=1}^{N-1} D_i = 0, D_N = 1 \right)$

The attacker may report truthfully or falsely in Phase I:

- If the attacker reports 0 in Phase I, then the announcement at the fusion center is $H_0$ (idle), and all honest SUs will transmit. It is easy to check that the attacker will also transmit and its expected reward is

$$R_A(s) = P_{N,(0)+(1)}^{l} \frac{1}{N} r_A - P_{N,(0)+(1)}^{B} C_b,$$

(7.35)

which is negative as required by the optimality of OR-rule.

- If the attacker reports 1 in Phase I, then the announcement at the fusion center is $H_1$ (busy), and all honest SUs will not transmit.

  - If the attacker chooses to transmit, its expected reward is

$$R_A(s) = P_{N,(0)+(1)}^{l} r_A - P_{N,(0)+(1)}^{B} (C_p + C_b),$$

(7.36)

which may or may not be negative.
– If the attacker waits, its expected reward equals 0 which is larger than (7.35).

To prevent attacks in this state, a high value of $C_b$ should be set to make (7.36) smaller than 0. This gives to

$$C_b > \frac{P_l P_{f,A}}{(1 - P_f)(1 - P_{m,A})} \left( \frac{1 - P_f}{P_m} \right)^{N-1} r_A - C_p,$$

where we denote the term in the right-hand side as threshold $C_{b}^{th2}$.

State $s = \left( \sum_{i=1}^{N-1} D_i = K \geq 1, D_N = \bar{M} \in \{0, 1\} \right)$

When at least one honest SU’s sensing decision is 1, then no matter what the attacker reports in Phase I, the fusion center always makes correct announcement $H_1$ (busy). All honest SUs will not transmit in Phase II.

• If the attacker chooses to transmit in Phase II, its expected reward is

$$R_A(s) = P_{N,(K)+\bar{M}}^I r_A - P_{N,(K)+\bar{M}}^B (C_p + C_b),$$

which may be positive or negative.

• If the attacker waits in Phase II, its expected reward equals 0.

To prevent all attacks in this state, a high value of $C_b$ should be set to make (7.38) smaller than 0. This gives to

$$C_b > \frac{P_l}{1 - P_I} \left( \frac{1 - P_f}{P_m} \right)^{N-1} \left( \frac{P_f P_m}{(1 - P_f)(1 - P_m)} \right)^K \left( \frac{1 - P_{f,A}}{P_{m,A}} \right)^{1-\bar{M}} \left( \frac{P_{f,A}}{1 - P_{m,A}} \right)^{\bar{M}} r_A - C_p,$$

(7.39)
which is maximized when $K = 1$ and $\tilde{M} = 0$ due to $P_m < 0.1$ and $P_f < 0.1$ as explained in footnote 9. To prevent all attacks in this state, we should require

$$C_b > \frac{P_I}{1 - P_I} \left( \frac{1 - P_f}{P_m} \right)^{N-1} \left( \frac{P_f P_m}{(1 - P_f)(1 - P_m)} \right) \frac{1 - P_{f,A} r_A}{P_{m,A}} - C_p,$$  

where the right-hand side is denoted as threshold $C_{b}^{th3}$.

To summarize, the requirement of $C_b$ to prevent attacks is $C_b > \max (C_{b}^{th1}, C_{b}^{th2}, C_{b}^{th3})$ in all possible states. It is easy to check that $C_{b}^{th1} > C_{b}^{th2}$ and $C_{b}^{th1} > C_{b}^{th3}$, and we can conclude the results in Theorem 35.

Observation 24. $C_{b}^{th}$ is increasing in both idle probability $P_I$ and the number of honest SUs $N - 1$.

A larger $P_I$ means a higher channel availability, and thus encourages the attacker to launch an attack so that it can exclusively utilize the channel more frequently. Also, as the honest SU number $N - 1$ increases, the more honest SUs’ sensing reports are overheard by the attacker. The attacker can estimate the actual channel state more accurately, and it is more likely to launch an attack.

Observation 25. Threshold $C_{b}^{th}$ is increasing in the attacker’s rate $r_A$, and it is decreasing in the attacker’s false alarm probability $P_{f,A}$ and missed detection probability $P_{m,A}$.

