Cooperative and Competitive Operator Pricing for Mobile Crowdsourced Internet Access

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***Abstract*—Mobile Crowdsourced Access (MCA) enables mobile users (MUs) to share their Internet connections by serving as tethers to other MUs, hence can improve the quality of service of MUs as well as the overall utilization of network resources. However, MCA can also reduce the revenue-generating mobile traffic and increase the network congestion for mobile network operators (MNOs), and thus has been blocked by some MNOs in practice. In this work, we reconcile the conflicting objectives of MNOs and MUs by introducing a pricing framework for MCA, where the direct traffic and tethering traffic are charged independently according to a data price and a tethering price, respectively. We derive the optimal data and tethering prices systematically for MUs with the α-fair utility in two scenarios with cooperative and competitive MNOs, respectively. We show that the optimal tethering prices are zero and the optimal usage-based data prices are identical for all MUs, in both the cooperative and competitive scenarios. Such optimal pricing schemes will lead to mutually beneficial results for MNOs and MUs. Our simulation results show that the proposed pricing scheme approximately triples both the MNOs' profit and the MUs' payoff when the MNOs cooperate, comparing to the case where MCA is blocked. Moreover, competition among MNOs will decrease MNOs' profit and further increase the MUs' payoff.**

I. Introduction

A. Background and Motivation

Global mobile data traffic has been experiencing explosive growth and is expected to reach 30.6 exabytes per month by 2020, approximately a 7.3-fold increase over 2015 [1]. However, the mobile network capacity is growing relatively slowly, which results in a huge gap between projected demand and supply at the global scale. On the other hand, the heterogeneity of networks and mobile users (MUs) leads to different levels of mismatch even at the same time and location. For example, a high-demand MU may not be fully satisfied in a low-capacity network, while a low-demand MU may underutilize his network resource in a high-capacity network. This creates opportunities for more effective resource allocation and sharing across networks and MUs.

One approach to achieve more effective overall network resource utilization is through the new paradigm of *User-Provided Network* (UPN), in which MUs serve as micro-operators and provide network connections (and resources) directly to each other [2], [3]. Examples of UPN include the services enabled by companies such as Karma [4] and Open Garden [5]. Specifically, Karma sells mobile devices (that convert 4G cellular signals to Wi-Fi signals) to its subscribers, and encourages the subscribers to operate as Wi-Fi hotspots and provide Internet access to non-subscribers. Open Garden, on the other hand, provides a mobile app to its MUs, and enables mobile devices to dynamically form mesh networks and flexibly tether data for each other to utilize the best Internet connections among the MUs. The Open Garden solution enables mobile multi-hop and multi-path connectivity sharing among MUs through Bluetooth and Wi-Fi direct links, and realizes *Mobile Crowdsourced Access* (MCA).

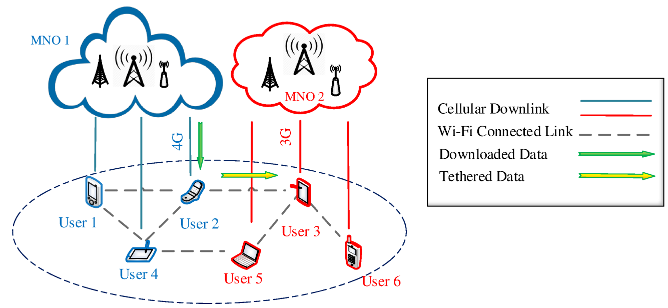

Fig. 1. Illustration of an Open-Garden-like MCA framework consisting of 2 MNOs and 6 MUs.

However, mobile network operators (MNOs) have not been entirely supportive of such emerging MCA and the associated forms of tethering. This is mainly due to the fact that MCA can effectively exploit the diversity of MUs and heterogeneous network environments, which can drive away revenue-generating traffic from MNOs (who charge MUs high usage-based data prices) or cause more network congestions for MNOs (who charge MUs low usage-based data prices). As an example, AT&T in the US has requested Google Play to block its subscribers' access to Open Garden [6].

