Make a Difference: Diversity-Driven Social Mobile Crowdsensing

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***Abstract*—In a mobile crowdsensing (MCS) application, *user diversity* and *social effect* are two important phenomena that determine its profitability, where the former improves the sensing quality, while the latter incentives the users' participation. In this paper, we consider a reward mechanism design for the service provider to achieve diversity in the collected data by exploiting the users' social relationship. Specifically, we formulate a two-stage decision problem, where the service provider first optimizes its rewards for profit maximization. The users then decide their effort levels through social network interactions as a participation game. The analysis is particularly challenging due to the users' interplay in both the diversity and social graphs, which leads to a non-convex bilevel optimization problem. Surprisingly, we find that the service provider can focus on one *superimposed graph* that incorporates the diversity and social relationship and compute the optimal reward as the Katz centrality in closed-form. Simulation results, based on the random graph and a real Facebook trace, show that the availability of network information improves both the service provider's profit and the users' social surplus over the incomplete information cases.**

I. Introduction

A. Motivations

With the advancement of consumer electronics in recent years, most of the mobile devices have embedded many *sensors*, such as cameras, microphones, accelerometers, global positioning systems (GPS), and thermometers, to gather data for various applications [1], [2]. This enables the rapid development of a new sensing paradigm, known as *mobile crowdsensing* (MCS), which involves numerous individuals using their mobile devices to collectively extract and share information related to a certain phenomenon of interest. Recently, commercial MCS applications have emerged that *service providers* utilize the wisdom of crowds to provide sensing services, such as Waze [3] for real-time traffic monitoring and Gigwalk [4] for mobile market research, to their customers. Various evidence has suggested that user diversity and social effect are two important phenomena that cannot be overlooked for the successful deployment of these MCS applications.

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The *user diversity* improves the sensing quality by conveying more valuable information and revealing deeper insights in the sensing tasks for the service provider. For example, in location dependent MCS applications, such as Waze [3], a service provider can monitor a larger area of road conditions by recruiting users who are spatially separated apart than those who are close together. As another example, in Gigwalk [4], when the service provider wants to conduct mobile market research of users' opinions on certain products, a group of participants with diverse backgrounds (e.g., age, gender, and education) may be able to provide more comprehensive opinions than a group of participants with a similar background. This is in line with the idea in [5]–[9], which suggested that diversity is one of the key conditions required to promote the wisdom of crowds, who are tightly connected nowadays through the online social networks.

In fact, with the *social effect* [10], [11] through these online social services, such as Facebook and Twitter, users can be motivated to participate more actively in the sensing activities. Besides providing explicit rewards, service providers are already taking advantage of this social effect as an extra *incentive* to boost the users' participation level [12]. For example, Waze allows its users to connect their accounts with Facebook or Twitter [3], while Gigwalk is encouraging its users to share their experiences with their friends via Facebook or Twitter [4]. To sum up, the social relationship through online platforms is an important part of the user's experience, which can be utilized by the service providers to promote their sensing campaigns.

Thus, to guarantee the quality and profitability of its sensing service, it is important for the service provider to design a diversity-driven and socially-aware reward mechanism, which exploits the users' social relationship to achieve diversity in the sensed data. While intuitively, a service provider should reward users with high diversity and social impact, how to systematically *integrate* the diversity and social effect in the reward mechanism under a *unified* framework is still an open question. For example, should the service provider recruit a diverse set of users with a different background to improve the quality of information or *seed* selected influential users with high social influence [11] to improve the overall participation level? More specifically, *for profit maximization, how should the service provider allocate rewards to the users by taking into account both the user diversity and social relationship in an integrated manner?*

B. Contributions

To address this question, we consider an MCS system with a service provider and multiple mobile users. First, the service provider optimizes the rewards to maximize its profit, which increases with both the users' individual contributions and *diversity*. Then, based on the rewards, the users decide their effort levels under the *social influences* from their friends. We model it as a two-stage decision process, where Stage I is the service provider's profit maximization problem, while Stage II is the users' participation game.

The analysis of this two-stage problem is particularly challenging as we need to consider the users' relationships in two graphs: a *diversity graph* on the users' differences, and a *social graph* on the users' social relationships. Our approach to address this challenge is to formulate a bilevel optimization problem, where we take into account Stage II's equilibrium conditions in Stage I's problem, and consider both the diversity and social effect in a single optimization problem. Despite the non-convexity of the bilevel problem, we derive the service provider's optimal reward and the users' unique Nash equilibrium in closed-form under a scenario of interest, and show some interesting engineering insights on the structure of the optimal reward.

We summarize our major contributions as follows:

- *Diversity-driven social MCS*: To the best of our knowledge, this is the first systematic study on the impacts of both the user diversity and social effect to the MCS reward mechanism design under a unified framework.
- *Closed-form solution to non-convex bilevel problem*: We derive the service provider's optimal reward and the users' unique Nash equilibrium in closed-form under a scenario of interest.
- *Insight on optimal reward design*: Surprisingly, the optimal reward is a weighted sum of the Katz centrality of other users in the *superimposed graph* of the social and diversity networks, where a larger weight is placed on a user with a higher diversity and social influence.
- *Win-win situation from network information*: Simulation results, based on a random network model and the Facebook trace [13], show that the social and diversity information results in a larger service provider's profit and users' social surplus over the incomplete information cases. In addition, the social effect is more important for profit maximization than the diversity under a highly asymmetric social relationship.

C. Related Works

There were a lot of studies in incentive mechanism design in MCS with different design objectives and knowledge of users' information [14]–[20]. However, they did not take into account the social effect or the user diversity.

The social effect has been considered in incentive mechanism design in crowdsourcing platforms in [21]–[23]. However, they did not take into account the user diversity, which is often an important factor that determines the system performance as discussed in [5], [6].

TABLE I
COMPARISONS BETWEEN CROWDSOURCING LITERATURE

Literature	Social Effect Considered?	Diversity Considered?
[14]–[20]	No	No
[21]–[23]	Yes	No
[7]–[9]	No	Yes
This Paper	Yes	Yes

The user diversity was considered in [7]–[9]. Wu *et al.* in [7] aimed to maximize the diversity of opinions given a budget constraint on the maximum number of workers. Cheng *et al.* in [8] studied the problem of assigning moving users to spatial tasks to maximize the spatial and temporal diversities. However, these studies mainly focused on the worker selection by the service provider, but did not consider the social effect of the users. In contrast, in this paper, we study both the reward optimization of the service provider and the participation decisions of the users. Wu *et al.* in [9] also incorporated the spatial and temporal diversities in the reward scheme. But it is a heuristic that did not consider the social effect.

