

Combining Spot and Futures Markets: A Hybrid Market Approach to Dynamic Spectrum Access

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Dynamic spectrum access (DSA) is a new paradigm of secondary spectrum utilization and sharing. It allows unlicensed secondary users (SUs) to exploit the under-utilized radio spectrum licensed to primary users opportunistically. Market mechanism is a widely-used promising means to regulate the consuming behaviours of users and, hence, achieve the efficient allocation and consumption of limited resources. In this paper, we propose and study a *hybrid* secondary spectrum market consisting of both the *futures market* and the *spot market*, in which SUs (buyers) purchase under-utilized licensed spectrum from a primary spectrum regulator (PR), either through predefined contracts via the futures market, or through spot transactions via the spot market. We focus on the optimal spectrum allocation among SUs in an exogenous hybrid market that maximizes the secondary spectrum utilization efficiency. The problem is challenging due to the stochasticity and asymmetry of network information. To solve this problem, we first derive an off-line optimal allocation policy that maximizes the ex-ante expected spectrum utilization efficiency based on the stochastic distribution of network information. We then propose an on-line VCG auction that determines the real-time allocation and pricing of every spectrum based on the realized network information and the pre-derived off-line policy. We further show that with the spatial frequency reuse, the proposed VCG auction is NP-hard; hence, it is not suitable for on-line implementation, especially in a large-scale market. To this end, we propose a heuristics approach based on an on-line VCG-like mechanism with polynomial-time complexity, and further characterize the corresponding performance loss bound analytically. We finally provide extensive numerical results to evaluate the performance of the proposed solutions.

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1. Introduction

1.1. Background and Motivations

Radio electromagnetic spectrum is a limited resource for wireless communications, and is becoming more congested and scarce with the explosive development of wireless services and networks. Nonetheless, extensive studies (e.g., FCC (2002), McHenry et al. (2006)) have shown that many spectrum bands are heavily *under-utilized* due to the significant variations in spectrum occupying time and space. This suggests that the traditional static licensing approach, i.e., granting exclusive spectrum usage right to a single commercial or government department over a long time and in a large area, is no longer efficient or suitable for future wireless networks. Dynamic spectrum access (DSA) has been recently recognized as a novel and promising approach to increase spectrum utilization efficiency and alleviate spectrum scarcity. The key idea is to enable unlicensed secondary users (SUs) to exploit the under-utilized radio spectrum licensed to primary users (PUs) in an opportunistic manner (see Haykin (2005), Akyildiz et al. (2006), Buddhikot (2007), Zhao & Sadler (2007)).¹ The long-term success of DSA requires many innovations in technology, economics, and policy. In particular, it is essential to design a secondary spectrum sharing mechanism that achieves a high secondary spectrum utilization efficiency, and meanwhile offers necessary incentives for PUs to open their licensed spectrum for secondary sharing and unlicensed access.²

Market mechanism is a widely-used promising means to regulate and coordinate the consuming behaviours of users and, hence, achieve the efficient allocation and consumption of limited resource. Therefore, market-driven secondary spectrum trading is a natural choice to achieve the efficient spectrum utilization, and also a promising approach to address the incentive issue (see Hossain et al. (2009), Tonmukayakul & Weiss (2009)). With secondary spectrum trading, SUs temporarily *purchase* the under-utilized licensed spectrum from a primary spectrum regulator (PR), such as FCC in the U.S. and Ofcom in the UK, before using these licensed spectrum.³ The secondary spectrum market differs from conventional markets (e.g., paintings, bonds, and electricity markets) in the following two aspects. First, *radio spectrum is spatially reusable*. That is, a spectrum can be potentially used by multiple SUs simultaneously, as long as these SUs do not generate mutual interferences (see Section 3.4 for details). Second, *the availability of spectrum is not deterministic*. To protect the subscribed PUs' benefits, a spectrum is only available to SUs when it is not occupied by any PU (i.e., when it is *idle*). Due to the stochastic nature of PUs' activities, the spectrum availability is often a random variable (see Section 3.1 for details). Based on the above two facts,

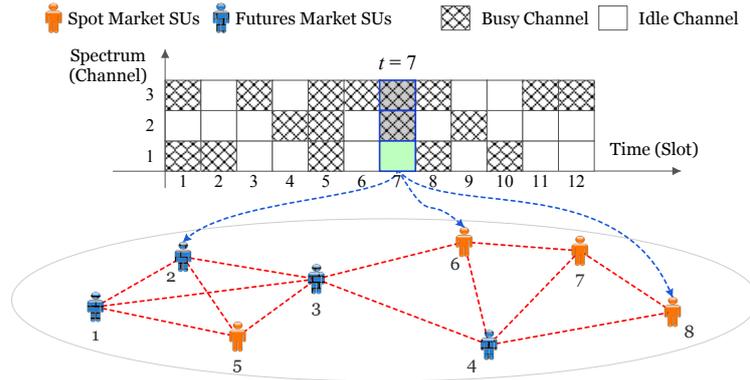


Figure 1 Illustration of a time-slotted secondary spectrum market. The SUs not connected by an edge are interference-free, and can use the same (idle) spectrum at the same time without mutual interference. For example, the idle (green) spectrum at time $t = 7$ is allocated to SUs $\{2, 6, 8\}$ simultaneously.

the mechanism design in such a secondary spectrum market needs to overcome a different set of challenges than in a conventional market.

1.2. Market Model and Problem Formulation

In this paper, we consider the secondary spectrum trading between one PR (seller) and multiple SUs (buyers) for all idle spectrum in a given time period which consists of multiple time slots.⁴ Moreover, we consider the *short-term* spectrum trading, i.e., the idle spectrum is traded on a slot-by-slot basis. Fig. 1 illustrates such a time-slotted secondary spectrum market within a period of 12 time slots. The blank and shadowed squares denote the idle spectrum and busy spectrum in each time slot, respectively. The edge connecting two SUs denotes the interference relationship between them. In other words, any two SUs without a connecting edge (e.g., SUs $\{4, 5\}$ or SUs $\{6, 8\}$) are interference-free (also called *independent*), and thus can use the same (idle) spectrum concurrently without any mutual interference. In this example, the idle spectrum at time $t = 7$ is allocated to SUs $\{2, 6, 8\}$ simultaneously, as these SUs are independent of each other.

More specifically, we consider a *hybrid* secondary spectrum market consisting of both the *futures market* and the *spot market*. In such a market, SUs can purchase spectrum either through predefined futures contracts via the futures market, or through spot transactions via the spot market. In the futures market, each SU reaches an agreement, called a futures contract or guaranteed contract, directly with the PR. The contract specifies not only the SU's total demand and payment in the given period, but also the PR's penalty if violating the contract.⁵ In the spot market, SUs buy spectrum in a real-time and on-demand manner through spot transactions, and multiple SUs may compete with each other for limited spectrum resource (e.g., through an auction). In the example of Fig. 1, we denote the contract users (blue) by 1, 2, 3, and 4, each requesting a pre-specific number of spectrum. Likewise, we denote the spot market users (red) by 5, 6, 7, and 8,

each requesting and competing for spectrum in an on-demand manner. It is easy to see that the futures market insures buyers (sellers) against uncertainties of future supply (demand) through predefined contracts, while the spot market allows buyers to compete for limited resource based on their real-time demands and preferences.⁶ Therefore, such a hybrid market has both the reliability of futures market and the flexibility of spot market; hence, it is highly desirable for Quality of Service (QoS) differentiations in secondary spectrum utilization. Specifically, an SU with elastic traffic (e.g., file transferring or FTP downloading) may be more interested in spot transactions to achieve a flexible resource-price tradeoff; whereas, an SU with inelastic traffic requiring specific data rates (e.g., Netflix video streaming or VoIP) may prefer the certainty of contracts in the futures market.

We consider a quasi-static market scenario, where both the network topology and the hybrid market structure are exogenously given and fixed in each time period, but may change across different time periods.⁷ In particular, we focus on the optimal spectrum allocation in a particular time period with a particular *exogenous* and fixed hybrid market, i.e., with given contract users (in the futures market) and given spot transaction users (in the spot market).⁸ Namely, we want to answer the following question: how to optimally allocate spectrum among the given contract users and spot market users to maximize the secondary spectrum utilization efficiency in that time period. Here, the secondary spectrum utilization efficiency (also called *spectrum efficiency*) is defined as the total social welfare generated by all SUs from utilizing idle spectrum in the whole time period (see Section 3.5 for the detailed definition).

This problem is challenging due to the *stochasticity* and *asymmetry* of network information. Here, network information mainly refers to the benefit (utility) of each SU from using each (idle) spectrum in each time slot. Stochasticity means that no one can observe the network information in the future, as the network changes randomly over time. Asymmetry means that the currently realized information of an SU is the private information, and cannot be observed by other entities, such as the PR and other SUs. Nevertheless, in the hybrid spectrum market, the allocation of spectrum in the whole time period must be jointly optimized due to the time-coupling constraint on the contract user demand (e.g., each contract user requires a specific number of spectrum in the whole period). This implies that solving the optimal spectrum allocation problem directly would require the complete network information in all time slots. As mentioned above, however, in practice the PR only has partial knowledge (e.g., the stochastic distribution) about the future network information due to the stochasticity of information. Moreover, it cannot observe all of the realized current network information, especially the private information of SUs, due to the asymmetry of information. Therefore, the key research problem becomes the following:

Table 1 Key results in this paper

Problem	Solution	Performance	Complexity	Section No.
E-SEM: Expected Spectrum Efficiency Maximization	Policy + VCG	Optimal	Exponential	5, 6
	Policy + VCG-like	Sub-optimal	Polynomial	5, 6, 7

PROBLEM 1. How should the PR optimally allocate the idle spectrum in the given period among the contract users and spot market users to maximize the spectrum efficiency, taking into consideration the spatial spectrum reuse, information stochasticity, and information asymmetry?

1.3. Solutions and Contributions

To solve this problem, we first derive an *off-line optimal policy* that maximizes the ex-ante expected spectrum efficiency based on the stochastic distribution of network information. We then design an *on-line VCG auction* that elicits SUs' private information realized in every time slot. Based on the elicited network information and the derived off-line policy, the VCG auction determines the real-time allocation and pricing of every spectrum. Such a solution technique (i.e., off-line policy and on-line auction) allows us to optimally allocate every spectrum in an on-line manner under stochastic and asymmetric information. We further show that with the spatial spectrum reuse, the proposed VCG auction relies on solving the maximum weight independent set (MWIS) problem, which is well-known to be NP-hard. Thus, it is not suitable for on-line implementation, especially in a large-scale market. This motivates us to further study low-complexity sub-optimal solutions. To this end, we propose a heuristics approach based on an on-line VCG-like mechanism with polynomial-time complexity, and further characterize the corresponding performance loss bound analytically. Our numerical results indicate that the heuristics approach exhibits good and robust performance (e.g., reaches at least 70% of the optimal efficiency in our simulations).

In summary, we list the key results and the corresponding section numbers in Table 1. It is important to note that the main contribution of this work is not the development of new auction theory, but rather the formulation of the hybrid spectrum market and the solution techniques (including the use of auction theory) to optimize the spectrum utilization in a given hybrid market. Specifically, the main contributions of this paper are as follows:

- *New modeling and solution technique:* We propose and study a hybrid spectrum market, which has both the reliability of futures market and the flexibility of spot market. Hence, it is highly desirable for QoS differentiations in secondary spectrum utilization. To the best of our knowledge, this is the first paper to study such a hybrid spectrum market with spatial spectrum reuse.
- *Optimal solution under stochastic and asymmetric information:* We analyze the optimal spectrum allocation in an exogenous hybrid market (in a particular time period) under stochastic and asymmetric information systematically. Our proposed solution consists of two parts: (i) an off-line

allocation policy that maximizes the ex-ante expected spectrum efficiency based on the stochastic network information; and (ii) an on-line VCG auction that determines the real-time allocation of every (idle) spectrum based on the realized network information and the pre-derived policy. Such a solution technique allows us to optimally allocate every spectrum in an on-line manner.

- *Heuristic solution with polynomial-time complexity:* We propose a heuristic approach based on an on-line VCG-like mechanism with polynomial-time complexity, and further characterize the corresponding performance loss bound analytically. This polynomial-time solution is particularly useful for achieving the efficient spectrum utilization in a large-scale network.

- *Performance evaluation:* We provide extensive numerical results to evaluate the performance of the proposed solutions. Our numerical results show that: (i) the proposed optimal allocation significantly outperforms the traditional greedy allocations, e.g., with an average increase of 20% in terms of the expected spectrum efficiency; and (ii) the proposed heuristics approach exhibits good and robust performance, e.g., reaching at least 70% of the optimal efficiency in our simulations.

The rest of this paper is organized as follows. After reviewing the literature in Section 2, we describe the system model in Section 3, and present the problem formulation in Section 4. Then we derive the off-line optimal policy in Section 5, and design the on-line VCG mechanisms in Section 6. In Section 7, we analyze the performance loss in the low-complexity heuristic solution. In Section 8, we provide the detailed simulation results. We finally conclude in Section 9.

2. Literature Review

2.1. Secondary Spectrum Trading for DSA

A major motivation of this work is to establish economic incentives and improve spectrum utilization efficiency in dynamic spectrum access (DSA) and cognitive radio networks (CRNs). There are several comprehensive surveys on the technical aspects of DSA and CRNs (see Haykin (2005), Akyildiz et al. (2006), Buddhikot (2007), Zhao & Sadler (2007)).

2.1.1. Secondary Spectrum Trading in Pure Spectrum Markets. Recent years have witnessed a growing body of literature on the economic analysis (in particular the incentive issues) of DSA (see, e.g., Hossain et al. (2009), Tonmukayakul & Weiss (2009)). Market-driven *secondary spectrum trading* is a promising paradigm to address the incentive issue and achieve high spectrum efficiency in DSA. The literature on secondary spectrum trading often considers *Pricing*, *Contract*, and *Auction*. Pricing and Contract are generally adopted by the *futures market*, wherein sellers and buyers enter into certain agreements, specifying the price, quality and demand in advance. In particular, Pricing is often used under information symmetry, where sellers and buyers possess the same market information (e.g., utility and quality) or hold the same degree of uncertainty about the

market information (see, e.g., Kloeck et al. (2005), Wang et al. (2008), Niyato & Hossain (2008)). Contract has been widely used in supply chain models, with a main focus on the supply chain coordination (see de Kok & Graves (2003) for a comprehensive survey). Contract is effective in a market under information asymmetry, where sellers and buyers possess different information. The seller (or buyer) can offer a selling (purchasing) contract to extract the buyer surplus (seller surplus) as much as possible (see, e.g., Gao et al. (2010), Duan et al. (2011), Kalathil & Jain (2010)). In contrast to Pricing and Contracts, Auctions are one of the most common mechanisms used by the *spot market*, wherein buyers compete to obtain commodities by offering different bids. Auctions are particularly suitable for the information asymmetry scenario. With a well-designed auction, buyers (bidders) have the incentive to bid for the resource in a truthful manner, and the seller can allocate the resource efficiently without knowing the buyers' private information in advance (see, e.g., Huang et al. (2006), Li et al. (2008), Gandhi et al. (2007), Wang et al. (2010), Zhou et al. (2008), Zhou & Zheng (2009)). However, the above work considers the secondary spectrum trading in a pure spectrum market (i.e., either spot or futures market). Instead, our setup incorporates the short-term secondary spectrum trading in a *hybrid* spectrum market.

2.1.2. Secondary Spectrum Trading in Hybrid Spectrum Markets Kasbekar et al. (2010) and Muthusamy et al. (2011) considered the secondary spectrum trading in a hybrid market. In their settings, primary sellers offer two types of contracts: the guaranteed-bandwidth contract and the opportunistic-access contract. The main difference between these two prior papers and our paper lies in the formulation of the guaranteed contract. Specifically, in Kasbekar et al. (2010) and Muthusamy et al. (2011), the guaranteed-bandwidth contract provides guaranteed access to a certain amount of bandwidth at *every* time slot. In our model, the guaranteed-delivery contract provides guaranteed access to a total amount of bandwidth in one time period; nevertheless, the bandwidth delivery at every time slot can be different, depending on the PUs' own demand. The main advantage of our approach is its flexibility in shifting secondary demand across time slots (to comply with the PUs' random demand). That is, it enables opportunistic delivery of a small (or large) bandwidth to SUs in those time slots that the PUs' own demand is high (or low). Additionally, our model is also more practically relevant to a wide range of applications, which do not require fixed data delivery per time slot, but demand a guaranteed average data rate over each time period. Furthermore, the underlying market models are also different. Kasbekar et al. (2010) and Muthusamy et al. (2011) assumed that the demand (supply) markets have infinite liquidity. That is, any bandwidth amount supplied by the seller can be sold out (any bandwidth amount demanded by the buyer can be bought from the market) at an "outside fixed price". In this sense, their market models are closely related to the ideal competitive market. We assume

Table 2 A summary of secondary spectrum trading literature

Market Type	Related Work
Futures Market	Pricing: Kloeck et al. (2005), Wang et al. (2008), Niyato & Hossain (2008). Contract: Gao et al. (2010), Duan et al. (2011), Kalathil & Jain (2010).
Spot Market	Auction: Huang et al. (2006), Gandhi et al. (2007), Li et al. (2008), Wang et al. (2010), Zhou et al. (2008), Zhou & Zheng (2009).
Hybrid Market	Kasbekar et al. (2010), Abhishek et al. (2012), Muthusamy et al. (2011), Gao et al. (2012a)

that the market price is endogenously determined by the associated seller and buyer (through, for example, an VCG mechanism). Thus, we essentially consider the monopoly market.

Abhishek et al. (2012) also considered a hybrid market, in which a cloud service provider sells its service to users via two different pricing schemes: pay-as-you-go (PAYG) and spot pricing. Under the PAYG, users are charged a fixed price per unit time. Under the spot pricing, users compete for services via using an auction. They focused on the optimal market formulation, that is, the service provider selects different PAYG prices such that different users will choose different pricing schemes or market types; consequently, different hybrid markets will be formulated. In our work, we focused on the optimal spectrum allocation in a *given* hybrid market. In other words, each SU is associated and fixed in a particular market (based on his or her application type), and the PR determines the optimal spectrum allocation among the given contract users and spot market users.

In Gao et al. (2012a), we studied the secondary spectrum trading in the same hybrid market. However, Gao et al. (2012a) did not consider spatial spectrum reuse, which is the key contribution of this paper. With spatial spectrum reuse, the same spectrum can be potentially used by multiple SUs simultaneously; thus, the total spectrum efficiency can be greatly improved. This new coupling in the spatial dimension creates many challenges (e.g., solving MWIS problems) in the optimal mechanism design, and makes the problem significantly different from those without spatial spectrum reuse. Precise mathematical modeling and understanding of the spatial coupling are often lacking in the wireless literature. One of the main contributions of this paper is to propose a low-complexity heuristic algorithm to tackle this issue, and to quantify the performance bound of the proposed algorithm. For convenience, we summarize the key literature in Table 2.