As the attacker’s rate $r_A$ increases, it values the exclusive transmission opportunity more. The attacker has a higher incentive to attack, and a higher $C_{b}^{th}$ is required to prevent the attack.
Figure 7.4: Direct punishment threshold $C_b^{th}$ for different $P_{f,A}$ and $P_{m,A}$ with $(P_f, P_m, N) = (0.05, 0.05, 11)$.

Figure 7.4 shows the threshold $C_b^{th}$ as a function of the attacker’s false alarm probability $P_{f,A}$ and missed detection probability $P_{m,A}$. Intuitively, as $P_{f,A}$ increases, the attacker has higher probability to overlook the channel access opportunity and it has less incentive to launch an attack. As $P_{m,A}$ increases, the attacker has a higher probability to trigger direct punishment and thus it is more conservative to attack. In both cases, the fusion center can to announce a lower $C_b$ to prevent all attacks.

### 7.7.2 Attack Prevention in Case.AT of Section 7.5

Section 7.5 focuses on Case.NT. Here we discuss Case.AT with multiple attackers ($M \geq 1$). Table 7.2 in Section 7.3 shows that with no punishment, the
attacker may still obtain a positive reward by transmission even when at least
one SU senses the channel busy in Case AT. Next, we discuss the attacker’s
action \( u(s) \) under any given state \( s = (\sum_{i \in N \setminus M} \bar{D}_i; \sum_{i \in M} D_i; \text{Punishment}) \).

- **State \( s = (\sum_{i \in N \setminus M} D_i; \sum_{i \in M} D_i = K, \text{off}) \):** Before the indirect
  punishment is triggered, the attackers may purposely change their attack
  behaviors (comparing to the actions in Table 7.2 in Section 7.3) in some
  states to deter punishment.

  - \( s = (0, 0, \text{off}) \): at least one attacker still chooses the action (busy,
    transmit) as in Table 7.2. In this state, the attackers obtain the
    largest expected aggregate reward \( P_{N,0}^L - MP_{N,0}^B C_p \) and induces the
    smallest missed detection probability \( P_{N,0}^B \) to trigger punishment.

  - \( s = (\sum_{i \in N \setminus M} D_i \geq 0, \sum_{i \in M} D_i \geq 0, \text{off}) \): the attackers may choose
    not to attack even if they can obtain a positive expected aggregate
    reward in current time slot, which is different from no punishment
    scenario in Table 7.2. This is because that the attackers fear to trigger
    the long-term indirect punishment, and they will attack only when
    missed detection probability to trigger punishment is low with small
    \( \sum_{i \in N} D_i \).

- **State \( s = (\text{Unknown}, \sum_{i \in M} D_i = K, \text{on}) \):** After the indirect punishment
  is triggered, the attackers know their own sensing results and will choose
  the actions as follows.
– **Weak Cooperation (Case.WC):** the attackers will choose the action (N/A, wait) even if all attackers sense the channel idle. Otherwise, they will receive a negative expected aggregate reward $P_{M,K}^I - MP_{M,K}^B C_p$.

– **Strong Cooperation (Case.SC):** the attackers will choose the action (N/A, transmit) when all attackers sense the channel idle. Even if at least one attackers senses the channel busy, they will still choose the action (N/A, transmit) if $P_{M,K}^I \geq MP_{M,K}^B C_p$.

**Lemma 7.** The attackers’ optimal long-term expected aggregate rewards in Case.WC and Case.SC are

$$LR_{WC}^H = LR_{SC}^H = \frac{M}{1 - \delta} Pr \left( \sum_{i \in \mathcal{N}} D_i = 0 \right) \left( P_{N,0}^I \frac{1}{N} - P_{N,0}^B C_p \right),$$

(7.41)

by behaving honestly, and

$$LR_{WC}^{DH} = \max_{0 \leq z \leq N} \sum_{k=0}^{z} Pr(\sum_{i \in \mathcal{N}} D_i = k) \frac{P_{N,k}^I - MP_{N,k}^B C_p}{1 - \delta(1 - \sum_{k=0}^{z} Pr(\sum_{i \in \mathcal{N}} D_i = k) P_{N,k}^B)}$$