Even before the emergence of MCA services such as Open Garden, MNOs have already deployed several methods to restrict tethering among MUs, mainly by direct prohibition or imposing an additional tethering fee. For example, the MNO "au" in Japan charges about \$5 per month for the tethering service [7]. In the US, MNOs such as AT&T and Verizon Wireless charged an additional \$15-\$30 per month for MU tethering before 2012 [8].[1] In the UK, some MNOs (such

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[1] Since Verizon Wireless had broken the "C Block rules", the FCC in the US has ruled against Verizon's tethering charging practice in 2012 [9].

as Virgin Mobile, The People's Operator, and iD Mobile) prohibit all tethering [10], while some remaining MNOs (such as Three) charge MUs an additional fee for upgrading to data plans that allow tethering [11].

Although the MNOs have taken the first step towards controlling and profiting from tethering, their current pricing and tethering policies have significant drawbacks that prevent their widespread acceptance of MCA. Namely, these existing policies neither capture the dynamics of the wireless mesh network nor consider the interaction among different MNOs. This motivates the following questions.

1) What type of data and tethering price scheme can maximize the profits of multiple MNOs, in both cooperative and competitive environments?
2) What is the impact of such a profit-maximizing policy on the MUs' surplus and the social welfare?

B. Solution Approach and Contributions

In this work, we study the optimal data and tethering pricing scheme in an Open-Garden-like MCA framework. An example of such an MCA framework with 2 MNOs and 6 MUs is illustrated in Fig. 1, where MNO 1 and its three subscribers are marked in blue and MNO 2 and its three subscribers are marked in red. In this example, MU 3 has a high data demand that cannot be entirely satisfied by her 3G downlink. Thus, MU 3 requests MU 2, who has a low demand but a high 4G downlink capacity, to download data for her.

To capture different interactions among MNOs, we consider both the cooperative scenario and the competitive scenario. In the former case, MNOs are cooperative and jointly decide the data and tethering pricing scheme to maximize their total profit.[2] In the latter case, MNOs are competitive and each determines the data and tethering pricing scheme independently to maximize its own profit.

Our goal is to understand the structure of the optimal data and tethering pricing schemes and their impact on the MCA service. Since MNOs usually choose the pricing policies over a slow time scale and MUs simply respond to these policies as price-takers, we model the interaction between MNOs and MUs as a two-stage Stackelberg game [13]. In the first stage, MNOs (leaders) determine the data and tethering prices. In the second stage, MUs (followers) jointly decide their traffic downloads (data rate) along with any tethering. We analyze the best responses for MNOs and for MUs systematically, based on which we derive the game equilibrium.

The main contributions of this paper are as follows.

- *Novel Pricing Framework.* As far as we know, we propose the first pricing framework for data downloading and tethering for MCA in which the MNO charges separate usage fees for both direct data downloads and tethered data (in Section IV).
- *Cooperative and Competitive Schemes.* We analyze the equilibria of the proposed pricing scheme under both scenarios of cooperative MNOs (in Section V) and competitive MNOs (in Section VI). With the widely used α-fair MU utility function, we show for both scenarios that the tethering price at the equilibrium is zero while the data price is identical even for heterogeneous MUs.
- *Performance Evaluation.* We show that both MUs and MNOs can benefit substantially from the proposed data and tethering price schemes, compared to the scenario where the MCA service is blocked (in Section VII). Our simulation results show that the cooperative scheme approximately triples the MNOs' profit and the MUs' payoff. The competitive scheme reduces the MNOs profits, while increases the MUs' payoff.

[2] An example of such MNO cooperation happens in the Swedish telecommunication market, where several MNOs (e.g., Orange, Hi3G, and Telia) share their mobile network infrastructure and licenses [12].

II. Related Work

Several existing works have been devoted to the pricing and incentive design for MCA. In [14], Gao *et al.* proposed a hybrid data pricing scheme motivated by Karma's UPN service, to incentivize MUs to operate as mobile Wi-Fi hotspots and provide Internet access for other MUs without direct Internet access. In [15], Khalili *et al.* further studied the user behavior dynamics and network evolution under such a hybrid data pricing scheme. However, such a hybrid pricing scheme cannot be directly applied in our model, as we consider the case where each MU can concurrently have direct mobile Internet access and receive tethered data from other MUs, which is substantially different from the model in [14].

