Subcarrier-Pair Based Resource Allocation for Cooperative AF Multi-Relay OFDM Systems

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***Abstract*— We study the joint allocation of three types of resources, namely, power, subcarriers and relay nodes, in cooperative two-hop multi-relay OFDM systems. Each relay adopts the amplify-and-forward (AF) protocol. The objective is to maximize the system transmission rate subject to individual power constraints on each node. We formulate such a problem as a subcarrier-pair based resource allocation that seeks the joint optimization of subcarrier pairing, subcarrier-pair-to-relay assignment, and power allocation. Using a dual decomposition method, we solve this problem efficiently in an asymptotically optimal manner. We further propose two suboptimal algorithms to trade off performance for complexity. Simulation results demonstrate that the proposed subcarrier-pair based resource allocation schemes significantly outperform the symbol based benchmark scheme. Moreover, it is shown that subcarrier pairing plays an important role in improving the system performance.**

I. Introduction

Relay-assisted cooperative communication has become very promising in various wireless systems [1]. It is able to boost the overall system performance including spectral efficiency, coverage area, and network lifetime. Efficient wireless resource allocation is critical to fully realize these benefits in cooperative communication systems. Resource allocation in orthogonal frequency division multiplexing (OFDM) based relay communication systems involves even more technical challenges. Compared with the traditional single-hop OFDM or OFDMA systems, we need to carefully coordinate the power and subcarrier allocations across different hops resulting from multiple relays. Compared with single-carrier relay systems, we can allocate multiple orthogonal subcarriers in every hop, which gives both more design freedoms and typically higher design complexity. In this paper, we study the joint relay selection, subcarrier assignment, and power control problem for a cooperative two-hop multi-relay OFDM system using the amplify-and-forward protocol. The objective is to maximize the transmission rate subject to an individual power constraint of each transmit node.

Recently, several results have been reported on relay selection in two-hop multi-relay systems [2]–[4]. A common selection strategy is to choose the relay with the best equivalent end-to-end channel gain. Similar strategy can be used in OFDM systems, where a relay is selected based on the channel condition of the whole OFDM symbol. However, such *symbol-based* relay selection may not be efficient since the differences of channel conditions among different subcarriers are not fully utilized. The *subcarrier-based* relay selection, which selects one best relay for each subcarrier, was then proposed in [3], [4] to exploit both node diversity and frequency diversity.

The subcarrier based relay selection usually assumes that signals received over one subcarrier is amplified (or decoded) and forwarded by a relay over the same subcarrier in the next hop. However this is not optimal in terms of system performance. A better performance can be achieved if subcarriers in the first and second hops are paired according to their channel conditions. Such a *subcarrier pairing* approach was proposed in [5]–[8]. In particular, the optimal subcarrier pairing criterion is studied in a two-hop single-relay OFDM system for both DF (decode-and-forward) and AF protocols in [5] and [7].

Relay selection along with subcarrier pairing is not sufficient to achieve the optimal performance in OFDM multi-relay systems. Power allocation is also essential. Authors in [6] proposed the optimal power allocation in a single-relay AF based OFDM system. A similar problem in a DF based OFDM system is studied in [5]. In [9], relay selection and power allocation are jointly optimized, but it did not consider subcarrier paring, the important issue discussed above.

In this paper we consider an AF-based two-hop multi-relay OFDM system, where we optimally and jointly allocate the three types of resources: power, subcarriers, and relay nodes. To the best of our knowledge, such joint optimization has not been considered in the literature. We formulate it as a joint relay selection, subcarrier pairing, and power allocation problem with an objective of maximizing the transmission rate under individual power constraints. Using a dual approach, the optimization problem can be solved in three phases. First, we find the optimal power allocation for any given strategy of subcarrier pairing and relay assignment. In the second phase, we determine the optimal relay assignment when subcarrier pairing is given. In the third phase, we obtain the optimal subcarrier paring using the Hungarian method. The overall complexity of the optimal algorithm is polynomial in the number of subcarriers and relay nodes. Based on the intuition derived from the optimal algorithm, we further propose two suboptimal algorithms that have lower complexity but can achieve close to optimal performances.