In this paper, we take into account both the diversity and social effect in the reward mechanism design in MCS. We show that the relative importance of these two factors in the profit maximization may change depending on the symmetry of the social influences.

The rest of the paper is organized as follows. We describe our two-stage system model in Section II, and derive the solutions in closed-form in Section III. We show our simulation results in Section IV and conclude the paper in Section V.

II. SYSTEM MODEL

In this section, we first model the social effect and the value of information to the service provider due to the user diversity. Next, we discuss the decisions of the service provider and mobile users in two stages: In Stage I, the service provider chooses rewards for profit maximization. In Stage II, the users decide their sensing efforts by taking into account the social influences from their friends in a participation game.

A. Decision Variables and Social Relationship

As shown in Fig. 1, we consider an MCS system with a service provider, who aims to collect measurements with the help of multiple users. Let $\mathcal{I} = \{1, \dots, I\}$ be the set of users. In the MCS campaign, user i exerts a *effort level* of $x_i \geq 0$ on the sensing task (e.g., the amount of energy and time spent). In return, the service provider gives a *reward*[1] $r_i \in \mathbb{R}$ per unit of effort that user i exerts on the sensing task. We let $\boldsymbol{x} = (x_1, \dots, x_I)$ be the effort profile of all the users, and $\boldsymbol{r} = (r_1, \dots, r_I)$ be the rewards allocated by the service provider. The users are connected through a social network defined as follows.

[1]Note that it is possible to have $r_i < 0$, where user i pays the service provider to participate in the sensing task. For example, for advertising purpose, if the service provider can invite some celebrities to join the MCS campaign, due to their strong social influences (as we will explain later), people may be willing to pay to work alongside them.

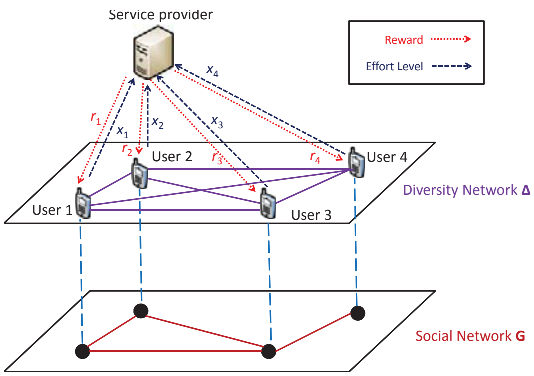

Fig. 1. In a diversity-driven social MCS system, the service provider gives rewards $\boldsymbol{r}$ to incentivize the users to exert a higher effort level $\boldsymbol{x}$ on their sensing tasks. The value of information is related to the user diversity as captured by the diversity network $\boldsymbol{\Delta}$, while the users exert peer effects among each other through the social network $\boldsymbol{G}$. We seek to understand how the service provider should optimize its rewards under this dual-network structure.

Definition 1 (Social network): A *social network* (or *social graph*) is represented by $\boldsymbol{G} = [g_{ij}]_{I\times I}$, where $g_{ij} \geq 0$ represents the *social influence* of user j on user i. For notational simplicity, we assume that $g_{ii} = 0, \forall\, i \in \mathcal{I}$.

B. Value of Information from User Diversity

We consider two factors that contribute to value of information (VoI). First, the VoI increases with each user's *individual contribution*. More specifically, the individual contribution of user i is $q_i x_i$, where $q_i \geq 0$ denotes user i's sensing capability. For example, a user with a high reputation in an MCS system can be assigned a larger value of q_i.

Second, the VoI increases with the users' *diversity* as captured by a diversity network defined as follows.

Definition 2 (Diversity network): A *diversity network* (or *diversity graph*) is represented by $\boldsymbol{\Delta} = [\delta_{ij}]_{I\times I}$, where $\delta_{ij} \geq 0, i, j \in \mathcal{I}$ is the difference between users i and j regarding the sensing requirements. By definition, we have $\delta_{ij} = \delta_{ji}, \forall\, i, j \in \mathcal{I}, i \neq j$ and $\delta_{ii} = 0, \forall\, i \in \mathcal{I}$.

For example, in location-dependent MCS applications, such as traffic monitoring, we may define δ_{ij} as the physical distance between users i and j. To model the effect that the diversity between users i and j are positive only when both users' effort levels are positive, we adopt a product-form function $\delta_{ij} x_i x_j$. Thus, considering all pair of users, the diversity[2] under effort levels $\boldsymbol{x}$ is $\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \delta_{ij} x_i x_j$.

Overall, the value of information is defined as follows.

Definition 3 (Value of information): The value of information under effort levels $\boldsymbol{x}$ is

$$\phi(\boldsymbol{x}) \triangleq \sum_{i\in\mathcal{I}} q_i x_i + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \delta_{ij} x_i x_j, \tag{1}$$

which is an increasing function in $\boldsymbol{x}$ with $\phi(\boldsymbol{0}) = 0$.

[2]Note that it is in the form of the *Rao's quadratic entropy* [24], which has been widely used to measure the diversity in a multi-species population in theoretical population biology.

C. Stage I: Service Provider's Profit Maximization

In Stage I, the service provider chooses the rewards to maximize its profit (i.e., VoI minus payment):

$$\underset{\boldsymbol{r}}{\text{maximize}} \quad \phi(\boldsymbol{x}(\boldsymbol{r})) - \sum_{i\in\mathcal{I}} r_i x_i(\boldsymbol{r}), \tag{2}$$

where $\boldsymbol{x}(\boldsymbol{r}) = (x_i(\boldsymbol{r}), i \in \mathcal{I})$ are the users' equilibrium effort levels under the reward vector $\boldsymbol{r}$, which will be discussed later in Section II-D. Here, $r_i x_i(\boldsymbol{r})$ is the payment to user i.

D. Stage II: Mobile Users' Participation Game

1) Payoff Function: Based on the reward decision in Stage I, each user i decides his effort level x_i in Stage II to maximize his payoff, which consists of three parts:

- Reward: User i receives a total reward $r_i x_i$ from the service provider in proportional to his effort level.
- Cost: User i has a participation cost (e.g., on energy or time spent) to perform the sensing task. We assume that it is a convex sensing cost $c_i x_i^2$ on his effort level with $c_i \geq 0$, which can be used to model the increasing marginal cost for every additional unit of effort exerted[3].
- Social satisfaction: By the *social effect* [10], [11], a user's payoff is positively influenced by the effort levels of his friends. We assume that the social satisfaction of user i due to his friend j takes the form $g_{ij} x_i x_j$, which is widely adopted in the social network literature [11], [25], [26] to model the bandwagon nature of the social effect.