2.2. Dual Sourcing in Supply Chain Management

The theoretic model used in our work is related to the *dual sourcing* problem in supply chain management. Specifically, with dual sourcing, a firm can procure a single commodity from a supplier via a long-term contract and/or from a spot market via short-term purchases (see Kleindorfer & Wu (2003)). In such a context, the long-term contract and short-term purchase in the supply chain model corresponds to the guaranteed contract and spot transaction in our model, respectively.

Note that the firm in the supply chain model buys commodities through dual sourcing, while the PR in our model sells commodities through guaranteed contracts and spot transactions. The long-term/guaranteed contract ensures consistent over time availability of commodities with guaranteed quality at a predetermined price; whereas, the short-term purchase/spot transaction provides high inventory flexibility, allowing firms to buy and sell commodities at any quantity with zero lead time, but at a random market price. The main advantage of dual sourcing is to hedge the future uncertainties in the supplier's commodity supply and the end users' demand, so as to hedge the financial risk in the supply chain (see Kleindorfer (2008a,b)).

The integration of long-term contracts and short-term purchases is of particular interest to the firm in a supply chain, due to the following two reasons: (i) sourcing competition can keep purchasing prices under control by the firm; and (ii) a wider supply base can mitigate the risk induced by the uncertainty at one supplier. Lee & Whang (2002) were the first to integrate, after sales, the spot market considerations within a newsvendor ordering framework. Peleg et al. (2002) studied the long-term and short-term integrated sourcing using a stylized two period model. Yi & Scheller-Wolf (2003) studied the nature of the optimal inventory policy when such a dual sourcing is used, in the presence of a fixed cost for the spot market participation. Wu & Kleindorfer (2005) proposed a general framework with integrated long-term and short-term contract decisions for non-storable commodities. Kouvelis et al. (2009) studied the problem of dual sourcing with financial hedging for storable commodities.

Our work differs from the above work in the following aspects. First, the spectrum in our model can be potentially used by multiple SUs simultaneously, while a traditional commodity can usually be used by one user only. Second, the availability of spectrum is stochastic, while the availability of a traditional commodity is usually deterministic. Third, we focus on the spectrum utilization efficiency maximization, rather than the financial risk hedging. Fourth, instead of designing the optimal contracts, we treat contracts as exogenously given, and focus on the problem of how to fulfill these contracts and cope with additional demand from spot markets. Finally, we consider stochastic and asymmetric network information.

3. System Model

3.1. System Description

We consider a dynamic spectrum access (DSA) system with one primary spectrum regulator (PR) and multiple secondary users (SUs). The PR has the authority to grant the secondary utilization of certain licensed frequency band, which is divided into K orthogonal *channels* using channelization methods, such as frequency division and code division. In Fig. 1, for example, we have $K = 3$. The frequency band is licensed to a set of subscribed licensed holders (also called primary users,

PUs), who access channels with the slotted transmission protocol which is widely-used in today’s wireless communication systems (e.g., GSM, WCDMA, and LTE). That is, the total time is divided into fixed-time intervals, called *time slots*, and each PU transmits over one or multiple channel(s) according to a synchronous time slot structure. Depending on the activities of PUs, some channels may be not used by any PU (i.e., idle) in some time slots, which can be potentially assigned for the secondary utilization of SUs.

We consider the *market-driven* dynamic spectrum access, also called secondary spectrum trading, in which SUs temporarily purchase the idle channels from the PR. More specifically, we consider the *short-term* secondary spectrum trading, in which the idle channel is traded on a slot-by-slot basis. Namely, the basic resource unit for trading is “a particular channel at a particular time slot”, referred to as a spectrum opportunity or *spectrum*. The main motivation for considering the *short-term* spectrum trading is as follows. The spectrum availability changes frequently and randomly over time due to the stochasticity of PUs’ activities; thus, a channel that is idle in a particular time slot may not always be idle in the future.

Let CH_{kt} denote the spectrum on the k -th channel at time slot t . Let $\xi_{kt} \in \{0, 1\}$ denote the state (availability) of spectrum CH_{kt} , with $\xi_{kt} = 1$ indicating that the spectrum is not used by any PU and thus is available for SUs, and $\xi_{kt} = 0$ otherwise. We consider the operations in a given period of T time slots. The total amount of spectrums in the period is referred to as the PR’s **spectrum supply**, denoted by $\mathcal{Q} \triangleq \{\text{CH}_{kt}\}_{k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$. The size of total supply is denoted by $S = |\mathcal{Q}| \triangleq K \cdot T$. The states of all spectrums in \mathcal{Q} are referred to as the **spectrum availability**, denoted by $\mathcal{X} \triangleq \{\xi_{kt}\}_{k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$.

Due to the uncertainty of PUs’ activities, the spectrum availability changes randomly across both time and frequency. Fig. 1 illustrates an example of such a system, where $K = 3$, $T = 12$, and $S = 36$. At the time slot $t = 1$, the spectrum CH_{21} is idle, while the spectrum CH_{11} and CH_{31} are busy, i.e., $\xi_{21} = 1$ and $\xi_{11} = \xi_{31} = 0$. Note that in practical wireless communication systems, T is usually very large. This is mainly due to two reasons. First, the length of each time slot in a wireless system is often quite small, e.g., in milliseconds or even in microseconds, which corresponds to the typical length of frame in many wireless communication systems (such as 4G LTE and Wi-Fi). In fact, the physical limit of choosing the time slot length is the so called *coherence time*, which is the time within which the channel condition does not change. In wireless communications, such coherence time is usually very small due to fast small scale multi-path fading. Second, the time scale of each time period is relatively large, e.g., in minutes or even hours, which corresponds to the validity period of contracts (to be defined in Section 3.3). Hence, the number of slots in each time period (i.e., T) is very large, e.g., $T = 6 \cdot 10^5$ when the length of slot is 1 millisecond and the length of period is 10 minutes.

Following are our assumptions on the spectrum availability and spectrum usage.

ASSUMPTION 1. *The spectrum availability is independently and identically distributed (i.i.d.) across time and frequency.⁹ Moreover, the spectrum availability is ergodic.*

ASSUMPTION 2. *Each SU can transmit over multiple channels simultaneously.¹⁰*

For convenience, we denote the availability of spectrum as a random variable $\xi \in \{0, 1\}$, and the probability mass function of ξ as $f_\xi(\cdot)$.¹¹ Suppose ρ is the idle probability of spectrum CH_{kt} . Then, $f_\xi(\xi_{kt}) = \rho$ if $\xi_{kt} = 1$, and $f_\xi(\xi_{kt}) = 1 - \rho$ if $\xi_{kt} = 0$.

3.2. Futures Market and Spot Market

Wireless applications can be broadly categorized as either *elastic* or *inelastic* depending on Quality-of-Service (QoS) requirements.¹² With elastic traffic, SUs have elastic demands for spectrum, in the sense that the tasks are not urgent in terms of time; thus, the QoS will not be significantly affected, even if the spectrum resource is limited and the transmission rate is low for a substantial amount of time. With inelastic traffic, SUs have inelastic demands for spectrum. In other words, the tasks can only function well when the data rate is guaranteed to be above certain thresholds such that the delay requirements are met; otherwise they will suffer significant performance loss. It is important to note that the information of the application type is usually explicitly represented in the headers of data packets, and can be easily extracted by the network operator through deep packet inspection. Hence, it is reasonable to assume that an SU cannot fake his or her application type arbitrarily. Examples of elastic traffic include FTP downloading, data backup, and cloud synchronization, and examples of inelastic traffic include VoIP, video streaming, and real-time data collection.

To accommodate the various requirements of elastic traffic and inelastic traffic (so as to achieve desirable QoS differentiations), we propose a *hybrid spectrum market* combining both the futures market and the spot market. Next we define the futures market and spot market formally.

Futures Market. In the futures market, each SU enters into an agreement, called a futures contract or *guaranteed contract*, directly with the PR. The contract specifies the key elements in trading, e.g., the SU's total demand and payment in the given period. Once the PR accepts a contract, it is committed to deliver the specified number of spectrum to the SU. If the PR fails to do so, it needs to pay certain *penalty* for compensating the SU's potential welfare loss. This penalty can be a unit price paid for every undelivered spectrum (called a soft contract) or simply a total payment for the violation of contract (called a hard contract).

Spot Market. In the spot market, each SU purchases spectrums in a real-time and on-demand manner through spot transactions. That is, an SU initiates a purchasing request only when he or she needs spectrum, and multiple SUs requesting the same spectrum would compete with each other for the spectrum, e.g., through an auction. The spectrum is delivered immediately to the

winner at a real-time market price, which depends on both the SUs' preferences and competitions. The winner's payment is also dependent on the SUs' preferences and competitions.

Although, in practice, it is more desirable to allow SUs to have the flexibility to choose which market to join, in this work we assume that *the SUs with inelastic traffic always choose the futures market, and the SUs with elastic traffic always choose the spot market.*¹³ In other words, we will leave out the SU's market selection problem, and instead focus exclusively on the spectrum allocation in the given hybrid markets in each time period. This setup helps us to concentrate on the technical challenges brought by spatial spectrum reuse and information stochasticity & asymmetry.

Next we provide the key assumptions on the hybrid market structure.

ASSUMPTION 3. *We assume a quasi-static market scenario, where the hybrid spectrum market is exogenously given and fixed in each time period, but may change across different time periods.*¹⁴

ASSUMPTION 4. *We assume that each SU has one application, and all SUs with inelastic traffic (elastic traffic) are in the futures market (spot market).*¹⁵

Based on these assumptions, we can divide SUs into two disjoint sets, each associated with one market. Let $\mathcal{N} \triangleq \{1, 2, \dots, N\}$ and $\mathcal{M} \triangleq \{1, 2, \dots, M\}$ denote the sets of SUs in the futures market and the spot market, respectively. For convenience, we will use the notation “ n ” to denote a contract user, and “ m ” to denote a spot market user. When needed, we will use the superscripts “ c ” and “ s ” to indicate variables related to the futures market and spot market, respectively.

For analytical convenience, we further assume that the size of futures market is much smaller than that of the spot market, i.e., $\mathcal{N} \ll \mathcal{M}$. This assumption is used to facilitate the computation of independent contract user sets in the later analysis.¹⁶

3.3. Futures Contract Definition

A contract could be quite complicated, depending on specific requirements of a contract user's application. In this work, we focus on a basic contract form, which consists of two parts: (i) the SU's demand and payment for spectrum in each period; and (ii) the PR's penalty when not delivering the demanded spectrums to the SU. Formally, we write the contract of an SU $n \in \mathcal{N}$ as:

$$\text{CTR}_n \triangleq \{B_n, D_n, \mathbf{J}_n\}, \quad (1)$$

where B_n is the SU's payment, D_n is the SU's demand in one period, and $\mathbf{J}_n = (J_n, \{\widehat{P}_n, \widehat{B}_n\})$ is the PR's penalty scheme. Here, $J_n \in \{0, 1\}$ indicates the contract type (soft or hard), \widehat{P}_n is the unit penalty for a soft contract ($J_n = 0$), and \widehat{B}_n is the total penalty for a hard contract ($J_n = 1$). Thus,

the **actual payment** of contract user n (denoted by R_n) depends on the number of spectrums he or she actually obtains (denoted by d_n), that is:

$$R_n = B_n - \mathbb{P}(d_n, D_n), \quad (2)$$

where $\mathbb{P}(d_n, D_n)$ is the PR's penalty if violating the contract, given by:

$$\mathbb{P}(d_n, D_n) \triangleq \begin{cases} [D_n - d_n]^+ \cdot \widehat{P}_n, & J_n = 0 \\ \mathbf{1}(D_n - d_n) \cdot \widehat{B}_n, & J_n = 1 \end{cases} \quad (3)$$

where $[x]^+ = \max\{0, x\}$ and $\mathbf{1}(x) = \max\{0, \frac{x}{|x|}\}$. That is, $[x]^+ = \mathbf{1}(x) = 0$ if $x \leq 0$, and $[x]^+ = x$ and $\mathbf{1}(x) = 1$ if $x > 0$. For convenience, we denote the set of all contracts in the futures market by $\mathcal{B} \triangleq \{\text{CTR}_n\}_{n \in \mathcal{N}}$. As each contract user is guaranteed to obtain the desired number of spectrum (otherwise he can get the PR's penalty as compensation), we refer to such a contract as the **guaranteed-delivery contract**.

The important assumptions on the guaranteed contract are listed below.

ASSUMPTION 5. *We assume that contract users care about the expected number of spectrums they obtain (hence the PR's penalty is based on the expected number of allocated spectrums).*

The main practical motivation for such an assumption is that most wireless applications in practice require an expected/average data rate during a certain time period. For example, video streaming concerns the average downloading rate in every minute, and VoIP concerns the average downloading/uploading rate in every second. When the actual data rate is occasionally less than the average rate, various coding and error concealment technologies can be employed, so that the SUs will not feel significant performance degradation. Thus, with a proper choice of the length of allocation period (e.g., one minute for video streaming or one second for VoIP), such a contract with the expected demand is suitable for most wireless applications. In fact, with the uncertainties of spectrum availability and the stochasticity of SU utility on each spectrum, it is practically impossible to guarantee a strict spectrum supply for each contract user. To the best of our knowledge, no such guarantee has been provided even in the latest communication standard. To make our analysis more complete, we also provide detailed theoretical analysis and numerical evaluation for the impact of this assumption on the performance in our technical report, Gao et al. (2012b).

3.4. Spatial Spectrum Reuse

As mentioned previously, an important characteristic of a spectrum market is that radio spectrum is *spatially reusable* subject to certain spatial interference constraints (see Tse & Viswanath (2005)). For example, a spectrum can be used by multiple SUs simultaneously if they are far enough apart

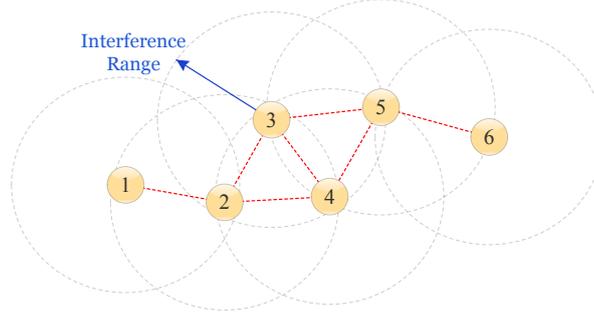


Figure 2 Conflict graph representation.

and do not interfere with each other. This leads to a fundamental difference between the spectrum market and conventional markets: the same spectrum can be potentially sold to multiple SUs.

In this work, we adopt a widely-used physical model to capture the interference relationships between SUs, namely, the *protocol interference model* (see Gupta & Kumar (2000)). Under the protocol interference model, multiple SUs can use the same spectrum simultaneously without mutual interference, if and only if none of the SUs falls inside the interference ranges of others (e.g., they are sufficiently far away from each other). The network under a protocol interference model can usually be represented by a *conflict graph*, with each vertex denoting an SU, and each edge indicating that two associated SUs fall inside the interference range of each other. Thus, any two SUs not connected by an edge (also called *independent users*) can use the same spectrum simultaneously, while any two connecting SUs cannot.

Fig. 2 illustrates the conflict graph representation for a network with 6 SUs, in which each dash circle denotes the interference range of the associated SU. In this example, SUs 1 and 3 are independent of each other (and therefore can use the same spectrum simultaneously), while SUs 1 and 2 are not. Fig. 1 also illustrates a similar conflict graph, where SUs 2, 6, and 8 are independent of each other, whereas SUs 1, 2, and 3 are not.

Let $G \triangleq (\mathcal{V}, \mathcal{E})$ denote the graph consisting of all SUs (and the associated edges, similarly hereafter), where $\mathcal{V} \triangleq \mathcal{M} \cup \mathcal{N}$ is the set of vertices (SUs), and $\mathcal{E} \triangleq \{\mathbf{e}_{ij} \mid i, j \in \mathcal{V}\}$ is the set of edges. Let $G^s \triangleq (\mathcal{V}^s, \mathcal{E}^s)$ denote the *spot market subgraph* consisting of all spot market users, and $G^c \triangleq (\mathcal{V}^c, \mathcal{E}^c)$ denote the *futures market subgraph* consisting of all contract users, that is, $\mathcal{V}^s \triangleq \mathcal{M}$, $\mathcal{V}^c \triangleq \mathcal{N}$, $\mathcal{E}^s \triangleq \{\mathbf{e}_{ij} \in \mathcal{E} \mid i, j \in \mathcal{M}\}$, and $\mathcal{E}^c \triangleq \{\mathbf{e}_{ij} \in \mathcal{E} \mid i, j \in \mathcal{N}\}$. For presentation convenience, we further introduce the concept of side market.

DEFINITION 1 (Side Market). The side market (subgraph) of a contract user set $\mathbf{n} \subseteq \mathcal{N}$, denoted by $G_{\mathbf{n}}^s = (\mathcal{V}_{\mathbf{n}}^s, \mathcal{E}_{\mathbf{n}}^s)$, is a subgraph of the spot market, consisting of all spot market users **except** the neighbors of contract users \mathbf{n} .

Here, the “neighbors” of \mathbf{n} refer to spot market users connecting to at least one contract user in the set \mathbf{n} . It is easy to see that $\mathcal{V}_{\mathbf{n}_1}^s \subseteq \mathcal{V}_{\mathbf{n}_2}^s \subseteq \mathcal{V}^s$ for any $\mathbf{n}_2 \subseteq \mathbf{n}_1 \subseteq \mathcal{N}$, since a neighbor of \mathbf{n}_2 must be a neighbor of \mathbf{n}_1 if $\mathbf{n}_2 \subseteq \mathbf{n}_1$. In Fig. 1, we have $\mathcal{V}^c = \{1, 2, 3, 4\}$, $\mathcal{V}^s = \{5, 6, 7, 8\}$, and examples of side markets include $\mathcal{V}_{\{1\}}^s = \{6, 7, 8\}$ and $\mathcal{V}_{\{1, 2, 3\}}^s = \{7, 8\}$. Obviously, $\mathcal{V}_{\{1, 2, 3\}}^s \subseteq \mathcal{V}_{\{1\}}^s \subseteq \mathcal{V}^s$.

Let $\mathcal{I}(G)$ and $\mathcal{C}(G)$ denote the sets of all independent sets and all cliques of a graph G , respectively.¹⁷ Then, the spatial interference constraint can be defined as follows.

DEFINITION 2 (Spatial Interference Constraint – SIC). Any two SUs in a clique of G cannot use the same spectrum simultaneously; on the other hand, all SUs in an independent set of G can use the same spectrum simultaneously.