The remainder of this paper is organized as follows. In

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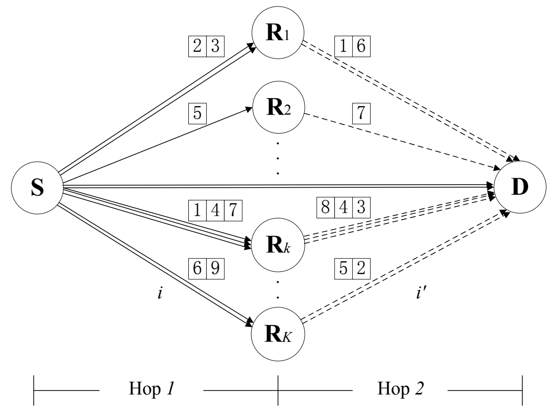

Fig. 1. Multi-relay assited cooperative OFDM system model

Section II we introduce the system model and describe the problem formulation. In Section III, we present the optimal solution using the dual method. In Section IV, we present sub-optimal algorithms with low complexity. Section V illustrates the performance of the proposed algorithms using simulation results. Finally we conclude this paper in Section VI.

II. System Model and Problem Formulation

We consider a multi-relay assisted cooperative OFDM system as shown in Fig. 1, where the source node communicates with the destination node via the help of K relay nodes. Each relay node operates in a half-duplex mode using the AF protocol. The $2K+1$ channels (the source to the destination, the source to the K relays, and the K relays to the destination) occupy the same bandwidth and experience independent and frequency-selective fading. The transmissions are based on OFDM, thus each channel is logically divided into N parallel orthogonal subcarriers with flat fading over each subcarrier. The transmission from the source to the destination is divided into frames, where each frame consists of several OFDM symbols. Each frame transmission is further divided into two time slots. In the first slot, the source transmits the signals on all subcarriers while the destination and all the relay nodes listen. In the second slot, each relay amplifies the received signals on a per-subcarrier basis, and forwards them to the destination. More specifically, suppose relay k receives the signal transmitted from the source on subcarrier i, amplifies it, and then forwards it on subcarrier i' in the second time slot. Here, i' may not be the same as i and they form a subcarrier pair (i, i'). To avoid interference among all the relays, each subcarrier pair can only be assigned to one relay. Each relay, on the other hand, can occupy more than one pair of subcarriers. In the extreme case where there is only one relay in the system, this relay occupies all subcarrier pairs. At the end of each transmission frame, the destination combines the received signals from both time slots and performs the optimal signal detection.

Let the noise power at the destination and relays over a subcarrier be denoted as σ_D^2 and $\sigma_{R,k}^2$ respectively, for $k \in \{1, \ldots, K\}$. The channel coefficients on subcarrier i associated with relay k in the first and second hops are denoted as $h_{i,k,1}$ and $h_{i,k,2}$, respectively, for $i \in \{1, \ldots, N\}$. We also denote the channel coefficient of the direct link connecting the source with the destination over subcarrier i as $h_{i,3}$. The *effective channel gains* can then be defined as $\alpha_{i,k,1} = |h_{i,k,1}|^2/\sigma_{R,k}^2$, $\alpha_{i,k,2} = |h_{i,k,2}|^2/\sigma_D^2$, and $\alpha_{i,3} = |h_{i,3}|^2/\sigma_D^2$ for all i and k. Assume that the subcarrier pair (i, i') is assigned to relay k. Denote $p_{i,k,1}$ and $p_{i',k,2}$ as the transmit power in the first and second hops along this path. Let the amplification factor of relay k over this subcarrier pair be

$$\beta_{i,i',k} = \sqrt{\frac{p_{i',k,2}}{|h_{i,k,1}|^2 p_{i,k,1} + \sigma_{R,k}^2}}\ . \tag{1}$$

Then, the end-to-end mutual information (in Nats/OFDM symbol) of the cooperative transmission assisted by relay k over subcarrier pair (i, i') can be expressed as [8]

$$R_{i,i',k} = \frac{1}{2}\ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{1 + \alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right). \tag{2}$$

Note that $R_{i,i',k}$ is not jointly concave in $p_{i,k,1}$ and $p_{i',k,2}$. To make the analysis tractable, we adopt the following approximation

$$R_{i,i',k} \approx \frac{1}{2}\ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right) \tag{3}$$

assuming that the signal amplified and forwarded by a relay is in the high SNR regime. Such approximation is common in the literature [8], [10]. It is also proved in [10] that even in the low SNR regime, the resource allocation by optimizing the rate in (3) can reach the truly optimal capacity very closely.