Let $\boldsymbol{x}_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_I)$ be the effort profile excluding user i. Overall, user i's payoff function is

$$U_i(x_i, \boldsymbol{x}_{-i}, r_i) \triangleq r_i x_i - c_i x_i^2 + \sum_{j\in\mathcal{I}} g_{ij} x_i x_j. \tag{3}$$

2) Participation Game: We formulate the users' effort level decisions as a non-cooperative game with positive externality, where if other users put in more efforts, a user can reduce its effort to reduce his cost but still obtains a good payoff.

Definition 4 (Participation game): A participation game is a tuple $\Omega = (\mathcal{I}, \mathcal{X}, \boldsymbol{U})$ defined by:

- Players: The set of users $\mathcal{I}$.
- Strategies: The set $\mathcal{X}_i = [0, \infty)$ for user i. The set of feasible strategy profile of all users is $\mathcal{X} = \mathcal{X}_1 \times \ldots \times \mathcal{X}_I$.
- Payoffs: The vector $\boldsymbol{U} = (U_i, \forall\, i \in \mathcal{I})$ contains the payoff functions of the users defined in (3).

Given the reward r_i and the strategy profile $\boldsymbol{x}_{-i}$ from other users, user i's *best response* is to maximize his payoff:

$$\underset{x_i \geq 0}{\text{maximize}} \quad U_i(x_i, \boldsymbol{x}_{-i}, r_i) = r_i x_i - c_i x_i^2 + \sum_{j\in\mathcal{I}} g_{ij} x_i x_j. \tag{4}$$

Notice that (4) is a *convex* optimization problem in x_i, and we can compute user i's best response in closed-form.

[3]We assume that the service provider has perfect information on cost c_i. When c_i is fixed, the service provider can estimate it after some interactions with the user. For example, given $\boldsymbol{r}$ and $\boldsymbol{G}$, after observing $\boldsymbol{x}$, the service provider can estimate c_i from (5) that we will discuss later.

Proposition 1: Given r_i and $\boldsymbol{x}_{-i}$, user i's best response is

$$x_i(r_i) = \max\Big\{0, \frac{r_i + \sum_{j\in\mathcal{I}} g_{ij}x_j}{2c_i}\Big\}. \tag{5}$$

Proof: Let λ_i be the dual variable of the inequality constraint $x_i \geq 0$ in problem (4). Since problem (4) is a *convex* problem [27], the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient and are given below:

$$x_i, \lambda_i \geq 0, \tag{6}$$

$$r_i - 2c_i x_i + \sum_{j\in\mathcal{I}} g_{ij}x_j + \lambda_i = 0, \text{ and} \tag{7}$$

$$\lambda_i x_i = 0. \tag{8}$$

Solving the KKT conditions for x_i, the results follows. ■

The intersection of all the users' best response correspondences is the Nash equilibrium (NE), where no user has the incentive to deviate from his effort choice given that other users choose their effort levels according to the equilibrium. We let $\boldsymbol{x}(\boldsymbol{r})$ in Section II-C be an NE achieved under $\boldsymbol{r}$.

Definition 5 (Nash equilibrium): Under a fixed reward vector $\boldsymbol{r}$, a strategy profile $\boldsymbol{x}^*$ is an NE of game Ω if

$$U_i(x_i^*, \boldsymbol{x}_{-i}^*, r_i) \geq U_i(x_i, \boldsymbol{x}_{-i}^*, r_i), \, \forall\, x_i \in \mathcal{X}_i, \forall\, i \in \mathcal{I}. \tag{9}$$

In the next section, we will analyze Stages I and II together by bilevel optimization.

III. Bilevel Optimization

In this section, we formulate a bilevel optimization problem for the two-stage decisions in Section III-A, and solve for the optimal reward and unique NE in closed-form in Section III-B. For the optimal reward, we reveal deeper insight into its structure related to the Katz centrality in a superimposed graph in Section III-C. The users' social surplus under the optimal solution is discussed in Section III-D.

A. Non-convex Bilevel Optimization Problem

By adding the necessary and sufficient KKT conditions of Stage II in (6)-(8) of all the users to the constraints of Stage I's problem (2), we are restricting the feasible set in Stage I to only the NE in Stage II. Thus, solving the two-stage problem is equivalent to solving the bilevel optimization problem

$$\begin{aligned}
\underset{\boldsymbol{r},\boldsymbol{x},\boldsymbol{\lambda}}{\text{maximize}} \quad & \pi(\boldsymbol{r},\boldsymbol{x}) \triangleq \sum_{i\in\mathcal{I}} q_i x_i + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} \delta_{ij}x_i x_j - \sum_{i\in\mathcal{I}} r_i x_i \\
\text{subject to} \quad & r_i - 2c_i x_i + \sum_{j\in\mathcal{I}} g_{ij}x_j + \lambda_i = 0, \, \forall\, i \in \mathcal{I}, \\
& \lambda_i x_i = 0, \, \forall\, i \in \mathcal{I}, \\
& x_i, \lambda_i \geq 0, \, \forall\, i \in \mathcal{I},
\end{aligned} \tag{10}$$

where $\pi(\boldsymbol{r},\boldsymbol{x})$ is the profit of the service provider with reward vector $\boldsymbol{r}$ and effort level vector $\boldsymbol{x}$.

By substituting the first and second equality constraints into the objective function, we can remove $\boldsymbol{r}$ and $\boldsymbol{\lambda}$ from the optimization variables and reformulate the problem as

$$\begin{aligned}
\underset{\boldsymbol{x}}{\text{maximize}} \quad & \sum_{i\in\mathcal{I}} q_i x_i + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}} (\delta_{ij} + g_{ij}) x_i x_j - 2\sum_{i\in\mathcal{I}} c_i x_i^2 \\
\text{subject to} \quad & x_i \geq 0, \, \forall\, i \in \mathcal{I}.
\end{aligned} \tag{11}$$

Notice that problem (11) is *non-convex*, because there are couplings in product-form[4] between the variables $\boldsymbol{x}$ in the second term in the objective function. Thus, it is very challenging to solve problem (11) in general. Nevertheless, under a practical scenario of interest, we are able to characterize the optimal solution in closed-form[5], and obtain some interesting insights.

B. Closed-Form Solutions to Stages I and II

If the users' costs are very small and the friends' social influences are very large, from (5), we can show that user i may unboundedly increase its effort level x_i, which is not practically interesting to consider. Instead, we focus on a scenario of interest with larger user costs and smaller social influences from friends that the following assumption is satisfied.

