Economics of Mobile Data Offloading

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Abstract— Mobile data offloading is a promising approach to alleviate network congestion and enhance quality of service (QoS) in mobile cellular networks. In this paper, we investigate the economics of mobile data offloading through third-party WiFi or femtocell access points (APs). Specifically, we consider a market-based data offloading solution, where macrocellular base stations (BSs) pay APs for offloading traffic. The key questions arising in such a marketplace are following: (i) how much traffic should each AP offload for each BS? and (ii) what is the corresponding payment of each BS to each AP? We answer these questions by using the non-cooperative game theory. In particular, we define a multi-leader multi-follower data offloading game (DOFF), where BSs (leaders) propose market prices, and accordingly APs (followers) determine the traffic volumes they are willing to offload. We characterize the subgame perfect equilibrium (SPE) of this game, and further compare the SPE with two other classic market outcomes: (i) the market balance (MB) in a perfect competition market (i.e., without price participation), and (ii) the monopoly outcome (MO) in a monopoly market (i.e., without price competition). Our results analytically show that (i) the price participation of BSs will drive market prices down, compared to those under the MB outcome, and (ii) the price competition (among BSs) will drive market prices up, compared to those under the MO outcome.

I. INTRODUCTION

We are witnessing an unprecedented worldwide growth of mobile data traffic, which is expected to reach 10.8 exabytes per month by 2016, an 18-fold increase over 2011 [1]. However, traditional network expansion methods by acquiring more spectrum licenses, deploying new macrocells of small size, and upgrading technologies (e.g., from WCDMA to LTE/LTE-A) are costly and time-consuming [2]. Clearly, network operators must find novel methods to resolve the mismatch between demand and supply growth, and mobile data offloading appears as one of the most attractive solutions.

Simply speaking, mobile data offloading is the use of complementary network technologies, such as WiFi and femtocell, for delivering data traffic originally targeted for cellular networks. A growing number of studies have been devoted to the potential performance benefits of mobile data offloading and the technologies to support it [3]–[12]. Specifically, the benefit of macrocellular data offloading through WiFi networks (called WiFi offloading) was studied and quantified using real data traces in [3]–[6]. It is shown that in a typical urban environment, WiFi can offload about 65% cellular traffic and save 55% battery energy for mobile users (MUs) [3].

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1As evidence, many network operators have already started to deploy their own WiFi or femtocell networks for data offloading (e.g., AT&T [17]), or initiate collaborations with other WiFi networks (as O2 did with BT [18]).
2For example, network operators can offer MUs certain rebate for compensating their QoS degradations (e.g., delay) induced by the data offloading.
is, data offloading is totally transparent to MUs.

Specifically, we consider the network-initiated mobile data offloading through third-party WiFi or femtocell APs, and focus on the economic interactions between network operators and APs. In particular, we focus on the incentives that network operators need to provide to APs in order to encourage cooperative data offloading. Figure 1 illustrates an instance of data offloading between 3 macrocellular base stations (BSs) and 6 APs. Each BS can offload traffic (of its MUs) to multiple APs, and each AP can admit traffic from multiple BSs.

We study the incentive design problem by adopting a market-based data offloading solution, where network operators share the data offloading benefits with APs through market mechanisms. Namely, network operators pay APs with certain monetary compensation for offloading traffic. Therefore, the key problems for such a market are to determine: (a) traffic offloading volumes: how much traffic should each AP offload for each BS? and (b) pricing rules: what is the corresponding payment of each BS to each AP? We answer these questions by using the non-cooperative game theory.

More specifically, we introduce a two-stage multi-leader multi-follower game, called data offloading game (DOFF), where BSs act as leaders and APs as followers. In the first stage, each BS proposes the price that it is willing to pay to each AP for offloading its traffic. In the second stage, each AP decides how much traffic to offload for each BS. We characterize the Nash equilibrium, NE, (or more precisely, the subgame perfect equilibrium, SPE) of this game. We show that under the SPE, (i) every AP rejects all BSs except those proposing the highest price, and (ii) the highest (BS) price to every AP equalizes the maximum marginal cost reduction (of all BSs) and the marginal payment (to the AP).

