Market Your Venue with Mobile Applications: Collaboration of Online and Offline Businesses

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***Abstract*—Many mobile applications (abbrev. apps) reward the users who physically visit some locations tagged as POIs (places-of-interest) by the apps. In this paper, we study the POI-based collaboration between apps and venues (e.g., restaurants and cafes). On the one hand, an app charges a venue and tags the venue as a POI, which attracts users to visit the venue and potentially increases the venue's sales. On the other hand, the venue can invest in the app-related infrastructure (e.g., Wi-Fi networks and smartphone chargers), which enhances the users' experience of using the app. However, the existing POI pricing schemes of the apps (e.g., Pokemon Go and Snapchat) cannot incentivize the venue's infrastructure investment, and hence cannot achieve the most effective app-venue collaboration. We model the interactions among an app, a venue, and users by a three-stage Stackelberg game, and design an optimal two-part pricing scheme for the app. This scheme has a *charge-with-subsidy* structure: the app first charges the venue for becoming a POI, and then subsidizes the venue every time a user interacts with the POI. Compared with the existing pricing schemes, our two-part pricing better incentivizes the venue's investment, attracts more users to interact with the POI, and achieves a much larger app revenue. We analyze the impacts of the app's and venue's characteristics on the app's optimal revenue, and show that the apps with small and large congestion effects should collaborate with opposite types of venues.**

I. Introduction

A. Motivations

Many popular mobile applications (abbrev. apps), especially the augmented reality apps, have tried to integrate users' digital experience with the physical world. For example, Pokemon Go (one of the most popular mobile games in 2016) tags some physical locations as "PokeStops" and "Gyms", and users can collect game items or participate in battles in the game when physically visiting these locations [1]. Snapchat (a popular image messaging app) provides users with various image filters, including "Geofilters", which can be used only when the users visit the specified physical locations [2]. Many other apps, such as Snatch [3] and Ingress [4], integrate users' digital experience and physical activities using similar approaches. We use POIs (places-of-interest) to refer to the physical locations where users can obtain rewards or unlock some features of the apps.

When the locations are some store or restaurant venues, the POI tags will benefit both the apps and the venues. On the one hand, the infrastructure at the venues enhances the users' experience of using the apps, which benefits the apps' businesses. For example, many apps (especially the augmented reality apps like Pokemon Go [5]) drain the smartphones' batteries quickly, which makes the smartphone chargers at the venues attractive to users. On the other hand, the POI tags significantly increase the customer traffic to the venues, which benefits the venues' businesses. This explains the increasing number of collaborations between apps (*online businesses*) and venues (*offline businesses*). For example, Pokemon Go collaborated with Sprint and McDonald's, tagging 10,500 Sprint stores in the U.S. and 3,000 McDonald's restaurants in Japan as POIs [6], [1]. In particular, Sprint stores provided smartphone charging stations for Pokemon Go players [6]. It was estimated that each of the McDonald's restaurants attracted up to 2,000 game players per day [1]. Furthermore, Wendy's (a restaurant chain) made its "Geofilters" in Snapchat, which drove 42,000 additional visitors within a week [2], and over 5,000 KFC restaurants in China were tagged as POIs by Yinyangshi [7].

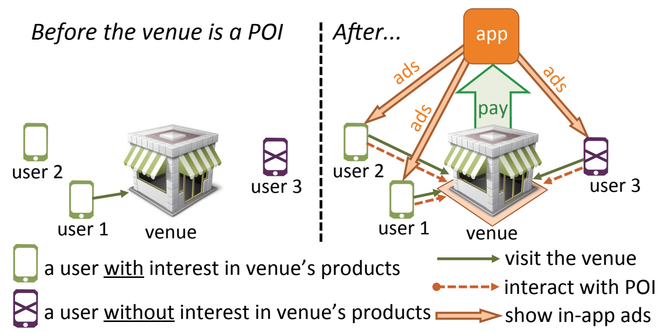

Fig. 1: An example of POI-based collaboration, where all users use the app.

We illustrate the collaboration between an app and a venue in Fig. 1 and discuss the key challenge of designing an effective business model to realize the full potential of the collaboration. As shown in the abovementioned examples, an app usually collaborates with a store/restaurant chain (e.g., Sprint, McDonald's, and KFC). Since the venues in a chain are typically located far from each other to avoid internal competitions, we can approximate the collaboration between the app and the chain by the collaboration between the app and a representative venue of the chain. In Fig. 1, before the venue becomes a POI, only the nearby users who are interested in the venue's products (e.g., user 1) will visit the venue.

After the venue pays the app and becomes a POI, more users (including those without interests in the venue's products) visit the venue to interact with the POI (e.g., participate in the battles held at the POI). The number of these visitors depends on the venue's investment in the app-related infrastructure. The app can also show location-dependent in-app advertisements to these visitors to obtain additional advertising revenue.

As Fig. 1 shows, the app and venue may not have fully aligned interests in attracting the users. The app delivers the advertisements to all users interacting with the POI (e.g., users $1 \sim 3$), and hence benefits from a high investment in the app-related infrastructure. The venue only gains profits from the users with interests in its products (e.g., users 1 and 2), and hence may not choose a high investment level. Therefore, the key challenge is to design the app's optimal pricing scheme, which (i) charges the venue for becoming a POI, and (ii) incentivizes the venue's investment in the app-related infrastructure.

Two pricing schemes commonly used by the apps today are the *per-player-only pricing* and *lump-sum-only pricing*. In the *per-player-only pricing* scheme, the apps charge a venue based on the number of users playing the apps at the venue. For example, Pokemon Go charged a venue up to \$0.50 per game player [1]. In the *lump-sum-only pricing* scheme, the apps (e.g., Snapchat) charge a venue a lump-sum fee, which is independent of the number of players at the venue. In this work, we will show that these existing pricing schemes cannot effectively incentivize the venues' investments. This motivates us to design a novel pricing scheme that induces the maximum infrastructure investments at the venues and increases the numbers of users interacting with the POIs.

B. Surveys

As the POI-based collaboration between apps and venues is relatively new, it is very important to obtain actual market data to understand the reality. There are several existing market surveys (e.g., Slant Marketing's [8] and ClickZ's [9]) about Pokemon Go players' engagements with the venues like restaurants, cafes, and bars. For example, in Slant Marketing's survey [8], 71% of the 500 respondents had visited these venues because of the POI features, and 51% of the respondents had visited at least one venue *for the first time* because of Pokemon Go. These data reveal the venues' potential benefits from becoming POIs.