Assumption 1: $4c_i > \sum_{j\in\mathcal{I}} (g_{ij} + g_{ji} + 2\delta_{ij}), \, \forall\, i \in \mathcal{I}$.

Let us define the sensing capability vector $\boldsymbol{q} = (q_1, \ldots, q_I)$ and a diagonal cost matrix $\boldsymbol{C} = [C_{ij}]_{I\times I}$, where $C_{ij} = c_i$ if $i = j$ and $C_{ij} = 0$ otherwise. With Assumption 1, although problem (11) is still non-convex, we find that the optimality conditions form a linear system that admits a unique solution. Thus, we can obtain the optimal reward of the service provider and the unique NE of the users in *closed-form*.

Theorem 1: Under Assumption 1, the optimal reward of the service provider in Stage I is

$$\boldsymbol{r}^* = \frac{\boldsymbol{q}}{2} + \Big(\boldsymbol{\Delta} + \frac{\boldsymbol{G}^T - \boldsymbol{G}}{2}\Big)(4\boldsymbol{C} - \boldsymbol{G} - \boldsymbol{G}^T - 2\boldsymbol{\Delta})^{-1}\boldsymbol{q}, \tag{12}$$

and the *unique* Nash equilibrium of the users in Stage II is

$$\boldsymbol{x}^* = (4\boldsymbol{C} - \boldsymbol{G} - \boldsymbol{G}^T - 2\boldsymbol{\Delta})^{-1}\boldsymbol{q} \succeq \boldsymbol{0}. \tag{13}$$

Here, $\boldsymbol{G}^T$ represents the transpose of matrix $\boldsymbol{G}$. The proof of Theorem 1 is given in Appendix B.

C. Reward as Katz Centrality under Homogeneous Costs

In this subsection, we show that the optimal reward in (12) is closely related to the *Katz centrality* [11], [28] of the graph $\boldsymbol{G} + \boldsymbol{G}^T + 2\boldsymbol{\Delta} = (\boldsymbol{G} + \boldsymbol{\Delta}) + (\boldsymbol{G} + \boldsymbol{\Delta})^T$. It takes into account the bi-directional network effects of both graphs $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ by adding them into the superimposed graph $\boldsymbol{G} + \boldsymbol{\Delta}$.

[4]Although the objective in problem (11) involves product-form functions, it is not a geometric programming problem [27, pp. 160], because the objective is not minimizing a posynomial.

[5]After solving $\boldsymbol{x}^*$ for problem (11), we can readily find $\boldsymbol{r}^*$ and $\boldsymbol{\lambda}^*$ that satisfy the constraints in problem (10) as it is just required that $r_i^* \in \mathbb{R}$.

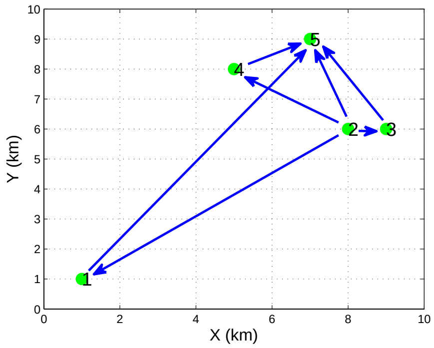

Fig. 2. An example of the diversity network and social network topologies.

Definition 6 (Katz centrality): For a graph $\boldsymbol{A} = [a_{ij}]_{I\times I}$, a scalar $\alpha \geq 0$, and a weight vector $\boldsymbol{w} = (w_1, \ldots, w_I)$, the Katz centrality is an I-dimensional vector defined as

$$\boldsymbol{\psi}(\boldsymbol{A}, \alpha, \boldsymbol{w}) = (\boldsymbol{I} - \alpha \boldsymbol{A})^{-1}\boldsymbol{w}. \quad (14)$$

Intuitively, under Katz centrality, a node in the graph has a high centrality (and is thus important) if it is connected with other important nodes or it is highly linked.

When all the users have the same sensing costs, we can see more clearly in Theorem 2 that the optimal reward depends on the weighted sum of the Katz centrality in the combined graph $\boldsymbol{G} + \boldsymbol{G}^T + 2\boldsymbol{\Delta}$ of the *other* users, as both the diversity and social relationship are defined relative to the *others*.

Theorem 2: Under Assumption 1, if all the users have the same sensing cost (i.e., $c_i = \hat{c}, \forall i \in \mathcal{I}$), then the optimal reward of the service provider in Stage I is

$$\boldsymbol{r}^* = \frac{\boldsymbol{q}}{2} + \frac{1}{4\hat{c}}\left(\boldsymbol{\Delta} + \frac{\boldsymbol{G}^T - \boldsymbol{G}}{2}\right)\boldsymbol{\psi}\left(\boldsymbol{G} + \boldsymbol{G}^T + 2\boldsymbol{\Delta}, \frac{1}{4\hat{c}}, \boldsymbol{q}\right). \quad (15)$$

The proof of Theorem 2 is given in Appendix C.

By representing the optimal reward in (15) in scalar form to show the insight of the result more clearly and noting that $\delta_{ii} = 0$ and $g_{ii} = 0$ for all $i \in \mathcal{I}$, we have

$$\begin{aligned} r_i^* &= \frac{q_i}{2} + \frac{1}{4\hat{c}} \sum_{j \in \mathcal{I}\backslash\{i\}} \left(\delta_{ij} + \frac{g_{ji} - g_{ij}}{2}\right)\psi_j\left(\boldsymbol{G} + \boldsymbol{G}^T + 2\boldsymbol{\Delta}, \frac{1}{4\hat{c}}, \boldsymbol{q}\right) \\ &= \underbrace{\frac{q_i}{2}}_{(a)} + \underbrace{\frac{1}{4\hat{c}} \sum_{j \in \mathcal{I}\backslash\{i\}} \delta_{ij}\psi_j}_{(b)} + \underbrace{\frac{1}{8\hat{c}} \sum_{j \in \mathcal{I}\backslash\{i\}} g_{ji}\psi_j}_{(c)} \underbrace{- \frac{1}{8\hat{c}} \sum_{j \in \mathcal{I}\backslash\{i\}} g_{ij}\psi_j}_{(d)}, \end{aligned} \quad (16)$$

where we write $\psi_j(\boldsymbol{G} + \boldsymbol{G}^T + 2\boldsymbol{\Delta}, \frac{1}{4\hat{c}}, \boldsymbol{q})$ as ψ_j in the second line of (16) for simplicity. Notice that the optimal reward consists of four components:

(a) Sensing capability.
(b) Diversity: Sum of the Katz centrality ψ_j of user j weighted by the difference δ_{ij} between users i and j.
(c) Social influence *on* others: The service provider should give some additional reward for user i's participation, as he will in turn motivate his friend j's participation through his social influence $g_{ji} \geq 0$.
(d) Social influence *from* others: Under the social influence $g_{ij} \geq 0$ from friend j, the service provider can pay less for user i's participation, so (d) is a negative term.

