Subcarrier-Pair Based Resource Allocation for Cooperative Multi-Relay OFDM Systems

Wenbing Dang, Meixia Tao, Hua Mu, and Jianwei Huang

***Abstract*—In this paper, we study the joint allocation of three types of resources, namely, power, subcarriers and relay nodes, in multi-relay assisted dual-hop cooperative OFDM systems. All the relays adopt the amplify-and-forward protocol and assist the transmission from the source to destination simultaneously but on orthogonal subcarriers. The objective is to maximize the system transmission rate subject to individual power constraints on each node or a total network power constraint. We formulate such a problem as a subcarrier-pair based resource allocation that seeks the joint optimization of subcarrier pairing, subcarrier-pair-to-relay assignment, and power allocation. Using a dual approach, we solve this problem efficiently in an asymptotically optimal manner. Specifically, for the optimization problem with individual power constraints, the computational complexity is polynomial in the number of subcarriers and relay nodes, whereas the complexity of the problem with a total power constraint is polynomial in the number of subcarriers. We further propose two suboptimal algorithms for the former to trade off performance for complexity. Simulation studies are conducted to evaluate the average transmission rate and outage probability of the proposed algorithms. The impact of relay location is also discussed.**

***Index Terms*—Cooperative communications, OFDM, subcarrier pairing, relay selection, and power allocation.**

I. Introduction

RELAY-ASSISTED communication has become very promising in various wireless systems, such as ad-hoc, mesh, and cellular networks [1]–[3]. It is able to boost the overall system performance by means of improving the spectral efficiency, extending the coverage area, and/or prolonging the network lifetime. To fully realize these benefits in cooperative communication systems, efficient wireless resource allocation is critical. In specific, the problem formulation may differ significantly in optimization objectives (transmission rate maximization, outage probability minimization), relaying protocols (decode-and-forward and amplify-and-forward), transmit power constraints (total power constraint, individual power constraints, and long term total power constraint), and system architectures (two-hop single-relay, two-hop multi-relay, and linear multi-hop). Optimal resource allocation in relay-assisted orthogonal frequency division multiplexing (OFDM) communication systems involves even more technical challenges. Compared with traditional single-hop OFDM or OFDMA systems, we need to carefully coordinate the power and subcarrier allocations across different hops resulting from multiple relays. Compared with the previous studies on single-carrier relay networks over flat fading channels, here we need to allocate multiple orthogonal frequency subcarriers in every hop, which gives us both more design freedoms and typically higher design complexity. In this paper, we study the joint relay selection, subcarrier assignment and power control problem for a two-hop multi-relay OFDM system using amplify-and-forward (AF) protocol. The objective is to maximize the transmission rate subject to an individual power constraint of each transmit node or a total network power constraint.

Recently, several results have been reported on relay selection in two-hop multi-relay systems [4]–[6]. A common selection strategy is to choose the relay with the best equivalent end-to-end channel gain. Considering mutual information as the performance measure, the end-to-end equivalent channel gain is calculated as the minimum of the channel gains of the first and second hops under decode-and-forward (DF) protocol or the harmonic mean of both channel gains under AF protocol. The similar strategy can be extended to OFDM systems, wherein we select one relay based on the channel condition of the whole OFDM symbol. However, such *symbol-based* relay selection, as illustrated in Fig. 1a, may not be efficient since the differences of channel conditions among different subcarriers are not fully utilized. Subcarrier-based relay selection as illustrated in Fig. 1b, which selects one best relay for each subcarrier, was then proposed in [5], [6] to exploit both node diversity and frequency diversity. Such scheme can be regarded as *subcarrier-to-relay assignment*.

The subcarrier-to-relay assignment usually assumes that signals received over one subcarrier, say subcarrier i, is amplified (or decoded) and forwarded by the relay also over subcarrier i in the next hop. However, this is not optimal in terms of system performance. A better performance can be achieved if subcarriers in the first and second hops are paired according to their channel conditions. An illustrative example is shown in Fig. 1c. The concept of *subcarrier pairing* was originally proposed in [7]–[9] independently for two-hop single-relay

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W. Dang and H. Mu were with the Department of Electronic Engineering, Shanghai Jiao Tong University. They are now with Colorado State University and Auburn University, respectively (e-mail: dwb87514@gmail.com, hzm0005@auburn.edu).

M. Tao is with the Department of Electronic Engineering, Shanghai Jiao Tong University, Shanghai, P. R. China 200240 (e-mail: mxtao@sjtu.edu.cn).

J. Huang is with the Department of Information Engineering, the Chinese University of Hong Kong, Hong Kong, P. R. China (e-mail: jwhuang@ie.cuhk.edu.hk).

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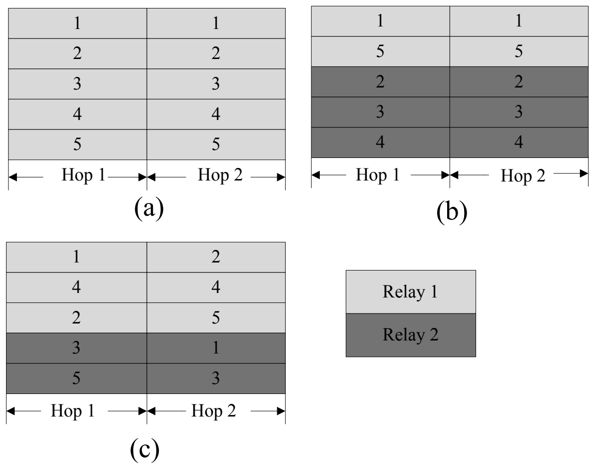

Fig. 1. Different relay selection schemes for a relay network consisting of $K = 2$ relays and $N = 5$ subcarriers: (a) OFDM symbol based relay selection, (b) subcarrier based relay selection, (c) subcarrier-pair based relay selection.

OFDM systems. In particular, it is proved in [9] and [10] that the *ordered subcarrier pairing* is optimal for AF protocol. Note, however that the aforementioned subcarrier pairing is done per subcarrier basis and requires that the signals received by the same relay on different subcarriers are processed individually. Therefore, such subcarrier-by-subcarrier based pairing may not be sufficient for DF protocol, where the information from one set of subcarriers in the first hop can be decoded and re-encoded jointly and then transmitted over a different set of subcarriers in the next hop. Naturally, combining subcarrier/subcarrier-set pairing with relay selection in multi-relay assisted OFDM systems can enhance the system performance. Their joint optimization is however challenging and less investigated in the literature. Authors in [11] and [12] have attempted to solve the subcarrier-to-relay assignment problem in multi-relay OFDM systems with single user and multiple users, respectively. However these results are for DF protocol only and cannot be applied in AF protocol.

Power allocation among subcarriers and among source and relay nodes also plays an important role in optimizing the performance of relay-based OFDM systems. Authors in [8] and [13] proposed the optimal power allocation and subcarrier pairing in a single-relay AF based OFDM system. A similar problem in a single-relay DF based OFDM system is studied in [7], [13]. In [14], the relay power allocation problem for a *parallel*-relay OFDM system, where all the relays successively send signals to the destination in pre-assigned non-overlapping time slots is considered. In [15], the authors investigate the joint optimization of power allocation, relay selection, and relay strategy selection, but do not consider subcarrier pairing. In [16], the authors also studied the joint power allocation and relay selection for a dual-hop multiuser system. However they only consider optimizing the relay power assuming a predetermined subcarrier allocation. Optimal power and time allocation under a long-term total power constraint in OFDM based linear multi-hop relay networks with DF protocol has been considered in [17], [18].