Our objective is to maximize the total end-to-end transmission rate subject to a set of constraints in the following. Denote $t_{i,i',k} \in \{0, 1\}$ as the binary variable for relay selection, with $t_{i,i',k} = 1$ indicating relay k is assigned the subcarrier pair (i, i') and $t_{i,i',k} = 0$ otherwise. It follows from the exclusive subcarrier-pair to relay assignment rule that:

$$\text{Relay assignment constraint :} \quad \sum_{k=1}^{K} t_{i,i',k} = 1,\ \forall i, i'. \tag{4}$$

Let $\rho_{i,i'} \in \{0, 1\}$ denote the indicator for subcarrier pairing, where $\rho_{i,i'} = 1$ means that subcarrier i in the first hop is paired with subcarrier i' in the second hop and $\rho_{i,i'} = 0$ otherwise. Since each subcarrier can be paired with one and only one subcarrier, the binary variables $\{\rho_{i,i'}\}$ must satisfy:

$$\text{Subcarrier pairing constraint :}$$
$$\sum_{i=1}^{N} \rho_{i,i'} = 1,\ \sum_{i'=1}^{N} \rho_{i,i'} = 1,\ \forall i, i' \tag{5}$$

Furthermore, the total transmission power at the source is limited and so is at each relay. Thus, the individual power constraints can be expressed as:

$$\text{Source power constraint :} \quad \sum_{i=1}^{N}\sum_{k=1}^{K} p_{i,k,1} \le P_S \tag{6}$$

$$\text{Relay power constraint}: \quad \sum_{i'=1}^{N} p_{i',k,2} \le P_{R,k}, \ \forall k \quad (7)$$

Thus, the variables to optimize in our problem are: power allocation $\boldsymbol{p} = \{p_{i,k,1}, p_{i',k,2}\}$ satisfying (6) (7), relay assignment $\boldsymbol{t} = \{t_{i,i',k}\}$ satisfying (4), and subcarrier pairing $\boldsymbol{\rho} = \{\rho_{i,i'}\}$ satisfying (5). We seek their joint optimization to maximize the system transmission rate. This is formulated as:

$$\max_{\{\boldsymbol{p},\boldsymbol{t},\boldsymbol{\rho}\}} \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} \rho_{i,i'} t_{i,i',k} R_{i,i',k} \quad (8)$$
$$s.t. \quad (4),(5),(6),(7).$$

III. Resource Allocation Using Dual Method

Obtaining the optimal solution $\{\boldsymbol{p}^*, \boldsymbol{t}^*, \boldsymbol{\rho}^*\}$ of (8) requires solving a mixed integer programming problem. The total $K^{N!}$ possibilities of subcarrier pairings and subcarrier-relay assignments significantly complicate the problem when *K* and *N* are large. Recently, it is shown in [11] that in multicarrier systems the duality gap of a non-convex resource allocation problem satisfying the time-sharing condition is neglectable as the number of subcarriers becomes sufficiently large. Since our optimization problem obviously satisfies the time-sharing condition, it can be solved by using the dual method and the solution is *asymptotically optimal*.

A. Optimizing the Dual Function

Define $\mathcal{D}$ as the set of all possible subcarrier pairings $\boldsymbol{\rho} = \{\rho_{i,i'}\}$ and subcarrier-relay assignments $\boldsymbol{t} = \{t_{i,i',k}\}$ satisfying (5) and (4), respectively. In addition, define $\mathcal{P}(\boldsymbol{\rho},\boldsymbol{t})$ as the set of all power allocations $\boldsymbol{p} = \{p_{i,k,1}, p_{i',k,2}\}$ for any given subcarrier pairing and relay assignment $(\boldsymbol{\rho},\boldsymbol{t})$ that satisfy $p_{i,k,1} \ge 0$, $p_{i',k,2} \ge 0$ for $\rho_{i,i'} t_{i,i',k} = 1$ and $p_{i,k,1} = p_{i',k,2} = 0$ for $\rho_{i,i'} t_{i,i',k} = 0$. Then, the dual function of problem (8) can be readily written as