We further compare the SPE with the outcomes of two other classic market settings: (i) the perfect competition market, and (ii) the monopoly market. The first case corresponds to the scenario where the market prices are determined by an external controller (e.g., a price regulator), and BSs and APs act as price-takers, i.e., do not estimate the impact of their strategies on the prices. In this sense, the market players do not participate directly in the determination of prices (no price participation). The second case arises when all BSs act as one single player, either because they are managed by the same operator or because they collude and create an effective monopoly. Thus, there is no price competition. We analytically show that the equilibrium price (arises in the SPE of our DOFF game) is upper-bounded by the market clearing price (arises in the equilibrium of the perfect competition market), and lower-bounded by the monopoly price (arises in the equilibrium of the monopoly market).

The main contributions of this work are the following:

- To the best of our knowledge, this is the first work that considers a mobile data offloading market with multiple macrocellular BSs and multiple APs, and applies game theoretic modeling and analysis of the equilibrium.
- We systematically study the economic interactions of BSs and APs. Specifically, we define a multi-leader multi-follower data offloading game–DOFF, with BSs as leaders and APs as followers; and analytically characterize the subgame perfect equilibrium–SPE.
- We compare the SPE of the DOFF game with the outcomes of perfect competition market (no price participation) and monopoly market (no price competition). Our results analytically show the impact of price participation and price competition on market prices.

The rest of this paper is organized as follows. In Section II, we present the system model. In Sections III, we provide the social optimality and market balance. In Section IV, we study the DOFF game in details. Finally, we conclude in Section V.

II. SYSTEM MODEL

A. System Description

We consider a system that includes a set $\mathcal{M} \triangleq \{1, \ldots, M\}$ of macrocellular base stations (BSs) and a set $\mathcal{N} \triangleq \{1, \ldots, N\}$ of third-party WiFi or femtocell access points (APs). BSs may have overlapping coverage areas, while APs are assumed to be non-overlapping with each other. Each BS serves a set of mobile users (MUs) randomly distributed within its coverage area. The traffic of an MU can be offloaded to an AP, only if it is covered by that AP (e.g., MU$_{11}$ and AP 1 in Figure 1). Figure 1 illustrates such a two-tier network.

Let $S_{mn}$ denote the total (maximum) BS $m$’s traffic that can be offloaded to AP $n$, i.e., those generated by all MUs of BS $m$ in the coverage area of AP $n$. Let $S_m$ denote the total BS $m$’s traffic that cannot be offloaded to any AP, i.e., those generated by all MUs of BS $m$ not covered by any AP. Obviously, $S_{mn} \equiv 0$ if there is no overlap between a BS $m$ and an AP $n$ (e.g., BS 1 and AP 2 in Figure 1). The traffic profile of BS $m$ is denoted by $S_m \equiv (S_{m0}, S_{m1}, \ldots, S_{mN})$.

Due to the uncertainty of MUs’ data usage and mobility, $S_m$ changes randomly over time. We consider a quasi-static network scenario, where $S_m, \forall m \in \mathcal{M}$, remains unchanged within every data offloading period (e.g., one minute in our simulation), while changes across periods.

We define the transmission efficiency of a communication link (between a BS and an MU, or an AP and an MU) as the average volume of traffic (Mbits) that can be supported by one unit of spectrum resource (MHz), denoted by $\theta$ (measured in Mbits/MHz). For convenience, we normalize the transmission efficiency within $\theta \in [0, 1]$. Since each AP’s coverage area is very small, we assume that: (i) the transmission efficiency
parameters between each AP and its covered MUs are identical, and attain the maximum value \( \theta = 1 \); and (ii) the transmission efficiency parameters between each BS \( m \) and its served MUs covered by the same AP (say \( n \)) are also identical, denoted by \( \theta_{mn} \in [0,1] \). We further denote \( \theta_{m0} \) as the average transmission efficiency between BS \( m \) and all of its served MUs whose traffic cannot be offloaded. The transmission efficiency profile of BS \( m \) is denoted by
\[
\theta_m \triangleq (\theta_{m0}, \theta_{m1}, ..., \theta_{mN}).
\]

We similarly assume that \( \theta_m, \forall m \) remains unchanged within every offloading period, while changes across periods.