TABLE II
CENTRALITY, OPTIMAL REWARDS, AND EFFORT LEVELS

User i	**1**	**2**	**3**	**4**	**5**
Centrality ψ_i	2.28	2.15	2.01	2.00	3.04
(a) $q_i/2$	0.5	0.5	0.5	0.5	0.5
(b) $\frac{1}{4\hat{c}}\sum_{j\in\mathcal{I}\backslash\{i\}}\delta_{ij}\psi_j$	0.28	0.13	0.15	0.14	0.14
(c) $\frac{1}{8\hat{c}}\sum_{j\in\mathcal{I}\backslash\{i\}}g_{ji}\psi_j$	0.32	0.45	0.32	0.32	0
(d) $-\frac{1}{8\hat{c}}\sum_{j\in\mathcal{I}\backslash\{i\}}g_{ij}\psi_j$	-0.045	0	-0.045	-0.045	-0.88
Optimal Reward r_i^*	1.05	1.08	0.92	0.91	-0.24
Effort x_i^* $(\times 10^{-3})$	9.5	9.0	8.4	8.3	12.6

Example 1: We illustrate the four components of the optimal reward discussed above for the diversity and social network topologies shown in Fig. 2. The users' locations are represented by the green dots. We consider a location-dependent MCS application, where the diversity between a pair of users i and j is defined by their physical separation:

$$\delta_{ij} = \epsilon\, \text{dist}(i, j), \forall i, j \in \mathcal{I}. \quad (17)$$

Here, $\text{dist}(i, j)$ is the distance between users i and j, and $\epsilon > 0$ is the diversity coefficient and we choose $\epsilon = 0.8$. A positive social influence is represented by a blue arrow, where we assume that $g_{i2} = 10, \forall i \in \mathcal{I}\backslash\{2\}$, $g_{5i} = 50, \forall i \in \mathcal{I}\backslash\{5\}$, and $g_{ij} = 0$ otherwise. We assume that all the users have the same sensing capability and cost such that $q_i = 1$ and $c_i = 60$ for all $i \in \mathcal{I}$.

In Table II, we show the four components of the optimal reward and have the following observations:

- User 1, who is physically isolated from other users, has the highest diversity. Thus, he receives the largest diversity component in (b).
- User 2, who has the strongest social influence on others (e.g., as a celebrity), receives the largest component in (c) and the largest reward.
- User 5, who is easily influenced by the others, has the highest centrality ψ_i and receives a negative reward due to the large negative component in (d). Interestingly, though, he exerts the highest level of effort among the users.

D. Users' Social Surplus under Optimal Reward

In the previous subsections, we mainly focused on the solution of the service provider's profit maximization problem. Here, we switch our attention to the users and characterize their aggregate benefit as the social surplus.

We define the *social surplus* as the sum of the payoffs in (3) of all the users:

$$\sigma(\boldsymbol{r}, \boldsymbol{x}) \triangleq \sum_{i \in \mathcal{I}} U_i(x_i, \boldsymbol{x}_{-i}, r_i). \quad (18)$$

We have the following proposition on its expression under the optimal reward $\boldsymbol{r}^*$ and the corresponding NE $\boldsymbol{x}^*$.

TABLE III
KEY ANALYTICAL RESULTS IN SECTION III

Results	Assumptions	Details
Theorem 1	Ass. 1	Closed-form optimal reward & unique NE
Theorem 2	Ass. 1 & equal costs	Reward as Katz centrality
Proposition 2	Ass. 1	Closed-form social surplus

Proposition 2: Under the optimal reward $\boldsymbol{r}^*$ in (12) and NE $\boldsymbol{x}^*$ in (13) due to Assumption 1, the users' social surplus is

$$\sigma(\boldsymbol{r}^*;\boldsymbol{x}^*) = \boldsymbol{q}^T\left((4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta})^{-1}\right)^T\boldsymbol{C}(4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta})^{-1}\boldsymbol{q}. \tag{19}$$

We can see that $\sigma(\boldsymbol{r}^*, \boldsymbol{x}^*)$ increases with the social connection $\boldsymbol{G}$, user diversity $\boldsymbol{\Delta}$, sensing capability $\boldsymbol{q}$, but decreases with the users' cost $\boldsymbol{C}$. From the objective function in problem (11), we can infer that these parameters also have similar impacts to the optimal profit $\pi(\boldsymbol{r}^*, \boldsymbol{x}^*)$. The proof of Proposition 2 is given in Appendix D.

We summarize our key results in this section in Table III.

IV. PERFORMANCE EVALUATIONS

In the previous sections, we assume that the service provider can obtain the full information of social network $\boldsymbol{G}$ and diversity network $\boldsymbol{\Delta}$, which may not be completely available in practice. To assess the performance of our proposed scheme under a practical scenario, we need to answer the following questions: *What are the impacts of incomplete network $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ information to the service provider? How about their impacts to the users? Which piece of information has a larger impact: $\boldsymbol{G}$ or $\boldsymbol{\Delta}$?*

In this section, we seek to answer the above questions by evaluating the service provider's profit and the users' social surplus under different levels of information on $\boldsymbol{G}$ and $\boldsymbol{\Delta}$. We obtain the following interesting insights: (i) The availability of *complete* $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ information increases *both* the service provider's profit and the users' social surplus over the no information scenario. However, it is not the case under *incomplete* $\boldsymbol{\Delta}$ information. (ii) Network $\boldsymbol{G}$ has a larger impact on the service provider's profit than network $\boldsymbol{\Delta}$ under a *highly asymmetric* social relationship, but the other way round when the social influences are more *balanced*.

A. Random Network and Simulation Settings

For the social network $\boldsymbol{G}$, we first consider the Erdos-Renyi random graph [11], where the social influence g_{ij} is positive with probability p. If that is the case, we assume that value of g_{ij} follows a truncated (on the range of $[0, \infty)$) normal distribution with mean μ and standard deviation σ [29]. We adopt $p = 0.8$, $\mu = 1$, and $\sigma = 0.2$ unless specified otherwise.

For the diversity network $\boldsymbol{\Delta}$, we consider a location-dependent MCS application that the user diversity is distance-based according to (17) with $\epsilon = 1$.