$$g(\boldsymbol{\beta}) \triangleq \max_{\substack{\boldsymbol{p}\in\mathcal{P}(\boldsymbol{\rho},\boldsymbol{t}) \\ \{\boldsymbol{\rho},\boldsymbol{t}\}\in\mathcal{D}}} L(\boldsymbol{p},\boldsymbol{t},\boldsymbol{\rho},\boldsymbol{\beta}), \quad (9)$$

where the Lagrangian is

$$L(\boldsymbol{p},\boldsymbol{t},\boldsymbol{\rho},\boldsymbol{\beta}) = \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} \ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right) + \beta_S\left(P_S - \sum_{i=1}^{N}\sum_{k=1}^{K} p_{i,k,1}\right) + \sum_{k=1}^{K}\beta_{R,k}\left(P_{R,k} - \sum_{i'=1}^{N} p_{i',k,2}\right), \quad (10)$$

with $\boldsymbol{\beta} = (\beta_S, \beta_{R,1}, \ldots, \beta_{R,K}) \succeq 0$ being the vector of the dual variables associated with the individual power constraints. The dual optimization problem is hence given by

$$\min_{\boldsymbol{\beta}} g(\boldsymbol{\beta}) \quad (11)$$
$$s.t. \quad \boldsymbol{\beta} \succeq 0.$$

Since a dual function is always convex by definition, gradient or subgradient-based methods [12] can be used to minimize $g(\boldsymbol{\beta})$ in (11) with guaranteed convergence. The details are skipped in this paper. In the following, we shall focus on finding a closed-form expression of the dual function $g(\boldsymbol{\beta})$.

B. Optimizing Primal Variables at a Given Dual Point

Computing the dual function $g(\boldsymbol{\beta})$ involves determining the optimal $\{\boldsymbol{p}^*, \boldsymbol{\rho}^*, \boldsymbol{t}^*\}$ at the given dual point $\boldsymbol{\beta}$. In the following of this subsection, we present the detailed derivation of the optimal primal variables in three phases. Before that, let us rewrite $g(\boldsymbol{\beta})$ in (9) as

$$g(\boldsymbol{\beta}) = \max_{\substack{\boldsymbol{p}\in\mathcal{P}(\boldsymbol{\rho},\boldsymbol{t}) \\ \{\boldsymbol{\rho},\boldsymbol{t}\}\in\mathcal{D}}} \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} L_{i,i',k} + \beta_S P_S + \sum_{k=1}^{K}\beta_{R,k}P_{R,k}, \quad (12)$$

where

$$L_{i,i',k} \triangleq \ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right) - \beta_S p_{i,k,1} - \beta_{R,k}p_{i',k,2}. \quad (13)$$

*1) **Optimal Power Allocation for Given Subcarrier Pairing and Relay Assignment**:* Here we analyze the optimal power allocation $\boldsymbol{p}^*$ for given subcarrier pairing and relay assignment $(\boldsymbol{\rho}, \boldsymbol{t})$. Suppose that a subcarrier pair (i, i') is valid and is assigned to relay k during a transmission frame, i.e. $\rho_{i,i'} t_{i,i',k} = 1$. Then the optimal power allocation over this subcarrier-pairing relay assignment unit (i, i', k) can be determined by solving the following problem:

$$\max L_{i,i',k} \quad (14)$$
$$s.t. \quad p_{i,k,1} \ge 0, \ p_{i',k,2} \ge 0.$$

It can be easily shown that $L_{i,i',k}$ is a concave function of $(p_{i,k,1}, p_{i',k,2})$. Applying KKT conditions [13], we obtain that the optimal power allocation follows

$$p^*_{i,k,1} = \begin{cases} c_{i,i',k}p^*_{i',k,2}, & \text{when } \ p^*_{i',k,2} > 0 \\ \left(\frac{1}{\beta_S} - \frac{1}{\alpha_{i,3}}\right)^+ & \text{,when } \ p^*_{i',k,2} = 0 \end{cases}, \quad (15)$$

and (16), (17) (see top of the next page). Here $(x)^+ \triangleq \max(0, x)$. The detailed derivation is omitted due to space limitation.