In this paper we consider an AF-based cooperative two-hop multi-relay OFDM system, where we optimally and jointly allocate the three types of resources: power, subcarriers, and relay nodes. To the best of our knowledge, such joint optimization has not been considered in the literature and is crucial for achieving the best system performance. The key contributions of this paper are as follows:

1) *New Problem Formulation*: we formulate a subcarrier-pair based resource allocation problem for cooperative two-hop multi-relay OFDM systems with AF protocol. Unlike many of the previous works in the literature which only explored partial resources, our formulation includes the relay selection, subcarrier pairing and power allocation altogether in a unified framework. The objective is to maximize the end-to-end transmission rate subject to individual or total power constraints.
2) *Optimal resource allocation algorithm*: We solve the optimization problems for both individual and total power constraints using a dual method in an asymptotically optimal manner. In particular, for the problem with individual power constraints, we show that it can be solved in three phases and the complexity is polynomial in the number of subcarriers and relay nodes. For the problem with a total power constraint, we show that it is much simpler and the complexity is only polynomial in the number of subcarriers.
3) *Suboptimal algorithms with low complexities*: Based on the intuition derived from the optimal algorithm, we further propose two suboptimal algorithms for the problem with individual power constraints. They have lower complexity but can achieve close to optimal performances.

The remainder of this paper is organized as follows. In Section II we introduce the system model and describe the constraints for resource allocation. The problem formulations and the associated optimal resource allocation algorithms under individual and total power constraints are presented in Section III and Section IV, respectively. In Section V, we present suboptimal algorithms with low complexity. Section VI demonstrates some simulation results to illustrate the performance of the proposed algorithms. Finally we conclude this paper in Section VII.

II. System Model

We consider a multi-relay assisted cooperative OFDM system shown in Fig. 2, where the source node communicates with the destination node via the help of K relay nodes. Each relay node operates in a time-division half-duplex mode using the AF protocol. The $2K + 1$ channels (the source to the destination, the source to the K relays, and the K relays to the destination) occupy the same bandwidth and experience independent, frequency-selective fading. The transmissions are based on OFDM modulation, thus each channel is logically divided into N orthogonal subcarriers with flat fading. All the channel state information in the network is assumed perfectly known at a central controller, which can be embedded with the source or the destination. The transmission from the source to the destination is on a time-frame basis with each frame consisting of multiple OFDM symbols. Each frame transmission is further divided into two time slots. In the first slot, the source transmits the signals on all subcarriers while the destination and all the relay nodes listen. In the second

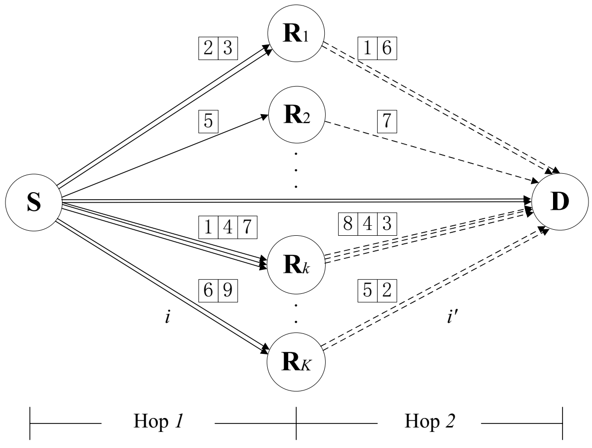

Fig. 2. Multi-relay assisted cooperative OFDM system model.

slot, the source remains silent while each relay amplifies the received signals on a subcarrier basis, and forwards them to the destination. More specifically, suppose relay k receives the signal transmitted from the source on subcarrier i, amplifies it, and then forwards it on subcarrier i' in the second time slot. Here, the subcarrier index i' may not be the same as i and they form a subcarrier pair (i, i'). To avoid interference among all the relays, each subcarrier pair can only be assigned to one relay. Each relay, on the other hand, can occupy more than one pair of subcarriers. In the extreme case where there is only one relay in the system, this relay occupies all subcarrier pairs. At the end of each transmission frame, the destination combines the received signals from both time slots and performs the optimal signal detection.

Let the noise power at the destination and relays over a subcarrier be denoted as σ_D^2 and $\sigma_{R,k}^2$ respectively, for $k \in \{1, \ldots, K\}$. The channel coefficients on subcarrier i associated with relay k in the first and second hops are denoted as $h_{i,k,1}$ and $h_{i,k,2}$, respectively, for $i \in \{1, \ldots, N\}$. The channel coefficient of the direct link connecting the source with the destination over subcarrier i is denoted as $h_{i,3}$. The *effective channel gains* can then be defined as $\alpha_{i,k,1} = |h_{i,k,1}|^2/\sigma_{R,k}^2$, $\alpha_{i,k,2} = |h_{i,k,2}|^2/\sigma_D^2$, and $\alpha_{i,3} = |h_{i,3}|^2/\sigma_D^2$ for all i and k. Assume that the subcarrier pair (i, i') is assigned to relay k. It is further assumed that the transmit powers in the first and second hops along this path are $p_{i,k,1}$ and $p_{i',k,2}$, respectively. The end-to-end mutual information (in Nats/OFDM symbol) of the cooperative transmission assisted by relay k over subcarrier pair (i, i') can be expressed as [13]

$$R_{i,i',k} = \frac{1}{2}\ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{1 + \alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right). \quad (1)$$

In (1), the factor $1/2$ accounts for the two time slots in each transmission frame, and the amplification factor of relay k over the subcarrier pair (i, i') is assumed by default to be

$$\beta_{i,i',k} = \sqrt{\frac{p_{i',k,2}}{|h_{i,k,1}|^2 p_{i,k,1} + \sigma_{R,k}^2}}.$$

Note that $R_{i,i',k}$ is not jointly concave in $p_{i,k,1}$ and $p_{i',k,2}$. To make the analysis more tractable, we adopt the following approximation

$$R_{i,i',k} \approx \frac{1}{2}\ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right). \quad (2)$$

This approximation is essentially based on the assumption that the signal amplified and forwarded by a relay is in the high signal-to-noise ratio (SNR) regime. Such approximation has been commonly used in the literature [13], [19]. It is also proved in [19] that even in the low SNR regime, the resource allocation by optimizing the rate in (2) can reach the truly optimal capacity very closely.

Our objective is to maximize the total end-to-end transmission rate subject to a set of constraints in the following. Denote $t_{i,i',k} \in \{0, 1\}$ as the binary variable for relay selection with $t_{i,i',k} = 1$ indicating relay k is assigned the subcarrier pair candidate (i, i') and $t_{i,i',k} = 0$ otherwise. It follows from the exclusive subcarrier-pair to relay assignment rule that:

$$\text{Relay assignment constraint}: \quad \sum_{k=1}^{K} t_{i,i',k} = 1, \ \forall i, i'. \quad (3)$$

Let $\rho_{i,i'} \in \{0, 1\}$ denote the indicator for subcarrier pairing, where $\rho_{i,i'} = 1$ means that subcarrier i in the first hop is paired with subcarrier i' in the second hop and $\rho_{i,i'} = 0$ otherwise. Since each subcarrier can be paired with one and only one subcarrier, the binary variables $\{\rho_{i,i'}\}$ must satisfy:

$$\text{Subcarrier pairing constraint}: \quad \sum_{i=1}^{N} \rho_{i,i'} = 1, \ \sum_{i'=1}^{N} \rho_{i,i'} = 1, \ \forall i, i'. \quad (4)$$

For the individual power constraint, the total transmission power at the source is limited and so is at each relay. This can be expressed as:

$$\text{Source power constraint}: \quad \sum_{i=1}^{N}\sum_{k=1}^{K} p_{i,k,1} \le P_S \quad (5)$$

$$\text{Relay power constraint}: \quad \sum_{i'=1}^{N} p_{i',k,2} \le P_{R,k}, \ \forall k. \quad (6)$$

For the total network power constraint, the sum of transmit powers at all the nodes is limited, i.e.,

$$\text{Total power constraint}: \quad \sum_{i=1}^{N}\sum_{k=1}^{K}\sum_{l=1}^{2} p_{i,k,l} \le P_T. \quad (7)$$

As such, the total variables to be optimized in our problem are: relay assignment $\boldsymbol{t} = \{t_{i,i',k}\}$ satisfying (3), subcarrier pairing $\boldsymbol{\rho} = \{\rho_{i,i'}\}$ satisfying (4), and power allocation $\boldsymbol{p} = \{p_{i,k,1}, p_{i',k,2}\}$ satisfying (5) and (6) or satisfying (7). Such optimization problem will be solved by the central controller and the resulting allocation information may be sent to each node via a separate channel.