For the Open-Garden-like MCA service, researchers proposed several incentive mechanisms in [16]–[18]. Specifically, in [16], [17], Iosifidis *et al.* proposed a distributed incentive mechanism for encouraging MCA service, and in [18], Syrivelis *et al.* designed a cloud-controlled MCA service and studied a coalitional game played among MUs. However, none of the above works considered the impact of MCA on MNOs or the strategic interactions among the MNOs.

Another cluster of related work studied pricing and incentive schemes for wireless mesh networks [19], [20], cooperative communication networks [21], and cooperative cognitive radio networks [22]–[24]. The focus of those studies is the incentive issue for user cooperation, without considering the interventions of operators. In this work, we focus on the pricing strategies of MNOs and the impact on the MUs' strategies.

TABLE I
MUs' Utility and System Parameters

MUs	MU 1	MU 2
Operational Cost (\$/MB)	0.5	2.5
Capacity (MBps)	8	3
Utility (\$)	$5 \cdot (x_1 + y_1)^{0.3}$	$10 \cdot (x_2 + y_2)^{0.3}$

III. An Illustrative Example

To introduce the key model features and demonstrate some key insights, we consider an illustrative model with two MNOs $\{1, 2\}$ and two MUs $\{1, 2\}$. MNO 1 provides 4G LTE service to MU 1, and MNO 2 provides 3G service to MU 2. Each

TABLE II
EQUILIBRIUM/OPTIMIZED RESULTS AND PERFORMANCE OF DIFFERENT SCHEMES

-	No-Tethering Pricing Scheme		Cooperative Scheme		Competitive Scheme		Social Welfare Maximization
MUs	MU 1	MU 2	MU 1	MU 2	MU 1	MU 2	-
Data Price ($/MB)	2.04	12	2.90	2.90	2.32	2.32	-
Tethering Price ($/MB)	-	-	**0**	**0**	**0**	**0**	-
Directly Downloaded Data (MB)	4.80	1.29	2.16	0	2.97	2	-
Received Tethered Data (MB)	-	-	0	5.84	0	6.03	-
MUs' Payoff ($)	43.9		54.3		59.7		-
MNOs' Profit ($)	13.1		19.3		14.11		-
Social Welfare ($)	57.0		73.5		73.8		77.8

MNO announces a usage-based *data price* and a usage-based *tethering price* to its subscriber. Two MUs are physically close by, so that they can share their Internet access (via tethering) through Wi-Fi Direct or Bluetooth. The MUs cooperatively determine the direct downloaded data $\{x_i\}_{i\in\{1,2\}}$ and the received tethered data $\{y_i\}_{i\in\{1,2\}}$ (y_i represents the amount of data MU $j(\neq i)$ downloads from the Internet and sends to MU i), in order to maximize their total payoff. Table I in Page 2 shows the system parameters and the MUs' utility functions.

We consider three pricing and tethering policies for MNOs:

- **No-Tethering Pricing (NTP) Scheme**: The MNOs block the tethering service (equivalently, setting the tethering price to be infinite). Hence, each MU can only download data from his own MNO. Each MNO optimizes its data price to maximize its profit.
- **Cooperative Data and Tethering Pricing Scheme**: Two MNOs cooperatively optimize the data prices and tethering prices to maximize their total profit.
- **Competitive Data and Tethering Pricing Scheme**: Each MNO sets its own data price and tethering price to maximize its own profit, taking into account the competition from the other MNOs.

Table II shows the optimal (equilibrium) prices, data downloading amount (per time slot), and various performance metrics for the preceding three schemes.

We observe that (i) for both cooperative and competitive schemes, the optimal/equilibrium tethering prices for two MUs are zero, and the data prices for two MUs are the same; and (ii) both cooperative and competitive schemes achieve a higher MU payoff and a higher MNO profit than the NTP scheme.

IV. SYSTEM MODEL

A. System Overview

We consider an MCA model of a set $\mathcal{N}=\{1,2,...,N\}$ of MNOs and a set $\mathcal{I}=\{1,2,...,I\}$ of MUs, as illustrated in Fig. 1. We assume each MU subscribes to only one MNO. Let $\mathcal{I}_n$ denote the set of subscribers of MNO n, and let $\sigma(i)$ denote the MNO to which MU i subscribes, i.e., $i\in\mathcal{I}_{\sigma(i)},\forall i\in\mathcal{I}$. In Fig. 1, for example, MNO 2's subscribers set is $\mathcal{I}_2=\{3,5,6\}$, and MU 2 subscribes to MNO, i.e., $\sigma(2)=1$.