Let \( x_{mn} \in [0, S_{mn}] \) denote the traffic offloaded from BS \( m \) to AP \( n \), and \( z_{mn} \in [0, +\infty) \) denote the corresponding payment from BS \( m \) to AP \( n \). Then, the traffic offloading matrix can be written as \( X \triangleq (x_{mn})_{m \in M, n \in N} \), and the payment matrix can be written as \( Z \triangleq (z_{mn})_{m \in M, n \in N} \).

### B. Macrocellular BS Modeling

Let \( C_m(b) \) denote the serving cost of BS \( m \) for \( b \) units of spectrum resource consumption. Such a serving cost may include the energy cost, operating cost, coordinating cost, etc. We generally assume that \( C_m(b) \) is strictly increasing and convex, that is, \( C''_m(b) > 0 \) and \( C''_m(b) \geq 0, \forall m \in M \).

Denote \( x_m \triangleq (x_{1m}, ..., x_{Nm}) \) and \( z_m \triangleq (z_{1m}, ..., z_{Nm}) \) as the traffic offloading profile and payment profile of BS \( m \), corresponding to the \( m \)-th rows of \( X \) and \( Z \), respectively. Given any \( x_m \), the BS \( m \)'s total spectrum resource consumption for delivering all remaining un-offloaded traffic is
\[
b_m(x_m, s_m, \theta_m) = \frac{S_m}{\theta_m} + \sum_{n=1}^{N} \frac{S_{mn} - x_{mn}}{\theta_{mn}}.
\]

For clarity, we will write \( b_m(x_m, s_m, \theta_m) \) as \( b_m \) hereafter. Therefore, the total cost of BS \( m \) (including both the serving cost and the payment to APs) under \( x_m \) and \( z_m \) is
\[
C_{m}^{TOT}(x_m, z_m) = C_m(b_m) + \sum_{n=1}^{N} z_{mn}.
\]

The payoff of every BS \( m \) is defined as the cost reduction achieved from data offloading. Formally,
\[
U_m^{BS}(x_m; z_m) = C_{m}^{TOT}(0; 0) - C_{m}^{TOT}(x_m; z_m) = R_m(x_m) - \sum_{n=1}^{N} z_{mn},
\]
where \( R_m(x_m) = C_m(b_0) - C_m(b_m) \) is the BS \( m \)'s serving cost reduction, and \( b_0 = b_m(0, s_m, \theta_m) = \sum_{n=0}^{N} \frac{S_{mn}}{\theta_{mn}} \) is the BS \( m \)'s total resource consumption without data offloading.

### C. AP Modeling

Each AP is owned by a private owner, and has its own traffic demand. Let \( \xi_n \) denote the AP \( n \)'s own traffic demand, which changes randomly over time. Denote \( f_n(\xi) \) and \( F_n(\xi) \) as the PDF and CDF of \( \xi_n \), respectively.\(^6\) Let \( B_n \) denote the total spectrum resource available for AP \( n \). Then, the expected profit of AP \( n \) (from its own traffic) is
\[
W_n(B_n) = (w_n - c_n) \cdot E_{\xi_n} [\min \{B_n, \xi_n\}],
\]
where \( w_n \) is the average unit revenue from its own traffic, \( c_n \) is the unit cost for its resource consumption, and \( E_{\xi_n} [\cdot] \) denotes the expectation with respect to \( \xi_n \).

Denote \( x_n \triangleq (x_{n1}, ..., x_{nM}) \) and \( z_n \triangleq (z_{n1}, ..., z_{nM}) \) as the traffic offloading profile and payment profile to AP \( n \), corresponding to the \( n \)-th columns of \( X \) and \( Z \), respectively. When admitting \( x_n \), the resource for BSs’ traffic is \( x_m \triangleq \sum_{m=1}^{M} x_{mn} \), and thus the resource left for its own traffic is \( B_n - x_n \). Obviously, a feasible \( x_n \) satisfies: \( x_n \leq B_n \). Under any feasible \( x_n \) and \( z_n \), the total profit of AP \( n \) is
\[
W_n^{TOT}(x_n; z_n) = W_n^{AP}(B_n - x_n) - c_n \cdot x_n + \sum_{m=1}^{M} z_{mn}.
\]

The payoff of every AP \( n \) is defined as the profit improvement achieved from data offloading, i.e.,
\[
U_n^{AP}(x_n; z_n) = W_n^{TOT}(x_n; z_n) - W_n^{TOT}(0; 0) = Q_n(x_n) + \sum_{m=1}^{M} z_{mn},
\]
where \( Q_n(x_n) = W_n^{AP}(B_n - x_n) - W_n^{AP}(B_n) - c_n x_n \) is the AP \( n \)'s own profit loss by offloading BSs’ traffic.