III. Optimal Resource Allocation Under Individual Power Constraints

After introducing all the constraints for resource allocation imposed on the network, in this section we seek the joint optimization of $\boldsymbol{p}$, $\boldsymbol{t}$, and $\boldsymbol{\rho}$ to maximize the system transmission

$$p^*_{i',k,2} = \begin{cases} \left(\dfrac{\alpha_{i,k,1}\alpha^2_{i',k,2} + (\alpha_{i,3} - \beta_S)(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})^2}{c_{i,i',k}\beta_S(\alpha_{i,k,1}c_{i,i',k} + \alpha_{i',k,2})(\alpha_{i,3}\alpha_{i,k,1}c_{i,i',k} + \alpha_{i,3}\alpha_{i',k,2} + \alpha_{i,k,1}\alpha_{i',k,2})} \right)^+, \\ \quad \text{if} \ \ \alpha_{i',k,2}\beta_S > \alpha_{i,3}\beta_{R,k} \\ 0, \quad \text{if} \ \ \alpha_{i',k,2}\beta_S \leq \alpha_{i,3}\beta_{R,k} \end{cases} \tag{16}$$

$$c_{i,i',k} = \frac{\alpha_{i',k,2}}{\alpha_{i,k,1}(\alpha_{i',k,2}\beta_S - \alpha_{i,3}\beta_{R,k})} \left(\sqrt{\beta_{R,k}(\alpha_{i,k,1}\alpha_{i',k,2}\beta_S - \alpha_{i,k,1}\alpha_{i,3}\beta_{R,k} + \alpha_{i',k,2}\alpha_{i,3}\beta_S)} + \alpha_{i,3}\beta_{R,k} \right) \tag{17}$$

rate under the individual power constraints. This is formulated as:

$$\max_{\{\boldsymbol{p},\boldsymbol{t},\boldsymbol{\rho}\}} \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} \rho_{i,i'} t_{i,i',k} R_{i,i',k} \quad \text{s.t.} \ \ (3), (4), (5), (6). \tag{8}$$

Obtaining the optimal solution $\{\boldsymbol{p}^*, \boldsymbol{t}^*, \boldsymbol{\rho}^*\}$ of (8) requires solving a mixed integer programming problem. The total $K^{N!}$ possibilities of subcarrier pairings and subcarrier-relay assignments significantly complicate the problem when K and N are large. Recently, it is shown in [20] that in multicarrier systems the duality gap of a non-convex resource allocation problem satisfying the time-sharing condition is negligible as the number of subcarriers becomes sufficiently large. Since our optimization problem obviously satisfies the time-sharing condition, it can be solved by using the dual method and the solution is *asymptotically* optimal. In the following subsections, we show that by using the dual method, the problem in (8) can be solved in three phases and the complexity is in polynomial time.

A. Optimizing the Dual Function

Define $\mathcal{D}$ as the set of all possible subcarrier pairings $\boldsymbol{\rho} = \{\rho_{i,i'}\}$ and subcarrier-relay assignments $\boldsymbol{t} = \{t_{i,i',k}\}$ satisfying (4) and (3), respectively. In addition, define $\mathcal{P}(\boldsymbol{\rho}, \boldsymbol{t})$ as the set of all power allocations $\boldsymbol{p} = \{p_{i,k,1}, p_{i',k,2}\}$ for the given subcarrier pairing and relay assignment $(\boldsymbol{\rho}, \boldsymbol{t})$ that satisfy $p_{i,k,1} \geq 0$, $p_{i',k,2} \geq 0$ for $\rho_{i,i'} t_{i,i',k} = 1$ and $p_{i,k,1} = p_{i',k,2} = 0$ for $\rho_{i,i'} t_{i,i',k} = 0$. Then, the Lagrange dual function of problem (8) can be readily written as

$$g(\boldsymbol{\beta}) \triangleq \max_{\substack{\boldsymbol{p}\in\mathcal{P}(\boldsymbol{\rho},\boldsymbol{t}) \\ \{\boldsymbol{\rho},\boldsymbol{t}\}\in\mathcal{D}}} L(\boldsymbol{p}, \boldsymbol{t}, \boldsymbol{\rho}, \boldsymbol{\beta}), \tag{9}$$

where the Lagrangian is

$$\begin{aligned} L(\boldsymbol{p}, \boldsymbol{t}, \boldsymbol{\rho}, \boldsymbol{\beta}) \ = \ & \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} \ln\Bigg(1 + \alpha_{i,3}p_{i,k,1} \\ & + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\Bigg) \\ & + \beta_S\Bigg(P_S - \sum_{i=1}^{N}\sum_{k=1}^{K} p_{i,k,1}\Bigg) \\ & + \sum_{k=1}^{K} \beta_{R,k}\Bigg(P_{R,k} - \sum_{i'=1}^{N} p_{i',k,2}\Bigg), \end{aligned} \tag{10}$$

with $\boldsymbol{\beta} = (\beta_S, \beta_{R,1}, \ldots, \beta_{R,K}) \succeq 0$ being the vector of the dual variables associated with the individual power constraints. The dual optimization problem is hence given by

$$\min_{\boldsymbol{\beta}} g(\boldsymbol{\beta}) \quad \text{s.t.} \ \ \boldsymbol{\beta} \succeq 0. \tag{11}$$

Since a dual function is always convex by definition [21], gradient or subgradient-based methods can be used to minimize $g(\boldsymbol{\beta})$ with guaranteed convergence. Let $\boldsymbol{p}^*(\boldsymbol{\beta})$ denote the optimal power allocation in (9) at dual point $\boldsymbol{\beta} = (\beta_S, \beta_{R,1}, \ldots, \beta_{R,K})$ (will be discussed in the next subsection), then a subgradient of $g(\boldsymbol{\beta})$ can be derived using a similar method as in [22]:

$$\triangle\beta_S = P_S - \sum_{i=1}^{N}\sum_{k=1}^{K} p^*_{i,k,1}(\boldsymbol{\beta}),$$

and

$$\triangle\beta_{R,k} = P_{R,k} - \sum_{i'=1}^{N} p^*_{i',k,2}(\boldsymbol{\beta}), \quad \forall k.$$

Denote $\triangle\boldsymbol{\beta} = (\triangle\beta_S, \triangle\beta_{R,1}, \ldots, \triangle\beta_{R,K})$. The dual variables are updated as $\boldsymbol{\beta}^{(l+1)} = \boldsymbol{\beta}^{(l)} + \epsilon^{(l)}\triangle\boldsymbol{\beta}$. Using the step size $\epsilon^{(l)}$ following the diminishing step size policy in [23], the subgradient method above is guaranteed to converge to the optimal dual variables $\boldsymbol{\beta}^*$. The computational complexity of such update method is polynomial in the number of dual variables $K+1$ [20].