The MUs cooperate and provide an Open-Garden-like MCA service to each other. More specifically, close-by MUs form one wireless mesh network (WMN), where all MUs are connected through Wi-Fi Direct.[3] For presentation clarity, we focus on the downlink. We further assume that there is a direct Wi-Fi connected link between any two MUs.

We consider a time period consisting of a set $\mathcal{T}=\{1,2,..,T\}$ of time slots.[4] Without loss of generality, we normalize the length of each time slot to be one. We consider a quasi-static mobility model, where each MU moves randomly across time slots, and remains at the same location within each time slot.

In each time slot, each MU can be either a *gateway*, or a *client*, or both. A gateway node downloads data directly from its MNO, and a client node consumes data for some mobile applications. Let C_i (MBps) be the maximum downloading speed[5] (capacity) that MU $i\in\mathcal{I}$ can achieve from her cellular downlink provided by the MNO.

We consider a linear average operational cost e_i of MNO $\sigma(i)$ for sending every MB to MU i. Without loss of generality, we rank the MUs in the increasing order of their downlink operational costs, i.e., $e_1<e_2<...<e_I$. We further consider a linear energy cost c per MB for the data transfer over the Wi-Fi Direct and cellular links [25].[6]

B. Mobile Network Operators

We introduce the MNOs' pricing decisions in this subsection. Denote $p_i\geq 0$ as the *data price* ($/MB) that MNO $\sigma(i)$ charges MU i for each downloaded MB. Here we allow the MNOs to charge different prices to different MUs, based on MUs' QoS requirements and network service types.[7]

We further consider a linear tethering price, i.e., the MNOs can charge for each tethered MB in additional to the basic data payment. Let $d_{i\leftarrow j}$ denote the *tethering price* ($/MB) that MNO $\sigma(j)$ charges MU j for each MB that MU j tethers to MU i. Note that the MNO can set a negative tethering

[3]For the case where the MUs form more than one disjointed WMNs, the operators can set different prices for different MUs, so that the pricing problem can be decomposed at each MNO. Thus, we only need to consider one completely connected WMN without loss of generality.

[4]The length of one time slot can be several to tens of minutes.

[5]We assume that the MUs are close to each other and the connected (Wi-Fi) links have high capacities. Hence the performance bottlenecks in the network are the cellular links.

[6]Note that the MU energy consumption per MB may depend on the gateways and the clients, but that is much smaller than the monetary cost. Thus, we consider an average downloading energy cost per MB for all MUs.

[7]Perfect price differentiation among MUs leads to maximum design flexibility for the MNOs. In practice, the MNOs may partially differentiate among MUs with a limited number of prices choices [26].

price, in which case MNO $\sigma(j)$ will give MU j a discount of $|d_{i\leftarrow j}|$ for the data tethered to MU i. This can happen if the MNO would like to encourage the MU to use another downlink channel with a lower operational cost. We further allow the MNO to differentiate not only the gateway MUs but also the clients,[8] *i.e.,* $d_{i\leftarrow j}$ can be different for every i and j.

Define $\boldsymbol{h} \triangleq \{h_{i\leftarrow j}\}_{i,j\in\mathcal{I}}$ as the hybrid price matrix, where $h_{i\leftarrow j}$ denotes the price for each MB MU j tethers to MU i, which includes both the data price and the tethering price,

$$h_{i\leftarrow j} \triangleq p_j + d_{i\leftarrow j}. \tag{1}$$

Here we define $d_{i\leftarrow i} \triangleq 0$ for each i.

With the preceding notation, the MNO n's profit is

$$V_n = \sum_{j\in\mathcal{I}_n}\sum_{i\in\mathcal{I}}(h_{i\leftarrow j} - e_j)x_{i\leftarrow j}, \tag{2}$$

where $x_{i\leftarrow j}$ is an MU traffic decision (to be defined next).