Finally, we list the key assumption on the network topology or graph G below.

ASSUMPTION 6. We assume that the network topology G is exogenously given, and remains unchanged during the whole period of interest.

This corresponds to a static or quasi-static network scenario, in which SUs do not move or move slowly. With this assumption, it is reasonable to assume that the network topology is public information to the PR.

3.5. Valuation

The *valuation* of an SU i over a spectrum CH_{kt} reflects the SU’s preference for the transmission rate (capacity) achieved on spectrum CH_{kt} . Specifically, it is determined by two factors: (i) the transmission rate of SU i over spectrum CH_{kt} (denoted by $r_{i,kt}$), reflecting how efficiently SU i utilizes spectrum CH_{kt} ; and (ii) the user preference of SU i (denoted by $\alpha_{i,kt}$), reflecting how eagerly SU i desires for spectrum CH_{kt} . To avoid confusion, we use notation $v_{m,kt}$ and $u_{n,kt}$ to denote the valuations of spot market user m and contract user n over spectrum CH_{kt} , respectively. Formally, we can write the valuations $v_{m,kt}$ and $u_{n,kt}$ as generic functions:

$$v_{m,kt} \triangleq g_m(r_{m,kt}, \alpha_{m,kt}) \quad \text{and} \quad u_{n,kt} \triangleq g_n(r_{n,kt}, \alpha_{n,kt}), \quad (4)$$

where $g_i(\cdot)$ is the generic valuation function of SU i , which generally increases in both the transmission rate and the user preference. Typical examples of valuation function include: (a) linear functions, e.g., $g_i(r, \alpha) = \alpha \cdot r$; (b) sigmoid functions, e.g., $g_i(r, \alpha) = \alpha \cdot \frac{1}{1 + e^{r_0 - r}}$; and (c) step functions, e.g., $g_i(r, \alpha) = \alpha$ if r exceeds a certain threshold r_0 , and 0 otherwise.

The user preference $\alpha_{i,kt}$ is a *subjective* feeling of SU i for spectrum CH_{kt} , and is related to factors such as traffic state, urgency, and importance. The transmission rate $r_{i,kt}$ is an *objective* attribute of spectrum CH_{kt} for SU i , and is related to a spectrum-specific *quality* q_{kt} and a user-specific channel coefficient $h_{i,kt}$. The q_{kt} is common for all SUs, regarding factors such as channel

bandwidth, noise level, and power constraint. The $h_{i,kt}$ is usually independent between different SUs, regarding factors such as path loss, shadow fading, and small scale fading (or multiple-path effect). Typically, $r_{i,kt}$ is given by the Shannon-Hartley theorem (see Shannon (2001)):

$$r_{i,kt} = B_k \cdot \log \left(1 + \frac{P_i \cdot |H_{i,kt}|^2}{\sigma_{kt}^2} \right), \quad (5)$$

where B_k is the channel bandwidth, P_i is the transmission power, σ_{kt}^2 is the noise level over spectrum CH_{kt} (captured by the common spectrum quality q_{kt}), and $|H_{i,kt}|^2$ is the channel gain of SU i over spectrum CH_{kt} (captured by the user-specific channel coefficient $h_{i,kt}$).

By (4) and (5), an SU's valuation (over a spectrum) is essentially a joint function of the common spectrum quality, user-specific channel coefficient, and user preference. For notational convenience, we will write it as $v_{m,kt} \triangleq g_m(q_{kt}, h_{m,kt}, \alpha_{m,kt})$ and $u_{n,kt} \triangleq g_n(q_{kt}, h_{n,kt}, \alpha_{n,kt})$. In general, the spectrum quality, user-specific channel coefficient, and user preference change randomly across time and frequency. For simplicity, the following assumptions are used in our work.

ASSUMPTION 7. *The spectrum quality, user-specific channel coefficient (of every SU), and user-preference (of every SU) are all i.i.d. across time and channels.*

Based on the above assumption, we can see that *the valuation of each SU is also i.i.d. across time and channels.*¹⁸ Let random variable \mathbf{q} denote the spectrum quality, \mathbf{h}_i and $\boldsymbol{\alpha}_i$ denote the user-specific channel coefficient and preference of SU i . Let random variables $\mathbf{v}_m \triangleq g_m(\mathbf{q}, \mathbf{h}_m, \boldsymbol{\alpha}_m)$ and $\mathbf{u}_n \triangleq g_n(\mathbf{q}, \mathbf{h}_n, \boldsymbol{\alpha}_n)$ denote the valuations of spot market user m and contract user n , respectively. We can further see that the valuations of different SUs (over the same spectrum) are partially correlated by the common spectrum quality \mathbf{q} . That is, *the valuation of each spectrum is partially correlated across users.* Such a valuation formulation generalizes both the case of independent user valuation (by removing the uncertainty of \mathbf{q}) and the case of completely correlated user valuation (by removing the uncertainty of \mathbf{h}_i and $\boldsymbol{\alpha}_i, \forall i$).

Let random vector $\boldsymbol{\Theta} \triangleq (\mathbf{v}_1, \dots, \mathbf{v}_M; \mathbf{u}_1, \dots, \mathbf{u}_N)$ denote the valuations (vector) of all SUs, and $\boldsymbol{\theta}_{kt} \triangleq (v_{1,kt}, \dots, v_{M,kt}; u_{1,kt}, \dots, u_{N,kt})$ denote a realization of $\boldsymbol{\Theta}$ on spectrum CH_{kt} . As will be shown later, in the spot market, the valuation $v_{m,kt}$ uniquely reflects the SU m 's maximum willingness-to-pay (or utility) for the spectrum CH_{kt} . In the futures market, however, the valuation $u_{n,kt}$ only partially reflects the SU n 's maximum willingness-to-pay (or utility) for the spectrum CH_{kt} , which depends also on the total number of spectrums that he obtains. Since each spectrum CH_{kt} is fully characterized by the utility vector $\boldsymbol{\theta}_{kt}$ and its availability ξ_{kt} , we also refer to $\{\boldsymbol{\theta}_{kt}, \xi_{kt}\}$ as the **information of spectrum ch_{kt}** . Formally,

DEFINITION 3 (Network Information). The information of each spectrum CH_{kt} consists of its availability and the valuations of all SUs over that spectrum, i.e., $\{\boldsymbol{\theta}_{kt}, \xi_{kt}\}$. The information of all spectrums in \mathcal{Q} is referred to as the network information, i.e., $\{\boldsymbol{\theta}_{kt}, \xi_{kt}\}_{k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$.

For convenience, we will refer to a spectrum with information $\{\theta_{kt}, \xi_{kt}\}$ as spectrum $\{\theta_{kt}, \xi_{kt}\}$. Note that the private information of a contract user n (or spot market user m) is defined as his or her valuation $u_{n,kt}$ (or $v_{m,kt}$) for each spectrum CH_{kt} . That is, we consider all other information of a contract user (e.g., the spectrum demand) as public information. It is important to note that in a broad sense, we can generalize our current analytical framework by allowing multi-dimensional private information, as we adopt a general VCG framework in the later analysis.

3.6. Spectrum Efficiency

When the PR leases an idle spectrum of a PU to an SU for secondary utilization, the SU achieves a certain benefit, called *SU-surplus*, which equals the difference between the total *utility* achieved from using these spectrums and the payment transferred to the related PU; the PU also achieves a certain benefit, called *PU-surplus*, which equals the difference between the payment from the SU and the cost of owning spectrum and sharing spectrum with unlicensed SUs.¹⁹ The **social surplus** generated by such a process is the sum of the SU-surplus and the PU-surplus, which equals the difference between the (SU) utility generated and the (PU) cost involved. That is:

$$\text{Social Surplus} = \text{SU Surplus} + \text{PU Surplus} = \text{SU Utility} - \text{PU Cost}.$$

Notice that the PU's cost is a constant (sunk cost) independent of the secondary spectrum allocation. Without loss of generality, we normalize it to zero. As a result, the social surplus is equivalent to the utility generated by the SU. Next we define the *utility* of each SU formally.

Utility of Spot Market User. In the spot market, based on the characteristics of elastic traffic, an SU achieves an immediate utility from any spectrum allocated, and such a utility is directly related to his or her valuation of the spectrum. Thus, the total utility of a spot market user m (denoted by w_m^s) can be directly defined as the total valuation achieved:²⁰

$$w_m^s = \sum_{k=1}^K \sum_{t=1}^T a_{m,kt}^s \cdot v_{m,kt} \cdot \xi_{kt}, \quad (6)$$

where $a_{m,kt}^s \in \{0, 1\}$ indicates whether to allocate a spectrum CH_{kt} to SU m . The factor ξ_{kt} means that only the idle spectrums can be used by SUs.

Utility of Contract User. In the futures market, however, an SU's utility is not only related to the valuation that he or she achieves, but also related to the total number of spectrums that he or she obtains. First, depending on the requirement of inelastic traffic, an SU achieves a fixed total utility if the number of allocated spectrums meets (or exceeds) the demand; otherwise the SU suffers certain utility loss. Such a utility is usually referred to as the *demand-related* utility, which coincides with the SU's payment defined in the contract, i.e., Eq. (2).²¹ For convenience, we

will use the notation D_n^\ddagger , B_n^\ddagger , \hat{P}_n^\ddagger , and \hat{B}_n^\ddagger to denote the spectrum demand, the fixed total utility when the demand is satisfied, the unit utility loss (for soft contract) and the fixed total utility loss (for hard contract) when the demand is not satisfied, respectively. Since we do not address the problem of contract design in this work, we simply consider that each contract coincides perfectly with the contract user's utility definition, i.e., $D_n^\ddagger = D_n$, $B_n^\ddagger = B_n$, $\hat{P}_n^\ddagger = \hat{P}_n$, and $\hat{B}_n^\ddagger = \hat{B}_n$. Thus, the total *demand-related* utility of a contract user n (denoted by w_n^c) has exactly the same form as his actual payment in the contract, that is:

$$w_n^c = B_n - \mathbb{P}(d_n, D_n) = B_n - \mathbb{P}\left(\sum_{k=1}^K \sum_{t=1}^T a_{n,kt}^c \cdot \xi_{kt}, D_n\right), \quad (7)$$

where $a_{n,kt}^c \in \{0, 1\}$ indicates whether to allocate a spectrum CH_{kt} to SU n , $d_n \triangleq \sum_{k=1}^K \sum_{t=1}^T a_{n,kt}^c \cdot \xi_{kt}$ is the total number of *idle* spectrums allocated to the SU n , and $\mathbb{P}(d_n, D_n)$ is the potential utility loss of the SU caused by the unmet demand, that is:

$$\mathbb{P}(d_n, D_n) = (1 - J_n) \cdot [D_n - d_n]^+ \cdot \hat{P}_n + J_n \cdot \mathbf{1}(D_n - d_n) \cdot \hat{B}_n. \quad (8)$$

which has the same form as the PR's potential penalty defined in (3).

Second, similar as spot market users, a contract user's utility is also related to the valuation that he or she can achieve on the allocated spectrum (or equivalently, the quality of the allocated spectrum). The reason is following: to achieve a desirable quality-of-service (QoS), a contract user needs to pay different levels of efforts (e.g., transmission powers) on spectrums with different qualities. For example, he may achieve the desirable QoS with a lower effort on a spectrum with a higher quality, hence lead to a higher utility. Such a utility is referred to as the *quality-based* utility. Similar as that of spot market users, the total *quality-based* utility of a contract user n (denoted by \tilde{w}_n^c) can be defined as the total valuation achieved:

$$\tilde{w}_n^c = \sum_{k=1}^K \sum_{t=1}^T a_{n,kt}^c \cdot u_{n,kt} \cdot \xi_{kt}. \quad (9)$$

To capture the above two factors in a contract user's utility, we define the utility of contract user n as a weighted sum of the demand-related utility w_n^c and the quality-based utility \tilde{w}_n^c , i.e.,

$$\tau_n \cdot w_n^c + (1 - \tau_n) \cdot \tilde{w}_n^c,$$

where $\tau_n \in [0, 1]$ is the factor weighting w_n^c and \tilde{w}_n^c . Such a utility definition is flexible in capturing contract users' different preferences regarding spectrum demand and spectrum quality.

The spectrum secondary utilization efficiency (or *spectrum efficiency*) is the aggregate utility of all SUs, also called *social welfare*. Specifically,

DEFINITION 4 (**Spectrum Efficiency**). The spectrum efficiency (or social welfare) is

$$W \triangleq \sum_{m=1}^M w_m^s + \sum_{n=1}^N (\tau_n \cdot w_n^c + (1 - \tau_n) \cdot \tilde{w}_n^c), \quad (10)$$

given any feasible spectrum allocation $\{a_{i,kt}\}_{i \in \mathcal{M} \cup \mathcal{N}, k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$.²²

Based on the above, the spectrum efficiency maximization (SEM) problem can be defined as follows: finding a feasible spectrum allocation $\{a_{i,kt}\}_{i \in \mathcal{M} \cup \mathcal{N}, k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$, such that the spectrum efficiency W defined in (10) is maximized.

4. Problem Formulation

We first formulate the spectrum efficiency maximization (SEM) problem under deterministic (complete) network information (Section 4.1), which is essentially a *matching* problem, and can be solved by many existing algorithms. Then we formulate the expected spectrum efficiency maximization (E-SEM) problem under stochastic network information (Section 4.2), which is an *infinite-dimensional* optimization problem. We will focus on solving the E-SEM problem in Section 5.

4.1. Problem Formulation under Deterministic Information

With complete and deterministic network information, i.e., $\{\theta_{kt}, \xi_{kt}\}_{k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$, the spectrum efficiency maximization (SEM) problem can be formulated as a *matching* between all spectrums and all SUs, and the optimal solution explicitly specifies the allocation of every spectrum to a particular set of (independent) SUs. Let $a_{m,kt}^s \in \{0, 1\}$ (or $a_{n,kt}^c \in \{0, 1\}$) indicate whether to allocate a spectrum CH_{kt} to spot market user m (or contract user n). Denote:

$$\mathbf{a}_{kt} \triangleq (a_{1,kt}^s, \dots, a_{M,kt}^s; a_{1,kt}^c, \dots, a_{N,kt}^c)$$

as the allocation (vector) of spectrum CH_{kt} , which contains the allocation of spectrum CH_{kt} to every SU. An *allocation strategy* consists of the allocations of all spectrums in the whole time period, denoted by $\mathbf{A}_{\text{DET}} \triangleq \{\mathbf{a}_{kt}\}_{k \in \{1, \dots, K\}, t \in \{1, \dots, T\}}$. Intuitively, the allocation strategy \mathbf{A}_{DET} is a mapping from every particular spectrum CH_{kt} to an allocation vector \mathbf{a}_{kt} .

By the Spatial Interference Constraint (SIC) given in Definition 2, an allocation strategy \mathbf{A}_{DET} is feasible, if and only if the allocation \mathbf{a}_{kt} for every spectrum CH_{kt} satisfies:

$$\text{SIC:} \quad \sum_{m \in \mathcal{C}_i \cap \mathcal{M}} a_{m,kt}^s + \sum_{n \in \mathcal{C}_i \cap \mathcal{N}} a_{n,kt}^c \leq \xi_{kt}, \quad \forall \mathcal{C}_i \in \mathcal{C}(\text{G}), \quad (11)$$

where \mathcal{C}_i is the i -th clique of graph G , and $\mathcal{C}_i^1 \triangleq \mathcal{C}_i \cap \mathcal{M}$ and $\mathcal{C}_i^2 \triangleq \mathcal{C}_i \cap \mathcal{N}$ denote the spot market user set and contract user set in \mathcal{C}_i , respectively. It is easy to see that $a_{m,kt}^s = a_{n,kt}^c \equiv 0, \forall m, n$, if

$\xi_{kt} = 0$ (i.e., CH_{kt} is not idle). Intuitively, (11) states that (i) only idle spectrums can be used by SUs, and (ii) every idle spectrum can be allocated to, at most, one SU in a clique.

According to the definition of spectrum efficiency in Definition 4 and the SIC in (11), the spectrum efficiency maximization (SEM) problem can be formally defined as follows:

$$\begin{aligned} \text{SEM: } \quad & \mathbf{A}_{\text{DET}}^* = \arg \max_{\mathbf{A}_{\text{DET}}} W \triangleq \sum_{m=1}^M w_m^s + \sum_{n=1}^N \left(\tau_n \cdot w_n^c + (1 - \tau_n) \cdot \tilde{w}_n^c \right) \\ & \text{s.t. } \quad a_{m,kt}^s \in \{0, 1\}, a_{n,kt}^c \in \{0, 1\}, \forall m \in \mathcal{M}, \forall n \in \mathcal{N}, \forall k \in \{1, \dots, K\}, \forall t \in \{1, \dots, T\}, \end{aligned} \quad (12)$$

SIC defined in (11), $\forall k \in \{1, \dots, K\}, \forall t \in \{1, \dots, T\}$,

$$D_n \geq d_n, \forall n \in \mathcal{N},$$

where $d_n \triangleq \sum_{k=1}^K \sum_{t=1}^T a_{n,kt}^c \cdot \xi_{kt}$ is the number of spectrums allocated to contract user n . The last constraint in (12) states that none of the contract users will get spectrums more than his or her demand. The reason for this is that over-allocation brings trivial welfare gain for contract users. It is easy to see that the above problem is an 0-1 integer linear programming (also called matching problem), and many algorithms, e.g., branch-and-bound algorithm and Kuhn-Munkres algorithm (see Munkres (1957)), have been developed to solve a matching problem effectively. Due to space limitations, we omit the details here.

4.2. Problem Formulation under Stochastic Information

In practice, the assumption of complete and deterministic information is often unreasonable, as the network changes randomly over time. Hence, in this section we consider the case in which the network information is stochastic and the PR only knows its distribution, but not realization. In this case, we formulate the *expected* spectrum efficiency maximization (E-SEM) problem based on the stochastic distribution (e.g., PDF or CDF) of network information.²³ The solution specifies the allocation of any spectrum under every possible information realization (rather than the explicit allocation of every particular spectrum). In this sense, it essentially defines a contingency plan (also called a policy) for allocating every spectrum under every possible information realization.

Based on the above, we can define the *allocation strategy* as a mapping from every possible information realization (rather than every particular spectrum, as under deterministic information) to an allocation vector consisting of the allocation probability to every SU. However, as shown in our technical report, Gao et al. (2012b), such a primitive definition is not appropriate due to the high complexity in determining the SIC constraint.

To this end, we propose a new definition for the allocation strategy. The basic idea is to *consider each independent SU set as a virtual player, and decide the spectrum allocation among the virtual players*. That is, we transform the primitive allocation strategy, regarding the allocation probability

(of every information realization) to every SU, into an equivalent strategy, regarding the allocation probability to every independent SU set.