B. Optimizing Primal Variables at a Given Dual Point

Computing the dual function $g(\boldsymbol{\beta})$ involves determining the optimal $\{\boldsymbol{p}^*, \boldsymbol{\rho}^*, \boldsymbol{t}^*\}$ at the given dual point $\boldsymbol{\beta}$. In the following of this subsection, we present the detailed derivation of the optimal primal variables in three phases. Before that, let us rewrite $g(\boldsymbol{\beta})$ in (9) as

$$g(\boldsymbol{\beta}) = \max_{\substack{\boldsymbol{p}\in\mathcal{P}(\boldsymbol{\rho},\boldsymbol{t}) \\ \{\boldsymbol{\rho},\boldsymbol{t}\}\in\mathcal{D}}} \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} L_{i,i',k} + \beta_S P_S + \sum_{k=1}^{K} \beta_{R,k} P_{R,k}, \tag{12}$$

where

$$\begin{aligned} L_{i,i',k} \ \triangleq \ & \ln\left(1 + \alpha_{i,3}p_{i,k,1} + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1} + \alpha_{i',k,2}p_{i',k,2}}\right) \\ & - \beta_S p_{i,k,1} - \beta_{R,k} p_{i',k,2}. \end{aligned} \tag{13}$$

$$H_{i,i',k}(\boldsymbol{\beta}) \triangleq \ln\left(\frac{\alpha_{i,3}\left(\alpha_{i,k,1}c_{i,i',k}+\alpha_{i',k,2}\right)^2+\alpha_{i,k,1}\alpha_{i',k,2}^2}{\beta_S\left(\alpha_{i,k,1}c_{i,i',k}+\alpha_{i',k,2}\right)^2}\right)-\left(\beta_S c_{i,i',k}+\beta_{R,k}\right)\cdot \left(\frac{\alpha_{i,k,1}\alpha_{i',k,2}^2+\left(\alpha_{i,3}-\beta_S\right)\left(\alpha_{i,k,1}c_{i,i',k}+\alpha_{i',k,2}\right)^2}{c_{i,i',k}\beta_S\left(\alpha_{i,k,1}c_{i,i',k}+\alpha_{i',k,2}\right)\left(\alpha_{i,3}\alpha_{i,k,1}c_{i,i',k}+\alpha_{i,3}\alpha_{i',k,2}+\alpha_{i,k,1}\alpha_{i',k,2}\right)}\right). \quad (20)$$

1) Optimal Power Allocation for Given Subcarrier Pairing and Relay Assignment: Here we analyze the optimal power allocation $\boldsymbol{p}^*$ for given subcarrier pairing and relay assignment $(\boldsymbol{\rho},\boldsymbol{t})$. Suppose that a subcarrier pair (i,i') is valid and is assigned to relay k in a frame of transmission time, i.e. $\rho_{i,i'}t_{i,i',k}=1$. Then the optimal power allocation over this subcarrier-pairing relay assignment unit $(i,\ i',\ k)$ can be determined by solving the following problem:

$$\max_{\{p_{i,k,1},p_{i',k,2}\}} L_{i,i',k} \quad \text{s.t.}\ \ p_{i,k,1}\geq 0,\ p_{i',k,2}\geq 0. \quad (14)$$

It can be easily shown that $L_{i,i',k}$ is a concave function of $(p_{i,k,1},p_{i',k,2})$. Applying Karush-Kuhn-Tucker (KKT) conditions [21], we obtain the optimal power allocation:

$$p_{i,k,1}^* = \begin{cases} c_{i,i',k}p_{i',k,2}^*, & \text{if } p_{i',k,2}^*>0 \\ \left(\frac{1}{\beta_S}-\frac{1}{\alpha_{i,3}}\right)^+, & \text{if } p_{i',k,2}^*=0 \end{cases}, \quad (15)$$

and $p_{i',k,2}^*$ is given in (16) at the top of the previous page, where $c_{i,i',k}$ is given in (17) and $(x)^+ \triangleq \max(0,x)$. A detailed derivation is given in Appendix I.

It indicates from (15) and (16) that for this particular path (i,i',k), if $\alpha_{i',k,2}\beta_S\leq\alpha_{i,3}\beta_{R,k}$, or equivalently $\frac{\alpha_{i',k,2}}{\beta_{R,k}}\leq\frac{\alpha_{i,3}}{\beta_S}$, no power is assigned in the second hop. This can be interpreted from an economic perspective. If we consider the dual variables β_S and $\beta_{R,k}$ as the power prices at the source and the kth relay node, respectively, then $\frac{\alpha_{i,3}}{\beta_S}$ can be viewed as the gain of SNR at the destination created by the source via direct link per dollar, and $\frac{\alpha_{i',k,2}}{\beta_{R,k}}$ the gain of SNR at the destination created by the kth relay per dollar. Given a fixed total payment of transmit power, to maximize the gain of SNR at the destination, all the payment should be given to the source if $\frac{\alpha_{i',k,2}}{\beta_{R,k}}\leq\frac{\alpha_{i,3}}{\beta_S}$. Otherwise if $\frac{\alpha_{i',k,2}}{\beta_{R,k}}>\frac{\alpha_{i,3}}{\beta_S}$, both the source and relay k should be assigned non-zero powers, which are related through the parameter $c_{i,i',k}$ in (15).

2) Optimal Relay Selection for Given Subcarrier Pairing: Substituting the optimal power allocation expression (15) and (16) into (13) to eliminate the power variables and then into (12), we can obtain an alternative expression of the dual function as

$$g(\boldsymbol{\beta}) = \max_{\{\boldsymbol{\rho},\boldsymbol{t}\}\in\mathcal{D}}\sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K}\rho_{i,i'}t_{i,i',k}H_{i,i',k}(\boldsymbol{\beta})+\beta_S P_S +\sum_{k=1}^{K}\beta_{R,k}P_{R,k}\ . \quad (18)$$

Here, the function $H_{i,i',k}(\boldsymbol{\beta})$ is defined as follows and plays an important role in relay selection as will be shown shortly.

If direct transmission (without the help of relay) is preferred for this particular path (i,i',k), we have

$$H_{i,i',k}(\boldsymbol{\beta}) \triangleq \left[\ln\left(\frac{\alpha_{i,3}}{\beta_S}\right)\right]^+ - \beta_S\left(\frac{1}{\beta_S}-\frac{1}{\alpha_{i,3}}\right)^+. \quad (19)$$

If cooperative transmission with AF relaying is preferred, then $H_{i,i',k}$ is given by (20) at the top of the page.

Based on (18), we next determine the optimal relay selection $\boldsymbol{t}^*$ at a given subcarrier pairing $\boldsymbol{\rho}$. Suppose $(i,\ i')$ is a valid subcarrier pair in the given subcarrier pairing scheme $\boldsymbol{\rho}$, i.e. $\rho_{i,i'}=1$. Then it is obvious from (18) that the optimal relay to be selected for this subcarrier pair should be the one having the maximum value of $H_{i,i',k}(\boldsymbol{\beta})$ in (19) or (20). That is:

$$t_{i,i',k}^* = \begin{cases} 1, & k=k(i,i')=\arg\max_k H_{i,i',k} \\ 0, & \text{otherwise} \end{cases} \quad (21)$$

Therefore, the function $H_{i,i',k}(\boldsymbol{\beta})$ defined in (19) or (20) serves as the optimal criterion for relay selection. An economic interpretation of this criterion is the following. The first term in (19) or (20) is the gain in end-to-end transmission rate when relay k is assigned to subcarrier pair (i,i'). As mentioned in the analysis of optimal power allocation above, if the dual variables β_S and $\beta_{R,k}$ are viewed as the power prices of the source and the k-th relay respectively, then $\beta_S p_{i,k,1}^*$ is the total cost of transmit power for the direct transmission and $\beta_S p_{i,k,1}^*+\beta_{R,k}p_{i',k,2}^*$ for cooperative transmission. Therefore, $H_{i,i',k}$ can be interpreted as the profit of transmitting over subcarrier pair (i,i') via relay k.

3) Optimal Subcarrier Pairing: Substituting (21) into (18), we obtain the corresponding dual function

$$g(\boldsymbol{\beta}) = \max_{\boldsymbol{\rho}\in\mathcal{D}}\sum_{i=1}^{K}\sum_{i'=1}^{K}\rho_{i,i'}H_{i,i'}+\beta_S P_S+\sum_{k=1}^{K}\beta_{R,k}P_{R,k}\ , \quad (22)$$

where $H_{i,i'}\triangleq H_{i,i',k(i,i')}(\boldsymbol{\beta})$. Now it remains to determine the optimal subcarrier pairing $\boldsymbol{\rho}$. Define the $N\times N$ profit matrix $\boldsymbol{H}=[H_{i,i'}]$. In order to maximize the objective in (22), we should pick exactly one element in each row and each column of matrix $\boldsymbol{H}$ such that the sum of profits is as large as possible. Clearly, this is a standard linear assignment problem and can be efficiently solved by the Hungarian method [24], whose complexity is $O(N^3)$.