C. Mobile Users

We model the MUs' utilities and payoffs in this subsection. Let $x_{i\leftarrow j} \geq 0$ denote the data downloaded by MU j and tethered to MU i (tethered data) if $j \neq i$, and the data MU i downloads for herself (directly downloaded data) if $j = i$. We define $\boldsymbol{x} \triangleq \{x_{i\leftarrow j}\}_{i,j\in\mathcal{I}}$ as the traffic matrix.

We consider the following α-fair utility function [28], [29] for MU i when consuming $\sum_{j\in\mathcal{I}} x_{i\leftarrow j}$ amount of data,

$$U_i\left(\sum_{j\in\mathcal{I}} x_{i\leftarrow j}\right) = \frac{\theta_i\left(\sum_{j\in\mathcal{I}} x_{i\leftarrow j}\right)^{1-\alpha}}{1-\alpha}, \tag{3}$$

where $\alpha \in [0, 1)$ and the scaling factor $\theta_i > 0$, representing MU i's willingness to pay.

Let $J(\cdot)$ denote the MUs' total payoff, given by

$$J(\boldsymbol{x};\boldsymbol{h}) = \sum_{i\in\mathcal{I}} U_i\left(\sum_{j\in\mathcal{I}} x_{i\leftarrow j}\right) - \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}}(h_{i\leftarrow j} + c)x_{i\leftarrow j}, \tag{4}$$

where c is the average linear energy cost for MUs.

D. Problem Formulation

MNOs decide the data and tethering prices at the beginning of the entire time period, and MUs decide the data traffic in each time slot. We assume that the system is ergodic across different time slots. Hence, for simplicity, we just need to consider one time slot. We model the interaction between MNOs and MUs as a two-stage Stackelberg game. Specifically, in the first stage, the MNOs simultaneously decide the hybrid price matrix $\boldsymbol{h}$. In the second stage, the MUs simultaneously decide the traffic matrix $\boldsymbol{x}$ and work cooperatively to maximize their total payoff. We note that the operation of the MCA relies on the MUs' cooperation and such cooperation can be imposed by a bargaining mechanism as in [16], which determines how MUs share the benefits of cooperation in a fair manner based on the Nash Bargaining Solution. However, MNOs only need to know MUs' traffic decision in order to determine the pricing schemes as shown in (2). Hence, we omit the details of the bargaining solution.

Depending on the relationships among MNOs, we consider both a cooperative model and a competitive model. The cooperative MNOs choose the hybrid pricing matrix together to maximize their total profit, whereas each competitive MNO sets the data and tethering price to maximize its own profit.

V. Cooperative MNOs

In this section, we consider the case where the MNOs cooperatively decide the hybrid price matrix $\boldsymbol{h}$ to maximize their total payoff. We will study the Subgame Perfect Equilibrium (SPE) of the two-stage Stackelberg game. Specifically, the MNOs decide the hybrid price matrix $\boldsymbol{h}$ in Stage I and MUs decide the traffic matrix $\boldsymbol{x}$ in Stage II.

We will derive the SPE by backward induction, i.e., given the hybrid pricing matrix $\boldsymbol{h}$, we characterize the MUs' traffic decision $\boldsymbol{x}^*(\boldsymbol{h})$ that maximizes the MUs' total payoff in Stage II, and then we characterize the MNOs' optimal hybrid pricing matrix $\boldsymbol{h}^*$ that maximizes the MNOs' profit.

We obtain the optimal traffic matrix $\boldsymbol{x}^*(\boldsymbol{h})$ by solving the MUs' payoff optimization problem (UOP) in Stage II,

$$\begin{aligned}
\text{(UOP)}\quad \max_{\boldsymbol{x}}\quad & \sum_{i\in\mathcal{I}} U_i\left(\sum_{j\in\mathcal{I}} x_{i\leftarrow j}\right) - \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}}(h_{i\leftarrow j} + c)x_{i\leftarrow j}\\
\text{s.t.}\quad & \sum_{i\in\mathcal{I}} x_{i\leftarrow j} \leq C_j, \qquad \forall j\in\mathcal{I}, \qquad (5)\\
& x_{i\leftarrow j} \geq 0, \qquad \forall i,j\in\mathcal{I}. \qquad (6)
\end{aligned}$$

Constraint (5) indicates that the sum of MU j's directly downloaded data and the data tethered to other MUs cannot exceed the capacity of her downlink C_j.