### D. Social Welfare

The **social welfare** is defined as the aggregate payoff of all BSs and APs, and denoted by
\[
\Psi(X; Z) = \sum_{m \in M} U_m^{BS}(x_m; z_m) + \sum_{n \in N} U_n^{AP}(x_n; z_n).
\]

We will also write \( \Psi(X; Z) \) as \( \Psi(X) \), since it is independent of the payment transfers between BSs and APs.

### III. Social Optimality and Market Balance

In this section, we first derive the centralized optimal solution (social optimality) which maximizes the social welfare. Then we study how to achieve the social optimality in a perfect competition market with price-taking BSs and APs.

#### A. Social Optimality

The social welfare optimization problem (SWO) is
\[
\text{(SWO)} \quad \max_X \quad \Psi(X)
\]
\[\text{s.t.} \quad \begin{aligned}
&(i) \quad 0 \leq x_{mn} \leq S_{mn}, \quad \forall m \in M, n \in N; \\
&(ii) \quad \sum_{m \in M} x_{mn} \leq B_n, \quad \forall n \in N.
\end{aligned}
\]

For convenience, we first introduce the following concepts.

**Definition 1** (Marginal Cost Reduction of BS \( m \)).

\[
MC_m^{BS}(x_m) \triangleq C'_m(b_m).
\]

**Definition 2** (Marginal Profit Loss of AP \( n \)).

\[
ML_n^{AP}(x_n) \triangleq w_n - (w_n - c_n) \cdot F_n(B_n - x_n).
\]

Intuitively, the marginal cost reduction \( MC_m^{BS} \) represents the decrease of BS \( m \)'s serving cost induced by reducing one additional unit of resource consumption. The marginal profit loss \( ML_n^{AP} \) represents the decrease of AP \( n \)'s profit induced by offloading one additional units of traffic for BSs.

Then, the first-order derivative of \( \Psi(X) \) can be written as:
\[
\frac{\partial \Psi(X)}{\partial x_{mn}} = \theta_{mn}^{-1} \cdot MC_m^{BS}(x_m) - ML_n^{AP}(x_n).
\]

We can further show that the SWO (8) is a convex optimization (see Appendix-A in [24]), and thus can be solved by the Karush-Kuhn-Tucker (KKT) conditions [23]. Formally,
Lemma 1 (Social Optimality). The socially optimal solution \( X^\circ \) is given by the following optimality conditions:

(A.1) \( \theta_m^1 \cdot \text{MC}^\text{BS}_m(x_m^\circ) - \text{ML}^\text{AP}_n(x_n^\circ) = -\mu_n^\circ + \lambda_n^\circ = 0, \)

(A.2) \( \mu_m^\circ \cdot (S_m - x_m^\circ) = 0, \lambda_m^\circ \cdot x_m^\circ = 0, \)

\( \eta_n^\circ \cdot (B_n - \sum_{m=1}^M x_m^\circ) = 0, \)

for all \( m \in M \) and \( n \in N \), where \( \mu_n^\circ, \lambda_n^\circ \) and \( \eta_n^\circ \) are the corresponding optimal Lagrange multipliers.

B. Market Balance in Perfect Competition Market

Now we consider a perfect competition market, also called a price-taking market, where market prices (pricing rules) are set by a central controller (e.g., price regulator) in order to match (balance) the market supply and demand. BSs and APs are price-takers, responding to the announced prices without considering the impact of their strategies on the market prices. The individual optimization problems for price-taking BSs (BSO) and APs (APO) are, respectively:

\[
\text{(BSO)} \max_{x_m \in \mathcal{X}_m} U^\text{BS}_m(x_m; z_m(x_m)), \text{ s.t. } x_m \in [0, S_m], \forall n \in N.
\]

\[
\text{(APO)} \max_{x_n} U^\text{AP}_n(x_n; z_n(x_n)), \text{ s.t. } \sum_{m \in M} x_m^\circ \leq B_n.
\]