It indicates from (15) and (16) that for this particular path (i, i', k), if $\alpha_{i',k,2}\beta_S \le \alpha_{i,3}\beta_{R,k}$, or equivalently $\frac{\alpha_{i',k,2}}{\beta_{R,k}} \le \frac{\alpha_{i,3}}{\beta_S}$, no power is assigned in the second hop. This can be interpreted from an economic perspective. If we consider the dual variables β_S and $\beta_{R,k}$ as the power prices at the source and the kth relay node, respectively, then $\frac{\alpha_{i,3}}{\beta_S}$ can be viewed as the gain of SNR at the destination created by the source via direct link per dollar, and $\frac{\alpha_{i',k,2}}{\beta_{R,k}}$ the gain of SNR at the destination created by the kth relay per dollar. Given a fixed total payment of transmit power, to maximize the gain of SNR at the destination, all the payment should be given to the source if $\frac{\alpha_{i',k,2}}{\beta_{R,k}} \le \frac{\alpha_{i,3}}{\beta_S}$. Otherwise if $\frac{\alpha_{i',k,2}}{\beta_{R,k}} > \frac{\alpha_{i,3}}{\beta_S}$, both the source and relay k should be assigned non-zero powers, which are related through the parameter $c_{i,i',k}$ in (17).

$$p^*_{i',k,2} = \begin{cases} \left(\frac{\alpha_{i,k,1}\alpha^2_{i',k,2} + (\alpha_{i,3} - \beta_S)(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})^2}{c_{i,i',k}\beta_S(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})(\alpha_{i,3}\alpha_{i,k,1}c_{i,i',k} + \alpha_{i,3}\alpha_{i',k,2} + \alpha_{i,k,1}\alpha_{i',k,2})} \right)^+, \text{if } \alpha_{i',k,2}\beta_S > \alpha_{i,3}\beta_{R,k} \\ 0, \quad \text{if } \alpha_{i',k,2}\beta_S \le \alpha_{i,3}\beta_{R,k} \end{cases} \tag{16}$$

$$c_{i,i',k} = \frac{\alpha_{i',k,2}}{\alpha_{i,k,1}(\alpha_{i',k,2}\beta_S - \alpha_{i,3}\beta_{R,k})} \left(\sqrt{\beta_{R,k}(\alpha_{i,k,1}\alpha_{i',k,2}\beta_S - \alpha_{i,k,1}\alpha_{i,3}\beta_{R,k} + \alpha_{i',k,2}\alpha_{i,3}\beta_S)} + \alpha_{i,3}\beta_{R,k} \right) \tag{17}$$

$$\begin{aligned} H_{i,i',k} \triangleq & \ln\left(\frac{\alpha_{i,3}(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})^2 + \alpha_{i,k,1}\alpha^2_{i',k,2}}{\beta_S(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})^2} \right) \\ & - (\beta_S c_{i,i',k} + \beta_{R,k}) \cdot \left(\frac{\alpha_{i,k,1}\alpha^2_{i',k,2} + (\alpha_{i,3} - \beta_S)(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})^2}{c_{i,i',k}\beta_S(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})(\alpha_{i,3}\alpha_{i,k,1}c_{i,i',k} + \alpha_{i,3}\alpha_{i',k,2} + \alpha_{i,k,1}\alpha_{i',k,2})} \right) \end{aligned} \tag{20}$$

*2) **Optimal Relay Selection for Given Subcarrier Pairing**:* Substituting (15) and (16) into (13), we eliminate the power control variables and obtain the alternative expression of (12)

$$g(\boldsymbol{\beta}) = \max_{\{\boldsymbol{\rho},\boldsymbol{t}\}\in\mathcal{D}} \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} \rho_{i,i'} t_{i,i',k} H_{i,i',k} + \beta_S P_S + \sum_{k=1}^{K} \beta_{R,k} P_{R,k} \ , \tag{18}$$

where $H_{i,i',k}$ is defined in the following.

If direct transmission without the help of relay is preferred (for this particular path (i, i', k)), we have:

$$H_{i,i',k} \triangleq \left[\ln\left(\frac{\alpha_{i,3}}{\beta_S} \right) \right]^+ - \beta_S \left(\frac{1}{\beta_S} - \frac{1}{\alpha_{i,3}} \right)^+ . \tag{19}$$

If cooperative transmission is preferred, then $H_{i,i',k}$ is given in (20) (see top of the page).