Because there is no prior survey about the dependence of users' experience on the POIs' infrastructure, we conducted a new survey involving 103 Pokemon Go players in North America, Europe, and Asia. We find that the infrastructure (including Wi-Fi networks, smartphone chargers, and air conditioners) at the POIs could enhance the game experience of 81% of the players.

In particular, our survey shows both the *network effect* and *congestion effect* among the players. The *network effect* means that when many players interact with the POI, each player's experience might increase, as the players can share the app's information and play the app together. The *congestion effect* means that the players need to compete for the limited infrastructure (e.g., Wi-Fi network access) at the POI, which might decrease each player's experience. In our survey, 64% of the players stated that their game experience could be improved if there are nearby players playing the game (i.e., *network effect*), and 59% of the players thought that the Wi-Fi speeds at the POIs affected their game experience (i.e., *congestion effect*).

C. Our Contributions

In this work, we analyze the intricate interactions among an app, a venue, and users. We design a *two-part pricing*, under which the app charges the venue based on a combination of a lump-sum fee and a per-player charge. We model the problem by a three-stage Stackelberg game: in Stage I, the app announces the two-part pricing; in Stage II, the venue decides whether to be a POI and how much to invest in the infrastructure; in Stage III, the users decide whether to visit the venue and whether to interact with the POI. The game's analysis is very challenging because of the coexistence of the *network effect* and *congestion effect*.

Our analysis provides the following practical insights.

1) Insight 1: charge with subsidy: The app's optimal two-part pricing scheme includes a *positive* lump-sum fee and a *negative* per-player charge, which implies that the app should first *charge* the venue for becoming a POI, and then *subsidize* the venue every time a user interacts with the POI. Furthermore, the amount of the per-player subsidy should equal the app's *unit advertising revenue*, which is the app's revenue from showing the in-app advertisements to one user.[1]

2) Insight 2: ideal POIs vary across apps: We investigate the impacts of the app's and venue's characteristics on the app's revenue, and show that different apps achieve their largest revenues when collaborating with different venues. If the app's congestion effect is large,[2] we prove that the app maximizes its revenue by collaborating with a venue whose offline products generate small utilities to the users (e.g., an ordinary cafe). If the app's congestion effect is small, the app should collaborate with a venue whose offline products generate large utilities to the users (e.g., a top-rated restaurant).

We summarize our major contributions as follows:

- *Survey of Users' Experience at POIs*: We conduct the first survey about the impact of POIs' infrastructure on the users' game experience and the externalities (network effect and congestion effect) among the users at POIs.
- *Theoretical Study of POI-Based Marketing*: Motivated by our survey, we model the interactions among the app, venue, and users as a three-stage game, and characterize

[1] Many apps, such as Snapchat [10], display advertisements to the apps' users and receive payments from the corresponding advertisers. For an app that does not show any in-app advertisement, the corresponding unit advertising revenue is 0 in our model.

[2] For example, if the app is bandwidth-consuming, it has a large congestion effect. This is because the users will easily experience the network congestion when the venue's Wi-Fi network is not fast enough.

their equilibrium strategies. To the best of our knowledge, this is the first theoretical study on the POI-based collaboration between online and offline businesses.

- *Design of Optimal Two-Part Pricing*: We design the optimal two-part pricing for the app, and show its *charge-with-subsidy* structure. In particular, the amount of the per-player subsidy equals the unit advertising revenue.
- *Analysis of Parameters' Influences*: We study the impacts of the app's and venue's characteristics on the app's revenue, and show that the ideal POIs' features could vary across apps. Our results provide the app with guidelines for selecting venues to collaborate with.
- *Comparison with State-of-the-Art Schemes*: We compare our two-part pricing scheme with the existing *lump-sum-only pricing* and *per-player-only pricing*, and show that our scheme achieves the largest app's revenue. In particular, (i) if the congestion effect is medium, our scheme significantly outperforms the lump-sum-only pricing; (ii) if the network effect is large, our scheme significantly outperforms the per-player-only pricing; (iii) if the unit advertising revenue is large, our scheme has an obvious improvement over both the state-of-the-art schemes.

The rest of the paper is organized as follows. In Section II, we introduce the model. In Sections III–V, we analyze the equilibrium strategies of the users, venue, and app. In Section VI, we provide the numerical results. We discuss the related work in Section VII, and conclude the paper in Section VIII.

II. Model

In this section, we introduce the strategies of the app, the representative venue of a chain, and the users, and formulate their interactions as a three-stage game.

A. App's Pricing

Since most popular apps (e.g., Pokemon Go and Snapchat) are free to users, we assume that the app does not charge the users. In our model, the app only decides the two-part pricing. We use $p \in \mathbb{R}$ to denote the *lump-sum fee* and $w \in \mathbb{R}$ to denote the *per-player charge*. When the venue becomes a POI, its payment to the app contains: (i) the lump-sum fee p, and (ii) the product between the per-player charge w and the number of users interacting with the POI. Note that both p and w can be negative, in which case the venue *receives* a payment from the app. The app's revenue has two components: (i) the venue's payment, and (ii) the advertising revenue from the in-app advertisements.

B. Venue's POI and Investment Choices

We use $r \in \{0,1\}$ to denote the venue's choice to become a POI ($r=1$) or not ($r=0$). Moreover, we use $I \geq 0$ to denote the venue's investment level on the app-related infrastructure, such as smartphone chargers and Wi-Fi networks.[3]

Before the venue chooses r and I, some app-related infrastructure might be initially available at the venue. We use parameter $I_0 \geq 0$ to denote the *initial investment level*. Hence, $I_0 + I$ is the *total investment level*.

C. Users' Types, Decisions, and Payoffs

We consider a continuum of users who use the app and seek to interact with a POI. We denote the mass of users by N.[4]

1) User's type: Each user is characterized by attributes l and c. The first attribute $l \in \{0,1\}$ indicates whether the user has an intrinsic interest in consuming the venue's *offline* products. We assume that ηN users have $l=1$ (will consume the offline products when visiting the venue), and the remaining $(1-\eta)N$ users have $l=0$. Hence, parameter $\eta \in [0,1]$ represents the popularity of the venue's offline products.

The second attribute c denotes the user's transportation cost for visiting the venue, and we assume that c is uniformly distributed in $[0, c_{\max}]$ [12]–[14].