Denote $\mathcal{I}_i \in \mathcal{I}(G)$ as the i -th independent set of the graph G , and $I \triangleq |\mathcal{I}(G)|$ as the total number of independent sets in the graph G . Let $a_i^{\text{IS}}(\theta, \xi) \in [0, 1]$ denote the allocation probability of a spectrum $\{\theta, \xi\}$ to independent SU set \mathcal{I}_i . The allocation probability vector of spectrum $\{\theta, \xi\}$ is:

$$\mathbf{a}_{\text{IS}}(\theta, \xi) \triangleq (a_1^{\text{IS}}(\theta, \xi), a_2^{\text{IS}}(\theta, \xi), \dots, a_I^{\text{IS}}(\theta, \xi)),$$

and the new allocation strategy can be formally defined as follows.

DEFINITION 5 (Allocation Strategy). An allocation strategy is a mapping from every information realization $\{\theta, \xi\}$ to a vector $\mathbf{a}_{\text{IS}}(\theta, \xi)$ consisting of the allocation probability of a spectrum $\{\theta, \xi\}$ to every independent SU set, denoted by $\mathbf{A}_{\text{IS}} \triangleq \{\mathbf{a}_{\text{IS}}(\theta, \xi)\}_{\theta \in \Theta, \xi \in \{0,1\}}$.

Since the SUs in an independent set can use the same spectrum simultaneously, we can easily find that strategy \mathbf{A}_{IS} is feasible, if the allocation $\mathbf{a}_{\text{IS}}(\theta, \xi)$ for every spectrum $\{\theta, \xi\}$ satisfies:

$$\sum_{i=1}^I a_i^{\text{IS}}(\theta, \xi) \leq \xi. \quad (13)$$

That is, the condition in (13) is *sufficient* for a feasible allocation strategy. The next question is whether it is able to represent all feasible allocations. The answer is YES, as shown below.

PROPOSITION 1. *Any feasible allocation can be represented by an allocation strategy (defined in Definition 5) subject to the condition in (13).*

For detailed proof, please refer to our technical report, Gao et al. (2012b). Proposition 1 implies that the condition in (13) is an effective SIC constraint. From Proposition 1, we can see that the main advantage of such a transformation is that we can systematically identify the necessary and sufficient conditions of a feasible allocation strategy (i.e., the SIC constraint), which is essential for the E-SEM problem formulation. Certainly, this will bring some challenges, among which the most critical one is that the problem of finding all independent sets is NP-hard. This motivates our later study on the approximate algorithm and the associated performance loss analysis.

Next we can provide the E-SEM problem formulation based on the new allocation strategy. Let $l_n^i \in \{0, 1\}$ (or $l_m^i \in \{0, 1\}$) indicate whether a contract user n (or spot market user m) belongs to the i -th independent set, i.e., $l_n^i = 1$ if $n \in \mathcal{I}_i$, and $l_n^i = 0$ otherwise. Then, the total allocation probability of spectrum $\{\theta, \xi\}$ to contract users n or spot market user m can be written as:

$$a_m^s(\theta, \xi) = \sum_{i=1}^I l_m^i \cdot a_i^{\text{IS}}(\theta, \xi) \quad \text{and} \quad a_n^c(\theta, \xi) = \sum_{i=1}^I l_n^i \cdot a_i^{\text{IS}}(\theta, \xi). \quad (14)$$

Given a feasible allocation strategy \mathbf{A}_{IS} , the expected welfare generated by a spot market m is:

$$\begin{aligned}\mathbb{E}[w_m^s] &= \sum_{k=1}^K \sum_{t=1}^T \sum_{\xi_{kt}=0}^1 f_{\xi}(\xi_{kt}) \cdot \xi_{kt} \int_{\theta_{kt}} a_m^s(\theta_{kt}, \xi_{kt}) \cdot v_m(\theta_{kt}) \cdot f_{\Theta}(\theta_{kt}) d\theta_{kt} \\ &= S \cdot \sum_{\xi=0}^1 f_{\xi}(\xi) \cdot \xi \int_{\theta} a_m^s(\theta, \xi) \cdot v_m(\theta) \cdot f_{\Theta}(\theta) d\theta \\ &= \rho S \cdot \int_{\theta} a_m^s(\theta, 1) \cdot v_m(\theta) \cdot f_{\Theta}(\theta) d\theta,\end{aligned}\tag{15}$$

where $v_m(\theta)$ is the spot market user m 's utility (i.e., the m -th element) in the utility vector $\theta = (v_1, \dots, v_M; u_1, \dots, u_N)$, and $f_{\Theta}(\theta)$ is the joint PDF of Θ , i.e., $f_{\Theta}(\theta) \triangleq \prod_{m=1}^M f_{v_m}(v_m) \prod_{n=1}^N f_{u_n}(u_n)$. The second equation follows because ξ and Θ are i.i.d. across time and channels. The third equation follows because (i) $f_{\xi}(\xi) = 1 - \rho$ if $\xi = 0$ and (ii) $f_{\xi}(\xi) = \rho$ if $\xi = 1$. Note that we write $dv_1 \dots dv_M du_1 \dots du_N$ as $d\theta$ and $\int_{v_1} \dots \int_{v_M} \int_{u_1} \dots \int_{u_N}$ as \int_{θ} for convenience.

The expected number of spectrums allocated to a contract user n is:²⁴

$$\begin{aligned}\mathbb{E}[d_n] &= \sum_{k=1}^K \sum_{t=1}^T \sum_{\xi_{kt}=0}^1 f_{\xi}(\xi_{kt}) \cdot \xi_{kt} \int_{\theta_{kt}} a_n^c(\theta_{kt}, \xi_{kt}) \cdot f_{\Theta}(\theta_{kt}) d\theta_{kt} \\ &= \rho S \cdot \int_{\theta} a_n^c(\theta, 1) \cdot f_{\Theta}(\theta) d\theta.\end{aligned}\tag{16}$$

Then, the expected demand-related utility generated by a contract user n is:

$$\mathbb{E}[w_n^c] = B_n - \mathbb{P}(\mathbb{E}[d_n], D_n),\tag{17}$$

where $\mathbb{P}(\mathbb{E}[d_n], D_n)$ is defined in (8). Note that the above utility formula is based on the expected number of spectrums that the contract user n obtains, rather than the explicit number as in (7). The major motivation for this assumption is discussed in Section 3.3.

The expected quality-based utility generated by a contract user n is:

$$\begin{aligned}\mathbb{E}[\tilde{w}_n^c] &= \sum_{k=1}^K \sum_{t=1}^T \sum_{\xi_{kt}=0}^1 f_{\xi}(\xi_{kt}) \cdot \xi_{kt} \int_{\theta_{kt}} a_n^c(\theta_{kt}, \xi_{kt}) \cdot u_n(\theta_{kt}) \cdot f_{\Theta}(\theta_{kt}) d\theta_{kt} \\ &= \rho S \cdot \int_{\theta} a_n^c(\theta, 1) \cdot u_n(\theta) \cdot f_{\Theta}(\theta) d\theta,\end{aligned}\tag{18}$$

where $u_n(\theta)$ denotes the contract user n 's valuation in the vector $\theta = (v_1, \dots, v_M; u_1, \dots, u_N)$.

The expected spectrum efficiency (expected social welfare) is the expected aggregate utility generated by all SUs, that is:

$$\mathbb{E}[W] \triangleq \sum_{m=1}^M \mathbb{E}[w_m^s] + \sum_{n=1}^N (\tau_n \cdot \mathbb{E}[w_n^c] + (1 - \tau_n) \cdot \mathbb{E}[\tilde{w}_n^c]).$$

Therefore, we have the following expected spectrum efficiency maximization (E-SEM) problem:

$$\begin{aligned}
 \text{E-SEM: } \quad & \mathbf{A}_{\text{IS}}^* = \arg \max_{\mathbf{A}_{\text{IS}}} \mathbb{E}[W] \triangleq \sum_{m=1}^M \mathbb{E}[w_m^s] + \sum_{n=1}^N \left(\tau_n \cdot \mathbb{E}[w_n^c] + (1 - \tau_n) \cdot \mathbb{E}[\tilde{w}_n^c] \right) \\
 \text{s.t. } \quad & a_i^{\text{IS}}(\theta, \xi) \in [0, 1], \quad \forall \theta \in \Theta, \forall \xi \in \{0, 1\}, \forall i \in \{1, \dots, I\}, \\
 & \sum_{i=1}^I a_i^{\text{IS}}(\theta, \xi) \leq \xi, \quad \forall \theta \in \Theta, \forall \xi \in \{0, 1\}, \quad (\text{SIC}) \\
 & D_n \geq \mathbb{E}[d_n], \quad \forall n \in \mathcal{N}.
 \end{aligned} \tag{19}$$

Obviously, the E-SEM problem (19) is an **infinite-dimensional** optimization problem (see Fattorini (1999)), in which the solution (or strategy) is not a number or a vector, but rather a continuous quantity. By the second constraint (SIC), we immediately have: $a_i^{\text{IS}}(\theta, \xi) \equiv 0$ if $\xi = 0$, $\forall \theta \in \Theta, \forall i = 1, \dots, I$. Therefore, in the rest of this paper, we will focus on solving $a_i^{\text{IS}}(\theta, 1), \forall \theta \in \Theta, \forall i = 1, \dots, I$, i.e., the allocation of idle spectrums, denoted by $\mathbf{A}_{\text{IS},1} \triangleq \{\mathbf{a}_{\text{IS}}(\theta, 1)\}_{\theta \in \Theta}$. **For notational convenience, we will write the tuple $\{\theta, 1\}$ as θ , and thus denote $\mathbf{A}_{\text{IS},1} \triangleq \{\mathbf{a}_{\text{IS}}(\theta)\}_{\theta \in \Theta}$.**

5. Off-line Optimal Policy

In this section, we solve the E-SEM problem given in (19). We first reduce the strategy space by removing some *irrelevant* strategies, so as to reduce the computational complexity.²⁵ Then we derive the optimal solution systemically using the primal-dual method. Due to space limits, we present all proofs in our technical report, Gao et al. (2012b).

5.1. Allocation Strategy Reduction

Now we simplify the E-SEM problem (19) by restricting our attention to a particular subset of the strategy space. Define the **weight** of each vertex in the graph G as the utility of the associated SU. Notice that the welfare generated by a spot market user is exactly his or her utility, and thus we have: if an idle spectrum θ (or spectrum $\{\theta, 1\}$) is allocated to an independent contract user set $\mathbf{n} \in \mathcal{I}(G^c)$, it must be allocated to an MWIS of the side market G_n^s at the same time. This observation helps us to eliminate some strategies without affecting the optimality of the solution.

Let $\mathcal{I}_i^1 \triangleq \mathcal{I}_i \cap \mathcal{M}$ and $\mathcal{I}_i^2 \triangleq \mathcal{I}_i \cap \mathcal{N}$ denote the spot market user set and contract user set in \mathcal{I}_i (i.e., the i -th independent set of graph G), respectively. Formally, we have:

PROPOSITION 2. *Suppose $\mathbf{A}_{\text{IS},1}^* = \{\mathbf{a}_{\text{IS}}^*(\theta)\}_{\theta \in \Theta}$, where $\mathbf{a}_{\text{IS}}^*(\theta) = (a_1^{\text{IS}*}(\theta), \dots, a_I^{\text{IS}*}(\theta))$, is an optimal solution of (19). Then for any $\theta \in \Theta$ we have: for every independent set $\mathcal{I}_i \in \mathcal{I}(G)$,*

(a) *If \mathcal{I}_i^1 is **not** an MWIS of the graph $G_{\mathcal{I}_i^2}^s$ (i.e., the side market (see Definition 1) of the contract user set \mathcal{I}_i^2), then $a_i^{\text{IS}*}(\theta) = 0$; or equivalently,*

(b) *If $a_i^{\text{IS}*}(\theta) > 0$, then \mathcal{I}_i^1 must be an MWIS of the graph $G_{\mathcal{I}_i^2}^s$.*

Proposition 2 implies that in an optimal solution, an idle spectrum θ will *never* be allocated to an independent set $\mathcal{I}_i = \mathcal{I}_i^1 \cup \mathcal{I}_i^2$ where \mathcal{I}_i^1 is *not* an MWIS of the graph $G_{\mathcal{I}_i^2}^s$. We refer to such an independent set \mathcal{I}_i as an *irrelevant independent set*. Obviously, we can ignore those allocation strategies that allocate spectrums to the irrelevant independent sets, and focus only on the allocation among the “relevant” independent sets \mathcal{I}_i (where \mathcal{I}_i^1 is an MWIS of the graph $G_{\mathcal{I}_i^2}^s$). In effect, we can restrict our attention to the independent sets of the futures market graph G^c , instead of the independent sets of the whole graph G : *if a spectrum is allocated to an independent contract user set (say \mathbf{n}) of G^c , it must be allocated to an MWIS of the side market graph $G_{\mathbf{n}}^s$ at the same time*. Consider the example in Fig. 1. Any idle spectrum will *not* be allocated to the independent set $\mathcal{I}_i = \{1, 6\}$, since $\{6\}$ is not an MWIS of $G_{\{1\}}^s$; otherwise, we can immediately increase the total welfare by reallocating the spectrum to another independent set $\mathcal{I}_j = \{1, 6, 8\}$.

The above result allows us to simplify the E-SEM problem. Denote $\mathcal{I}_i^c \in \mathcal{I}(G^c)$ as the i -th independent contract user set of the futures market graph G^c , and $I_c \triangleq |\mathcal{I}(G^c)|$ as the total number of independent sets of the graph G^c . Let $a_i^{\text{ISC}}(\theta) \in [0, 1]$ denote the allocation probability of an idle spectrum θ to the i -th independent contract user set \mathcal{I}_i^c .²⁶ Formally, we can define the **reduced** allocation strategy as $\mathbf{A}_{\text{ISC}} \triangleq \{\mathbf{a}_{\text{ISC}}(\theta)\}_{\theta \in \Theta}$, where

$$\mathbf{a}_{\text{ISC}}(\theta) \triangleq (a_0^{\text{ISC}}(\theta), a_1^{\text{ISC}}(\theta), \dots, a_{I_c}^{\text{ISC}}(\theta))$$

is the reduced allocation probability vector of an idle spectrum θ , consisting of the allocation probability of spectrum $\{\theta, 1\}$ to every **independent contract user set** of the graph G^c . Note that the 0-th independent contract user set is simply defined as the empty set, i.e., $\mathcal{I}_0^c = \emptyset$.

Similar to (13), a reduced allocation strategy \mathbf{A}_{ISC} is feasible, if and only if for every information realization $\{\theta, 1\}$, the reduced allocation probability vector $\mathbf{a}_{\text{ISC}}(\theta)$ satisfies:

$$\text{SIC:} \quad \sum_{i=0}^{I_c} a_i^{\text{ISC}}(\theta) \leq 1. \quad (20)$$

By Proposition 2, if an idle spectrum θ is allocated to the i -th independent contract user set \mathcal{I}_i^c , it must be allocated to an MWIS of the side market graph $G_{\mathcal{I}_i^c}^s$ (denoted by \mathcal{I}_i^s) at the same time. Let $z_i(\theta)$ denote the total weight of \mathcal{I}_i^s . Obviously, $z_i(\theta)$ is the maximum total welfare generated by spot market users in the side market $G_{\mathcal{I}_i^c}^s$ (over an idle spectrum θ), referred to as the **side welfare** of \mathcal{I}_i^c .²⁷ Thus, given a feasible strategy \mathbf{A}_{ISC} , the expected side welfare of \mathcal{I}_i^c is:

$$\begin{aligned} \mathbb{E}[W_i^s] &= \sum_{k=1}^K \sum_{t=1}^T \sum_{\xi_{kt}=0}^1 f_{\xi}(\xi_{kt}) \cdot \xi_{kt} \int_{\theta_{kt}} a_i^{\text{ISC}}(\theta_{kt}, \xi_{kt}) \cdot z_i(\theta_{kt}) \cdot f_{\Theta}(\theta_{kt}) d\theta_{kt} \\ &= \rho S \cdot \int_{\Theta} a_i^{\text{ISC}}(\theta) \cdot z_i(\theta) \cdot f_{\Theta}(\theta) d\theta. \end{aligned} \quad (21)$$

The expected social welfare generated by the spot market is the sum of the expected side welfare of all independent contract user sets (including the empty set), that is:

$$\mathbb{E}[W^s] = \sum_{i=0}^{I_c} \mathbb{E}[W_i^s] = \sum_{i=0}^{I_c} \left(\rho S \cdot \int_{\Theta} a_i^{\text{ISC}}(\theta) \cdot z_i(\theta) \cdot f_{\Theta}(\theta) d\theta \right). \quad (22)$$

Similar to (14), the allocation probability of an idle spectrum θ to a contract users n is:

$$a_n^c(\theta) = \sum_{i=0}^{I_c} l_n^i \cdot a_i^{\text{ISC}}(\theta) = \sum_{i=1}^{I_c} l_n^i \cdot a_i^{\text{ISC}}(\theta). \quad (23)$$

The 2nd equality follows because $l_n^0 \equiv 0, \forall n \in \mathcal{N}$ (since $\mathcal{I}_0^c = \emptyset$). Furthermore, the expected number of spectrums allocated to contract user n (i.e., $\mathbb{E}[d_n]$) and the expected utility generated by contract user n (i.e., $\mathbb{E}[w_n]$ and $\mathbb{E}[\tilde{w}_n]$) are given by (16)-(18) directly. The expected spectrum efficiency is:

$$\mathbb{E}[W] = \sum_{i=0}^{I_c} \mathbb{E}[W_i^s] + \sum_{n=1}^N \left(\tau_n \cdot \mathbb{E}[w_n^c] + (1 - \tau_n) \cdot \mathbb{E}[\tilde{w}_n^c] \right), \quad (24)$$

Based on the reduced allocation strategy \mathbf{A}_{ISC} and the associated expected spectrum efficiency formula in (24), we can rewrite the E-SEM in (19) into the following equivalent form:

$$\begin{aligned} \text{E-SEM(EQ):} \quad & \mathbf{A}_{\text{ISC}}^* = \arg \max_{\mathbf{A}_{\text{ISC}}} \mathbb{E}[W] = \sum_{i=0}^{I_c} \mathbb{E}[W_i^s] + \sum_{n=1}^N \left(\tau_n \cdot \mathbb{E}[w_n^c] + (1 - \tau_n) \cdot \mathbb{E}[\tilde{w}_n^c] \right) \\ & \text{s.t. (i)} \quad a_i^{\text{ISC}}(\theta) \in [0, 1], \quad \forall \theta \in \Theta, \forall i \in \{0, 1, \dots, I_c\}, \\ & \quad \text{(ii)} \quad \sum_{i=0}^{I_c} a_i^{\text{ISC}}(\theta) \leq 1, \quad \forall \theta \in \Theta, \quad (\text{SIC}) \\ & \quad \text{(iii)} \quad D_n \geq \mathbb{E}[d_n], \quad \forall n \in \mathcal{N}. \end{aligned} \quad (25)$$

Notice that the independent sets in the futures market graph G^c are much less than in the whole graph G , since the number of independent sets increases exponentially with the number of vertices. Thus, by (25), we can greatly reduce the complexity of the E-SEM problem in (19). Recall the example in Fig. 1, there are a total of 26 independent sets in graph G , while only 6 in graph G^c .