Based on (18), we next determine the optimal relay selection $\boldsymbol{t}^*$ at a given subcarrier pairing $\boldsymbol{\rho}$. Suppose $(i,\ i')$ is a valid subcarrier pair in the given subcarrier pairing scheme $\boldsymbol{\rho}$, i.e. $\rho_{i,i'} = 1$. Then it is obvious from (18) that the optimal relay to be selected for this subcarrier pair should be the one having the maximum value of $H_{i,i',k}$ in (19) or (20). That is:

$$t^*_{i,i',k} = \begin{cases} 1, & k = k^*(i,i') = \arg\max_k H_{i,i',k} \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

Therefore, the function $H_{i,i',k}$ defined in (19) or (20) serves as the optimal criterion for relay selection. When there exist more than one relay maximizing $H_{i,i',k}$, we randomly pick one relay for the subcarrier pair (i, i'). This does not affect the optimality of the dual function. An economic interpretation of this criterion is as follows. The first term in (19) or (20) is the gain in end-to-end transmission rate when relay k is assigned to subcarrier pair (i, i'). As mentioned earlier, if the dual variables β_S and $\beta_{R,k}$ are viewed as the power prices of the source and the k-th relay respectively, then $\beta_S p^*_{i,k,1}$ is the total cost of transmit power for the direct transmission and $\beta_S p^*_{i,k,1} + \beta_{R,k} p^*_{i',k,2}$ for cooperative transmission. Therefore, $H_{i,i',k}$ can be interpreted as the profit of transmitting over subcarrier pair (i, i') via relay k.

*3) **Optimal Subcarrier Pairing**:* Substituting (21) into (18), we obtain the corresponding dual function

$$g(\boldsymbol{\beta}) = \max_{\boldsymbol{\rho}\in\mathcal{D}} \sum_{i=1}^{N}\sum_{i'=1}^{N} \rho_{i,i'} H_{i,i'} + \beta_S P_S + \sum_{k=1}^{K} \beta_{R,k} P_{R,k} \ , \tag{22}$$

where $H_{i,i'} \triangleq H_{i,i',k^*(i,i')}$. Now it remains to determine the optimal subcarrier pairing $\boldsymbol{\rho}$. Define the $N \times N$ profit matrix $\boldsymbol{H} = [H_{i,i'}]$. In order to maximize the objective in (22) over the set $\mathcal{D}$, we should pick exactly one element in each row and each column of matrix $\boldsymbol{H}$ such that the sum of profits is as large as possible. Clearly, this is a standard linear assignment problem and can be efficiently solved by the Hungarian method [14], whose complexity is $O(N^3)$.

Let $\pi(i)$ denote the subcarrier index in the second hop optimally paired with subcarrier i in the first hop, for $i = 1, \ldots, N$. Then, the optimal subcarrier pairing variables can be expressed as

$$\rho^*_{i,i'} = \begin{cases} 1, & i' = \pi(i) \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

Combining the above three phases togethers, we have obtained the optimal primal variables $\{\boldsymbol{p}^*, \boldsymbol{t}^*, \boldsymbol{\rho}^*\}$ for given dual variables $\boldsymbol{\beta}$. Since the complexity of dual updates can be polynomial in K [11], say in the order of K^α, the overall complexity of solving the dual problem (11) is $O(N^3K^\alpha)$.

C. Refinement of power allocation

Having obtained the optimal dual point $\boldsymbol{\beta}^*$, we now determine the optimal solution to the original primal problem (8). Due to the non-zero duality gap for finite N, the optimal $\boldsymbol{t}^*(\boldsymbol{\beta}^*)$, $\boldsymbol{\rho}^*(\boldsymbol{\beta}^*)$ and $\boldsymbol{p}^*(\boldsymbol{\beta}^*)$ may be infeasible. To solve this issue, we first obtain the solution for the subcarrier pairing and relay assignment $\{\boldsymbol{t}^*(\boldsymbol{\beta}^*), \boldsymbol{\rho}^*(\boldsymbol{\beta}^*)\}$ using the method above, and then refine the power allocation $\boldsymbol{p}$ to meet the power constraints in the primal problem. This approach is asymptotically optimal due to vanishing duality gap when N

is sufficiently large [11]. It can be shown easily that the power refinement follows the same expression as in (15) and (16).