Let $\pi(i)$ denote the subcarrier index in the second hop optimally paired with subcarrier i in the first hop, for $i=1,\ldots,N$. Then, the optimal subcarrier pairing variables can be expressed as

$$\rho_{i,i'}^* = \begin{cases} 1, & i'=\pi(i) \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

Combining the above three phases together, we have obtained the optimal primal variables $\{\boldsymbol{p}^*,\boldsymbol{t}^*,\boldsymbol{\rho}^*\}$ for given dual variables $\boldsymbol{\beta}$. Suppose the complexity of dual variable updates mentioned in Section III-A is in the order of K^α. The computational complexity of solving the dual problem (11) is $O(N^3K^\alpha)$.

C. Refinement of Power Allocation

Having obtained the optimal dual point $\boldsymbol{\beta}^*$, we now need to determine the optimal solution to the original primal problem (8). Due to the non-zero duality gap for finite N, the optimal $\boldsymbol{t}^*(\boldsymbol{\beta}^*)$, $\boldsymbol{\rho}^*(\boldsymbol{\beta}^*)$ and $\boldsymbol{p}^*(\boldsymbol{\beta}^*)$ may be infeasible. To solve this issue, we first obtain the solution for the subcarrier pairing and relay assignment $\{\boldsymbol{t}^*(\boldsymbol{\beta}^*), \boldsymbol{\rho}^*(\boldsymbol{\beta}^*)\}$ using the method in the previous subsection, and then refine the power allocation $\boldsymbol{p}$ to meet the power constraints (5) and (6) in the primal problem. This approach is asymptotically optimal due to the vanishing duality gap when N is sufficiently large.

Let $\mathcal{S}_k$ denote the set of active subcarrier pairs $(i, \pi(i))$ assigned on relay k. By substituting the value of $\{\boldsymbol{t}^*(\boldsymbol{\beta}^*), \boldsymbol{\rho}^*(\boldsymbol{\beta}^*)\}$ into (8), the power refinement problem can be equivalent to the following power allocation problem, which is convex:

$$\max \sum_{k=1}^{K} \sum_{(i,\pi(i))\in\mathcal{S}_k} R_{i,\pi(i),k} \tag{24}$$

$$\text{s.t.} \sum_{k=1}^{K} \sum_{(i,\pi(i))\in\mathcal{S}_k} p_{i,k,1} \le P_S \tag{25}$$

$$\sum_{(i,\pi(i))\in\mathcal{S}_k} p_{\pi(i),k,2} \le P_{R,k},\ \forall k \text{ with } \mathcal{S}_k \neq \emptyset\ . \tag{26}$$

Define $\mathcal{D}$ as the set of all non-negative $\{p_{i,k,1}\}$ and $\{p_{\pi(i),k,2}\}$. The Lagrangian of problem (24) over $\mathcal{D}$ is

$$\begin{aligned} J(\boldsymbol{p},\boldsymbol{\eta}) = & \sum_{k=1}^{K} \sum_{(i,\pi(i))\in\mathcal{S}_k} \ln\Bigg(1+\alpha_{i,3}p_{i,k,1} \\ & + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{\pi(i),k,2}p_{\pi(i),k,2}}{\alpha_{i,k,1}p_{i,k,1}+\alpha_{\pi(i),k,2}p_{\pi(i),k,2}}\Bigg) \\ & + \eta_S\left(P_S - \sum_{k=K} \sum_{(i,\pi(i))\in\mathcal{S}_k} p_{i,k,1}\right) \\ & + \sum_{k=1}^{K} \eta_{R,k}\left(P_{R,k} - \sum_{(i,\pi(i))\in\mathcal{S}_k} p_{\pi(i),k,2}\right). \end{aligned} \tag{27}$$

Then the dual function is

$$g(\boldsymbol{\eta}) \triangleq \max_{\boldsymbol{p}\in\mathcal{D}} J(\boldsymbol{p},\boldsymbol{\eta}). \tag{28}$$

Employing KKT conditions to compute the dual function at given dual point, it can be readily seen that the optimal power allocation follows the same expression as in (15) and (16). We rewrite them as follows for ease of presentation:

$$p^*_{i,k,1} = \begin{cases} c_{i,\pi(i),k}p^*_{\pi(i),k,2}, & \text{if } p^*_{\pi(i),k,2} > 0 \\ \left(\frac{1}{\eta_S} - \frac{1}{\alpha_{i,3}}\right)^+, & \text{if } p^*_{\pi(i),k,2} = 0 \end{cases},$$

and $p^*_{\pi(i),k,2}$ and $c_{i,i',k}$ are shown at the top of the next page. The Lagrange multipliers η_S and $\eta_{R,k}$ are chosen to meet the power constraints (25) and (26) respectively and can be updated by the subgradient method.

It can be easily seen that the computational complexity of the power refinement process is far more smaller than that of solving the dual problem. Thus the overall complexity of our joint optimal resource allocation algorithm is $O(N^3K^\alpha)$.

IV. Optimal Resource Allocation Under Total Power Constraint

In the previous section, we solved the joint optimization of relay selection, subcarrier pairing and power allocation under the individual power constraints (5) and (6). In this section, we solve the same problem except that the total power constraint (7) is considered. The optimization problem is presented as follows.

$$\max_{\{\boldsymbol{p},\boldsymbol{t},\boldsymbol{\rho}\}} \sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{k=1}^{K} \rho_{i,i'}t_{i,i',k}R_{i,i',k} \quad \text{s.t. } (3), (4), (7). \tag{29}$$

Here the primal variables $\boldsymbol{\rho}$ again represent the subcarrier pairing, $\boldsymbol{t}$ for the relay selection, and $\boldsymbol{p}$ for the power allocation. Problem (29) can be solved by the similar method in the previous section. However, the computational complexity can be much lower as elaborated below.

Given a potential transmission path (i, i', k), suppose the sum power allocated in both hops is given by $s_{i,i',k}$, which can take an arbitrary value between 0 and P_T. That is,

$$p_{i,k,1} + p_{i',k,2} = s_{i,i',k}. \tag{30}$$

We study the following problem optimizing $p_{i,k,1}$ and $p_{i',k,2}$ to maximize the end-to-end transmission rate over (i, i', k):

$$\begin{aligned} \max_{\{p_{i,k,1},p_{i',k,2}\}} & \frac{1}{2}\ln\Bigg(1+\alpha_{i,3}p_{i,k,1} \\ & + \frac{\alpha_{i,k,1}p_{i,k,1}\alpha_{i',k,2}p_{i',k,2}}{\alpha_{i,k,1}p_{i,k,1}+\alpha_{i',k,2}p_{i',k,2}}\Bigg) \\ \text{s.t. } & (30) \end{aligned} \tag{31}$$

The solution of (31) has been derived in [13] as

$$p^*_{i,k,1} = \begin{cases} \frac{\alpha_{i',k,2}\left(d_{i,i',k}+\alpha_{i,3}\right)s_{i,i',k}}{d_{i,i',k}\left(d_{i,i',k}+\alpha_{i',k,2}\right)}, \text{if } \alpha_{i',k,2} > \alpha_{i,3} \\ s_{i,i',k}, \text{ if } \alpha_{i',k,2} \le \alpha_{i,3} \end{cases} \tag{32}$$

and

$$p^*_{i',k,2} = \begin{cases} \frac{\alpha_{i,k,1}\left(\alpha_{i',k,2}-\alpha_{i,3}\right)s_{i,i',k}}{d_{i,i',k}\left(d_{i,i',k}+\alpha_{i',k,2}\right)}, \text{if } \alpha_{i',k,2} > \alpha_{i,3} \\ 0, \text{ if } \alpha_{i',k,2} \le \alpha_{i,3} \end{cases} \tag{33}$$

where $d_{i,i',k} \triangleq \sqrt{\alpha_{i,k,1}\alpha_{i',k,2} - \alpha_{i,k,1}\alpha_{i,3} + \alpha_{i',k,2}\alpha_{i,3}}$. Substituting (32) and (33) into (31), we obtain the maximum rate for the transmission path (i, i', k) subject to the power assignment $s_{i,i',k}$ as

$$R^*_{i,i',k} = \begin{cases} \frac{1}{2}\ln\left(1+\frac{\alpha_{i',k,2}\left(d_{i,i',k}+\alpha_{i,3}\right)^2}{\left(d_{i,i',k}+\alpha_{i',k,2}\right)^2}s_{i,i',k}\right), \\ \qquad \text{if } \alpha_{i',k,2} > \alpha_{i,3} \\ \frac{1}{2}\ln\left(1+\alpha_{i,3}s_{i,i',k}\right), \text{if } \alpha_{i',k,2} \le \alpha_{i,3} \end{cases} \tag{34}$$