For each set of system parameter choices, we run the simulations 10000 times with randomized social network and diversity network, and show the average value. Unless specified otherwise, we assume that there are $I = 100$ users, who are randomly placed in a 10 km $\times$ 10 km region. All the users have the same sensing capability and cost such that $q_i = 1$ and $c_i = c = 100$ for all $i \in \mathcal{I}$.

Let $\boldsymbol{x}^a$ be the *actual* NE achieved by the users through their interactions in the *actual* social network $\boldsymbol{G}$. On the other hand, we let $\boldsymbol{x}^*$ be the *estimated* NE of the service provider computed by plugging its available information of $\boldsymbol{G}$, $\boldsymbol{\Delta}$, $\boldsymbol{C}$, and $\boldsymbol{q}$ into (13). Since the service provider may not have accurate information on these parameters, in particular $\boldsymbol{G}$ and $\boldsymbol{\Delta}$, it is possible that $\boldsymbol{x}^a \neq \boldsymbol{x}^*$. In our simulation, we study the value of network information $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ by considering that the service provider has various levels of information on $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ in computing $\boldsymbol{r}^*$ and $\boldsymbol{x}^*$ by (12) and (13), respectively. We consider the following information scenarios:

- Full information: The service provider has perfect knowledge about social graph $\boldsymbol{G}$ and diversity graph $\boldsymbol{\Delta}$ in computing $\boldsymbol{r}^*$ and $\boldsymbol{x}^*$. Thus, we have $\boldsymbol{x}^a = \boldsymbol{x}^*$.
- $\boldsymbol{G} = 0$: The service provider only has knowledge of network $\boldsymbol{\Delta}$, but not $\boldsymbol{G}$. It computes $\boldsymbol{r}^*$ and $\boldsymbol{x}^*$ by assuming $\boldsymbol{G} = 0$, so $\boldsymbol{x}^a \neq \boldsymbol{x}^*$ in general.
- $\boldsymbol{\Delta} = 0$: The service provider only has knowledge of network $\boldsymbol{G}$, but not $\boldsymbol{\Delta}$. Thus, it computes $\boldsymbol{r}^*$ and $\boldsymbol{x}^*$ by assuming $\boldsymbol{\Delta} = 0$, so $\boldsymbol{x}^a \neq \boldsymbol{x}^*$ in general.
- $\boldsymbol{G} = \boldsymbol{\Delta} = 0$: The service provider knows nothing about both $\boldsymbol{G}$ and $\boldsymbol{\Delta}$, so it assumes $\boldsymbol{G} = 0$ and $\boldsymbol{\Delta} = 0$ in computing $\boldsymbol{r}^*$ and $\boldsymbol{x}^*$. We have $\boldsymbol{x}^a \neq \boldsymbol{x}^*$ in general.

B. Win-Win from Network Information

Impact of link probability on profit: In Fig. 3, we plot the service provider's profit against link probability p for $I = 100$ users and cost $c = 100$. It can be seen that the service provider's profits under all levels of information increase with p, as the service provider can give less reward but still induce the same effort levels under a tighter social connection. Also, the differences in profit between the full information case and any of the other cases increases with p. It suggests that the impact of the network information $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ on the service provider's profit is more significant when the users are more tightly connected socially. Furthermore, the gap between the full information and $\boldsymbol{\Delta} = 0$ cases is larger than that between the full information and $\boldsymbol{G} = 0$ cases. It implies that the $\boldsymbol{\Delta}$ information is more important than the $\boldsymbol{G}$ information for the service provider to maximize its profit under a more *balanced* social relationship, when all the social influences g_{ij} follow the same distribution. Also, from (16), as the optimal reward under the $\boldsymbol{G} = \boldsymbol{\Delta} = 0$ case is very similar to the $\boldsymbol{\Delta} = 0$ case, the profits obtained are similar.

Impact of link probability on social surplus: In Fig. 4, we plot the users' social surplus against p for $I = 100$ and $c = 100$. It can be observed that as p increases, the peer effect is more significant, so the social surplus increases.

To sum up, besides increasing the service provider's profit as shown in Fig. 3, the availability of the information about $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ increases the users' social surplus in Fig. 4 as well. Thus, this network information allows a *win-win* situation for

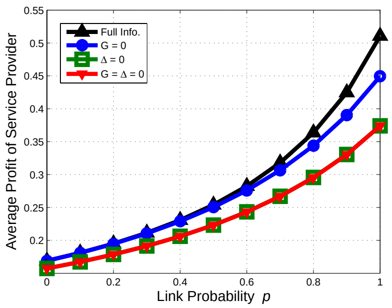

Fig. 3. The service provider's profit vs link probability p.

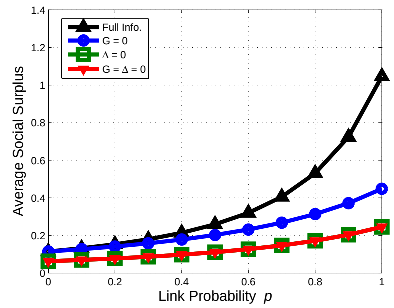

Fig. 4. The users' social surplus vs link probability p.

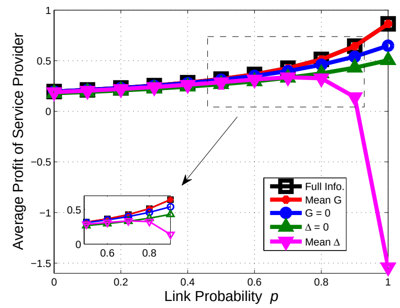

Fig. 5. The service provider's profit vs link probability p.

both the service provider and the users, and it is not simply used by the service provider to extract revenue from the users.

C. Incomplete Network Information

In the aforementioned simulations, we have considered the cases that the service provider has either full information or no information at all for the social network $\boldsymbol{G}$ or the diversity network $\boldsymbol{\Delta}$. In this subsection, we further consider the cases between these two extremes, where the service provider has the information on the mean values of $\boldsymbol{G}$ or $\boldsymbol{\Delta}$.

For the case with mean $\boldsymbol{G}$ information, the service provider assumes that the social network equals $\boldsymbol{G}^{\text{avg}} = [g_{ij}^{\text{avg}}]_{I\times I}$, where $g_{ii}^{\text{avg}} = 0, \forall i \in \mathcal{I}$ and $g_{ij}^{\text{avg}} = \mu p, \forall i,j \in \mathcal{I}, i \neq j$, where μ is the mean of the social tie and p is the link probability. We assume that the service provider knows $\boldsymbol{\Delta}$ accurately in this mean $\boldsymbol{G}$ scheme.