2) User's decision and payoff: We denote a user's decision by d, which has three possible values: $d=0$ (do not visit the venue), $d=1$ (visit the venue but do not interact with the POI), and $d=2$ (visit the venue and interact with the POI). Under the venue's choices r and I, a type-(l,c) user's payoff is

$$\Pi^{\text{user}}(l,c,d,r,I) = \begin{cases} 0, & \text{if } d=0, \\ Ul - c, & \text{if } d=1, \\ Ul - c + V + \theta \bar{y}(r,I)N - \frac{\delta}{I+I_0}\bar{y}(r,I)N, & \text{if } d=2. \end{cases} \quad (1)$$

When $d=0$, the user's payoff is 0.[5] When $d=1$, the user's payoff is the difference between Ul and the transportation cost c. Recall that the user consumes the offline products during its visit if and only if $l=1$. Here, $U>0$ denotes the user's utility of consuming the offline products.

Compared with $d=1$, the user's payoff under $d=2$ contains additional terms because of the interaction with the POI. We use $V>0$ to denote a user's base utility of interacting with the POI. The term $\theta \bar{y}(r,I)N$ corresponds to the network effect, which increases with the number of users interacting with the POI [13], [14]. Here, parameter $\theta \geq 0$ is the *network effect factor*, describing the strength of the network effect. We use function $\bar{y}(r,I) \in [0,1]$ to denote the fraction of users choosing $d=2$ (i.e., interacting with the POI), given the venue's choices r and I. The $\bar{y}(r,I)$ depends on all users' equilibrium decisions, and will be computed in Section III.

[3]Cellular technologies (e.g., LTE technology) suffer from building penetration loss [11] and may have poor indoor performance. Hence, it is necessary for the venue to offer high-quality Wi-Fi service, which guarantees users' wireless connection and enhances users' game experience.

[4]We assume that the number of users using the app is relatively small, compared with the number of users who do not use the app. In this case, the users who do not use the app are not affected by whether the venue is a POI, and these users are not considered in our model.

[5]Even if the users do not interact with the POI (i.e., $d=0$ or 1), they might still use the app. However, in this case, the app's usage will be much smaller than that when the users interact with the POI. Furthermore, the users who do not interact with the POI might use the app at different locations. Therefore, we do not consider the congestion effect and network effect among these users. Without loss of generality, we normalize these users' utilities of using the app to 0 in (1).

Stage I

The app announces $(p, w) \in \mathbb{R} \times \mathbb{R}$.

$\Downarrow$

Stage II

The venue chooses $r \in \{0,1\}$ and $I \geq 0$.

$\Downarrow$

Stage III

Each type-(l, c) user decides $d \in \{0,1,2\}$.

Fig. 2: Three-Stage Game.

The term $-\frac{\delta}{I+I_0}\bar{y}(r, I)\, N$ corresponds to the congestion effect of sharing the app-related infrastructure, where parameter $\delta > 0$ is the *congestion effect factor*. The congestion level $\frac{\delta}{I+I_0}\bar{y}(r, I)\, N$ increases with the number of users interacting with the POI, and decreases with the total investment level $I+I_0$.[6] As we can see in Section III, when $I+I_0$ approximates 0, we have $\bar{y}(r, I) = 0$. This implies that no user will interact with the POI at the equilibrium when there is no app-related infrastructure (e.g., no wireless network).

3) Fractions $\bar{x}(r, I)$ and $\bar{y}(r, I)$: We use function $\bar{x}(r, I) \in [0,1]$ to denote the fraction of users that have $l = 1$ and visit the venue (i.e., choose $d = 1$ or 2), given the venue's choices r and I. Function $\bar{x}(r, I)$ corresponds to the fraction of users consuming the venue's *offline* products, hence the venue wants to increase $\bar{x}(r, I)$. Recall that $\bar{y}(r, I)$ is the fraction of users interacting with the POI (i.e., choosing $d = 2$) at the equilibrium, hence the app wants to increase $\bar{y}(r, I)$. The difference between $\bar{x}(r, I)$ and $\bar{y}(r, I)$ reveals that the venue and app have overlapping but not fully aligned interests in attracting the users.

D. Three-Stage Stackelberg Game

We formulate the interactions among the app, venue, and users by a three-stage Stackelberg game, as illustrated in Fig. 2. Since the app has the market power and decides whether to tag the venue as a POI, we assume that the app is the leader and first-mover in the game.

We analyze the three-stage game by backward induction. We assume that the users' maximum transportation cost $c_{\max}$ is large so that $c_{\max} > U+V+\theta N$ [13], [14]. This captures a general case where some users are located far from the venue and will not visit it even if it becomes a POI.

III. Stage III: Users' Decisions

Given the app's pricing (p, w) in Stage I and the venue's choices of r and I in Stage II, each type-(l, c) user solves the following problem in Stage III.

Problem 1. *A type-(l, c) user decides d^* by solving*

$$\max \Pi^{\text{user}}(l, c, d, r, I) \tag{2}$$

$$\text{var. } d \in \begin{cases} \{0,1\}, & \text{if } r = 0, \\ \{0,1,2\}, & \text{if } r = 1, \end{cases} \tag{3}$$

[6]References [15] and [16] used similar congestion effect models. The model captures the fact that the marginal reduction in the congestion level decreases with the investment, and also makes the three-stage game's analysis tractable.

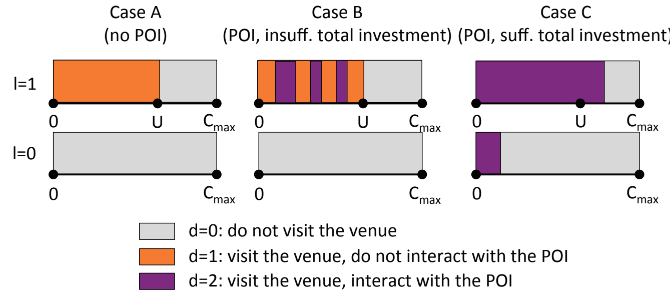

Fig. 3: Users' equilibrium decisions in Stage III. Since there are infinitely many $\hat{\mathcal{C}}$ satisfying Proposition 2, users have infinitely many equilibriums in Case B, and we show one example here.

where the payoff function $\Pi^{\text{user}}(l, c, d, r, I)$ is given in (1).

Here, (3) implies that the user can interact with the POI if and only if the venue is a POI. Based on the venue's choices of r and I, we show the users' optimal decisions in the following three propositions, and illustrate them in Fig. 3.