Let \( x_{m}^\dagger \triangleq (x_{m1}^\dagger, ..., x_{mN}^\dagger) \) denote the optimal individual decision of BS \( m \) (i.e., the solution of BSO), and \( x_{n}^\dagger \triangleq (x_{n1}^\dagger, ..., x_{nM}^\dagger) \) denote the optimal individual decision of AP \( n \) (i.e., the solution of APO). Intuitively, we can also view \( x_m^\dagger \) as the demand of BS \( m \) for AP \( n \)’s resource, and \( x_m^\circ \) as the supply of AP \( n \)’s resource intended for BS \( m \).

Definition 3 (Market Balance - MB). The offloading market is considered balanced when \( x_m^\dagger = x_m^\circ, \forall m \in M, n \in N \).

By interpreting the Lagrange multipliers as shadow prices, we define the following market clearing prices:

\[
p_n^\circ \triangleq \text{ML}_n^\text{AP}(x_n^\circ) + \eta_n^\circ, \quad \forall n \in N,
\]

each associated with one AP. It is easy to show that under the above market clearing prices, we have: \( x_m^\dagger = x_m^\circ \) for all \( m \in M \) and \( n \in N \). Therefore:

Observation 1 (Market Balance). The market clearing prices (12) lead to the market balance. In addition, under the market balance, the social welfare is maximized.

IV. Game Theoretic Analysis in the Free Market

In this section, we consider a free market, where there is no central controller, and both traffic offloading volumes and pricing rules are determined freely by BSs and APs. In this case, some players (e.g., BSs in our analysis) are given the authority to set market prices, and thus become price-setters. This is the so-called price participation. Naturally, price competition may occur between the price-setters. We are interested in addressing the following problems: what is the impact of price participation and price competition on the market outcome and on the social welfare achieved.

A. Data Offloading Game Formulation

We formulate the interactions between BSs and APs in the free market as a two-stage non-cooperative game, called data offloading game (DOFF). In the first stage (Stage-I), every BS proposes a price to every AP within its coverage area. In the second stage (Stage-II), every AP indicates the traffic volume it will be interested in offloading for every BS who has proposed to it. In the context of multi-stage game, we refer to BSs as leaders, and APs as followers. Obviously, under such a game situation, BSs are price-setters, while APs are still price-takers.11

Let \( N_m \) denote the set of APs within the coverage area of BS \( m \), and \( M_n \) denote the set of BSs whose coverage areas contain AP \( n \). In the example of Figure 1, \( N_1 = \{ A_1, B_1, A_2, A_3 \} \), and \( M_6 = \{ B_1, B_2, B_3 \} \). The strategy of BS \( m \) is the price profile \( p_m = (p_m)_{n \in N_m}, \) where \( p_m \) is the price BS \( m \) proposes to AP \( n \); and the strategy of AP \( n \) is the traffic profile \( x_n = (x_{m})_{m \in M_n}, \) where \( x_{m} \) is the traffic offloading volume AP \( n \) responds to BS \( m \).

Definition 4 (Subgame Perfect Equilibrium - SPE). A strategy profile \( \{ (p_m)_{m \in M}, (x_n)_{n \in N} \} \) is a subgame perfect equilibrium if it represents a Nash equilibrium at each stage, i.e.,

\[
\begin{align*}
I: & \quad p_m = \arg \max_{p_m} U^\text{BS}_m(x_m^\star(p_m); p_m; x_n \circ(p_m)), \\
II: & \quad x_n = x_n^\star(p_n) = \arg \max_{x_n} U^\text{AP}_n(x_n; p_n \circ x_n),
\end{align*}
\]

for all BSs \( m \in M \) at Stage-I and APs \( n \in N \) at Stage-II.12

B. An Illustrative Model with One BS and One AP

We first look at an illustrative model with one BS and one AP. In this case, the DOFF game degenerates to a conventional Stackelberg game. We use this simple model to illustrate (i) the best individual decisions of BSs and ASs, and (ii) the impact of price participation on the market outcome.

For notational consistency, we denote the BS as \( M = \{ m \} \) and \( N = \{ n \} \). We solve the SPE of this game by the backward induction method.