Observing (34), it is interesting that the maximum rate achieved over the relay-assisted transmission path can be

$$p^*_{\pi(i),k,2} = \begin{cases} \left(\dfrac{\alpha_{i,k,1}\alpha^2_{\pi(i),k,2} - (\alpha_{i,3} - \eta_S)\left(\alpha_{i,k,1}c_{i,\pi(i),k} + \alpha_{\pi(i),k,2}\right)^2}{c_{i,\pi(i),k}\beta_S\left(\alpha_{i,k,1}c_{i,\pi(i),k} + \alpha_{\pi(i),k,2}\right)\left(\alpha_{i,3}\alpha_{i,k,1}c_{i,\pi(i),k} + \alpha_{i,3}\alpha_{\pi(i),k,2} + \alpha_{i,k,1}\alpha_{\pi(i),k,2}\right)} \right)^+, \\ \quad \text{if} \ \ \alpha_{\pi(i),k,2}\eta_S > \alpha_{i,3}\eta_{R,k} \\ 0, \quad \text{if} \ \ \alpha_{\pi(i),k,2}\eta_S \le \alpha_{i,3}\eta_{R,k} \end{cases}$$

$$c_{i,i',k} = \frac{\alpha_{i',k,2}}{\alpha_{i,k,1}\left(\alpha_{i',k,2}\eta_S - \alpha_{i,3}\eta_{R,k}\right)}\left(\sqrt{\eta_{R,k}\left(\alpha_{i,k,1}\alpha_{i',k,2}\eta_S - \alpha_{i,k,1}\alpha_{i,3}\eta_{R,k} + \alpha_{i',k,2}\alpha_{i,3}\eta_S\right)} + \alpha_{i,3}\beta_{R,k}\right).$$

equivalent to the rate on a point-to-point link. This motivates the definition of *equivalent end-to-end channel gain* given the optimal power allocation in [13]

$$\overline{\alpha}_{i,i',k} = \begin{cases} \dfrac{\alpha_{i',k,2}\left(d_{i,i',k} + \alpha_{i,3}\right)^2}{\left(d_{i,i',k} + \alpha_{i',k,2}\right)^2}, & \text{if} \ \ \alpha_{i',k,2} > \alpha_{i,3} \\ \alpha_{i,3}, & \text{if} \ \ \alpha_{i',k,2} \le \alpha_{i,3} \end{cases}. \quad (35)$$

The equivalent end-to-end channel gain helps simplifying the optimization of power allocation. For each potential transmission path (i, i', k), now we only need to optimize the new power variables $\{s_{i,i',k}\}$, rater than $\{p_{i,k,1}, p_{i',k,2}\}$. To further simplify (29), we have the following lemma regarding relay selection, the proof of which is simple and ignored.

Lemma 1: For each subcarrier pair candidate (i, i'), the optimal relay assignment is to select the relay node that has the largest equivalent end-to-end channel gain $\overline{\alpha}_{i,i',k}$. That is,

$$t^*_{i,i',k} = \begin{cases} 1, & k = k(i,i') = \arg\max_k \overline{\alpha}_{i,i',k} \\ 0, & \text{otherwise} \end{cases} \quad (36)$$

Let $s_{i,i'} = s_{i,i',k(i,i')}$ and denote $\boldsymbol{s} = \{s_{i,i'}\}$. Then the original problem (29) can be simplified as

$$\max_{\{\boldsymbol{s},\boldsymbol{\rho}\}} \sum_{i=1}^{N}\sum_{i'=1}^{N} \rho_{i,i'} \ln(1 + \overline{\alpha}_{i,i'}s_{i,i'}) \quad (37)$$
$$\text{s.t.} \quad \sum_{i=1}^{N} \rho_{i,i'} = 1, \ \sum_{i'=1}^{N} \rho_{i,i'} = 1, \ \forall i, i'$$
$$\sum_{i=1}^{N}\sum_{i'=1}^{N} s_{i,i'} \le P_T$$

By using the similar dual method and Hungarian method as in Section III, this new problem can be readily solved. But compared with the algorithm in Section III, solving (37) only involves the update of one, instead of $K + 1$, dual variable. Therefore, the complexity is only $O(N^3)$.

V. Suboptimal Methods

In Section III we derived the (asymptotically) optimal resource allocation algorithm under the individual power constraints, which may not be computationally efficient in practical systems for large N and K. In this section we propose two suboptimal algorithms with reduced complexity.

A. Suboptimal Algorithm 1: Equal Power Allocation based subcarrier-pairing and relay assignment

In this suboptimal algorithm, we first determine the subcarrier pairing and relay assignment based on metrics obtained by assuming equal power allocation, rather than the metrics $H_{i,i',k}$ defined in (19) or (20). The procedure is as follows.

Without loss of generality, we let all the transmit nodes be subject to the same individual power constraint, i.e.

$$P_S = P_{R,k} = P, \quad \forall k.$$

We firstly set the transmit power distributed equally over all subcarriers in both time slots:

$$p_{i,k,1} = \frac{1}{N}P, \quad p_{i',k,2} = \frac{K}{N}P, \forall i, k. \quad (38)$$

Let us substitute (38) into the rate function (1). For each possible subcarrier pair candidate (i, i'), we select the relay that maximizes the transmission rate $R_{i,i',k}$ and denote it as $k(i, i')$. The Hungarian method can be again employed to determine the optimal subcarrier pairing, as in the optimal algorithm.

Once the subcarrier pairing and relay assignment are obtained, the power allocation over subcarriers at each transmit node remains to be uniform as in (38), except that for each relay node, the individual power P is equally distributed over its assigned subcarriers in the second hop.

Compared with the optimal algorithm, this suboptimal algorithm does not need to update the dual variables. Hence its complexity is $O(N^3)$ only.

B. Suboptimal Algorithm 2: Fixed Subcarrier pairing

Similar to the previous suboptimal schemes [5], [6], we let the subcarrier pairing be pre-fixed, rather than seeking the optimal subcarrier pairing. Without loss of generality, the subcarrier pairing is arranged as

$$\pi(i) = i, \ \forall i.$$

This indicates that the signals transmitted by the source on one subcarrier is forwarded on the same subcarrier by a relay to the destination. The rest of this suboptimal algorithm is to determine the joint optimal relay assignment and power allocation. As the key difference with the optimal algorithm is that there is no need to optimize the subcarrier pairing, the relay assignment in this suboptimal algorithm can be performed as $k(i, i) = \arg\max_k H_{i,i,k}, \forall i$.

Compared with suboptimal algorithm 1, suboptimal algorithm 2 still needs to update the dual variables to compute the metrics $H_{i,i,k}$. Since the number of steps to update the dual variables is $O(K^\alpha)$ as assumed earlier, and it requires the complexity of $O(NK)$ to perform all the relay selection and power allocation at each step, the overall complexity of the suboptimal algorithm with fixed subcarrier pairing is $O(NK^{\alpha+1})$.