(7.42)

$$LR_{SC}^{DH} = \max_{0 \leq z \leq N} \sum_{k=0}^{z} Pr(\sum_{i \in \mathcal{N}} D_i = k) \left[ \frac{P_{N,k}^I - MP_{N,k}^B C_p}{1 - \delta(1 - \sum_{k=0}^{z} Pr(\sum_{i \in \mathcal{N}} D_i = k) P_{N,k}^B)} \right] + \frac{\delta}{1 - \delta} \frac{P_{M,0}^B Pr(\sum_{i \in \mathcal{M}} D_i = 0)(P_{M,0}^I - MP_{M,0}^B C_p)}{1 - \delta(1 - \sum_{k=0}^{z} Pr(\sum_{i \in \mathcal{N}} D_i = k) P_{N,k}^B)},$$

(7.43)

by behaving dishonestly. Here the superscript “H” indicates honest behaviors of attackers and “DH” indicates dishonest behaviors. We denote the value of $z$ that achieves the maximum of (7.42) in Case.WC or (7.43) in Case.SC as $z^*$. The attackers’ optimal policy $u^*$ has a threshold structure as follows.
• If $\sum_{i \in N} D_i \leq z^*$: At least one attacker will take the action (busy, transmit) before the indirect punishment is triggered. After the indirect punishment is triggered,

  - Case.WC: all attackers will take the action (N/A, wait).

  - Case.SC: all attackers will take the action (N/A, transmit) if $P_{M,K}^I > MP_{M,K}^B C_p$ where $K$ attackers sense the channel busy. Otherwise, they will take the action (N/A, wait).

• If $\sum_{i \in N} D_i > z^*$: The attackers will take the action (busy, wait) before the indirect punishment is triggered. After the indirect punishment is triggered,

  - Case.WC: all attackers will take the action (N/A, wait).

  - Case.SC: all attackers will take the action (N/A, transmit) if $P_{M,K}^I > MP_{M,K}^B C_p$ where $K$ attackers sense the channel busy. Otherwise, they will take the action (N/A, wait).

We can show in Lemma 7 that $LR_{DH}^{WC} < LR_{DH}^{H}$ in Case.WC and $LR_{DH}^{HC} < LR_{DH}^{SC}$ in Case.SC when $\delta$ is close to 1. Thus we can find an appropriate discount factor threshold to ensure that $LR_{DH}^{WC} < LR_{DH}^{H}$ for Case.WC and $LR_{DH}^{SC} < LR_{DH}^{HC}$ for Case.SC.

**Theorem 36.** For multiple attackers in Case.AT, there exists a threshold $\delta^{th} \in (0,1)$ such that no attacks will happen if $\delta > \delta^{th}$. 
7.7.3 Proof of Lemma 6

In Case NT, the attackers will only attack with the action (busy, transmit) when all SUs sense the channel idle when the indirect punishment is not triggered. But they may or may not transmit after the punishment is triggered.

Case WC (Weak Cooperation)

In Case WC, the attackers will not transmit even when all attackers sense the channel idle after the punishment is triggered.

If the attackers behave as honest SUs, the indirect punishment will never be triggered. They share the spectrum opportunities with the honest SUs when all SUs sense the channel idle. The attackers’ long-term expected aggregate reward is

\[
LR_{WC} = \sum_{t=0}^{\infty} \delta^t \Pr \left( \sum_{i \in N} D_i = 0 \right) \left( P_{N,0}^I \frac{1}{N} - P_{N,0}^B C_p \right) M,
\]

which can be rewritten as in (7.22).

If the attackers attack with the action (busy, transmit) when all SUs sense the channel idle \( \left( \sum_{i \in N} D_i = 0 \right) \), then in time slot \( t = 0 \) the indirect punishment will be triggered with the missed detection probability \( P_{N,0}^B \). If no collision happens with the probability \( P_{N,0}^I \), the attack will not be detected and the attackers will attack again if all SUs sense the channel idle in the next time slot.