1) Stage-II: Given the BS’s price \( p_m \), the AP’s optimization problem (APO) and the corresponding solution are

\[
\text{(APO)} \max_{x_m} U^\text{AP}_n(x_m; p_m, x_m), \text{ s.t. } x_m \in [0, B_n].
\]

Lemma 2 (AP’s Optimal Decision).

\[
x_m^\star = \begin{cases} 
0 & \text{if } p_m < \hat{c}_n \\
B_n - F^{-1}(1) \left( \frac{w_n - c_n}{w_n - c_n} \right) & \text{if } p_m \in [\hat{c}_n, w_n] \\
B_n & \text{if } p_m > w_n
\end{cases}
\]

where \( \hat{c}_n = c_n + \frac{c_n - w_n}{1 - F^{-1}(1)} \).

11This is due to the two-stage modeling assumption (i.e., BSs and APs are not making decisions simultaneously). Note that for a data offloading market, the two-stage assumption is more reasonable than assuming that BSs and APs make decisions at the same time. The key reason is that BSs usually have more market power, and thus have the priority to determine market prices.

12Here \( P_{-m} \triangleq (p_1, ..., p_{m-1}, p_{m+1}, ..., p_N) \) and \( p \odot x \) represents the inner product of two vectors \( p \) and \( x \). Recall that (i) \( p_m = (p_m, ..., p_M) \), and \( x_m = (x_{m1}, ..., x_{mN}) \) are all prices and traffic offloading volumes related to BS \( m \), and (ii) \( p_m = (p_m, ..., p_M) \), and \( x_m = (x_{m1}, ..., x_{mN}) \) are all prices and traffic offloading volumes related to AP \( n \). By (I), every \( x_{m}^\star \) is a function of \( \mathbf{p}^\star \). Thus, \( x_{m}^\star \) is a function of \( \mathbf{p}^\star = (p_n)_{n \in N}, \) which can also be written as \( \mathbf{p}^\star = (p_n)_{m \in M} = (p_{-m}, p_m). \)
1) Stage-II: First, we study every AP's optimal decision $x_n^*$ when receiving multiple price proposals from BSs. Let $p_n \triangleq (p_{mn})_{m \in M_n}$ denote the prices AP $n$ receives. The optimization problem (APO) for AP $n$ is

$$\text{(APO)} \quad \max_{x_n \in \mathcal{X}_n} U_n^B(x_n; p_n \odot x_n) \quad \text{s.t.} \quad (i) \quad x_{mn} \geq 0, \forall m \in M_n; \quad (ii) \quad \sum_{m \in M_n} x_{mn} \leq B_n. \quad (17)$$

Note that (17) is a direct extension of the APO in Section III-B. Next we provide some additional properties for the solution of (17). Denote $p_n^+ \triangleq \max_{m \in M_n} p_{mn}$ as the highest price of all BSs, $M_n^+ \triangleq \{m \in M_n \mid p_{mn} = p_n^+\}$ as the set of BSs proposing the highest price, and $X_n^+ \triangleq \sum_{m \in M_n^+} x_{mn}$ as the total resource for those BSs proposing the highest price.

**Lemma 4.** The optimal solution $x_n^*$ of problem (17) satisfies:

(a) $x_{mn}^* = 0, \forall m \notin M_n^+$;

(b) $X_n^* \text{ is given by (13) by replacing } p_{mn} \text{ as } p_n^+.$

Lemma 4 shows that only the highest price is accepted by the AP. We further notice that any feasible division of $X_n^*$ among BSs $M_n^+$ leads to the same payoff for the AP. That is, the allocation of $X_n^*$ is irrelevant to the AP's payoff. This implies that the conditions in Lemma 4 are also sufficient.