5.2. Necessary Conditions

For exposition convenience, we will omit the superscript ‘‘ISC’’ in $a_i^{\text{ISC}}(\theta)$, as long as there is no confusion caused. Suppose $\mathbf{A}_{\text{ISC}}^* = \{\mathbf{a}_{\text{ISC}}^*(\theta)\}_{\theta \in \Theta}$ where $\mathbf{a}_{\text{ISC}}^*(\theta) = (a_0^*(\theta), a_1^*(\theta), \dots, a_{I_c}^*(\theta))$ is the optimal solution to (25). The following necessary conditions hold.

LEMMA 1 (Necessary Condition I). *For any $\theta \in \Theta$, we have: $\sum_{i=0}^{I_c} a_i^*(\theta) = 1$.*

Intuitively, Lemma 1 states that every idle spectrum will be allocated to SUs. This is because it can always produce certain positive utility when used by some SUs, e.g., spot market users.

LEMMA 2 (Necessary Condition II). *For any contract user n , we have: $\mathbb{E}[d_n] \leq D_n$.*

Intuitively, Lemma 2 states that none of the contract users will get spectrums (on average) more than his or her demand. This is because spectrum over-allocation to a contract user brings trivial utility gain for the contract user (see the definition of contract user’s utility in Eq. [17]).

5.3. Optimal Solution

We focus on the optimal solution for soft contracts (with unit utility loss for every under-allocated spectrum). The analysis for hard contracts (with a fixed total utility loss for the unsatisfaction in demand) is similar and shown in our technical report, Gao et al. (2012b).

By Lemma 1, we can replace the decision variable $a_0(\theta)$ by $a_0(\theta) = 1 - \sum_{i=1}^{I_c} a_i(\theta)$. By Lemma 2, we can rewrite the potential welfare loss in (8) as $\mathbb{P}(d_n, D_n) = (D_n - \mathbb{E}[d_n]) \cdot \widehat{P}_n$ (for soft contracts). Furthermore, by the 2nd constraint of the problem (25), we can simplify the 1st constraint as $a_i(\theta) \geq 0, \forall i$. Thus, the optimization problem (25) can be rewritten as:

$$\begin{aligned} \mathbf{A}_{\text{ISC}'}^* &= \arg \max_{\mathbf{A}_{\text{ISC}'}} \left(\rho S \int_{\Theta} (1 - \sum_{i=1}^{I_c} a_i(\theta)) z_0(\theta) f_{\Theta}(\theta) d\theta + \sum_{i=1}^{I_c} \rho S \int_{\Theta} a_i(\theta) z_i(\theta) f_{\Theta}(\theta) d\theta \right. \\ &\quad \left. + \sum_{n=1}^N [\tau_n \cdot (B_n - (D_n - \mathbb{E}[d_n]) \cdot \widehat{P}_n) + (1 - \tau_n) \cdot \rho S \int_{\Theta} a_n^c(\theta) u_n(\theta) f_{\Theta}(\theta) d\theta] \right) \\ &= \arg \max_{\mathbf{A}_{\text{ISC}'}} \left(F + \rho S \cdot \sum_{i=1}^{I_c} \int_{\Theta} H_i(\theta) a_i(\theta) f_{\Theta}(\theta) d\theta \right), \\ \text{s.t.} \quad & \text{(i)} \quad a_i(\theta) \geq 0, \forall \theta \in \Theta, \forall i \in \{1, \dots, I_c\}; \\ & \text{(ii)} \quad \sum_{i=1}^{I_c} a_i(\theta) \leq 1, \forall \theta \in \Theta; \\ & \text{(iii)} \quad \mathbb{E}[d_n] \leq D_n, \forall n \in \mathcal{N}; \end{aligned} \quad (26)$$

where

- $\mathbf{A}_{\text{ISC}'} = \{\mathbf{a}_{\text{ISC}'}(\theta)\}_{\theta \in \Theta}$ with $\mathbf{a}_{\text{ISC}'}(\theta) = (a_1(\theta), \dots, a_{I_c}(\theta))$ is the reduced allocation strategy;
- $\mathbb{E}[d_n] = \rho S \int_{\Theta} a_n^c(\theta) f_{\Theta}(\theta) d\theta = \sum_{i=1}^{I_c} l_n^i \cdot \rho S \int_{\Theta} a_i(\theta) \cdot f_{\Theta}(\theta) d\theta$ is the expected number of spectrums allocated to the contract user n ;
- $F = \rho S \cdot \int_{\Theta} z_0(\theta) f_{\Theta}(\theta) d\theta + \sum_{n=1}^N \tau_n \cdot (B_n - \widehat{P}_n \cdot D_n)$ is a constant independent of the allocation;
- $H_i(\theta) = -z_0(\theta) + z_i(\theta) + \sum_{n=1}^N l_n^i \cdot (\tau_n \cdot \widehat{P}_n + (1 - \tau_n) \cdot u_n(\theta))$ is an information-specific variable.

To analytically solve the optimization problem (26), we first study the dual problem of (26) using the primal-dual method (see Dantzig (1998) for details). Then we determine the optimal dual variables, from which we further derive the optimal primal solution.

5.3.1. Primal-Dual Method We introduce Lagrange multipliers (also called *dual variables* or *shadow prices*) $\mu_i(\theta)$, $\eta(\theta)$, and λ_n for three constraints in (26). The Lagrangian is:

$$\begin{aligned} \text{LAG} &= F + \rho S \cdot \sum_{i=1}^{I_c} \int_{\Theta} H_i(\theta) a_i(\theta) f_{\Theta}(\theta) d\theta + \sum_{i=1}^{I_c} \int_{\Theta} \mu_i(\theta) a_i(\theta) f_{\Theta}(\theta) d\theta \\ &\quad + \int_{\Theta} \eta(\theta) \left(1 - \sum_{i=1}^{I_c} a_i(\theta) \right) f_{\Theta}(\theta) d\theta + \sum_{n=1}^N \lambda_n \left(D_n - \sum_{i=1}^{I_c} l_n^i \cdot \rho S \int_{\Theta} a_i(\theta) \cdot f_{\Theta}(\theta) d\theta \right), \end{aligned}$$

which can be written as $\text{LAG} \triangleq \int_{\Theta} \mathcal{W}(\theta) f_{\Theta}(\theta) d\theta$, where $\mathcal{W}(\theta)$, called Sub-Lagrangian, is given by

$$\mathcal{W}(\theta) \triangleq F + \eta(\theta) + \sum_{n=1}^N \lambda_n D_n + \sum_{i=1}^{I_c} \left(\rho S \cdot H_i(\theta) + \mu_i(\theta) - \eta(\theta) - \rho S \cdot \sum_{n=1}^N l_n^i \lambda_n \right) \cdot a_i(\theta).$$

We define the *marginal welfare* (MW) as the first partial derivative of $\mathcal{W}(\theta)$ over each independent contract user set's allocation probability $a_i(\theta)$. Formally,

DEFINITION 6 (**Marginal Welfare – MW**).

$$\dot{\mathbf{w}}_i(\theta) \triangleq \frac{\partial \mathcal{W}(\theta)}{\partial a_i(\theta)} = \rho S \cdot H_i(\theta) + \mu_i(\theta) - \eta(\theta) - \rho S \cdot \sum_{n=1}^N l_n^i \cdot \lambda_n. \quad (27)$$

By Euler-Lagrange conditions for optimality (see Dantzig (1998)), if an inner optimal solution exists, the optimal solution $\mathbf{A}_{\text{ISC}}^*(\theta)$ must satisfy the conditions: $\dot{\mathbf{w}}_i(\theta) = 0, \forall i$; otherwise, the optimal solution occurs at the boundaries. From Definition 6, we can see that the MW $\dot{\mathbf{w}}_i(\theta)$ is independent of $\mathbf{A}_{\text{ISC}}(\theta)$. Thus, the optimal solution $\mathbf{A}_{\text{ISC}}^*(\theta)$ must occur at the boundaries, that is:

$$a_i^*(\theta) = \begin{cases} 0, & \dot{\mathbf{w}}_i(\theta) < 0 \\ 1, & \dot{\mathbf{w}}_i(\theta) > 0 \end{cases} \quad (28)$$

By (27) and (28), the optimal solution is determined by the dual variables $\mu_i(\theta), \eta(\theta)$, and λ_n . By the primal-dual method (Dantzig (1998)), every dual variable must be nonnegative and is positive only when the associated constraint is tight, i.e., the following dual constraints hold:

$$\begin{cases} \text{(D.1)} & \mu_i^*(\theta) \geq 0, a_i^*(\theta) \geq 0, \mu_i^*(\theta) \cdot a_i^*(\theta) = 0, \quad \forall i \in \{1, \dots, I_c\}, \forall \theta \in \Theta; \\ \text{(D.2)} & \eta^*(\theta) \geq 0, 1 - \sum_{i=1}^{I_c} a_i^*(\theta) \geq 0, \eta^*(\theta) \cdot (1 - \sum_{i=1}^{I_c} a_i^*(\theta)) = 0, \quad \forall \theta \in \Theta; \\ \text{(D.3)} & \lambda_n^* \geq 0, D_n - \sum_{i=1}^{I_c} l_n^i \cdot \rho S \int_{\Theta} a_i^*(\theta) \cdot f_{\Theta}(\theta) d\theta \geq 0, \\ & \lambda_n^* \cdot (D_n - \sum_{i=1}^{I_c} l_n^i \cdot \rho S \int_{\Theta} a_i^*(\theta) \cdot f_{\Theta}(\theta) d\theta) = 0, \quad \forall n \in \mathcal{N}. \end{cases}$$

By the duality principle, the primal problem (26) is equivalent to the problem of finding a set of optimal dual variables $\mu_i^*(\theta), \eta^*(\theta)$, and λ_n^* , such that all dual constraints are satisfied (dual problem). The optimality of this approach is guaranteed as the objective and constraints in (26) are linear, which implies that (26) is a convex optimization problem.

For convenience, we further introduce the concepts of *inner marginal welfare* (I-MW) and *core marginal welfare* (C-MW), which will be frequently used in the later discussion.

DEFINITION 7 (**Inner Marginal Welfare – I-MW**).

$$\dot{\mathbf{w}}_i^{\text{IN}}(\theta) \triangleq \rho S \cdot H_i(\theta) - \eta(\theta) - \rho S \cdot \sum_{n=1}^N l_n^i \cdot \lambda_n. \quad (29)$$

DEFINITION 8 (**Core Marginal Welfare – C-MW**).

$$\dot{\mathbf{w}}_i^{\text{Co}}(\theta) \triangleq \rho S \cdot H_i(\theta) - \rho S \cdot \sum_{n=1}^N l_n^i \cdot \lambda_n. \quad (30)$$

It is easy to see that $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) = \dot{\mathbf{w}}_i(\theta) - \mu_i(\theta)$ and $\dot{\mathbf{w}}_i^{\text{Co}}(\theta) = \dot{\mathbf{w}}_i(\theta) - \mu_i(\theta) + \eta(\theta)$. That is, both I-MW and C-MW are variations of MW by ignoring certain non-critical dual variables in the MW. This is also the reason that we refer to them as “inner” and “core” marginal welfares.

5.3.2. Optimal Shadow Prices and Optimal Solution Now we study the optimal shadow prices (dual variables) and optimal primal solution.

We start by showing the necessary conditions for the optimal $\mu_i^*(\theta)$. Specifically,

LEMMA 3 (Feasible Range of $\mu_i^*(\theta)$). *The optimal shadow prices $\mu_i^*(\theta)$ must satisfy:*

- (a) $\mu_i^*(\theta) = 0$, if $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) \geq 0$, and (b) $\mu_i^*(\theta) \in [0, |\dot{\mathbf{w}}_i^{\text{IN}}(\theta)|]$, if $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) < 0$.

Notice that $\dot{\mathbf{w}}_i(\theta) = \dot{\mathbf{w}}_i^{\text{IN}}(\theta) + \mu_i^*(\theta)$. Lemma 3 implies that (a) $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) \geq 0$ if and only if $\dot{\mathbf{w}}_i(\theta) \geq 0$, and (b) $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) < 0$ if and only if $\dot{\mathbf{w}}_i(\theta) \leq 0$. That is, $\mu_i^*(\theta)$ never changes the sign of the MW $\dot{\mathbf{w}}_i(\theta)$, which implies that $\mu_i^*(\theta)$ has no impact on the optimal solution (28). Intuitively, this is because the optimal solution given by (28) never violates the first constraint (associated with shadow prices $\mu_i(\theta)$) of the primal problem (26). To better understand Lemma 3, we provide an illustration of $\dot{\mathbf{w}}_i^{\text{IN}}(\theta)$, $\dot{\mathbf{w}}_i(\theta)$, and the feasible range of $\mu_i^*(\theta)$ (shown as the shadow area) in Fig. 3, in which Region II corresponds to case (a), and Regions I and III correspond to case (b) in Lemma 3. In Region II, the optimal $\mu_i^*(\theta)$ is 0, and thus $\dot{\mathbf{w}}_i(\theta) = \dot{\mathbf{w}}_i^{\text{IN}}(\theta) \geq 0$. In Region I or III, the optimal $\mu_i^*(\theta)$ is in the shadow area, and thus $\dot{\mathbf{w}}_i(\theta) = \dot{\mathbf{w}}_i^{\text{IN}}(\theta) + \mu_i^*(\theta) \leq 0$.

Next we show the necessary conditions for the optimal $\eta^*(\theta)$. Denote:

$$\dot{W}_{1\text{st}}(\theta) \triangleq \max_{i \in \{1, \dots, I_C\}} \dot{\mathbf{w}}_i^{\text{Co}}(\theta) \quad \text{and} \quad \dot{W}_{2\text{nd}}(\theta) \triangleq \max_{i \in \{1, \dots, I_C\} / \{i_1\}} \dot{\mathbf{w}}_i^{\text{Co}}(\theta)$$

as the highest and second-highest C-MWs among all independent contract user sets, respectively, where $i_1 \triangleq \arg \max_{i \in \{1, \dots, I_C\}} \dot{\mathbf{w}}_i^{\text{Co}}(\theta)$. The feasible range of $\eta^*(\theta)$ is shown as follows.

LEMMA 4 (Feasible Range of $\eta^*(\theta)$). *The optimal shadow prices $\eta^*(\theta)$ must satisfy:*

- (a) $\eta^*(\theta) \in [\max(0, \dot{W}_{2\text{nd}}(\theta)), \dot{W}_{1\text{st}}(\theta)]$, if $\dot{W}_{1\text{st}}(\theta) > 0$, and (b) $\eta^*(\theta) = 0$, if $\dot{W}_{1\text{st}}(\theta) \leq 0$.

Notice that $\dot{\mathbf{w}}_i(\theta) = \dot{\mathbf{w}}_i^{\text{Co}}(\theta) + \mu_i^*(\theta) - \eta^*(\theta)$ and $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) = \dot{\mathbf{w}}_i^{\text{Co}}(\theta) - \eta^*(\theta)$. That is, the shadow price $\eta^*(\theta)$ can be viewed as an *identical* reduction on all C-MWs (at a particular θ). Lemma 4 suggests that a feasible $\eta^*(\theta)$ would lie between $\dot{W}_{1\text{st}}(\theta)$ and $\max(0, \dot{W}_{2\text{nd}}(\theta))$, with which there is, at most, one independent contract user set (the i_1 -th one $\mathcal{I}_{i_1}^C$, if exists) having positive I-MW, i.e., $\dot{\mathbf{w}}_i^{\text{IN}}(\theta) \leq 0, \forall i \neq i_1$. We therefore have $\dot{\mathbf{w}}_i(\theta) \leq 0$ and $a_i^*(\theta) = 0, \forall i \neq i_1$ by Lemma 3 and (28). Thus, the coupling constraint (ii) will be satisfied. To better understand Lemma 4, we provide an illustration of $\dot{\mathbf{w}}_i^{\text{Co}}(\theta)$, $\dot{\mathbf{w}}_i(\theta)$, and $\eta^*(\theta)$ (shown as the shadow areas) in Fig. 4. Here we have two independent contract user sets, $i = 1, 2$. The upper dash curve denotes $\dot{W}_{1\text{st}}(\theta)$, the highest C-MW of two independent sets. The lower dash curve denotes $\dot{W}_{2\text{nd}}(\theta)$, the second highest (i.e., lowest) C-MW of two independent sets. Regions I and II correspond to case (a) where $\dot{W}_{1\text{st}}(\theta) > 0$, and the remaining regions correspond to case (b) where $\dot{W}_{1\text{st}}(\theta) \leq 0$. It is easy to see that in Region I (or II), the 1st (or 2nd) independent set realizes the highest positive C-MW, i.e., $\dot{W}_{1\text{st}}(\theta) = \dot{\mathbf{w}}_1^{\text{Co}}(\theta) > 0$

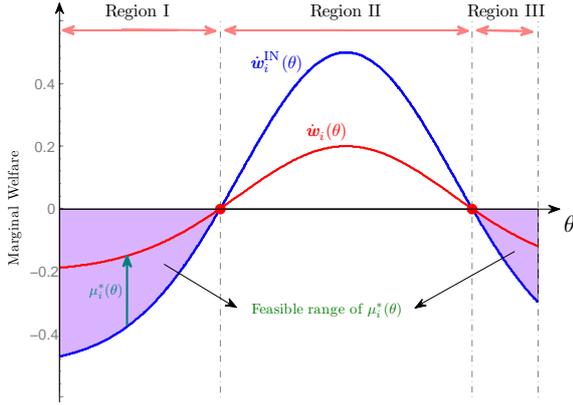


Figure 3 Illustration of $\mu_i^*(\theta)$ in Lemma 3

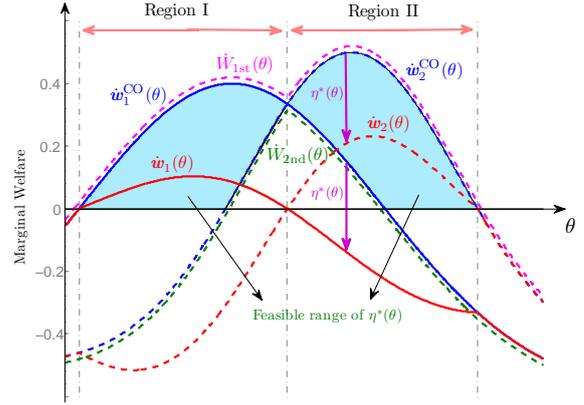


Figure 4 Illustration of $\eta^*(\theta)$ in Lemma 4

(or $\dot{W}_{1st}(\theta) = \dot{w}_2^{Co}(\theta) > 0$), and thus given any feasible $\eta^*(\theta)$ in the shadow area, the 1st (or 2nd) independent set has the unique positive marginal welfare.