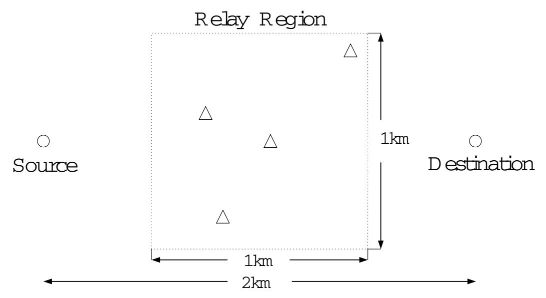

Fig. 3. Relay node distribution for the setup in Figs. 4, 5 and 6.

VI. Simulation Results

In this section we present some simulation results to evaluate the performance of the proposed optimal and suboptimal resource allocation algorithms. As the individual power constraint is more likely to happen in practical systems than the total power constraint, only the algorithms dealing with individual power constraints will be examined throughout this section. In addition, all the relays and the source are assumed subject to a same power constraint for simplicity.

The Stanford University Interim (SUI)-6 channel model is employed, in which the central frequency is given at 1.9GHz to emulate a broadband wireless network, and each channel is a 6-tap channel. The signal fading follows the Ricean distribution with K factor of 1 on the first tap while a Rayleigh distribution on the other five taps. The noise spectrum density is set to 4.14×10^{-21}W/Hz. The total bandwidth is fixed to 1MHz and the root mean square (rms) delay is $0.305\ \mu s$. We choose $N = 32$ thus all subcarriers can be regarded to experience flat-fading. This number is also large enough for the duality gap to vanish. The distance between the source and destination is 2km. The path-loss exponent α is fixed to 3.5. No shadowing is considered here.

As a benchmark, the performance of the OFDM symbol-based relay selection (i.e. select the single relay node with the maximum averaged channel gain over the whole frequency band) is also presented. This baseline approach operates as follows: (i) For each relay node, sort the subcarriers in each hop individually in descending order according to channel gains over them. (ii) Pair the sorted subcarriers on the two hops one by one and compute the total transmission rate under the equal power assumption for each relay. Note that such sorting and pairing approach is optimal for single-relay OFDM systems [9], [10]. (iii) Select the relay that maximizes the rate, and then perform optimal power allocation on the subcarrier pairs.

We first consider the relay node distribution shown in Fig. 3, where the multiple relays are placed randomly in a square region between the source and the destination.

Figs. 4 and 5 compare the average end-to-end spectral efficiency achieved by different schemes when there are $K = 8$ and $K = 16$ relays in the network, respectively. The results are based upon an average over 100 realizations of relay distribution. Note that, to verify the diminishing duality-gap at $N = 32$ subcarriers, the solution of the dual optimization problem, $g(\boldsymbol{\beta}^*)$ in (11) is also presented in the two figures. It defines the upper bound of the truly optimal solution of the primal problem.

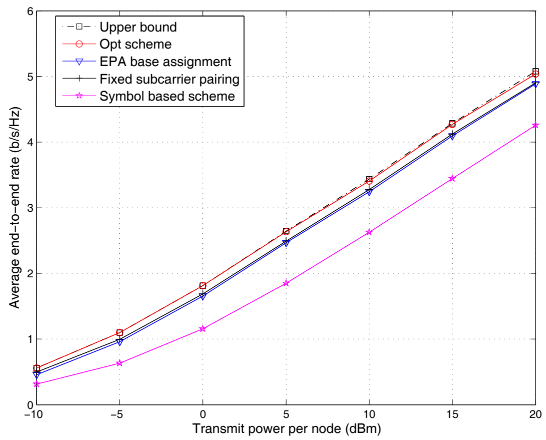

Fig. 4. Average end-to-end transmission rate versus transmit power per node with $K = 8$ relays.

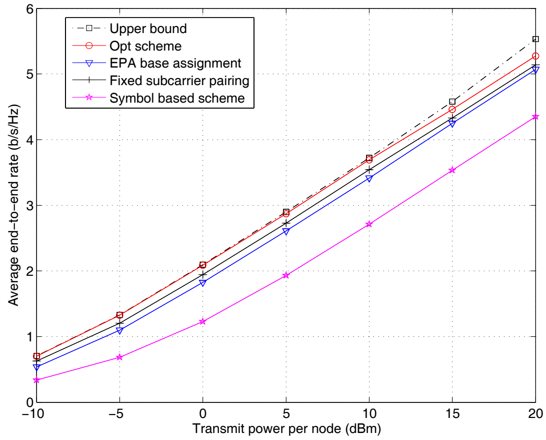

Fig. 5. Average end-to-end transmission rate versus transmit power per node with $K = 16$ relays.

From these results, we first observe that the proposed asymptotically optimal algorithm using the dual-method has almost the same performance as the upper bound. This confirms that $N = 32$ subcarriers are indeed sufficient to approach the truly optimal performance. Then we observe that the proposed subcarrier-pair based resource allocation algorithms significantly outperform the baseline scheme, symbol-based relay selection. In particular, at 5dBm transmit power per node and with 8 relays, the optimal algorithm achieves about 40% improvement in the spectral efficiency. Comparing the two suboptimal methods with the optimal one in Fig. 4, one can see that to achieve the same spectral efficiency, they only incur marginal power loss, even though their computational complexity is dramatically lower than that of the optimal scheme.

From Fig. 5 with 16 relays, we also observe that the fixed subcarrier pairing scheme (suboptimal algorithm 2) performs slightly better than the EPA based scheme (suboptimal algorithm 1). However note that the computational complexity of the former increases for large K due to the updates of dual variables.

Using the same relay setup in Fig. 3, we also examine the information outage probability of the different schemes.

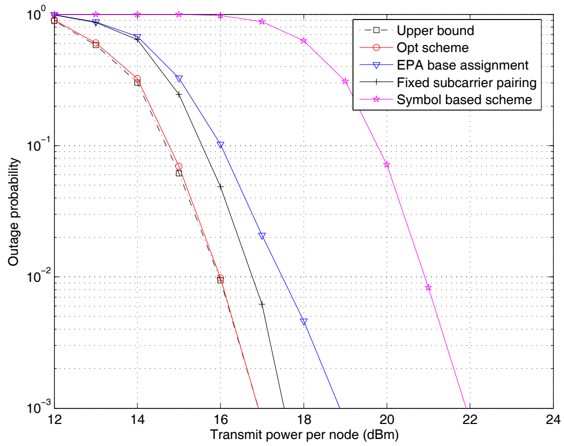

Fig. 6. Outage probability versus transmit power per node with $K = 8$ relays. Target rate = 4 bits/sec/Hz.

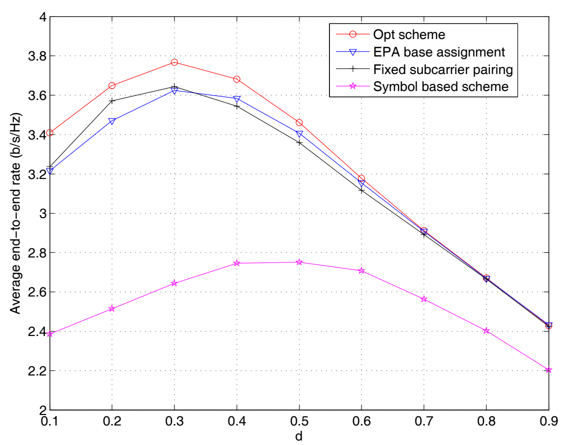

Fig. 7. Average end-to-end transmission rate versus relay location. Power per node = 10dBm, with $K = 8$ relays.

The results are shown in Fig. 6, where the relay number is $K = 8$ and the target transmission rate is $R_0 = 4$ bit/s/Hz. As expected, the proposed subcarrier-pair based schemes (both optimal and suboptimal) can dramatically reduce the outage probability compared with the benchmark scheme. Among the two suboptimal schemes, the EPA based scheme performs slightly worse than the fixed subcarrier pairing scheme. Nevertheless, the complexity of the EPA based scheme is much lower as no dual updates are needed.