Although the AP $n$'s benefit does not depend on the detailed allocation of $X_n^*$ (among $M_n^+$), an explicit allocation mechanism is still necessary for analyzing the BSs' best decisions. We adopt the following proportional allocation:

$$\begin{align*}
x_{mn}^* &\triangleq \psi_{mn} \cdot X_n^*, \quad \forall m \in M_n^+, \quad (18) \\
\psi_{mn} &\triangleq \frac{d_{mn}}{\sum_{k \in M_n^+} d_{mn}}, \quad \text{and } d_{mn} \text{ is the BS } m \text{'s traffic offloading demand (to AP } n \text{) under the price } p_n^+. \quad (18)
\end{align*}$$

where $\psi_{mn}$ is the traffic portion BS $m$ is entitled to, and $d_{mn}$ is the BS $m$'s traffic offloading demand to BS $n$. We further show that problem (14) is a convex optimization, and thus can be solved by KKT analysis. Formally,

**Lemma 3 (BS's Optimal Decision).** The BS $m$'s optimal price $p_{mn}^*$ satisfies the following conditions:

(D.1) $\theta_{mn}^{-1} \cdot MC_m(x_m^*) - MC_m^B(p_{mn}) - \mu_{mn}^* + \lambda_{mn}^* = 0,$

(D.2) $\mu_{mn}^* \cdot (S_m - x_m^*) = 0, \quad \lambda_{mn}^* \cdot x_m^* = 0,$

where $\mu_{mn}^*$ and $\lambda_{mn}^*$ are the optimal Lagrange multipliers.

3) Observation: Figure 2 illustrates the SPE and MB, from which we can see: $x_m^* \leq x_m^0$, and $p_{mn}^* \leq p_{mn}^0$. Therefore, we have

**Observation 2.** The price participation (of BSs) drives the market price down from the market clearing price (as shown by the SPE and MB in Figure 2).

C. A General Model with Multiple BSs and Multiple APs

We now return to the general model with multiple BSs and multiple APs. Similarly, we apply the backward induction.

Intuitively, if the BS proposes a large enough price $p_{mn}$ (i.e., $p_{mn} > w_n$), AP $n$ will sell all of its resource to the BS. On the other hand, if the price $p_{mn}$ is smaller than a threshold $\tilde{c}_n$, AP $n$ will sell none of its resource to the BS. In this sense, we refer to $\tilde{c}_n$ as the critical price of AP $n$.

2) Stage-I: Knowing the AP’s response, the BS’s optimization problem (BSO) in Stage-I is

$$\text{(BSO)} \quad \max_{p_{mn}} U_m^B(x_m(p_{mn}); p_{mn}) \quad \text{s.t.} \quad 0 \leq x_m(p_{mn}) \leq S_m. \quad (14)$$

For convenience, we introduce the following concept.

**Definition 5 (Marginal Payment of BS to AP).** The marginal payment $MP_m(p_{mn})$ represents the increase of BS’s total payment (not unit price) in order to have AP $n$ offload one additional unit of traffic.$^{13}$

Then, the first-order derivative of $U_m^B(\cdot; \cdot)$ can be written as

$$\frac{\partial U_m^B}{\partial p_{mn}} = \left(\theta_{mn}^{-1} \cdot MC_m(x_m^*) - MP_m(p_{mn})\right) \cdot \frac{\partial x_m^*}{\partial p_{mn}}. \quad (16)$$

We can further show that problem (14) is a convex optimization, and thus can be solved by KKT analysis. Formally,

**Lemma 4.** The optimal solution $x_m^*$ of problem (17) satisfies:

(a) $x_m^* = 0, \forall m \notin M_n^+$;

(b) $x_m^*$ is given by (13) by replacing $p_{mn}$ as $p_n^+$.

Lemma 4 shows that only the highest price is accepted by the AP. We further notice that any feasible division of $X_n^*$ among BSs $M_n^+$ leads to the same payoff for the AP. That is, the allocation of $X_n^*$ is irrelevant to the AP's payoff. This implies that the conditions in Lemma 4 are also sufficient.

Although the AP $n$'s benefit does not depend on the detailed allocation of $X_n^*$ (among $M_n^+$), an explicit allocation mechanism is still necessary for analyzing the BSs' best decisions. We adopt the following proportional allocation:

$$\begin{align*}
x_{mn}^* &\triangleq \psi_{mn} \cdot X_n^*, \quad \forall m \in M_n^+, \quad (18) \\
\psi_{mn} &\triangleq \frac{d_{mn}}{\sum_{k \in M_n^+} d_{mn}}, \quad \text{and } d_{mn} \text{ is the BS } m \text{'s traffic offloading demand (to AP } n \text{) under the price } p_n^+. \quad (18)
\end{align*}$$

where $\psi_{mn}$ is the traffic portion BS $m$ is entitled to, and $d_{mn}$ is the BS $m$'s traffic offloading demand to BS $n$. We further show that problem (14) is a convex optimization, and thus can be solved by KKT analysis. Formally,