By Lemmas 3 and 4, we can find that the optimal solution given by (28) is equivalent to such a solution that allocates spectrums to the independent contract user set with the highest and positive C-MW (and meanwhile the associated side market). Denote $\mathbf{i}_1 \triangleq \{i \in \{1, \dots, I_C\} \mid \dot{w}_i^{Co}(\theta) = \dot{W}_{1st}(\theta)\}$ as the collection of independent sets with the highest C-MW (over a spectrum $\{\theta, 1\}$). Formally, we have the following optimal solution.

LEMMA 5 (Optimal Allocation Strategy). For any idle spectrum θ , the optimal allocation strategy $\mathbf{A}_{ISC}^*(\theta)$ is given by: (i) if $i \notin \mathbf{i}_1$, then $a_i^*(\theta) = 0$; and (ii) if $i \in \mathbf{i}_1$, then

- (ii.a) $a_i^*(\theta) \in [0, 1]$ such that $\sum_{i \in \mathbf{i}_1} a_i^*(\theta) = 1$, if $\dot{W}_{1st}(\theta) > 0$;
- (ii.b) $a_i^*(\theta) \in [0, 1]$ such that $\sum_{i \in \mathbf{i}_1} a_i^*(\theta) \leq 1$, if $\dot{W}_{1st}(\theta) = 0$;
- (ii.c) $a_i^*(\theta) = 0$, if $\dot{W}_{1st}(\theta) < 0$;

Lemma 5 states that in the optimal solution, only the independent contract user sets with the highest positive C-MW may obtain an idle spectrum θ . If the C-MW is negative for all independent contract user sets (i.e., $\dot{W}_{1st}(\theta) < 0$), the spectrum is allocated to the 0-th independent contract user set (empty set) and the associated side market (the whole spot market). For example, in Fig. 4, we have $\mathbf{i}_1 = \{1\}$ in Regions I and I' (on the left side of Region I) and $\mathbf{i}_1 = \{2\}$ in Regions II and II' (on the right side of Region II). Thus, we have $a_1^*(\theta) = a_2^*(\theta) = 0$ in Regions I' and II' since $\dot{W}_{1st}(\theta) < 0$; $a_1^*(\theta) = 1, a_2^*(\theta) = 0$ in Region I; and $a_1^*(\theta) = 0, a_2^*(\theta) = 1$ in Region II.

When SUs have continuous and heterogeneous utilities, the realization of θ such that $\dot{W}_{1st}(\theta) = 0$ (i.e., case (b) in Lemma 5) or $|\mathbf{i}_1| > 1$ (i.e., multiple independent sets having the same highest C-MP) has a zero size support, and therefore can be ignored. Thus, the optimal allocation given by Lemma 5 is equivalent to:

$$a_i^*(\theta) = 1 \quad \text{if and only if} \quad \dot{w}_i^{Co}(\theta) > 0 \quad \text{and} \quad \dot{w}_i^{Co}(\theta) > \max_{j \neq i} \dot{w}_j^{Co}(\theta). \quad (31)$$

Recall the example in Figure 4. There are two distinct realizations of θ such that $\dot{W}_{1st}(\theta) = 0$, i.e., the boundary points between Region I and I' and between Region II and II'. In addition, there is only one distinct realization of θ such that $|\dot{z}_1| > 1$, i.e., the boundary point between Region I and II. Obviously, these realizations have a zero size support.

Now let us look into (31) to gain further insights. First, notice that:

$$\dot{\mathbf{w}}_i^{\text{Co}}(\theta) = \rho S \cdot \left(-z_0(\theta) + z_i(\theta) + \sum_{n=1}^N l_n^i \cdot (\tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot u_n(\theta)) - \sum_{n=1}^N l_n^i \cdot \lambda_n \right).$$

The first term $z_0(\theta)$ is the side welfare (from a spectrum $\{\theta, 1\}$) of the 0-th independent contract user set (empty set). That is, it is the welfare generated by the spot market users, when the spectrum is not allocated to any contract user. In this sense, it can be viewed as an *outside option* with regard to the futures market. The second term $z_i(\theta)$ is the side welfare of the i -th independent contract user set. That is, it is the welfare generated by the spot market users (in the associated side market), when the spectrum is allocated to the i -th independent contract user set. The third term $\sum_{n=1}^N l_n^i \cdot (\tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot u_n(\theta))$ denotes the direct welfare *improvement* (also called *welfare gain*) generated by the contract users in the i -th independent contract user set from a spectrum $\{\theta, 1\}$.²⁸ The last term $\sum_{n=1}^N l_n^i \cdot \lambda_n^*$ is the shadow prices of the i -th independent contract user set, which imposes an identical “shade” on the C-MW of the i -th independent set over all spectrums. It is used to rule out the relatively low welfare spectrums and assign the high welfare spectrums to the i -th independent contract user set.

Let $\Theta_i^+ \triangleq \{\theta \mid \dot{\mathbf{w}}_i^{\text{Co}}(\theta) > 0 \ \& \ \dot{\mathbf{w}}_i^{\text{Co}}(\theta) > \max_{j \neq i} \dot{\mathbf{w}}_j^{\text{Co}}(\theta)\}$ denote the set of spectrum information θ with which the i -th independent contract user set having the unique highest and positive C-MP. Since Θ_i^+ is determined by the shadow prices (vector) $\boldsymbol{\lambda}^* \triangleq (\lambda_1^*, \dots, \lambda_N^*)$, we can write Θ_i^+ as a function of $\boldsymbol{\lambda}^*$, denoted by $\Theta_i^+(\boldsymbol{\lambda}^*)$. The optimal solution is therefore equivalent to: $a_i^*(\theta) = 1$ if and only if $\theta \in \Theta_i^+(\boldsymbol{\lambda}^*)$. Accordingly, the set of spectrums allocated to a contract user n can be written as: $\Theta_n^+ = \bigcup_{i=1}^I l_n^i \odot \Theta_i^+$, where $l_n^i \odot \Theta_i^+ = \emptyset$ if $l_n^i = 0$ and $l_n^i \odot \Theta_i^+ = \Theta_i^+$ if $l_n^i = 1$. Furthermore, the expected number of spectrums allocated to a contract user n can be written as: $\mathbb{E}[d_n] = \rho S \int_{\theta \in \Theta_n^+(\boldsymbol{\lambda}^*)} f_{\Theta}(\theta) d\theta$. According to the dual constraints (D.3), the optimal shadow price λ_n^* is either 0 if the associated constraint (iii) is not tight, or otherwise a non-negative value that makes the associated constraint tight. Formally,

LEMMA 6 (Optimal Shadow Price λ_n^*). *The optimal shadow price λ_n^* is given by:*

$$\lambda_n^* = \max \left\{ 0, \arg_{\lambda_n} \rho S \int_{\theta \in \Theta_n^+(\boldsymbol{\lambda}_{-n}^*, \lambda_n)} f_{\Theta}(\theta) d\theta = D_n \right\}, \quad \forall n \in \mathcal{N},$$

where $\boldsymbol{\lambda}_{-n}^*$ is the vector of all independent contract user sets' shadow prices except that of the i -th independent set, i.e., $\boldsymbol{\lambda}_{-n}^* \triangleq (\lambda_1^*, \dots, \lambda_{n-1}^*, \lambda_{n+1}^*, \dots, \lambda_N^*)$.

After obtaining the optimal shadow price vector λ^* and substituting the optimal λ^* into the C-MWs of all independent contract user sets, we can easily determine the optimal allocation of any spectrum under any information $\{\theta, 1\}$ according to Lemma 5 or (31). Intuitively, the shadow price λ_n^* imposes a *vertical* (downward) shift on the C-MPs of those independent sets containing the contract user n , such that $\mathbb{E}[d_n]$ meets the demand D_n .

5.4. The Optimal Policy

We now summarize the *optimal policy* based on the optimal solution derived above. Simply speaking, a policy is a set of pre-defined **allocation rules**, specifying explicitly or implicitly the allocation of any spectrum under every possible information realization. We will show that the optimal solution in Lemma 5 is equivalent to such a policy that allocates every spectrum to an MWIS of the graph G with well-defined weights. Formally, we show this in the following theorem.

THEOREM 1 (Optimal Policy). *For any idle spectrum with information realization $\{\theta, 1\}$,*

- *Define the **weight** of each vertex of graph G in the following way:*
 - *The weight of each spot market user m is: v_m ;*
 - *The weight of each contract user n is: $\tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot u_n(\theta) - \lambda_n^*$;*
- *Allocate the spectrum $\{\theta, 1\}$ to an **MWIS** of the weighted graph G defined above.*

By the optimal policy in Theorem 1, every spectrum will be allocated to an MWIS of G , where the weight of each vertex (SU) is defined as his *current* valuation directly for spot market users or a function of his *current* valuation for contract users. That is, the weight is related to the realized actual information of the current spectrum and the optimal policy only, but independent of the information of future or past spectrums. In this sense, we disentangle the time-coupled spectrum allocation problem (which determines the spectrum allocations in all time slots jointly based on the complete network information) into a set of sub-problems. Each sub-problem determines the spectrum allocation in a particular time slot based on the currently realized information and the pre-derived optimal shadow prices. The coupling relationship of allocations in different time slots is captured by the pre-derived optimal shadow prices (in Lemma 6).

6. On-line VCG Mechanisms

In the previous section, we have shown in Theorem 1 that the E-SEM problem can be disentangled into a set of sub-problems, each determining the allocation of one particular spectrum. More specifically, every spectrum is allocated to an MWIS of the graph G , in which the weight of each vertex is related only to the utility of the associated SU over the spectrum, while independent of his or her utilities over other spectrums or the utility of other SUs.

Obviously, if the PR can observe all of the realized information of one spectrum, it is straightforward to determine the optimal allocation of that spectrum, based on Theorem 1. As mentioned previously, however, the SU's valuation is private information, and cannot be observed by the PR. That is, the network information is asymmetric between the PR and SUs. In this case, the real-time allocation of every spectrum essentially depends on the SUs' reports (also called *bids*) about their valuations. SUs may have incentives to falsely report their valuations to enlarge their welfare. Therefore, an incentive compatible mechanism is necessary for credibly eliciting the SUs' private valuations and then optimally allocating every spectrum.

6.1. VCG Mechanism

We first propose a VCG auction, which is incentive-compatible for SUs to truthfully report their private valuations, and is optimal for the real-time allocation of every spectrum. Essentially, a VCG auction charges each SU a fee if his or her report is *pivotal*, i.e., if his or her report changes the optimal allocation so as to harm other SUs. Let \tilde{v}_m and \tilde{u}_n denote the reports (i.e., bids) of spot market user m and contract user n . Denote $\tilde{\theta} \triangleq (\tilde{v}_1, \dots, \tilde{v}_M; \tilde{u}_1, \dots, \tilde{u}_N)$ as the bid vector of all SUs. Formally, the VCG based spectrum allocation and charging mechanism is shown as follows.

VCG AUCTION For any idle spectrum with a bid vector $\tilde{\theta}$,

- Define the **weight** of each vertex of graph G in the following way:
 - The weight of each spot market user m is: $x_m \triangleq \tilde{v}_m$;
 - The weight of each contract user n is: $x_n \triangleq \tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot \tilde{u}_n(\theta) - \lambda_n^*$;
- Allocate the spectrum to an **MWIS** of the above weighted graph G :
 - Denote $\mathcal{I}^* \in \mathcal{I}(G)$ as the allocated independent set, i.e., $\sum_{j \in \mathcal{I}^*} x_j \geq \sum_{j \in \mathcal{I}_i} x_j, \forall \mathcal{I}_i \in \mathcal{I}(G)$;
- Charge each SU k in the following way:
 - Denote $\mathcal{I}_{-k}^* \in \mathcal{I}(G_{-k})$ as an MWIS of G_{-k} , the graph consisting of all SUs except k ;
 - Charge the SU k a critical price defined as: $p_k \triangleq \sum_{j \in \mathcal{I}_{-k}^*} x_j - \sum_{j \in \mathcal{I}^* / \{k\}} x_j$.

The above mechanism is a standard VCG auction, and thus it leads to truth-telling. The novelty of our VCG auction is that the bid of each contract user n is not exogenously given, but rather related to the optimal shadow prices λ_n^* derived in the previous section. Hence, such a solution (i.e., the off-line policy and the on-line VCG auction) allows us to optimally allocate every spectrum in an on-line manner under stochastic and asymmetric network information.

It is notable that the above spectrum auction with spatial spectrum reuse is mathematically equivalent to the auction with single-minded buyers proposed in Nisan (2007). Specifically, in an auction with single-minded buyers, each bidder requests a specific bundle of items, and achieves a positive revenue if obtaining this bundle of items, and zero otherwise. We can view any two

bidders with non-conflicting requirements (i.e., there is no intersection between their requested bundles of items) as non-conflicting (independent) buyers, based on which we can construct such a conflict graph in which conflicting bidders are connected by an edge and non-conflicting bidders are disconnected. Accordingly, a set of independent bidders (i.e., those not connected by edges) can be chosen as auction winners at the same time. This is equivalent to the single-slot spectrum auction in our work. We would like to emphasize that our work is not restricted to the study of the single-slot spectrum auction (in a particular time slot). In fact, we focus on the auction of spectrum in a period of T time slots. This multi-slot spectrum allocation problem is essentially a combination of T coupled single-slot spectrum auction problems (which are coupled by the optimal shadow prices derived in advance). This is also the key difference between our work and the single-minded auction in Nisan (2007).

Next we provide some comments on this mechanism. First, as shown in Section 3.3, the contract user's actual payment is pre-defined, and determined only by the number of spectrums that he or she obtains, while independent of the utility. In the above VCG auction, however, the contract user's payment depends on his or her valuation. Actually, we can view a contract user's payment in the VCG auction as a subsidy that encourages the contract user to report truthfully. Second, if an SU k is not allocated a spectrum (i.e., $k \notin \mathcal{I}^*$), then we have $\mathcal{I}_{-k}^* = \mathcal{I}^*$, and thus his payment is $p_k = \sum_{j \in \mathcal{I}_{-k}^*} x_j - \sum_{j \in \mathcal{I}^* / \{k\}} x_j = 0$. Third, the above VCG auction relies on the solving of MWIS problems, which are well-known to be NP-hard. Thus, the VCG auction may not be suitable for large networks. This motivates us to further study a low-complexity mechanism instead.

6.2. VCG-like Mechanism

Due to the spatial spectrum reuse, the VCG auction in Mechanism 1 requires a solution to the MWIS problem, which is NP-hard. Thus, the above VCG auction is not suitable for on-line implementation. To this end, we further propose a low-complexity *VCG-like mechanism*, based on approximate MWIS algorithms with polynomial-time complexity. Formally, the VCG-like spectrum allocation and charging mechanism works in the following way.

VCG-LIKE MECHANISM For any idle spectrum with a bid vector $\tilde{\theta}$,

- Define the **weight** of each vertex in the graph G in the following way:
 - The weight of each spot market user m is: $x_m \triangleq \tilde{v}_m$;
 - The weight of each contract user n is: $x_n \triangleq \tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot \tilde{u}_n(\theta) - \lambda_n^*$;
- Allocate each spectrum in the following greedy way: Define the survival SU set $\mathcal{N}^\dagger = \mathcal{N}$
 - Do until $\mathcal{N}^\dagger = \emptyset$: Allocate the spectrum to the maximum weight SU in \mathcal{N}^\dagger , and
 Remove this SU as well as the neighboring SUs from \mathcal{N}^\dagger ;

- Denote $\mathcal{I}^\dagger \in \mathcal{I}(G)$ as the allocated independent SU set;
- Charge each SU k in the following way:
 - Denote $\mathcal{I}_{-k}^\dagger \in \mathcal{I}(G_{-k})$ as the allocated SU set using the above allocation mechanism on G_{-k} ;
 - Charge the SU k a critical price defined as: $p_k \triangleq \sum_{j \in \mathcal{I}_{-k}^\dagger} x_j - \sum_{j \in \mathcal{I}^\dagger / \{k\}} x_j$.

The key difference between this VCG-like mechanism and the previous VCG auction lies in the allocation strategy. Specifically, in the VCG auction, every spectrum is allocated to an MWIS of SUs; whereas, in the VCG-like mechanism, every spectrum is allocated to a set of SUs based on a greedy algorithm. Some important observations are as follows. First, the above VCG-like mechanism has a polynomial-time complexity. Second, the above VCG-like mechanism leads to truth-telling (under this greedy allocation mechanism). This can be easily proved by the monotonicity property and bid-independent property (see Krishna (2009)). Third, the above VCG-like mechanism is *not* optimal due to the lack of optimal algorithm for MWIS calculations. The welfare loss caused by this non-optimal VCG-like mechanism will be studied in the next section.

7. Welfare Loss Analysis

Now we study the welfare loss induced by the approximate MWIS algorithms. We first notice that the E-SEM problem in (25) depends only on the stochastic distribution information, and the optimal solution (policy) can be derived in an *off-line* manner. That is, we can enumerate all possible scenarios and calculate the corresponding solution beforehand. In this sense, we can always apply the precise MWIS algorithm to compute the off-line optimal policy exactly. Hence, the approximate MWIS algorithms only affect the real-time allocation of every spectrum (which must be determined in an *on-line* manner).

In the previous section, we have proposed a specific on-line VCG-like mechanism (based on a greedy approximate MWIS algorithm) for the real-time allocation of every spectrum. Now we characterize the welfare loss under *generic* VCG-like mechanisms, or equivalently, under generic approximate MWIS algorithms. This quantitative analysis helps us to achieve a desirable tradeoff between performance and complexity.

7.1. Precise and Approximate C-MWs

We first examine how an approximate MWIS algorithm affects the achieved performance. By Lemma 5, the optimal allocation of every spectrum θ relies on the *accurate calculations* of $z_i(\theta), \forall i = 0, 1, \dots, I_C$, i.e., the side welfare of the i -th independent contract user set. With the above VCG-like mechanism, however, we can only get an approximate (degraded) side welfare $\tilde{z}_i(\theta)$ with certain performance guarantee. This degradation will potentially change the relationship between C-MWs, and further change the spectrum allocation. For example, a relatively larger (smaller) degradation

on the side welfare $z_i(\theta)$ will lead to a smaller (larger) C-MW, and therefore a smaller (larger) allocation probability to the i -th independent contract user set.