Proposition 1 (Case A: No POI)**.** *When $r = 0$, the unique equilibrium at Stage III is*

$$d^*(l, c, r, I) = \begin{cases} 1, & \text{if } c \in [0, Ul), \\ 0, & \text{if } c \in [Ul, c_{\max}], \end{cases} \tag{4}$$

where $l \in \{0,1\}$ and $c \in [0, c_{\max}]$.[7] *Moreover, $\bar{x}(r, I) = \eta\frac{U}{c_{\max}}$ and $\bar{y}(r, I) = 0$.*

When the venue is not a POI, only the users with intrinsic interests on the offline products (i.e., $l = 1$) and small transportation costs (i.e., $c < U$) will visit the venue.

We define $I_{\text{th}} \triangleq \frac{\delta}{\theta + \frac{V c_{\max}}{\eta U N}}$ as the *threshold investment level*. We say the total investment is *sufficient* if $I + I_0 > I_{\text{th}}$, and it is *insufficient* otherwise. In Propositions 2 and 3, we show that after becoming a POI, the venue attracts new visitors if and only if the total investment is sufficient.

Proposition 2 (Case B: POI with insufficient total investment)**.** *When $r = 1$ and $I + I_0 \leq I_{\text{th}}$, the unique form of equilibrium at Stage III is*

$$d^*(l, c, r, I) = \begin{cases} 2, & \text{if } c \in \hat{\mathcal{C}} \text{ and } l = 1, \\ 1, & \text{if } c \in [0, Ul) \setminus \hat{\mathcal{C}}, \\ 0, & \text{if } c \in [Ul, c_{\max}], \end{cases} \tag{5}$$

where $l \in \{0,1\}$, $c \in [0, c_{\max}]$, and $\hat{\mathcal{C}} \subseteq [0, U)$ can be any set that satisfies $\eta \int_0^U \frac{1}{c_{\max}} \mathbb{1}_{\{c \in \hat{\mathcal{C}}\}} dc = \frac{V}{\left(\frac{\delta}{I+I_0} - \theta\right) N}$.[8] *Moreover, $\bar{x}(r, I) = \eta\frac{U}{c_{\max}}$ and $\bar{y}(r, I) = \frac{V}{\left(\frac{\delta}{I+I_0} - \theta\right) N}$.*

[7]At the equilibrium, the user whose l and c satisfy $c = Ul$ has the same payoff under choices $d = 0$ and $d = 1$. This user's decision does not affect the computation of $\bar{x}(r, I)$ (and the analysis of Stage II and Stage I). This is because c follows a continuous distribution, and the probability for a user to have $c = Ul$ is zero. Without affecting the analysis, we assume that such a user always chooses $d = 0$ to simplify the presentation. Similar assumptions are made in Proposition 2 and Proposition 3.

[8]Here, $\mathbb{1}_{\{\cdot\}}$ is the indicator function, which equals 1 if the event in the braces is true, and equals 0 if the event is false.

$$H_1(w) \triangleq \begin{cases} -\frac{N}{c_{\max}} b\eta^2 U + \frac{N}{c_{\max}-N\theta}\left(\sqrt{(V+\eta U)(b\eta - w)} - \sqrt{\delta k}\right)^2 + kI_0, & \text{if } w < w_1, \\ -wN\eta\frac{U}{c_{\max}} - kI_{\text{th}} + kI_0, & \text{if } w_1 \le w \le w_0, \\ -\frac{V}{\frac{\delta}{I_0}-\theta} w, & \text{if } w > w_0. \end{cases} \tag{7}$$

$$H_2(w) \triangleq \begin{cases} -\frac{N}{c_{\max}} b\eta^2 U + \frac{N}{c_{\max}-N\theta}\left(\sqrt{(V+\eta U)(b\eta - w)} - \sqrt{\delta k}\right)^2 + kI_0, & \text{if } w < w_2, \\ -\frac{V}{\frac{\delta}{I_0}-\theta} w, & \text{if } w \ge w_2. \end{cases} \tag{8}$$

$$H_3(w) \triangleq \begin{cases} -\frac{N}{c_{\max}} b\eta^2 U + \frac{N}{c_{\max}-N\theta}\left(\sqrt{(V+\eta U)(b\eta - w)} - \sqrt{\delta k}\right)^2 + kI_0, & \text{if } w < w_3, \\ (b\eta - w) N \frac{Vc_{\max}I_0 - \eta UN\delta + \eta UN\theta I_0}{c_{\max}^2 I_0 + c_{\max}N\delta - c_{\max}N\theta I_0} - wN\eta\frac{U}{c_{\max}}, & \text{if } w \ge w_3. \end{cases} \tag{9}$$

When the venue becomes a POI and the total investment is insufficient, the app-related infrastructure at the venue does not allow many users to interact with the POI. In this case, a user's net payoff of interacting with the POI at the equilibrium is zero ($V+\theta\bar{y}(r,I)N - \frac{\delta}{I+I_0}\bar{y}(r,I)N = 0$) because of the congestion. Compared with Case A, the venue whose r and I satisfy Case B does not attract any new visitors. In Case B, the app-related infrastructure simply enables some of the initial visitors (i.e., the visitors when the venue is not a POI) to interact with the POI. We use $\hat{\mathcal{C}}$ to denote the set of these visitors' transportation costs in (5), and $\hat{\mathcal{C}}$ need not be an interval. In Fig. 3's example, the set of transportation costs of the initial visitors who interact with the POI consists of three intervals (i.e., the three purple intervals).

Proposition 3 (Case C: POI with sufficient total investment)**.** *When $r = 1$ and $I + I_0 > I_{\text{th}}$, the unique equilibrium at Stage III is*

$$d^*(l,c,r,I) = \begin{cases} 2, & \text{if } c \in \left[0, Ul + \frac{Vc_{\max}(I+I_0) - \eta UN\delta + \eta UN\theta(I+I_0)}{c_{\max}(I+I_0)+N\delta - N\theta(I+I_0)}\right), \\ 0, & \text{if } c \in \left[Ul + \frac{Vc_{\max}(I+I_0) - \eta UN\delta + \eta UN\theta(I+I_0)}{c_{\max}(I+I_0)+N\delta - N\theta(I+I_0)}, c_{\max}\right], \end{cases} \tag{6}$$

where $l \in \{0,1\}$ and $c \in [0, c_{\max}]$. Moreover, $\bar{x}(r,I) = \eta\frac{U}{c_{\max}} + \eta\frac{Vc_{\max}(I+I_0) - \eta UN\delta + \eta UN\theta(I+I_0)}{c_{\max}^2(I+I_0)+c_{\max}N\delta - c_{\max}N\theta(I+I_0)}$ *and* $\bar{y}(r,I) = \eta\frac{U}{c_{\max}} + \frac{Vc_{\max}(I+I_0) - \eta UN\delta + \eta UN\theta(I+I_0)}{c_{\max}^2(I+I_0)+c_{\max}N\delta - c_{\max}N\theta(I+I_0)}$.