IV. Suboptimal Methods

In the previous section we derived the (asymptotically) optimal resource allocation algorithm, which may not be computationally efficient in practical systems for large N and K. In this section we propose two suboptimal algorithms with reduced complexity.

A. Suboptimal Algorithm 1: Equal Power Allocation (EPA) based subcarrier-pairing and relay assignment

This suboptimal algorithm determines the subcarrier pairing and relay assignment based on metrics obtained by assuming equal power allocation, rather than the metrics $H_{i,i',k}$ defined in (19) or (20). The procedure is as follows.

Without loss of generality, we let all the transmit nodes be subject to the same individual power constraint, i.e., $P_S = P_{R,k} = P$, $\forall k$. We firstly let the transmit power be distributed equally over all subcarriers in both time slots:

$$p_{i,k,1} = \frac{1}{N}P, \quad p_{i',k,2} = \frac{K}{N}P. \tag{24}$$

Let us substitute (24) into (2). For each possible subcarrier pair candidate (i, i'), we select the relay that maximizes the transmission rate $R_{i,i',k}$ and denote it as $k(i, i')$. The Hungarian method can be employed to determine the optimal subcarrier pairing, as in the optimal algorithm.

Once the subcarrier pairing and relay assignment are obtained, the power allocation over subcarriers at each transmit node remains to be uniform as in (24), except that for each relay node, the individual power P is equally distributed over its assigned subcarriers in the second hop.

Compared with the optimal algorithm, this suboptimal algorithm does not need to update the dual variables. Hence its complexity is $O(N^3)$ only.

B. Suboptimal Algorithm 2: Fixed Subcarrier pairing

Similar to the previous suboptimal schemes [3], [4], we let the subcarrier pairing be pre-fixed, rather than seeking the optimal subcarrier paring. Without loss of generality, the subcarrier pairing is arranged as

$$\pi(i) = i, \quad \forall i. \tag{25}$$

That is, the signals transmitted by the source on one subcarrier is forwarded on the same subcarrier by a relay to the destination. The rest of this suboptimal algorithm is to determine the joint optimal relay assignment and power allocation. As the key difference with the optimal algorithm is that there is no need to optimize the subcarrier pairing, the relay assignment in this suboptimal algorithm can be performed as $k(i, i) = \arg\max_k H_{i,i,k}$, $\forall i$.

Compared with suboptimal algorithm 1, suboptimal algorithm 2 still needs to update the dual variables in order to compute the metrics $H_{i,i,k}$. Since the number of steps to update the dual variables is $O(K^\alpha)$ as assumed in the previous section, and it requires the complexity of $O(NK)$ to perform all the relay selection and power allocation at each step, the overall computational complexity of the suboptimal algorithm with fixed subcarrier pairing is $O(NK^{\alpha+1})$.

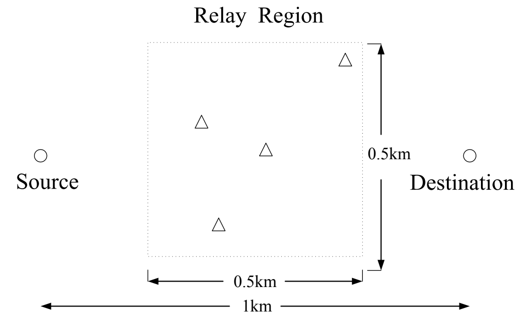

Fig. 2. Relay node distribution for the set up in Fig. 3

V. Simulation Results

We consider a network consisting of one source, one destination, and $K = 4$ relay nodes. The Stanford University Interim (SUI)-6 channel model is employed, in which the central frequency is given at 1.9GHz. The individual power constraint for each transmit node is assumed to be the same, and the noise power is normalized to 1. The total bandwidth is fixed to 1MHz. The path-loss exponent is fixed to 3.5. No shadowing is considered. We choose $N = 16$ so that all subcarriers will experience flat fading. In our simulations it was found that $N = 16$ is large enough to approach the optimal performance.