By focusing on time slot \( t = 0 \), the attackers’ long-term expected aggregate
reward is

\[ LR_{WC}^{DH} = Pr \left( \sum_{i \in N} D_i > 0 \right) \delta LR_{WC}^{DH} + Pr \left( \sum_{i \in N} D_i = 0 \right) \]

\[ \cdot \left[ P_{I,0}^I (1 + \delta LR_{WC}^{DH}) - P_{N,0}^B (MC_p) \right]. \]

We can then recursively rewrite \( LR_{WC}^{DH} \) as in (7.23).

**Case SC (Strong Cooperation)**

In **Case SC**, the attackers will still transmit when all attackers sense the channel idle after the punishment is triggered.

If the attackers behave as honest SUs, no indirect punishment will be triggered and they will receive the same long-term expected aggregate reward in (7.22).

If the attackers attack with the action (busy, transmit) when all SUs sense the channel idle in time slot \( t = 0 \), we can derive the attackers’ long-term expected aggregate reward similar as the **Case WC**,

\[ LR_{SC}^{DH} = Pr \left( \sum_{i \in N} D_i > 0 \right) \delta LR_{SC}^{DH} + Pr \left( \sum_{i \in N} D_i = 0 \right) \left[ P_{I,0}^I (1 + \delta LR_{SC}^{DH}) \right. \]

\[ - P_{N,0}^B MC_p + P_{N,0}^B \frac{\delta}{1 - \delta} Pr \left( \sum_{i \in M} D_i = 0 \right) \left( P_{M,0}^I - MP_{M,0}^B C_p \right) \right]. \]

The only difference here is that the attackers can still obtain a positive expected aggregate reward after the punishment is triggered. Then we can recursively rewrite \( LR_{SC}^{DH} \) as in (7.24).
Chapter 8

Conclusion and Future Work

8.1 Conclusion

This thesis exploits the interactions between economic and technological decisions made by wireless network operators. A network operator’s decisions may involve the choices and timings of technology adoptions, the amounts of wireless resources to invest, and the prices to set for his services. These decisions are coupled with each other and need to be jointly optimized. In a practical wireless network, an operator faces limited resources, immature technology, and market competition, and we take these challenges into account in the modeling and analysis. This thesis focuses on such issues in the following two types of networks. We first study the economics of cellular networks, which have the largest market occupancy among all wireless technologies. We then look at the economics of cognitive radio networks, which represent one of the
main trends for the development of wireless technologies in the near future. Note that cognitive radios can also apply to cellular networks to improve the spectrum utilization.

In the first part of this thesis, we study a cellular operator’s economic and technological decisions related to network upgrade, service differentiation, and social applications. First, we study operators’ timing decisions of 4G network upgrade in a competitive market. We show that operators should select different upgrade times to avoid severe competition, which matches well with many industry observations. The availability of 4G upgrade increases operator competition and may decrease operators’ profits. Second, to resolve indoor users’ poor signal receptions in 4G service, we study an operator’s economic incentive to deploy femtocell service on top of his existing macrocell service. We need to perform network optimization by considering how two differentiated services share the limited spectrum bands and how to guarantee users’ payoffs by introducing femtocell service. Finally, we are interested in understanding how an operator can provide economic incentives for heterogeneous smartphone users to collaborate in social applications (e.g., data acquisition and distributed computing). Such incentive mechanisms are difficult to design as smartphone users are heterogeneous in types (e.g., sensitivities to privacy loss, energy and computing efficiencies) and the operator does not have complete information on this.
In the second part of this thesis, we study how a secondary operator’s investment flexibility, sensing uncertainty, and sensing security in cognitive radio networks affect his economic and technological decisions. First, we consider that a secondary operator can obtain spectrum resource in short-term via both sensing and dynamic spectrum leasing, which are much more flexible than the current static licensing of spectrum to quickly respond to the market changes. We are interested in understanding an operator’s optimal decision in balancing the low cost of sensing and the reliability of leasing. We propose that an operator should sense first to learn primary users’ activity level and then lease and price based on that knowledge. Second, we study operators’ competition in both investment and pricing, and show that such competition can significantly increase users’ payoffs by providing more resource in the market at a lower price. Finally, the operator wants to improve the sensing accuracy by using collaborative spectrum sensing instead of simple sensing, but such an approach is vulnerable to data falsification attacks. We design effective attack-prevention mechanisms for the challenging case where attackers can cooperate with each other and eavesdrop other users’ sensing data to mislead the fusion center.