When the venue becomes a POI and the total investment is sufficient, the infrastructure enables all of the venue's visitors to interact with the POI and obtain positive net payoffs of interacting with the POI at the equilibrium ($V+\theta\bar{y}(r,I)N - \frac{\delta}{I+I_0}\bar{y}(r,I)N > 0$). Compared with Case A and Case B, the venue whose r and I satisfy Case C attracts new visitors, including users without intrinsic interests on the offline products.

IV. Stage II: Venue's POI and Investment Choices

In Stage II, the venue solves the following problem by responding to the app's pricing (p, w) in Stage I and anticipating the users' decisions $d^*(l,c,r,I)$ in Stage III.

Problem 2. *The venue makes the POI choice r^* and investment choice I^* by solving*

$$\max \Pi^{\text{venue}}(r,I,p,w) \triangleq bN\bar{x}(r,I) - kI - r(p + wN\bar{y}(r,I)) \tag{10}$$

$$\text{var. } r \in \{0,1\}, I \ge 0. \tag{11}$$

Here, $b > 0$ is the venue's profit due to one user's consumption of the offline products, and $k > 0$ denotes the unit investment cost.

In (10), $\Pi^{\text{venue}}(r,I,p,w)$ is the venue's payoff, the term $bN\bar{x}(r,I)$ is the venue's aggregate profit due to its offline products' sales,[9] the term kI is the investment cost [15], and the term $r(p + wN\bar{y}(r,I))$ is the payment to the app under the two-part pricing. Recall that $\bar{x}(r,I)$ and $\bar{y}(r,I)$ are the fractions of users consuming the offline products and interacting with the POI, respectively, and they are given in Propositions 1, 2, and 3 in Section III.

We define the *threshold congestion effect factor* δ_{th} as

$$\delta_{\text{th}} \triangleq \frac{(Vc_{\max} + \theta\eta UN)(b\eta(Vc_{\max} + \theta\eta UN) - \theta I_0 c_{\max} k)}{kc_{\max}\eta U(c_{\max} - \theta N)}.$$

In the following, we analyze three situations with different I_0 and δ, and derive the venue's corresponding optimal choices.

A. Situation I: Small Initial Investment ($I_0 \le I_{\text{th}}$) and Large Congestion Effect ($\delta > \delta_{\text{th}}$)

In order to facilitate the presentation, we define $w_0 \triangleq -k\frac{(\delta - \theta I_0)c_{\max}}{Vc_{\max}+\theta\eta UN} < 0$ and $w_1 \triangleq b\eta - \frac{\delta k (V+\eta U)c_{\max}^2}{(Vc_{\max}+\theta\eta UN)^2}$. With $\delta > \delta_{\text{th}}$, we can show that $w_0 > w_1$. Based on w_0, w_1, and function $H_1(w)$ defined in (7), we derive Proposition 4 and illustrate it in Fig. 4 (the illustrations of Propositions 5 and 6 are omitted because of the space limit).

[9] To simplify the model, we assume that each user makes the visiting decision once. In practice, a user may visit the venue (e.g., a cafe) regularly. In this case, we use b to represent the venue's *overall* profit due to one user's regular consumption of the offline products within a certain time period.

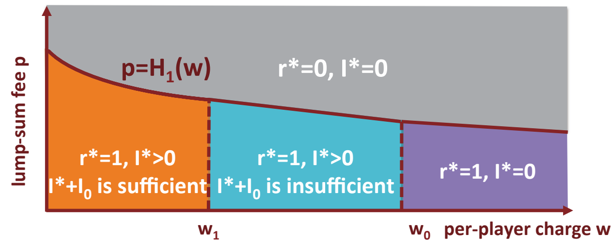

Fig. 4: Situation I of Venue's POI Decision r^* and Investment I^* in Stage II.

Proposition 4. *When $I_0 \leq I_{\rm th}$ and $\delta > \delta_{\rm th}$, the venue's optimal choices are*

$$(r^*(p,w), I^*(p,w)) = \begin{cases} (0,0), & \text{if } p > H_1(w), \\ \left(1, \frac{N}{c_{\max}-N\theta}\sqrt{\frac{\delta(V+\eta U)(b\eta-w)}{k}} - \delta\frac{N}{c_{\max}-N\theta} - I_0\right), & \text{if } p \leq H_1(w), w < w_1, \\ (1, I_{\rm th} - I_0), & \text{if } p \leq H_1(w), w_1 \leq w \leq w_0, \\ (1,0), & \text{if } p \leq H_1(w), w > w_0. \end{cases}$$

First, we see that the venue will become a POI (i.e., $r^*(p,w)=1$) if and only if p and w satisfy $p \leq H_1(w)$ (i.e., the orange, blue, and purple parts in Fig. 4). This means that $H_1(w)$ is the maximum lump-sum fee under which the venue will be a POI in Situation I, given the per-player charge w.

Second, we discuss the venue's investment $I^*(p,w)$. When $p > H_1(w)$, the venue does not become a POI, and hence chooses $I^*(p,w) = 0$. When $p \leq H_1(w)$, $I^*(p,w)$ is independent of p, and is decreasing in w. Specifically, $I^*(p,w)$ has three different expressions based on the value of w: (a) when $w < w_1$, the venue chooses $I^*(p,w) = \frac{N}{c_{\max}-N\theta}\sqrt{\frac{\delta(V+\eta U)(b\eta-w)}{k}} - \delta\frac{N}{c_{\max}-N\theta} - I_0 > I_{\rm th} - I_0$. According to Proposition 3 and our analysis in Stage III, the venue achieves a sufficient total investment, which attracts new visitors; (b) when $w_1 \leq w \leq w_0$, the venue chooses $I^*(p,w) = I_{\rm th} - I_0$. According to Proposition 2 and our analysis in Stage III, the venue's total investment is insufficient. The investment enables all the initial visitors to interact with the POI but cannot attract new visitors; (c) when $w > w_0$, the per-player charge is large, and the venue does not invest.

B. Situation II: Small Initial Investment ($I_0 \leq I_{\rm th}$) and Small Congestion Effect ($\delta \leq \delta_{\rm th}$)

We define w_2 as the $w \in [w_0, w_1]$ (since $\delta \leq \delta_{\rm th}$, it can be easily shown that $w_0 \leq w_1$) that satisfies the equation $\frac{N}{c_{\max}-N\theta}\left(\sqrt{(V+\eta U)(b\eta-w)} - \sqrt{\delta k}\right)^2 - \frac{N}{c_{\max}} b\eta^2 U + kI_0 + \frac{V}{\frac{\delta}{I_0}-\theta} w = 0$. We can prove that w_2 is unique. Based on w_2 and $H_2(w)$ defined in (8), we have the following proposition.