Regarding the average user diversity, when the users are uniformly placed in a $l \times l$ region, the minimum and maximum possible distances between two users are 0 and $\sqrt{2}l$, respectively. For the case with mean $\boldsymbol{\Delta}$ information, the service provider assumes that the diversity network equals $\boldsymbol{\Delta}^{\text{avg}} = [\delta_{ij}^{\text{avg}}]_{I\times I}$, where $\delta_{ii}^{\text{avg}} = 0, \forall i \in \mathcal{I}$ and $\delta_{ij}^{\text{avg}} = \frac{\sqrt{2}l}{2}, \forall i,j \in \mathcal{I}, i \neq j$. We assume that the service provider knows $\boldsymbol{G}$ accurately in this mean $\boldsymbol{\Delta}$ scheme.

In Fig. 5, we plot the service provider's profit against p for $I = 100$ and $c = 100$. For the mean $\boldsymbol{G}$ scheme, it achieves a similar service provider's profit as the full information case, which suggests that the mean social network information is sufficient for reward optimization. However, for the mean $\boldsymbol{\Delta}$ scheme, it results in a larger profit than the $\boldsymbol{\Delta} = 0$ scheme when $p \leq 0.6$. However, when $p > 0.6$, the profit decreases with p and even reaches a negative value when $p = 1$. It is due to the large reward at large p, which leads to the excessive payment to the users. Thus, an inaccurate estimation of $\boldsymbol{\Delta}$ can harm the service provider significantly.

D. Highly Asymmetric Social Effect

In this subsection, we consider a more asymmetric social relationship, where g_{ij} follows the same truncated normal distribution as discussed in Section IV-A for $i < j$, but $g_{ij} = 0$ otherwise. In other words, a user with a higher user index has more social influences on the other users.

In Fig. 6, we plot the service provider's profit against the mean social tie μ for $I = 100$ and $c = 100$. When $\mu \leq 2$, we can see that the diversity network $\boldsymbol{\Delta}$ is more important than the social network $\boldsymbol{G}$ for the service provider to maximize its profit. When $\mu > 2$, the social influences become highly asymmetric, we observe the opposite phenomenon that $\boldsymbol{G}$ plays a more important role than $\boldsymbol{\Delta}$. It is because under a highly asymmetric social relationship, from (16), the allocated rewards for the $\boldsymbol{G} = 0$ case can be very different from the optimal rewards with full information.

E. Facebook Social Network Trace

In this subsection, instead of using the Erdos-Renyi random graph, we consider the Facebook trace [13] for $\boldsymbol{G}$. More specifically, we consider an ego network, which corresponds to a user's friend list in Facebook, with 150 users. In the trace, a link exists between a pair of users if they are friends. We assume that the social influence g_{ij} follows the same truncated normal distribution as in the random graph model.

Impact of number of users on profit: In Fig. 7, we plot the service provider's profit against the total number of users I for $c = 60$. We can see that the service provider's profit increases with I under all the schemes, as there is a higher chance to successfully recruit users with more diversity. Also, we observe that the profit gain of the full information case is more significant when I is larger, when $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ carry more information. Moreover, as the chance of social connection between a random pair of users in a large social network is usually low, we notice that the service provider should perform a more confined recruitment among peer influence groups [30] (e.g., the friend lists in Facebook) to realize the full benefits of peer effects in the reward mechanism design.

Impact of number of users on social surplus: In Fig. 8, we plot the users' social surplus against I for $c = 60$. For the full information and $\boldsymbol{G} = 0$ schemes, we observe that the social surplus increases with I. It is because the service provider experiences a higher level of diversity with more users and provides larger rewards, which leads to a higher social surplus. However, for the $\boldsymbol{\Delta} = 0$ and $\boldsymbol{G} = \boldsymbol{\Delta} = 0$ cases, from (16), the optimal reward for each user is roughly the same over I when the positive social influences g_{ij} follow the same distribution, so the social surplus increases very slowly with I.

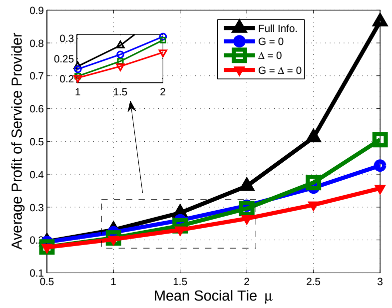

Fig. 6. The service provider's profit vs mean social tie μ.

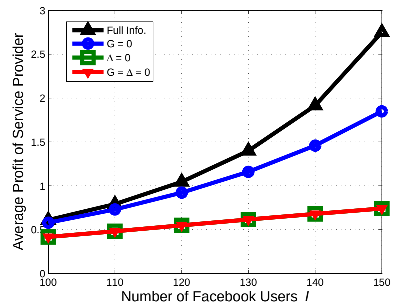

Fig. 7. The service provider's profit vs I in Facebook trace.

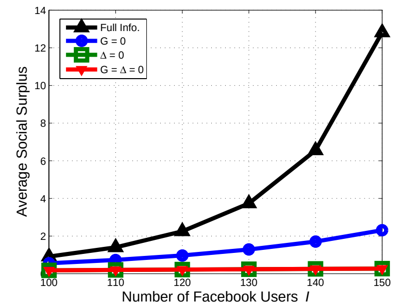

Fig. 8. The users' social surplus vs I in Facebook trace.

V. Conclusion

In this paper, we studied the diversity-driven and socially-aware reward mechanism design in mobile crowdsensing, where the service provider takes advantage of the users' social relationship to improve the value of information due to the user diversity. We aimed to understand how the service provider should optimize its rewards under a dual-graph structure, where the users' social relationship and diversity are modeled by social graph $\boldsymbol{G}$ and diversity graph $\boldsymbol{\Delta}$, respectively.

Despite the non-convexity of the corresponding bilevel problem, we derived the service provider's optimal rewards and the users' unique Nash equilibrium in closed-form under a scenario of interest. Surprisingly, a user's optimal reward is proportional to the weighted sum of the Katz centrality of other users in the superimposed graph $\boldsymbol{G}+\boldsymbol{\Delta}$. Simulation results, based on a random graph model and the Facebook trace, showed that the availability of *complete* $\boldsymbol{G}$ and $\boldsymbol{\Delta}$ information is win-win for the service provider and the users, but it is not the case under *incomplete* $\boldsymbol{\Delta}$ information. Furthermore, the social effect plays a more important role than the diversity when the social relationship is highly asymmetric.

To the best of our knowledge, this is the first systematic study on the reward mechanism in MCS considering both the user diversity and social effect. As the first attempt in this direction, we assumed that the service provider has perfect information on the users' costs. For future work, it is interesting to study the incentive mechanism with private user information, which is challenging due to couplings among the users' decisions. We will also generalize the model to consider multiple tasks instead of a single task as in this paper.