Next, we consider a different relay distribution. We assume that the multiple relay nodes in the network form a cluster and lie approximately on the line connecting the source and destination. The radius of the relay cluster is much smaller than the distance from the source to the destination. Then, the channel statistics experienced by different relays can be assumed identical. Let d denote the distance ratio of the source-relay link to the source-destination link. Fig. 7 presents the average end-to-end spectral efficiency achieved by different schemes when d varies.

For the optimal scheme, the maximum transmission rate is achieved at about $d = 0.3$. This is in contrast to the baseline approach where the maximum occurs around $d = 0.4$. The performance of the EPA based assignment overshadows that of the fixed subcarrier pairing scheme when $d > 0.3$.

VII. Conclusion

In this paper, we formulate the subcarrier-pair based resource allocation problem for a cooperative multi-relay OFDM system with AF protocol. It finds the joint optimization of subcarrier pairing, relay assignment and power allocation with the objective of maximizing the transmission rate. Both individual and total power constraints are investigated. By utilizing the dual method, the mixed integer programming problems are solved efficiently with polynomial complexity. Besides, it is found that the complexity of the problem with a total power constraint is much lower than that of the problem with individual power constraints. To reduce the complexity of the problem with individual power constraints, we further propose two suboptimal approaches. The simulation results show that the suboptimal algorithm which decouples the power allocation with the subcarrier pairing and relay assignment performs closely to the optimal one, while having much less complexity.

Appendix A
Derivation of Optimal Solution $\boldsymbol{p}^*$ in (15) and (16)

The derivative of $L_{i,i',k}$ with respect to variable $p_{i,k,1}$, replaced by p_1 here for simplicity is given by

$$\frac{\partial L}{\partial p_1} = -\beta_S + \frac{\alpha_3\left(\alpha_1 p_1 + \alpha_2 p_2\right)^2 + \alpha_1\alpha_2^2 p_2^2}{\left(\alpha_1 p_1 + \alpha_2 p_2\right)} \cdot \frac{1}{\alpha_1 p_1 + \alpha_2 p_2 + p_1\left(\alpha_1\alpha_3 p_1 + \alpha_2\alpha_3 p_2 + \alpha_1\alpha_2 p_2\right)}. \quad (39)$$

Similarly, we have

$$\frac{\partial L}{\partial p_2} = -\beta_R + \frac{\alpha_1^2\alpha_2 p_1^2}{\left(\alpha_1 p_1 + \alpha_2 p_2\right)} \cdot \frac{1}{\alpha_1 p_1 + \alpha_2 p_2 + p_1\left(\alpha_1\alpha_3 p_1 + \alpha_2\alpha_3 p_2 + \alpha_1\alpha_2 p_2\right)}. \quad (40)$$

We first discuss the case that both p_1^* and p_2^* are positive. Equating (39) and (40) to zero, we obtain

$$\beta_R\left[\alpha_3\left(\alpha_1 p_1 + \alpha_2 p_2\right)^2 + \alpha_1\alpha_2^2 p_2^2\right] = \beta_S\alpha_1^2\alpha_2 p_1^2.$$

This yields

$$p_1^* = c p_2^*, \quad (41)$$

where

$$c = \frac{\alpha_2}{\alpha_1\left(\alpha_2\beta_S - \alpha_3\beta_R\right)} \cdot \left(\sqrt{\beta_R\left(\alpha_1\alpha_2\beta_S - \alpha_1\alpha_3\beta_R + \alpha_2\alpha_3\beta_S\right)} + \alpha_3\beta_R\right).$$

In order to let $p_1^* > 0$, the factor c should be positive, thus we have $\alpha_2\beta_S > \alpha_3\beta_R$. Substituting (41) into (39) we get

$$p_2^* = \frac{\alpha_1\alpha_2^2 + \left(\alpha_3 - \beta_S\right)\left(\alpha_1 c + \alpha_2\right)^2}{c\beta_S\left(\alpha_1 c + \alpha_2\right)\left(\alpha_3\alpha_1 c + \alpha_3\alpha_2 + \alpha_1\alpha_2\right)}. \quad (42)$$

If the value of (42) is negative, then p_2^* should be set to zero. When $\alpha_2\beta_S \le \alpha_3\beta_R$, it can be also proved that $p_2^* = 0$. For these cases, the optimal power allocation in the first hop should follow the expression of conventional water-filling approach :

$$p_1^* = \left(\frac{1}{\beta_S} - \frac{1}{\alpha_3} \right)^+.$$

Thus, the optimality of solution $\boldsymbol{p}^*$ in (15) and (16) is proved.

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Wenbing Dang (S'09) received the B.S. degree in electronic engineering from Shanghai Jiao Tong University, Shanghai, China, in 2009. Since Fall 2009, he has been pursuing the Ph.D. degree in electrical and computer engineering at Colorado State University, Fort Collins. His current research interests lie in the area of wireless communication, radar waveform design, and statistical signal processing.

Meixia Tao (S'00-M'04) received the B.S. degree in Electronic Engineering from Fudan University, Shanghai, China, in 1999, and the Ph.D. degree in Electrical and Electronic Engineering from Hong Kong University of Science & Technology in 2003.

She is currently an Associate Professor at the Department of Electronic Engineering, Shanghai Jiao Tong University, China. From Aug. 2003 to Aug. 2004, she was a Member of Professional Staff at Hong Kong Applied Science & Technology Research Institute Co. Ltd. From Aug 2004 to Dec. 2007, she was with the Department of Electrical and Computer Engineering at National University of Singapore as an Assistant Professor. Her current research interests include cooperative transmission, physical layer network coding, resource allocation of OFDM networks, and MIMO techniques.

Dr. Tao is an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, an Associate Editor for the IEEE COMMUNICATIONS LETTERS and a Guest Editor for the EURASIP JOURNAL ON WIRELESS COMMUNICATIONS AND NETWORKING. She served as Track/Symposium Co-Chair for APCC2009, ChinaCom 2009, IEEE ICCCN'07, and IEEE ICCCAS'07. She also served as Technical Program Committee member for various conferences, including IEEE GLOBECOM (2007, 2009), IEEE ICC (2006-2010), IEEE WCNC (2007-2010), and IEEE VTC (2006-Fall, 2008-Spring, 2009-Spring). Dr. Tao is the recipient of the Publication Awards in the Institution of Engineers, Singapore, in 2005. She received the IEEE ComSoC Asia-Pacific Outstanding Young Researcher Award in 2009.

Hua Mu received the M.S. degree in Electronic Engineering from Shanghai Jiao Tong University, Shanghai, China, in 2008. Since May 2009, she has been working towards the Ph.D. degree at Department of Electrical and Computer Engineering, Auburn University, Auburn, USA. Her current research interests lie in the area of wireless communication and signal processing for communications.

Jianwei Huang is an Assistant Professor in the Department of Information Engineering at the Chinese University of Hong Kong. He received Ph.D. in Electrical and Computer Engineering from Northwestern University in 2005, and worked as a Postdoc Research Associate in the Department of Electrical Engineering at Princeton University during 2005-2007.

Dr. Huang conducts research in the area of nonlinear optimization and game theoretical analysis of communication networks, with current focus on network economics, cognitive radio networks, broadband communication networks, and multimedia over wireless. He is the recipient of the IEEE ComSoc Asia-Pacific Outstanding Young Researcher Award in 2009, Asia Pacific Conference on Communications Best Paper Award in 2009, and Walter P. Murphy Fellowship at Northwestern University in 2001.

Dr. Huang is an Associate Editor of (Elsevier) JOURNAL OF COMPUTER & ELECTRICAL ENGINEERING, the Director of IEEE MMTC E-letter, and the Lead Guest Editor for various journals including IEEE JOURNAL OF SELECTED AREAS IN COMMUNICATIONS, JOURNAL OF COMMUNICATIONS, and JOURNAL OF ADVANCES IN MULTIMEDIA. He serves as the TPC co-chair of the International Conference on Game Theory for Networks 2009, the TPC co-chair of IEEE Globecom Wireless Communications Symposium 2010, and the TPC co-chair of the International Wireless Communications and Mobile Computing Conference Mobile Computing Symposium 2010.