Proposition 5. *When $I_0 \leq I_{\rm th}$ and $\delta \leq \delta_{\rm th}$, the venue's optimal choices are*

$$(r^*(p,w), I^*(p,w)) = \begin{cases} (0,0), & \text{if } p > H_2(w), \\ \left(1, \frac{N}{c_{\max}-N\theta}\sqrt{\frac{\delta(V+\eta U)(b\eta-w)}{k}} - \delta\frac{N}{c_{\max}-N\theta} - I_0\right), & \text{if } p \leq H_2(w), w < w_2, \\ (1,0), & \text{if } p \leq H_2(w), w \geq w_2. \end{cases}$$

First, the venue becomes a POI if and only if $p \leq H_2(w)$. Second, when $p \leq H_2(w)$, the venue's optimal investment level $I^*(p,w)$ has two different expressions: (a) when $w < w_2$, the venue achieves a sufficient total investment and attracts new visitors; (b) when $w \geq w_2$, the venue does not invest because of the large per-player charge. *In Situation II, whenever $I^*(p,w) > 0$, the total investment $I^*(p,w) + I_0$ is always sufficient, which is different from Situation I.* This is because the congestion effect factor δ in Situation II is smaller than that in Situation I, which makes it easier for the venue to attract new visitors.

C. Situation III: Large Initial Investment ($I_0 > I_{\rm th}$)

We define $w_3 \triangleq b\eta - \frac{k((c_{\max}-\theta N)I_0+\delta N)^2}{\delta(V+\eta U)N^2}$ and define $H_3(w)$ in (9), based on which we have the following proposition.

Proposition 6. *When $I_0 > I_{\rm th}$, the venue's optimal choices are*

$$(r^*(p,w), I^*(p,w)) = \begin{cases} (0,0), & \text{if } p > H_3(w), \\ \left(1, \frac{N}{c_{\max}-N\theta}\sqrt{\frac{\delta(V+\eta U)(b\eta-w)}{k}} - \delta\frac{N}{c_{\max}-N\theta} - I_0\right), & \text{if } p \leq H_3(w), w < w_3, \\ (1,0), & \text{if } p \leq H_3(w), w \geq w_3. \end{cases}$$

First, the venue becomes a POI if and only if $p \leq H_3(w)$. Second, the venue chooses a positive investment level if and only if $p \leq H_3(w)$ and $w < w_3$. In Situation III, the initial investment level I_0 is above $I_{\rm th}$, so the total investment is always sufficient, regardless of w. Therefore, as long as the venue becomes a POI, it attracts new visitors.

D. Summary of Three Situations

We summarize the three situations' main features as follows.

- In Situation I (small initial investment and large congestion effect), even if the venue's equilibrium investment level $I^*(p,w)$ is positive, it may not attract new visitors.
- In Situation II (small initial investment and small congestion effect), the venue attracts new visitors if and only if its equilibrium investment level is positive.
- In Situation III (large initial investment), the venue always attracts new visitors after becoming a POI, regardless of its equilibrium investment level.

V. Stage I: App's Pricing

A. Problem Formulation

In Stage I, the app solves Problem 3, anticipating the venue's and users' decisions in Stages II and III, respectively.

Problem 3. *The app determines (p^*, w^*) by solving*

$$\max R^{\rm app}(p,w) \triangleq r^*(p,w)\Big(p + wN\bar{y}(r^*(p,w), I^*(p,w))\Big) + \phi N\bar{y}(r^*(p,w), I^*(p,w)) \quad (12)$$

$$\text{var.} \quad p, w \in \mathbb{R}. \quad (13)$$

Here, $\phi \geq 0$ is the unit advertising revenue, representing the app's advertising revenue when a user interacts with the POI.

$R^{\rm app}(p,w)$ is the app's revenue, which has two components: the venue's payment based on the two-part pricing, and the app's advertising revenue. Function $\bar{y}(r, I)$ is given in Propositions 1, 2, and 3. Functions $r^*(p, w)$ and $I^*(p, w)$ are given in Propositions 4, 5, and 6.

B. Optimal Two-Part Pricing

We show the app's optimal two-part pricing in Theorem 1.

Theorem 1. *The app's optimal two-part pricing is*

$$w^* = -\phi,\ p^* = \tilde{H}(-\phi), \tag{14}$$

where function $\tilde{H}(w), w \in \mathbb{R}$, is defined as

$$\tilde{H}(w) \triangleq \begin{cases} H_1(w), & \text{if } I_0 \leq I_{\text{th}} \text{ and } \delta > \delta_{\text{th}}, \\ H_2(w), & \text{if } I_0 \leq I_{\text{th}} \text{ and } \delta \leq \delta_{\text{th}}, \\ H_3(w), & \text{if } I_0 > I_{\text{th}}. \end{cases} \tag{15}$$

The per-player charge $w^* \leq 0$ and the lump-sum fee $p^* \geq 0$.

From Propositions 4, 5, and 6, $\tilde{H}(w)$ is the maximum lump-sum fee under which the venue will be a POI, given the w.

We first discuss the intuitions behind Theorem 1. With $w^* \leq 0$, the app *pays* the venue based on the number of users interacting with the POI. This incentivizes the venue to invest in the app-related infrastructure, which attracts more users to interact with the POI. When $w^* = -\phi$, the venue will be incentivized to choose an investment level that maximizes the summation of the app's revenue and the venue's payoff. Meanwhile, the app sets $p^* = \tilde{H}(-\phi)$, which is the maximum lump-sum fee the venue will accept under $w^* = -\phi$. With $p^* = \tilde{H}(-\phi)$, the app extracts all the venue's surplus. Hence, we can see that w^* and p^* maximize the app's revenue.

Theorem 1 brings the following practical insights. The app should announce a *charge-with-subsidy* scheme to the venue: (i) in order to become a POI, the venue needs to pay the app $\tilde{H}(-\phi)$; (ii) every time a user interacts with the POI, the app pays the venue ϕ (unit advertising revenue).

C. App's Revenue and Venue's Payoff

Under the prices p^* and w^*, the app's revenue and the venue's payoff are given in the following corollary.