There are some directions to generalize the studies in this thesis. First, we want to study the multiple operators’ heterogeneity in the competitive market. For example, we may consider multiple heterogenous operators who have
different approaches to obtain resources, and users have different valuations towards the services provided by different operators. Second, a network operator sometimes does not have complete information about the market (e.g., users’ demands, other operators’ decisions, and users’ types in social applications), and it is interesting to study how he can learn such information and adjust his decisions over time. Next we provide some concrete thoughts on the possible generalization for each chapter in this thesis.

8.2 Extensions of 4G Network Upgrade in Chapter 2

In Chapter 2, we only consider duopoly competition, operators’ pure strategies in 4G upgrades, and their short upgrade duration. We can consider the following to extend our results.

First, we could consider an oligopoly market with more than two competitive operators. Intuitively, multiple operators under inter-network switching would still select asymmetric upgrade times to avoid severe competition. Compared to duopoly, the operator who is the last to upgrade loses more market share to the others and but enjoys the smallest upgrade cost and the largest network effect.

Second, it is also interesting to study operators’ usage of 4G plan announcements before actual upgrades. One operator (who actually decides to upgrades later than what he announces) can prevent some of his users switching to other
networks. We can use a signaling game to study operators’ announcements.

Third, we can study operators’ mixed strategies in 4G upgrades, where one operator chooses a probability distribution of upgrading at different times. Although theoretically interesting, mixed strategies are often hard to implement in practical market [67].

Fourth, we can study the case where a 4G upgrade takes a certain period of time including 4G technology trials and integration with other services (e.g., 2G and 3G).

Finally, we can consider a dynamic 4G market where the total number of users is not fixed during the transition from 3G to 4G standard. For example, there would be more young people entering the market and they may prefer the popular 4G to traditional 3G. In this case, one operator has more incentive to deploy 4G service to attract these new arrivals.

8.3 Extensions of Femtocell Service Provision in Chapter 3

In Chapter 3, we only consider “separate carriers” scheme of spectrum sharing between femtocell and macrocell services, and only focus monopoly market. There are several directions to extend the results in Chapter 3.

We can further consider the “shared carriers” scheme, where femtocell service and macrocell service share part of or the whole spectrum. We need to
optimize the pricing and spectrum allocation decisions by trading off the increased spectrum availabilities and mutual interferences. The results here also lay the foundation of further study of market competition between multiple operators, where some or all of them provide the femtocell service on top of their existing macrocell service. With competition, users can switch operators and services, and an operator needs to consider other operators’ femtocell adoption time and quality in his own decisions.

8.4 Extensions of Smartphone Collaboration on Social Applications in Chapter 4

In Chapter 4, we only consider the operator’s threshold-based revenue model in data acquisition and the operator does not learn users’ types over time. There are some possible ways to extend the results in Chapter 4.

For the data acquisition applications, for example, we can consider a flexible revenue model instead of a threshold one. For example, Google can still benefit if a few users take pictures of some critical events. In this case, the client’s valuation of users’ helps may be concavely increasing in their efforts, and a user wants to participate early in collaboration. It is also interesting to study the repeated collaborations between the client and users in distributed computing. Though users’ types can change over time (e.g., their remaining battery levels), client can update his beliefs about users’ types by observing
users’ choice history.

8.5 Extensions of Monopoly Spectrum Market in Chapter 5

In Chapter 5, we have made several simplifying assumptions, including perfect spectrum sensing, operator’s knowledge of complete market information, and the similar time scales for leasing and sensing).

Some assumption (e.g., imperfect sensing) can be (easily) generalized without affecting the main insights. We can incorporate imperfect spectrum sensing (i.e., miss-detection and false-positive) into the model, which will change the uncertainty of the spectrum sensing. Given that our results work for any distribution of the sensing realization $\alpha$, it is likely that such generalization does not change the major insights.

Generalizations of some other assumptions, however, lead to more challenging new problems.

- *Incomplete information of secondary users:* when the operator does not know the information of the users, the system needs to be modeled as a dynamic game with incomplete information. More elaborate economic models such as screening and signaling [53] become relevant.