Appendix

A. Inverse-Nonnegative Matrix

The proof of Theorem 1 in Appendix B makes use of the result in this section.

Definition 7: A square matrix $\boldsymbol{A}$ is *inverse-nonnegative* if it is nonsingular and $\boldsymbol{A}^{-1}\geq\boldsymbol{0}$ [31].

Lemma 1: If $4c_i>\sum_{j\in\mathcal{I}}(g_{ij}+g_{ji}+2\delta_{ij})$, $\forall\, i\in\mathcal{I}$, then the matrix $4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta}$ is inverse-nonnegative.

Proof: Let $\boldsymbol{A}=4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta}$, so

$$a_{ij}=\begin{cases}4c_i-2g_{ii}-2\delta_{ii}=4c_i, & \text{if } i=j,\\ -g_{ij}-g_{ji}-2\delta_{ij}, & \text{if } i\neq j.\end{cases}\tag{20}$$

Since $a_{ii}=4c_i>0$, all the diagonal elements are positive. Moreover, from the given condition, we have

$$|a_{ii}|=4c_i>\sum_{j\in\mathcal{I}}(g_{ij}+g_{ji}+2\delta_{ij})=\sum_{j=1,j\neq i}^{I}|a_{ij}|,\ \forall\, i\in\mathcal{I},\tag{21}$$

so $\boldsymbol{A}$ is *strictly diagonally dominant* [31]. From [31, pp. 9-18, Facts 2(a), 2(n), 2(o)], $\boldsymbol{A}$ is an *M-matrix*, which is inverse-nonnegative. ■

B. Proof of Theorem 1

From the KKT conditions [32] of problem (11), for the optimal solution $\boldsymbol{x}^*$, we have

$$x_i^*\geq 0,\ \forall\, i\in\mathcal{I}\ \text{and}\tag{22}$$

$$q_i+2\sum_{j\in\mathcal{I}}\delta_{ij}x_j^*+\sum_{j\in\mathcal{I}}(g_{ij}+g_{ji})x_j^*-4c_ix_i^*=0,\ \forall\, i\in\mathcal{I},\tag{23}$$

where (23) is the first order condition in x_i of the Lagrangian. In matrix form, we have

$$(4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta})\boldsymbol{x}^*=\boldsymbol{q}.\tag{24}$$

Thus,

$$\boldsymbol{x}^*=(4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta})^{-1}\boldsymbol{q}\succeq\boldsymbol{0},\tag{25}$$

where the matrix $4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta}$ is inverse-nonnegative under Assumption 1 by Lemma 1. In problem (10), we can choose $\boldsymbol{\lambda}^*=\boldsymbol{0}$ such that the equality constraint in problem

(10) can be written in matrix form as

$$
\begin{aligned}
\boldsymbol{r}^* &= (2\boldsymbol{C}-\boldsymbol{G})\boldsymbol{x}^* \\
&= (2\boldsymbol{C}-\boldsymbol{G})\Big(2\boldsymbol{C}-\boldsymbol{G}-\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}-\boldsymbol{\Delta}\Big)^{-1}\frac{\boldsymbol{q}}{2} \\
&= (2\boldsymbol{C}-\boldsymbol{G})\Bigg(\Big(\boldsymbol{I}-\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)(2\boldsymbol{C}-\boldsymbol{G})^{-1}\Big)(2\boldsymbol{C}-\boldsymbol{G})\Bigg)^{-1}\frac{\boldsymbol{q}}{2} \\
&= \Big(\boldsymbol{I}-\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)(2\boldsymbol{C}-\boldsymbol{G})^{-1}\Big)^{-1}\frac{\boldsymbol{q}}{2} \\
&= \Big(\boldsymbol{I}+\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)\Big(2\boldsymbol{C}-\boldsymbol{G}-\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}-\boldsymbol{\Delta}\Big)^{-1}\Big)\frac{\boldsymbol{q}}{2} \\
&= \frac{\boldsymbol{q}}{2}+\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)(4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta})^{-1}\boldsymbol{q}. \qquad (26)
\end{aligned}
$$

The fourth equality is due to $(\boldsymbol{AB})^{-1}=\boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$. The fifth equality is due to the matrix inversion lemma [33, pp. 19]. ■

C. Proof of Theorem 2

From (12), we obtain

$$
\begin{aligned}
\boldsymbol{r}^* &= \frac{\boldsymbol{q}}{2}+\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)(4\boldsymbol{C}-\boldsymbol{G}-\boldsymbol{G}^T-2\boldsymbol{\Delta})^{-1}\boldsymbol{q} \\
&= \frac{\boldsymbol{q}}{2}+\frac{1}{4\hat{c}}\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)\Big(\boldsymbol{I}-\frac{1}{4\hat{c}}(\boldsymbol{G}+\boldsymbol{G}^T+2\boldsymbol{\Delta})\Big)^{-1}\boldsymbol{q} \\
&= \frac{\boldsymbol{q}}{2}+\frac{1}{4\hat{c}}\Big(\boldsymbol{\Delta}+\frac{\boldsymbol{G}^T-\boldsymbol{G}}{2}\Big)\boldsymbol{\psi}(\boldsymbol{G}+\boldsymbol{G}^T+2\boldsymbol{\Delta},\frac{1}{4\hat{c}},\boldsymbol{q}), \qquad (27)
\end{aligned}
$$

where the second equality is due to $\boldsymbol{C}=\hat{c}\boldsymbol{I}$ when $c_i=\hat{c}$, $\forall i\in\mathcal{I}$. The third equality is by the definition in (14). ■

D. Proof of Proposition 2

From (3), we have

$$
\begin{aligned}
U_i(x_i^*,\boldsymbol{x}_{-i}^*,r_i^*) &= x_i^*(r_i^*+\sum_{j\in\mathcal{I}}g_{ij}x_j^*)-c_ix_i^{*2} \\
&= x_i^*(2c_ix_i^*)-c_ix_i^{*2}=c_ix_i^{*2}, \qquad (28)
\end{aligned}
$$

where the second line is due to the best response in (5). Overall, from (18), we have

$$
\sigma(\boldsymbol{r}^*,\boldsymbol{x}^*)=\sum_{i\in\mathcal{I}}c_ix_i^{*2}=\boldsymbol{x}^{*T}\boldsymbol{C}\boldsymbol{x}^*. \qquad (29)
$$

By substituting $\boldsymbol{x}^*$ in (13) into (29), the result follows. ■

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