Corollary 1. *Under p^* and w^*, we have*

$$R^{\text{app}}(p^*, w^*) = \tilde{H}(-\phi) \geq 0, \tag{16}$$

$$\Pi^{\text{venue}}(r^*(p^*, w^*), I^*(p^*, w^*), p^*, w^*) = bN\eta \frac{U}{c_{\max}}. \tag{17}$$

Based on (12) and $w^* = -\phi$, the app's payment to the venue due to the negative per-player charge cancels out the app's total advertising revenue. Hence, the app's optimal revenue equals its lump-sum fee, i.e., $R^{\text{app}}(p^*, w^*) = p^* = \tilde{H}(-\phi)$.

From (17), the venue's payoff under the app's optimal pricing is $bN\eta \frac{U}{c_{\max}}$, which equals the venue's payoff when it does not become a POI. This is because we assume that the app has the market power. In this case, the app can extract all the venue's surplus via pricing. We can also consider a more general bargaining-based negotiation model between the app and venue in Stage I. The bargaining formulation only changes the profit split between the app and venue, and does not affect the venue's choices in Stage II and the users' decisions in Stage III. Under the bargaining model, the venue's payoff increases with its bargaining power and could be much larger than $bN\eta \frac{U}{c_{\max}}$.

D. Parameters' Influences on App's Revenue

In this section, we analyze the influence of U (i.e., the user's utility of consuming the offline products) on the app's optimal revenue $R^{\text{app}}(p^*, w^*)$, which provides guidelines for the app regarding which type of venues to collaborate with. The influences of many other parameters, such as V, θ, and δ, are intuitive, and hence the results are omitted here.

The influence of U is summarized as follows.

Proposition 7. *When $\delta \leq \frac{(b\eta N\theta + \phi c_{\max}) I_0}{b\eta N}$, $R^{\text{app}}(p^*, w^*)$ increases with $U \in (0, \infty)$.*

Proposition 8. *When $\delta > \frac{(b\eta N\theta + \phi c_{\max}) I_0}{b\eta N}$, $R^{\text{app}}(p^*, w^*)$ decreases with U for $U \in \left(0, \frac{\delta k (b\eta + \phi)}{\eta \left(\frac{N\theta b\eta}{c_{\max}} + \phi\right)^2} - \frac{V}{\eta}\right]$, and increases with U for $U \in \left[\frac{\delta k (b\eta + \phi)}{\eta \left(\frac{N\theta b\eta}{c_{\max}} + \phi\right)^2} - \frac{V}{\eta}, \infty\right)$.*

To better illustrate the above propositions, we discuss the following three types of impacts when U increases. (i) (*positive impact*) The venue attracts more users with intrinsic interests on the offline products, which increases the number of users interacting with the POI. This enables the app to obtain more advertising revenue. (ii) (*positive impact*) More users visit the venue before it becomes a POI. After the venue becomes a POI and makes sufficient investment, all of these initial visitors interact with the POI, which generates a large *network effect* and attracts more visitors. This potentially increases the venue's payment to the app. (iii) (*negative impact*) The threshold investment level $I_{\text{th}} = \frac{\delta}{\theta + \frac{V c_{\max}}{\eta U N}}$ increases, and the venue should invest more to achieve a sufficient total investment, which overcomes the *congestion* and attracts new visitors. This potentially reduces the venue's payment to the app.

In Proposition 7, the congestion effect $\delta \leq \frac{(b\eta N\theta + \phi c_{\max}) I_0}{b\eta N}$. In this case, the third impact (related to the congestion effect) is dominated by the first two impacts. Hence, the increase of U always improves $R^{\text{app}}(p^*, w^*)$.

In Proposition 8, different impacts dominate in different regions of U, and hence lead to more complicated and interesting results. Based on our assumption in Section II-D, U is upper-bounded by $c_{\max} - V - \theta N$. If δ is very large, the threshold $\frac{\delta k (b\eta + \phi)}{\eta \left(\frac{N\theta b\eta}{c_{\max}} + \phi\right)^2} - \frac{V}{\eta}$ will exceed the upper bound, and $R^{\text{app}}(p^*, w^*)$ will decrease with U for $U \in (0, c_{\max} - V - \theta N)$.

Based on the above discussions, we obtain the following practical insights. First, if the congestion effect is small, the app achieves a large revenue when collaborating with a venue with a large U (e.g., a top-rated restaurant). Second, if the congestion effect is very large, the app achieves a large revenue when collaborating with a venue with a small U (e.g., an ordinary cafe). Hence, the apps with small and large

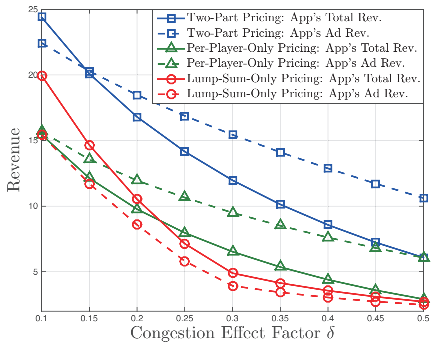

Fig. 5: Pricings' Comparison Under Different δ.

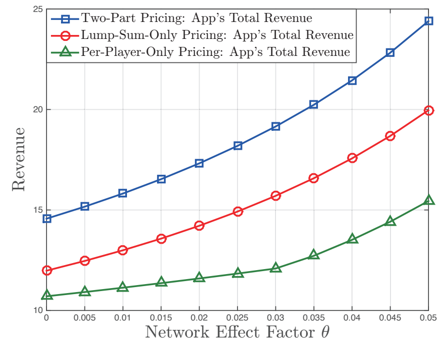

Fig. 6: Pricings' Comparison Under Different θ.

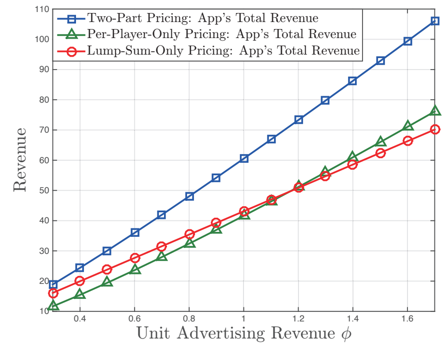

Fig. 7: Pricings' Comparison Under Different ϕ.

congestion effects should collaborate with opposite types of venues, i.e., *the ideal POIs' features vary across apps.*

VI. Numerical Results

In this section, we compare our proposed two-part pricing scheme with two state-of-the-art pricing schemes: the *per-player-only pricing* (e.g., used by Pokemon Go), where the app charges the venue only based on the per-player charge $w^*_{\text{only}} = \arg\max_{w\in\mathbb{R}} R^{\text{app}}(0, w)$; the *lump-sum-only pricing* (e.g., used by Snapchat), where the app charges the venue only based on the lump-sum fee $p^*_{\text{only}} = \arg\max_{p\in\mathbb{R}} R^{\text{app}}(p, 0)$.