As a benchmark, the performance of the OFDM symbol-based relay selection (i.e. selecting the single relay node with the maximum averaged channel gain over the whole frequency band) is also presented. This baseline approach operates as follows: (i) For each relay node, sort the subcarriers in each hop individually in the descending order according to channel gains over them. (ii) Pair the sorted subcarriers on the two hops one by one and compute the total transmission rate under the equal power assumption for each relay. Such sorting and pairing approach is optimal for single-relay OFDM systems [7]. (iii) Select the relay that maximizes the rate, and then perform the optimal power allocation on the subcarrier pairs.

We first evaluate the average end-to-end spectral efficiency at different transmit power levels. All the $K = 4$ relay nodes are assumed to distribute randomly in the square region as shown in Fig. 2. Fig. 3 shows the results averaged over 100 realizations of relay distribution. It is observed that the proposed subcarrier-pair based optimal resource allocation significantly outperforms the baseline scheme, symbol based relay selection. In particular, at 20dBW transmit power per node, it achieves about 40% improvement in the end-to-end spectral efficiency. Comparing the two suboptimal methods with the optimal one, one can see that to achieve the same spectral efficiency, the suboptimal methods only incurs less than 1dB power loss. By taking a closer look at Fig. 3, it is also found that at high power region, the EPA based scheme (suboptimal algorithm 1) performs slightly better than the fixed subcarrier pairing scheme (suboptimal algorithm 2).

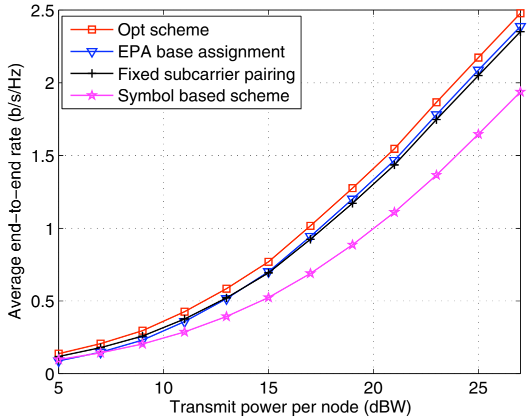

Fig. 3. Comparison of average end-to-end transmission rate versus transmit power per node.

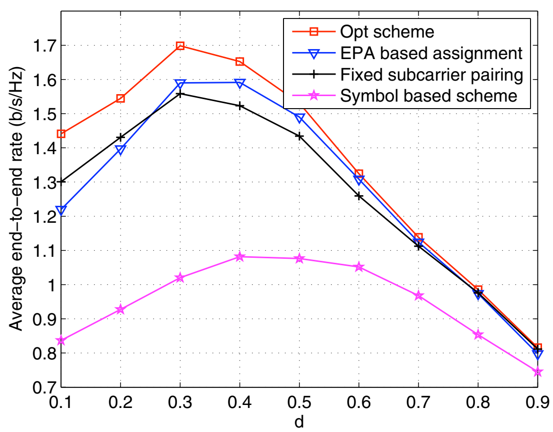

Fig. 4. Comparison of average end-to-end transmission rate versus relay location. Power per node = 20dBW.

Fig. 4 illustrates the average end-to-end transmission rates with respect to different locations of relays. All the $K = 4$ relay nodes in the network are assumed to form a cluster and lie approximately on the line connecting the source and destination. The radius of the relay cluster is much smaller than the distance from the source to the destination. The variable d denotes the distance ratio of the source-relay link to the source-destination link. For the optimal scheme, the maximal transmission rate is achieved at about $d = 0.3$ and it is 70% higher than that of the symbol based benchmark scheme. The performance of the EPA based assignment overshadows that of the fixed subcarrier pairing scheme when $d \geq 0.3$. This further demonstrates the importance of subcarrier pairing in improving the end-to-end transmission rate.

VI. Conclusion

In this paper, we study the joint optimal subcarrier pairing, relay assignment and power allocation for a cooperative multi-relay OFDM system. We utilize the dual method to solve the optimization problem with polynomial complexity. To further reduce computational complexity, we propose two suboptimal approaches. The simulation results show that the suboptimal algorithm which decouples the power allocation with the subcarrier pairing and relay assignment performs closely to the optimal one, while having much less complexity. Future work may investigate subcarrier-pair based joint user scheduling and resource allocation in cooperative multi-relay multi-user OFDM networks.

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