Without loss of generality, we treat the performance ratio (i.e., the ratio of the achieved performance and the optimal performance) of the adopted approximate MWIS algorithm as a random variable drawn from $[0, 1]$. Let random variable ϵ_i denote the performance ratio of the MWIS problem of calculating $z_i(\theta)$, and $\epsilon \triangleq (\epsilon_0, \epsilon_1, \dots, \epsilon_{I_c})$ denote the random vector consisting of the performance ratios of all $I_c + 1$ MWIS problems. Let $\epsilon_i \in [0, 1]$ denote a particular realization of the random variable ϵ_i , and $\epsilon \triangleq (\epsilon_0, \epsilon_1, \dots, \epsilon_{I_c})$ denote a particular realization of the random vector ϵ . For simplicity and without loss of generality, we assume that all performance ratios in vector ϵ are independent of each other, but they are not necessarily identically distributed.

For any spectrum $\{\theta, 1\}$, the *precise* C-MW of the i -th independent contract user set is

$$\dot{w}_i^{\text{Co}}(\theta) = \rho S \cdot \left(-z_0(\theta) + z_i(\theta) + \sum_{n=1}^N l_n^i \cdot (\tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot u_n(\theta) - \lambda_n^*) \right), \quad (32)$$

while the *approximate* C-MW under certain performance ratio realization of ϵ is:

$$\tilde{w}_i^{\text{Co}}(\theta) = \rho S \cdot \left(-\epsilon_0 \cdot z_0(\theta) + \epsilon_i \cdot z_i(\theta) + \sum_{n=1}^N l_n^i \cdot (\tau_n \cdot \hat{P}_n + (1 - \tau_n) \cdot u_n(\theta) - \lambda_n^*) \right). \quad (33)$$

7.2. Welfare Loss and Performance Ratio

Now we study the welfare loss and the achievable performance ratio. We can divide the whole spectrum range Θ into $I_c + 1$ parts based on the allocated independent contract user set under the optimal allocation policy: (i) Θ_i , $i = 1, \dots, I_c$, the spectrums intended for the i -th independent contract user set; and (ii) Θ_0 , the spectrums intended for the spot market only. Formally,

$$\begin{cases} \Theta_i = \{\theta \in \Theta \mid \dot{w}_i^{\text{Co}}(\theta) > 0 \ \& \ \dot{w}_i^{\text{Co}}(\theta) > \max_{j \neq i} \dot{w}_j^{\text{Co}}(\theta)\}, & i = 1, \dots, I_c. \\ \Theta_0 = \{\theta \in \Theta \mid \dot{w}_i^{\text{Co}}(\theta) < 0, \forall i = 1, \dots, I_c\}. \end{cases}$$

In the following, we will first derive the welfare loss (upper-bound) in each part of Θ . Then we characterize the achievable performance ratio (lower-bound).

7.2.1. Welfare Loss in Θ_i We first consider the welfare loss on spectrums in Θ_i . Under the optimal allocation policy based on precise C-MWs, every idle spectrum θ in Θ_i is allocated to the i -th independent contract user set (and the associated side market) with an instantaneous welfare:

$$z_i(\theta) + \sum_{n=1}^N l_n^i \cdot (1 - \tau_n) \cdot u_n(\theta).$$

The first term denotes the side welfare, and the second term denotes the (quality-based) utility of the i -th independent contract user set. Note that we ignore the demand-related utility of contract

user n , since it is constant and not affected by the mis-allocation; we also ignore the potential utility loss induced by under-allocation here, which will be analyzed specifically later.

Obviously, with precise C-MWs (using optimal MWIS algorithms), the i -th independent contract user set has the highest positive C-MW among all independent sets. With approximate C-MWs (using approximate MWIS algorithms), however, there are possibly three different outcomes, each leading to a different allocation and thus a different welfare loss. Specifically,

(1) *The i -th independent contract user set itself has the highest and positive approximate C-MW, i.e., $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta) > 0$ and $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta) > \max_{j \neq i} \tilde{\mathbf{w}}_j^{\text{Co}}(\theta)$.* In this case, the spectrum will still be allocated to the i -th independent contract user set with a degraded instantaneous welfare $\epsilon_i \cdot z_i(\theta) + \sum_{n=1}^N l_n^i \cdot (1 - \tau_n) \cdot u_n(\theta)$. Thus, the welfare loss can be written as:

$$\psi_i(\theta) \triangleq (1 - \epsilon_i) \cdot z_i(\theta).$$

(2) *The i' -th (with $i' \neq i$) independent contract user set has the highest and positive approximate C-MW, i.e., $\tilde{\mathbf{w}}_{i'}^{\text{Co}}(\theta) > 0$ and $\tilde{\mathbf{w}}_{i'}^{\text{Co}}(\theta) > \max_{j \neq i'} \tilde{\mathbf{w}}_j^{\text{Co}}(\theta)$.* In this case, the spectrum will be mistakenly allocated to the i' -th independent set instead with an instantaneous welfare $\epsilon_{i'} \cdot z_{i'}(\theta) + \sum_{n=1}^N l_n^{i'} \cdot (1 - \tau_n) \cdot u_n(\theta)$. Thus, the welfare loss can be written as:

$$\psi_{i'}(\theta) \triangleq z_i(\theta) - \epsilon_{i'} \cdot z_{i'}(\theta) + \sum_{n=1}^N (l_n^i - l_n^{i'}) \cdot (1 - \tau_n) \cdot u_n(\theta).$$

(3) *All independent contract user sets have the negative approximate C-MWs, i.e., $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta) < 0, \forall i = 1, \dots, I_C$.* In this case, the spectrum will be mistakenly allocated to the spot market only with an instantaneous welfare $\epsilon_0 \cdot z_0(\theta)$. Thus, the welfare loss can be written as:

$$\psi_0(\theta) \triangleq z_i(\theta) - \epsilon_0 \cdot z_0(\theta) + \sum_{n=1}^N l_n^i \cdot (1 - \tau_n) \cdot u_n(\theta).$$

Denote $\mathbf{e}_i^i(\theta) \triangleq \{\epsilon \in \epsilon \mid \tilde{\mathbf{w}}_i^{\text{Co}}(\theta) > 0 \ \& \ \tilde{\mathbf{w}}_i^{\text{Co}}(\theta) > \max_{j \neq i} \tilde{\mathbf{w}}_j^{\text{Co}}(\theta)\}$ as the set of performance ratio ϵ such that the i -th independent contract user set itself has the highest and positive approximate C-MW (i.e., case [1] mentioned above), $\mathbf{e}_i^{i'}(\theta) \triangleq \{\epsilon \in \epsilon \mid \tilde{\mathbf{w}}_{i'}^{\text{Co}}(\theta) > 0 \ \& \ \tilde{\mathbf{w}}_{i'}^{\text{Co}}(\theta) > \max_{j \neq i'} \tilde{\mathbf{w}}_j^{\text{Co}}(\theta)\}$ as the set of ϵ such that the i' -th (where $i' \neq i$) independent contract user set has the highest and positive approximate C-MW (i.e., case [2] mentioned above), and $\mathbf{e}_i^0(\theta) \triangleq \{\epsilon \in \epsilon \mid \tilde{\mathbf{w}}_i^{\text{Co}}(\theta) < 0, \forall i = 1, \dots, I_C\}$ as the set of ϵ such that all independent contract user sets have negative approximate C-MWs (i.e., case [3] mentioned above).²⁹ It is easy to see that $\bigcup_{j=0}^{I_C} \mathbf{e}_i^j$ equals ϵ , i.e., the whole range of ϵ . Here, the subscript and superscript in $\mathbf{e}_i^j(\theta)$ denote the indexes of allocated independent contract user sets under precise C-MWs and under approximate C-MWs, respectively. For convenience, we will omit the parameter θ in $\mathbf{e}_i^j(\theta)$, as long as there is no confusion caused.

Based on the above, the average welfare loss on every spectrum in Θ_i is formally given by

$$\begin{aligned} & \int_{\theta \in \Theta_i} \left(\int_{\epsilon \in \mathbf{e}_i^1} \psi_i(\theta) \cdot f_\epsilon(\epsilon) d\epsilon + \int_{\epsilon \in \mathbf{e}_i^0} \psi_0(\theta) \cdot f_\epsilon(\epsilon) d\epsilon + \sum_{i' \neq i} \int_{\epsilon \in \mathbf{e}_i^{i'}} \psi_{i'}(\theta) \cdot f_\epsilon(\epsilon) d\epsilon \right) \cdot f_\Theta(\theta) d\theta \\ = & \int_{\theta \in \Theta_i} \left(\int_{\epsilon \in \mathbf{e}_i^1} \psi_i(\theta) f_\epsilon(\epsilon) d\epsilon + \int_{\epsilon \in \mathbf{e}_i^0} (\psi_0(\theta) - \psi_i(\theta)) f_\epsilon(\epsilon) d\epsilon + \sum_{i' \neq i} \int_{\epsilon \in \mathbf{e}_i^{i'}} (\psi_{i'}(\theta) - \psi_i(\theta)) f_\epsilon(\epsilon) d\epsilon \right) f_\Theta(\theta) d\theta \\ \leq & \int_{\theta \in \Theta_i} \left((1 - \bar{\epsilon}_i) \cdot z_i(\theta) - \int_{\epsilon \in \mathbf{e}_i^0} Y_i \cdot f_\epsilon(\epsilon) d\epsilon + \sum_{i' \neq i} \int_{\epsilon \in \mathbf{e}_i^{i'}} (Y_{i'} - Y_i) \cdot f_\epsilon(\epsilon) d\epsilon \right) \cdot f_\Theta(\theta) d\theta \\ = & \int_{\theta \in \Theta_i} (1 - \bar{\epsilon}_i) \cdot z_i(\theta) \cdot f_\Theta(\theta) d\theta - Y_i \cdot \beta_i^0 + \sum_{i'=1}^{I_c} (Y_{i'} - Y_i) \cdot \beta_i^{i'}, \end{aligned}$$

where

- $\bar{\epsilon}_i = \mathbb{E}[\epsilon_i] \triangleq \int_{\epsilon \in \mathbf{e}_i} \epsilon_i f_\epsilon(\epsilon) d\epsilon$ is the average performance ratio of the approximate MWIS algorithm in calculating $z_i(\theta)$, i.e., the expectation of random variable ϵ_i ;
- $Y_i = \sum_{n=1}^N l_n^i \cdot X_n$ is the sum of X_n of all contract users in the i -th independent set, where $X_n \triangleq \hat{P}_n - \lambda_n^*$ is a constant associated with contract user n only;
- $\int_{\epsilon \in \mathbf{e}_i^0} f_\epsilon(\epsilon) d\epsilon$ is the probability of $\epsilon \in \mathbf{e}_i^0(\theta)$, i.e., the probability of mis-allocating a spectrum $\theta \in \Theta_i$ to the 0-th independent contract user set (or the spot market only) under $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta), \forall i$.
- $\beta_i^0 = \int_{\theta \in \Theta_i} \int_{\epsilon \in \mathbf{e}_i^0} f_\epsilon(\epsilon) f_\Theta(\theta) d\epsilon d\theta$ is the average mis-allocation probability of spectrums intended for the i -th independent contract user set to the 0-th one. That is, β_i^0 is the average probability of transferring spectrums from the i -th independent set to the 0-th one;
- $\int_{\epsilon \in \mathbf{e}_i^{i'}} f_\epsilon(\epsilon) d\epsilon$ is the probability of $\epsilon \in \mathbf{e}_i^{i'}(\theta)$, i.e., the probability of mis-allocating a spectrum $\theta \in \Theta_i$ to the i' -th independent set under approximate C-MWs $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta), \forall i$.
- $\beta_i^{i'} = \int_{\theta \in \Theta_i} \int_{\epsilon \in \mathbf{e}_i^{i'}} f_\epsilon(\epsilon) f_\Theta(\theta) d\epsilon d\theta$ is the average mis-allocation probability of spectrum intended for the i -th independent contract user set to the i' -th one. That is, $\beta_i^{i'}$ is the average probability of transferring spectrums from the i -th independent set to the i' -th one;
- The inequality follows because: (i) $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta) < 0$ if $\epsilon \in \mathbf{e}_i^0$, and $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta) < \tilde{\mathbf{w}}_{i'}^{\text{Co}}(\theta)$ if $\epsilon \in \mathbf{e}_i^{i'}$; and (ii) $\tilde{\mathbf{w}}_i^{\text{Co}}(\theta) = \psi_0(\theta) - \psi_i(\theta) + Y_i$. Thus, we have: $\psi_0(\theta) - \psi_i(\theta) < -Y_i$, and $\psi_{i'}(\theta) - \psi_i(\theta) < Y_{i'} - Y_i$.

Thus, we can obtain an upper-bound of the expected welfare loss on every spectrum in Θ_i as

$$\text{Loss}^{(i)} \leq \Phi_i - Y_i \cdot \beta_i^0 + \sum_{i'=1}^{I_c} (Y_{i'} - Y_i) \cdot \beta_i^{i'}, \quad (34)$$

where $\Phi_i \triangleq \int_{\theta \in \Theta_i} (1 - \bar{\epsilon}_i) \cdot z_i(\theta) f_\Theta(\theta) d\theta$ is the average side welfare loss of the i -th independent contract user set (over every spectrum in Θ_i).

7.2.2. Welfare Loss in Θ_0 Using a similar analysis to that in (34), we can obtain an upper-bound of the expected welfare loss on every spectrum in Θ_0 as:

$$\text{Loss}^{(0)} \leq \Phi_0 + \sum_{i=1}^{I_c} Y_i \cdot \beta_0^i. \quad (35)$$

where $\Phi_0 \triangleq \int_{\theta \in \Theta_0} (1 - \bar{\epsilon}_0) \cdot z_0(\theta) f_\Theta(\theta) d\theta$ is the average side welfare loss of the 0-th independent contract user set (over every spectrum in Θ_0), and $\beta_0^i = \int_{\theta \in \Theta_0} \int_{\epsilon \in \mathbf{e}_i^0} f_\epsilon(\epsilon) f_\Theta(\theta) d\epsilon d\theta$ is the average probability of transferring spectrums from the 0-th independent contract user set to the i -th one.

By (34) and (35), the average welfare loss on every spectrum is bounded by:

$$\begin{aligned}
\sum_{i=0}^{I_c} \text{Loss}^{(i)} &\leq \sum_{i=0}^{I_c} \Phi_i + \sum_{i=1}^{I_c} \left(\sum_{i'=1}^{I_c} (Y_{i'} - Y_i) \cdot \beta_i^{i'} - Y_i \cdot \beta_i^0 \right) + \sum_{i=1}^{I_c} Y_i \cdot \beta_0^i \\
&= \sum_{i=0}^{I_c} \Phi_i + \sum_{i=1}^{I_c} \left(\sum_{i'=1}^{I_c} (Y_i \cdot \beta_{i'}^i - Y_i \cdot \beta_i^{i'}) - Y_i \cdot \beta_i^0 \right) + \sum_{i=1}^{I_c} Y_i \cdot \beta_0^i \quad (36) \\
&= \sum_{i=0}^{I_c} \Phi_i - \sum_{i=1}^{I_c} Y_i \cdot \Delta_i \triangleq \sum_{i=0}^{I_c} \Phi_i - \sum_{n=1}^N X_n \cdot \gamma_n,
\end{aligned}$$

where

- $\Delta_i \triangleq \sum_{i'=1}^{I_c} \beta_i^{i'} + \beta_i^0 - \sum_{i'=1}^{I_c} \beta_{i'}^i - \beta_0^i$ is the *change* on average allocation probability of a spectrum to the i -th independent contract user set, in which the first two terms denote the allocation probability decrease (induced by mis-allocating the spectrums intended for the i -th independent set to others), and the last two terms denote the allocation probability increase;
- $\gamma_n \triangleq \sum_{i=1}^{I_c} l_n^i \cdot \Delta_i$ is the *change* on average allocation probability of a spectrum to contract user n , which is the sum of the probability changes of all related independent sets;
- The second line follows because $\sum_{i=1}^{I_c} \sum_{i'=1}^{I_c} Y_{i'} \beta_i^{i'} = \sum_{i'=1}^{I_c} \sum_{i=1}^{I_c} Y_i \beta_{i'}^i$ by exchanging the indices of two summing operations, and the last line follows because $Y_i = \sum_{n=1}^N l_n^i \cdot X_n$.

7.2.3. Welfare Loss by Spectrum Under-Allocation It is important to note that the welfare loss in (36) is induced by the **non-optimal utilization** of spectrum due to the mis-allocation, without including the potential utility loss of contract users (defined in Eq. [8]) induced by the spectrum **under-allocation** due to the mis-allocation.

Now we study the welfare loss of contract users induced by the spectrum under-allocation. From (8) we can easily find that such a potential welfare loss is determined by the expected number of spectrums that each contract user obtains. By Lemma 2, none of the contract users can obtain spectrums more than his or her demand. Thus, any negative change on a contract user n 's average allocation probability γ_n would impose certain welfare loss for contract user n , while any positive change on γ_n may or may not bring welfare gain for contract user n (depending on whether the contract user already obtained the demanded spectrums). Formally, the average welfare loss induced by the spectrum under-allocation (over every spectrum) is bounded by:

$$\text{Loss}^{\text{WL}} \leq \sum_{n=1}^N \hat{P}_n \cdot \left[\sum_{i=1}^{I_c} l_n^i \cdot \Delta_i \right]^+ = \sum_{n=1}^N \hat{P}_n \cdot [\gamma_n]^+. \quad (37)$$

7.2.4. Overall Welfare Loss Combining (36) and (37), the expected overall welfare loss on every spectrum is bounded by:

$$\begin{aligned}
\text{Loss}^{\text{ALL}} &= \sum_{i=0}^{I_c} \text{Loss}^{(i)} + \text{Loss}^{\text{WL}} \\
&\leq \sum_{i=0}^{I_c} \Phi_i - \sum_{n=1}^N X_n \cdot \gamma_n + \sum_{n=1}^N \hat{P}_n \cdot [\gamma_n]^+ \quad (38) \\
&= \sum_{i=0}^{I_c} (1 - \bar{\epsilon}_i) \cdot \mathbb{E}[Z_i]^* - \sum_{n=1}^N X_n \cdot \gamma_n + \sum_{n=1}^N \hat{P}_n \cdot [\gamma_n]^+,
\end{aligned}$$

where $\mathbb{E}[Z_i]^* \triangleq \int_{\theta \in \Theta_i} z_i(\theta) f_{\Theta}(\theta) d\theta$. Since $\mathbb{E}[Z_i]^* = \frac{1}{\rho S} \cdot \mathbb{E}[W_i^S]^*$, where $\mathbb{E}[W_i^S]^*$ is the expected side welfare of the i -th independent set over all S spectrums (defined in Eq. [22]) under the optimal allocation, $\mathbb{E}[Z_i]^*$ represents the expected side welfare of the i -th independent set over every spectrum.