1) Impact of congestion effect: In Fig. 5, we compare the three schemes under different congestion effect factor δ. We choose $N = 200$, $c_{\max} = 24$, $U = 3$, $V = 5$, $I_0 = 0.6$, $k = 3$, $b=1$, $\eta=0.2$, $\theta=0.05$, and $\phi=0.4$. We change δ from 0.1 to 0.5, and plot the app's total revenues R^{app} (solid curves) and advertising revenues (dash curves) under different schemes.

First, we observe that the two-part pricing always achieves the largest app's total revenue (solid blue curve). For example, the two-part pricing improves the app's total revenue over the per-player-only pricing by at least 55% for all δ's values shown in Fig. 5. Second, the two-part pricing always achieves the largest app's advertising revenue (dash blue curve), which implies that it also achieves the highest number of users interacting with the POI. This is because the two-part pricing has the lowest per-player charge, and can best incentivize the venue to invest in the app-related infrastructure and relieve the congestion.

When δ is medium (e.g., $0.2 \leq \delta \leq 0.35$), the two-part pricing significantly improves the app's total revenue compared with the lump-sum-only pricing. To understand this, note that the solid blue curve could be below the dash blue curve under the two-part pricing. This means that the app pays the venue to incentivize the investment. Under the lump-sum-only pricing, however, the app cannot incentivize investment by paying the venue. Hence, when δ is medium, the two-part pricing relieves the congestion, and significantly outperforms the lump-sum-only pricing. When δ further increases (e.g., $\delta > 0.35$), the congestion cannot be efficiently relieved even with the venue's investment, and the gap between the app's total revenues under the two-part pricing and lump-sum-only pricing decreases.

2) Impact of network effect: In Fig. 6, we compare the three pricing schemes under different network effect θ. We let $\delta = 0.1$, and change θ from 0 to 0.05. The other parameters are the same as in Fig. 5. When θ is large, the two-part pricing significantly outperforms the per-player-only pricing (the performance gap increases with θ). This is because a large network effect enables the POI to attract many visitors, and the venue is willing to pay the app for becoming a POI. Under the two-part pricing, the app can set a large lump-sum fee to obtain a large venue's payment. Under the per-player-only pricing, the app cannot set a large per-player charge to obtain a large venue's payment, since a large per-player charge will reduce the venue's investment, the number of users interacting with the POI, and the app's advertising revenue.

3) Impact of advertising revenue: In Fig. 7, we compare the three pricing schemes under different unit advertising revenue ϕ. We let $\delta = 0.1$, and change ϕ from 0.3 to 1.7. The other parameters are the same as in Fig. 5. When ϕ is large, the two-part pricing significantly outperforms the other two schemes. This is because the two-part pricing best incentivizes the venue's investment, and hence results in the highest number of users interacting with the POI. When ϕ is large, the two-part pricing achieves a much larger app's total revenue than the other two schemes.

VII. Related Work

A. Interaction between online and offline businesses

First, there have been many references studying the *online/offline competitions*. For example, Forman *et al.* in [17] investigated the competition between online and offline retailers of the same products, considering the consumers' heterogeneous physical locations. Viswanathan *et al.* in [18] analyzed the impacts of the network effect and consumers' switching costs on the competition between the online and offline firms. Balasubramanian in [19] investigated how the online retailers selectively inform the consumers about the online channel when competing with the offline retailers.

Second, there are few references focusing on the *online/offline collaborations*. Yu *et al.* in [20] studied a situation where the online advertisers sponsor the venues' public Wi-Fi services, and deliver mobile advertisements to the venues' visitors. Yu *et al.* in [21] considered the mobile network operators and venues' collaborations in deploying public Wi-Fi hotspots, which offload the cellular networks' traffic and attract visitors to the venues. References [22] and [23] are related to the empirical studies of Pokemon Go's impacts on the offline businesses. For example, Pamuru *et al.* in [22] collected consumers' reviews of 2,032 restaurants in Houston, and investigated the correlation between the reviews and whether the restaurants are covered by the POIs ("PokeStops"). Different from [22] and [23], *our work provides the first analytical modeling and analysis for the collaboration between online apps and offline businesses.*

B. Two-Part Pricing

Since the studies in [24] and [25], there have been many references analyzing the two-part pricing and its applications. In this pricing scheme, a seller can use the per-unit charge to induce the buyer's efficient (e.g., welfare-maximizing) consumption, and use the lump-sum fee to extract the buyer's surplus. The two-part pricing is particularly useful in the POI-based collaboration, where the app (seller) induces the venue's (buyer's) investment via the per-player charge. Different from the pricing schemes in the references that include positive per-unit charges (e.g., [24]–[27]), our optimal two-part pricing includes a *negative* per-player charge. This is because the investment cost is paid by the venue rather than the app, and the app needs to subsidize the venue's investment via a negative per-player charge.

VIII. Conclusion

The economics of the online apps and offline venues' collaboration is a fast-emerging research area, and we provided the first modeling and analysis of the POI-based collaboration. We designed a *charge-with-subsidy* pricing scheme, which can significantly improve the apps' revenues and the users' engagements with the venues, compared with the state-of-the-art pricing schemes. We showed that the ideal POIs' features could vary across apps, and we provided useful guidelines for the apps to optimally select venues to collaborate with.

Our work opens up exciting directions for future works. First, our work assumes that different users have the same values of network effect factor and congestion effect factor. It is more practical to consider the users with heterogeneous sensitivities to the network effect and congestion effect. Second, our work focuses on the collaboration between an app and a store/restaurant chain's representative venue. For future research, it is interesting to consider the collaboration between an app and multiple venues of different owners in the same area. However, this extension is very complicated and challenging, since the users decide the visits by comparing both the qualities of the venues' offline products and the venues' investment levels on the app-related infrastructure (related to the qualities of the online products).

Acknowledgment

The authors would like to thank Jinghan Hao, for helping conduct the survey about Pokemon Go players' game experience. This work is supported in part by the General Research Funds Projects (CUHK 14219016 and CityU 11527316) established under the University Grant Committee of the Hong Kong Special Administrative Region, China, and in part by a research grant from Science Foundation Ireland (SFI) under Grant Number 17/CDA/4760.

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