**Lemma 3 (BS’s Optimal Decision).** The BS $m$’s optimal price $p_{mn}^*$ satisfies the following conditions:

(D.1) $\theta_{mn}^{-1} \cdot MC_m(x_m^*) - MP_m(p_{mn}) - \mu_{mn}^* + \lambda_{mn}^* = 0,$

(D.2) $\mu_{mn}^* \cdot (S_m - x_m^*) = 0, \quad \lambda_{mn}^* \cdot x_m^* = 0,$

where $\mu_{mn}^*$ and $\lambda_{mn}^*$ are the optimal Lagrange multipliers.

3) Observation: Figure 2 illustrates the SPE and MB, from which we can see: $x_m^* \leq x_m^0$, and $p_{mn}^* \leq p_{mn}^0$. Therefore, we have

**Observation 2.** The price participation (of BSs) drives the market price down from the market clearing price (as shown by the SPE and MB in Figure 2).

C. A General Model with Multiple BSs and Multiple APs

We now return to the general model with multiple BSs and multiple APs. Similarly, we apply the backward induction.

$^{13}$To achieve this, the BS needs to increase the price by $\Delta p_{mn} = \frac{\partial p_{mn}}{\partial x_{mn}} \cdot \Delta x_{mn}$. This leads to an additional payment that consists of two parts: (i) $p_{mn} + \Delta p_{mn}$ by offloading an additional unit of traffic at the new price $p_{mn} + \Delta p_{mn}$, and (ii) $x_{mn} \cdot \Delta p_{mn}$ for the existing $x_{mn}$ units of offloading traffic.

1) Stage-II: First, we study every AP $n$’s optimal decision $x_n^*$ when receiving multiple price proposals from BSs. Let $p_n \triangleq (p_{mn})_{m \in M_n}$ denote the prices AP $n$ receives. The optimization problem (APO) for AP $n$ is

$$\text{(APO)} \quad \max_{x_n \in \mathcal{X}_n} U_n^B(x_n; p_n \odot x_n) \quad \text{s.t.} \quad (i) \quad x_{mn} \geq 0, \forall m \in M_n; \quad (ii) \quad \sum_{m \in M_n} x_{mn} \leq B_n. \quad (17)$$

Note that (17) is a direct extension of the APO in Section III-B. Next we provide some additional properties for the solution of (17). Denote $p_n^+ \triangleq \max_{m \in M_n} p_{mn}$ as the highest price of all BSs, $M_n^+ \triangleq \{m \in M_n \mid p_{mn} = p_n^+\}$ as the set of BSs proposing the highest price, and $X_n^+ \triangleq \sum_{m \in M_n^+} x_{mn}$ as the total resource for those BSs proposing the highest price.

**Lemma 4.** The optimal solution $x_m^*$ of problem (17) satisfies:

(a) $x_{mn}^* = 0, \forall m \notin M_n^+$;

(b) $X_n^*$ is given by (13) by replacing $p_{mn}$ as $p_n^+.$

Lemma 4 shows that only the highest price is accepted by the AP. We further notice that any feasible division of $X_n^*$ among BSs $M_n^+$ leads to the same payoff for the AP. That is, the allocation of $X_n^*$ is irrelevant to the AP's payoff. This implies that the conditions in Lemma 4 are also sufficient.
The increasing traffic demand and the network operators’ growing needs for reducing their capital and operational expenditures (CAPEX and OPEX) are expected to boost third-party offloading solutions. Studying the economic interactions of the involved entities is an important step towards the realization of this promising solution. In this paper, we study the economics of mobile data offloading through third-party WiFi or femtocell APs. We propose a market-based offloading scenario, and study the market outcome by using the non-cooperative game theory. We further study the impact of price participation and competition on the market outcome. Our analysis sheds light to many different aspects of this interesting problem. There are many directions for future work in this area. For example, it would be useful to study the price of anarchy of the resulting SPE. Another interesting direction is to study the offloading market under incomplete information.

REFERENCES