- *Time scale separation:* it is possible that dynamic leasing is performed at a different (much larger) time scale compared with spectrum sensing.
In that case, the operator has to make the leasing decision first, and then make several sequential sensing decisions. This leads to a dynamic decision model with more stages and tight couplings across sequential decisions.

- **Operator competition:** There may be multiple C-MVNOs providing services in the same geographic area. In that case, the operators need to attract the users through price competition. Also, if they sense and lease from the same primary operator, the operators may have overlapping or conflicting resource requests. Although we have obtained some preliminary results along this line in [56], more studies are definitely desirable.

### 8.6 Extensions of Competitive Spectrum Market in Chapter 6

In Chapter 6, we only consider leasing availability of operators, users’ same valuation towards different operators’ services, and duopoly competition. There are several ways to extend the results in Chapter 6.

First, we can consider the case where the operators can also obtain resource through spectrum sensing [65, 77]. Compared with leasing, sensing is cheaper but the amount of useful spectrum is less predictable due to the primary users’ stochastic traffic. With the possibility of sensing, we need to consider a four-stage dynamic game model.
Second, we can consider the case where users might experience different channel conditions when they choose different providers, e.g., when they need to communicate with the base stations of the operators. Competition under such channel heterogeneity has been partially considered by Gajic et al. [61] without considering the cost of spectrum acquisition.

Third, we can consider a more general and realistic model for user’s transmission. For example, a user’s payoff and demand are affected by the received signal strength indicator (RSSI) to its receiver and whether the transmitter is able to transmit the data to the receiver with the allocated bandwidth and time given. Also, we can consider detailed signal fading and its effect on users’ changing demand, and different allocated frequencies may have different power requirements.

Fourth, we can study the multiple-operator (oligopoly) case, where the analysis becomes much more complicated without closed-form solutions. For example, when we do backward induction analysis at Stage II, all possible combinations of multiple operators’ leasing decisions in Stage I need to be considered. Nevertheless, we can still infer some intuitions about the oligopoly case based on our duopoly analysis. For example, operators’ competition will be more severe and their equilibrium symmetric prices will be closer to leasing costs with oligopoly. Also, operators will be more conservative in leasing decisions in Stage I, since each operator is expected to serve fewer users in Stage III.
Finally, each operator’s profit will decrease in the number of operators. It will be useful to verify these intuitions and discover additional new insights in the oligopoly case.

Finally, we can consider the case where the operator improves its profit by price differentiation, \textit{i.e.}, charging different users different prices based on their channel conditions and transmission power. The key issue is to achieve the best tradeoff between pricing complexity and profit improvement. A similar tradeoff has been studied for a monopoly network service provider in Li \textit{et al.} [76].

\textbf{8.7 Extensions of Security Protection in Collaborative Spectrum Sensing in Chapter 7}

In Chapter 7, we only consider fast change of PU’s traffic, fusion center’s unawareness of all SUs’ transmission characteristics, attackers’ rationality, and perfect communications between SUs and the fusion center. There are several possible ways to extend the results in Chapter 7.

First, we can consider the case where PU’s traffic changes slowly over time (\textit{e.g.}, TV transmitters), where we need to consider the correlation between spectrum occupancies over different time slots. In this case, we should to use a much more complicated MDP model than the one in Section 7.5.

Second, we can study the case that the fusion center knows all SUs’ trans-
mission characteristics (e.g., modulation and coding schemes) and can monitor attackers’ sensing information communication in Phase I and attackers’ transmissions in Phase II. In this case, the fusion center may be able to identify attackers. However, the attackers can change their modulation and coding schemes (e.g., as some honest SUs) or secure their transmissions (via MAC-layer encryptions) to avoid being identified.

Third, we can study the denial-of-service attacks. Throughout this paper, we consider that attackers are rational and are only interested in maximizing their own rewards. For denial of service attacks, however, the attackers’ objective is to let honest SUs lose transmission opportunities or break down the effectiveness of collaborative sensing.

Finally, we can consider imperfect control channel between SUs and the fusion center (e.g., some SUs receive false announcement from the fusion center). In that case, an indirect punishment can be triggered due to channel communication errors instead of attacks. We need to design the indirect punishment which will resume collaborative sensing after a period of time (instead of an infinitely long punishment).
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