7.2.5. Achievable Performance Ratio Let $\bar{\epsilon} \triangleq \min_i \bar{\epsilon}_i$ denote the minimum expected performance ratio of all MWIS algorithms applied. Subtracting the overall welfare loss from the optimal welfare (defined in Eq. [24]), we have the following average achievable welfare

$$\begin{aligned} W_F^{\text{ACH}} &= W_F^{\text{OPT}} - \rho S \cdot \text{LOSS}^{\text{ALL}} \\ &= \sum_{i=0}^{I_c} \mathbb{E}[W_i^S]^* + \sum_{n=1}^N (\tau_n \mathbb{E}[w_n^c]^* + (1 - \tau_n) \mathbb{E}[\tilde{w}_n^c]^*) - \rho S \cdot \text{LOSS}^{\text{ALL}} \\ &\geq \sum_{i=0}^{I_c} \bar{\epsilon} \cdot \mathbb{E}[W_i^S]^* + \sum_{n=1}^N (\tau_n \mathbb{E}[w_n^c]^* + (1 - \tau_n) \mathbb{E}[\tilde{w}_n^c]^*) + \rho S \cdot \sum_{n=1}^N (X_n \gamma_n - \hat{P}_n [\gamma_n]^+), \end{aligned} \quad (39)$$

where $\rho S \cdot \text{LOSS}^{\text{ALL}}$ is the overall welfare loss over all S spectrums, and $W_F^{\text{OPT}} \triangleq \mathbb{E}[W]^* = \sum_{i=0}^{I_c} \mathbb{E}[W_i^S]^* + \sum_{n=1}^N (\tau_n \mathbb{E}[w_n^c]^* + (1 - \tau_n) \mathbb{E}[\tilde{w}_n^c]^*)$ is the overall welfare (spectrum efficiency) under the optimal allocation (using precise C-MWs), wherein $(\tau_n \mathbb{E}[w_n^c]^* + (1 - \tau_n) \mathbb{E}[\tilde{w}_n^c]^*)$ is the expected welfare of the contract user n under the optimal allocation. Note that $\rho S \cdot \gamma_n$ is the negative change of the expected number of spectrums allocated to contract user n .

Let us elaborate on the insights behind (39). First, there is a welfare loss ratio $(1 - \bar{\epsilon})$ on the spot market (either the whole spot market, or the side spot market of a particular independent contract user set), shown by the first term in (39). This welfare loss ratio is the same as the performance loss ratio of the adopted approximate MWIS algorithm. Second, there is an average welfare loss $t_n \triangleq X_n \cdot \gamma_n - \hat{P}_n \cdot [\gamma_n]^+$ for each contract user n . Obviously, if $\gamma_n > 0$, which implies that the number of spectrums for contract user n is decreased (under approximate C-MWs), we have: $t_n = X_n \cdot \gamma_n - \hat{P}_n \cdot \gamma_n = -\lambda_n^* \cdot \gamma_n \leq 0$; and if $\gamma_n < 0$, which implies the number of spectrum for contract user n is increased (under approximate C-MWs), we also have: $t_n = X_n \cdot \gamma_n < 0$.

Finally, we have the following bound for the relative welfare ratio (WR):

$$\text{WR} = \frac{W_F^{\text{ACH}}}{W_F^{\text{OPT}}} \geq \bar{\epsilon} + \sum_{n=1}^N \frac{(1 - \bar{\epsilon}) \cdot (\tau_n \mathbb{E}[w_n^c]^* + (1 - \tau_n) \mathbb{E}[\tilde{w}_n^c]^*) + \rho S \cdot t_n}{\mathbb{E}[W]^*}. \quad (40)$$

Note that the above discussion concerns the theoretical *lower-bound* of the achieved performance ratio. Even if this bound is poor, the actual welfare may still be maintained at a good level. We will show by our simulation results (Figs. 7 and 8) that even with the worst MWIS algorithm in our simulations, the actual welfare ratio is more than 70%; whereas, the theoretical bound is lower than $\bar{\epsilon}$ (i.e., 50%). Moreover, in our technical report, Gao et al. (2012b), we will show that the above welfare ratio is *conservatively* tight, in the sense that there does not exist another bound that is always better than our proposed one in any case.

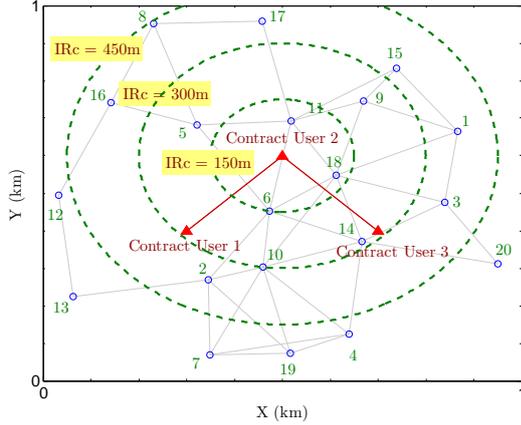


Figure 5 Network topology.

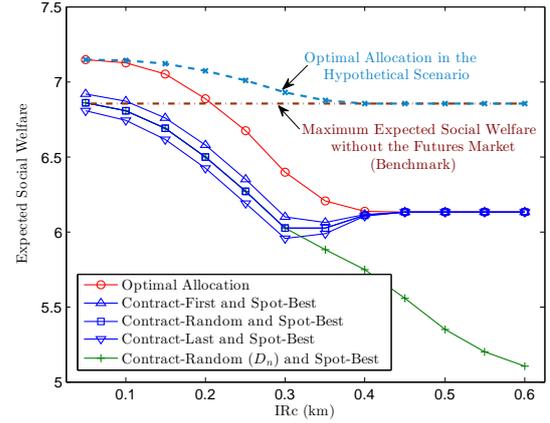


Figure 6 Expected social welfare.

8. Simulation Results

We perform simulations in MATLAB. Unless otherwise stated, we will use the following default setting: (i) all spot market SUs are randomly distributed in a square area with $1000\text{m} \times 1000\text{m}$; (ii) all contract SUs are placed on some predetermined locations in the square area; and (iii) all SUs' valuations are uniformly distributed in $[0, 1]$, i.e., $F_{v_m}(x) = F_{u_n}(x) = x, \forall x \in [0, 1]$. Let IRs and IRc denote the interference ranges of the spot market users and contract users, respectively.

8.1. Optimal Social Welfare $\mathbb{E}[W]^*$

Fig. 5 presents a realization of the random network topology with 20 randomly distributed spot market users and 3 contract users located at $(300\text{m}, 400\text{m})$, $(500\text{m}, 600\text{m})$ and $(700\text{m}, 400\text{m})$, respectively. The interference range of spot market users is $\text{IRs} = 300\text{m}$. The ellipses denote different values of IRc, i.e., the interference range of contract users. Given a particular IRc, the spot market users outside the ellipse(s) of a contract user (set) form the corresponding side market. The edges between contract users denote their interference relationships under a certain IRc (e.g., when $\text{IRc} = 300\text{m}$). In Fig. 5's example, contract users 1 and 2 (or 2 and 3) interfere with each other, while contract users 1 and 3 do not. Therefore, there are totally four independent contract user sets: $\{1\}$, $\{2\}$, $\{3\}$, and $\{1, 3\}$.

Fig. 6 presents the expected social welfare *vs* IRc under different allocation strategies. Note that each value in the figure is the average of 1,000 simulations, and each simulation corresponds to a particular realization of the random network topology. The dashed straight line denotes the maximum expected social welfare *without* the futures market (benchmark), which is achieved by optimally allocating each spectrum to the spot market (i.e., to an MWIS of the spot market). Obviously, this benchmark is independent of the interference range of contract users (IRc). The

upper dash curve (with mark “×”) denotes the maximum expected social welfare in a hypothetical hybrid market, which consists of both the spot market and the futures market, but ignores the potential utility loss of contract users (due to the spectrum under-allocation) in the futures market. The red curve (with mark “○”) denotes the maximum expected social welfare under the optimal allocation strategy in our proposed hybrid market. The lower three blue curves denote the expected social welfares under allocation strategies “Contract-Last and Spot-Best” (with mark “△”), “Contract-Random and Spot-Best” (with mark “□”), and “Contract-Last and Spot-Best” (with mark “▽”), respectively. Under the “Contract-Last/Random/First and Spot-Best” strategies, the PR allocates $\mathbb{E}[D_n]$ spectrums with the lowest, arbitrary, and highest utilities to each contract user n , respectively. Here, $\mathbb{E}[D_n]$ is the expected number of spectrum contract user n achieves under the optimal allocation (derived in our paper). The green curve (with mark “+”) denotes the expected social welfare under such an allocation strategy that randomly allocates D_n (i.e., the demand of contract user n) spectrums to each contract user n , called “Contract-Random (D_n) and Spot-Best”. Note that under all of the above four strategies, when a spectrum is allocated to a contract user (or a set of independent contract users) in the futures market, it will also be allocated to an MWIS of the associated side (spot) market at the same time. This is the reason that we call them “Spot-Best”.

Fig. 6 shows that our proposed optimal allocation strategy outperforms other strategies significantly. The maximum expected social welfare under the proposed optimal strategy is on average 20% higher than that under “Contract-Random (D_n) and Spot-Best”. This performance gap becomes even larger with more contract users. Moreover, the performances under “Contract-First/Random/Last and Spot-Best” are very close to each other. This is because different SUs’ valuations for the same spectrum are independent of each other; hence, a spectrum that brings a high or low valuation for a contract user may not bring high or low valuations for spot market users. When IR_c is small, the performance under “Contract-Last and Spot-Best” is a bit higher than that under “Contract-Random and Spot-Best”, which is further a bit higher than that under “Contract-First and Spot-Best”. This is because allocating a spectrum to a spot market user can generate a higher effective welfare than allocating a spectrum (with the same valuation) to a contract user (as the latter is discounted by τ). When IR_c is large, the performances under “Contract-First/Random/Last and Spot-Best” are identical, and all converge to the maximum expected social welfare achieved in the proposed optimal allocation. This is because $\mathbb{E}[D_n] \rightarrow 0$ in this case; thus, all spectrums are optimally allocated to the spot market as in our proposed optimal allocation. Notice that this maximum expected social welfare is less than the benchmark (without the futures market), due to the utility loss of contract users in our hybrid market. Finally, the performance under “Contract-Random (D_n) and Spot-Best” is same as that under “Contract-Random

and Spot-Best” when IRc is small; whereas, it is worse than that under “Contract-Random and Spot-Best” when IRc is large. This reason is as follows. When IRc is small, $\mathbb{E}[D_n] = D_n$ (i.e., the contract user demand is fully satisfied) due to the weak conflict between spot market users and contract users); when IRc is large, $\mathbb{E}[D_n] \rightarrow 0$ due to the severe conflict between spot market users and contract users.

Fig. 6 further shows that the maximum expected social welfare with the futures market (under the proposed optimal strategy) may be smaller than the maximum expected social welfare without the futures market (benchmark), when, for example, IRc is large. That is, accepting contract users in the futures market may reduce the total achieved social welfare, compared to that under a pure spot market. Notice that this welfare gap can serve as a guidance used to decide whether or not to accept futures contracts prior to the spectrum allocation optimization considered in this work. For example, in this figure, it is more efficient to accept futures contracts when IRc is smaller than 0.2, while it is more efficient in a pure spot market when IRc is larger than 0.2. The reason that a hybrid market may be worse than a pure spot market is as follows. As long as a contract is accepted, the PR has the responsibility to fulfill the contract; otherwise, the contract user will suffer certain utility loss, which harms the entire social welfare. In the benchmark case, however, we consider the spot market only, while removing all contract users as well as their impacts on the social welfare. That is, in this benchmark case, the PR will not allocate any spectrum to contract users and there will be no penalty for this.

For a fair and more meaningful comparison, we propose the above hypothetical hybrid market scenario in our simulation. Specifically, in the hypothetical scenario, there is no utility loss for contract users when their demands are not satisfied. Clearly, the benchmark solution (ignoring both the futures market and utility loss for contract users) is a feasible solution of operating such a hypothetical scenario (considering the futures market but ignoring the utility loss for contract users), but not a feasible solution of operating our proposed hybrid market (considering both the futures market and utility loss for contract users). We can see that the maximum social welfare in the hypothetical scenario is always higher than the maximum that can be achieved in the our proposed hybrid market and the maximum that can be achieved in the benchmark case. More specifically, when the IRc is small, the maximum social welfare in the hypothetical scenario is close to that in our proposed hybrid market. This is because with a small IRc , the needs of contract users are likely to be fulfilled in both the proposed and the hypothetical hybrid markets, hence leading to a small difference between these two market scenarios. When the IRc is large, however, the maximum social welfare in the hypothetical scenario is close to that in the benchmark case. This is because with a large IRc , the needs of contract users are likely to be violated (i.e., assigned few or even no spectrum by the PR) in both the proposed and the hypothetical hybrid markets, hence

leading to a large difference between these two market scenarios (due to the welfare loss in our proposed scenarios), but a small difference between the hypothetical scenario and the benchmark case (as in these two cases, spectrums are allocated to the spot market only, without any welfare loss from contract users in the futures market).

8.2. Welfare Loss by Approximate MWIS Algorithms

Now we study the welfare loss and the relative welfare ratio (WR) under the approximate MWIS algorithms. We take the performance ratio of the adopted approximate MWIS algorithm as a random variable ϵ drawn from $[e_0, 1]$ uniformly and independently, where $e_0 \in [0, 1]$. Accordingly, the average performance ratio is $\bar{\epsilon} = 0.5 \cdot (1 + e_0)$.

Figs. 7 and 8 present the achieved WR and the analytical WR bound given in (40) under different approximate MWIS algorithms (with different average performance ratio $\bar{\epsilon}$). The upper (red) five curves denote the achieved WR when IRc increases from 100m to 500m, and the lower (blue) five curves denote the analytical WR bounds when IRc increases from 100m to 500m. From Figs. 7 and 8 we can see that WR decreases with IRc. This is because a larger IRc means a smaller side welfare $z_i(\theta)$ for each independent contract user set \mathcal{I}_i^C , and the side welfare $z_0(\theta)$ becomes relatively more significant. Thus, the C-MW $\dot{w}_i^{Co}(\theta)$ defined in Definition 8 is mainly determined by $z_0(\theta)$, and the welfare loss is mainly due to the degradation of $z_0(\theta)$. Additionally, with certain degradation of $z_0(\theta)$, the spectrum intended for the 0-th independent contract user set (i.e., the spot market only) may be mis-allocated to another independent set (e.g., the i -th one) with a welfare loss $z_0(\theta) - \epsilon_i \cdot z_i(\theta) + \sum_{n=1}^N l_n^i (1 - \tau_n) u_n(\theta)$. Obviously, the smaller the side welfare $z_i(\theta)$, the higher the possible welfare loss. We can also see that the gap between the relative WR (either the achieved WR or the analytical WR bound) under different values of IRc becomes smaller, when IRs becomes larger. For example, when IRs=100m (in Fig. 7), the gap between the achieved WR under IRc=100m and IRc=500m is $80\% - 70\% = 10\%$ in the worst case ($\bar{\epsilon} = 0.5$); while when IRs = 300m (in Fig. 8), this gap is only $78\% - 72\% = 6\%$. This is because a larger IRs implies a lower level of $z_i(\theta)$; thus, the impact of IRc on $z_i(\theta)$ becomes less significant on the C-MWs.

Figs. 7 and 8 show that even with the worst approximate MWIS algorithm in our simulations (i.e., $\bar{\epsilon} = 0.5$ when ϵ drawn from $[0, 1]$), the achieved social welfare can reach more than 70% of the optimal social welfare. This implies that the optimal allocation (policy) proposed in this work is robust to the performance degradation of the approximate MWIS algorithms. We will further show in our technical report, Gao et al. (2012b), that the above welfare ratio bound is conservatively tight, in the sense that there does not exist another bound that is *always* better than our proposed one in any case.

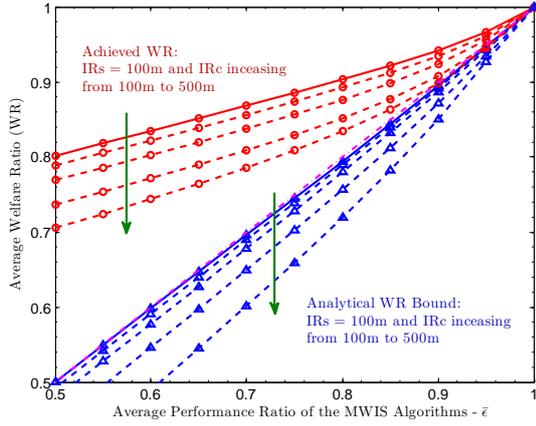


Figure 7 The achieved WR and analytical WR bound with IRs=100m.

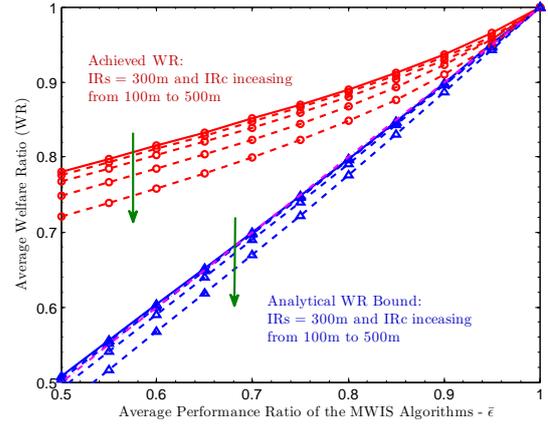


Figure 8 The achieved WR and analytical WR bound with IRs=300m.

9. Conclusion

We study the short-term secondary spectrum trading in a hybrid spectrum market, and focus on the spectrum efficiency maximization with spatial spectrum reuse under stochastic and asymmetric information. We first compute an off-line optimal policy that specifies the allocation of any spectrum under every possible information realization, and then design an on-line VCG auction that elicits SUs' private information and determines the real allocation of every spectrum. Such a solution technique allows us to optimally allocate every spectrum in an on-line manner, based on the policy derived and the information elicited. We further propose a heuristics solution based on a low complexity VCG-like mechanism, and quantify the associated spectrum efficiency loss.

We would like to point out that the problem considered in this work (i.e., spectrum allocation in an exogenous hybrid market in which every SU is associated and fixed in a particular market) can be viewed as a sub-problem of the whole market optimization problem, which consists of both the hybrid market formulation and the spectrum allocation. Our analysis of spectrum allocation in a given market provides a very important first step towards the “social optimal design” for the whole model. In our future work, we will study the problem of joint market formulation and spectrum allocation, in which every SU can make strategic decisions about the market selection, and can further negotiate with the PR for the contract details if choosing the futures market.

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