Given $\boldsymbol{x}^*(\boldsymbol{h})$, we can derive the optimal hybrid pricing matrix $\boldsymbol{h}^*$ by solving the operators' profit maximization problem (OPP) in Stage I,

$$\text{(OPP)}\quad \max_{\boldsymbol{h}}\quad \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}}(h_{i\leftarrow j} - e_j)x^*_{i\leftarrow j}(\boldsymbol{h}).$$

A. Traffic and Demand in Stage II

In this subsection, we characterize $\boldsymbol{x}^*(\boldsymbol{h})$ by solving (UOP). We can show that (UOP) is a convex problem. Hence, the KKT conditions are sufficient and necessary for global optimality [30]. The KKT conditions for (UOP) are given by

$$\theta_i\left(\sum_{l\in\mathcal{I}} x_{i\leftarrow l}\right)^{-\alpha} - h_{i\leftarrow j} - c - \lambda_j + \beta_{i\leftarrow j} = 0\ \forall i,j, \tag{7}$$

$$\lambda_j\left(\sum_{i\in\mathcal{I}} x_{i\leftarrow j} - C_j\right) = 0\ \ \forall j, \tag{8}$$

$$x_{i\leftarrow j}\beta_{i\leftarrow j} = 0\ \ \forall i,j, \tag{9}$$

$$\lambda_i \geq 0\ \ \forall i, \tag{10}$$

$$\beta_{i\leftarrow j} \geq 0\ \ \forall i,j. \tag{11}$$

[8] In practice, there are several ways for the MNOs to detect whether and for whom an MU is tethering, such as through MAC address inspection [27].

Thus, we have the following lemma.

Lemma 1. *Given any hybrid pricing matrix* $\boldsymbol{h}$ *and* $h_{i\leftarrow k} > h_{i\leftarrow j}$*, the optimal traffic matrix* $\boldsymbol{x}^*$ *satisfies*

$$x^*_{i\leftarrow k} > 0 \quad \text{only if} \quad \sum_{l\in\mathcal{I}} x^*_{l\leftarrow j} = C_j. \tag{12}$$

That is, MUs purchase the data from a lower price downlink first (and fully utilize that downlink's capacity) before purchasing data from a higher price downlink.

Note that if there exist downlinks $j \neq k$ such that $h_{i\leftarrow j} = h_{i\leftarrow k}$, then (UOP) is not strictly convex and there may be more than one global optimal solution that satisfies the KKT conditions, which implies the MNOs' total profit can be a multi-valued function of $\boldsymbol{h}$. Hence, for analytical simplification, we assume that each user prioritizes the utilization of the downlink with a lower operational cost (i.e., a smaller index), when facing multiple downlinks with the same hybrid price.[9]

The preceding analysis has assumed an arbitrary matrix $\boldsymbol{h}$. In the following, we will focus on the properties of the MUs' traffic $\boldsymbol{x}^*(\boldsymbol{h}^*)$, given the MNOs choose equilibrium price matrix $\boldsymbol{h}^*$.

Theorem 1. *Given the equilibrium hybrid price matrix* $\boldsymbol{h}^*$*, the solution* $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*)$ *satisfying the KKT conditions must also satisfy*

$$\lambda_i^* = 0, \qquad\qquad \forall i \in \mathcal{I}, \tag{13}$$

$$\theta_i \left(\sum_{l\in\mathcal{I}} x^*_{i\leftarrow l}\right)^{-\alpha} \begin{cases} = h^*_{i\leftarrow j} + c, & \text{if} \quad x^*_{i\leftarrow j} > 0 \\ \leq h^*_{i\leftarrow j} + c, & \text{if} \quad x^*_{i\leftarrow j} = 0 \end{cases}, \tag{14}$$

where $\boldsymbol{\lambda} \triangleq \{\lambda_i\}_{i\in\mathcal{I}}$ *are the dual variables corresponding to constraints* (5).

The intuition is that for any given $\boldsymbol{h}$, if $\lambda_j^* > 0$ is optimal, then the MNOs can always increase $h_{i\leftarrow j}, \forall i \in \mathcal{I}$, to increase their profit without affecting the MUs' traffic decisions. By Theorem 1 and our earlier assumption, it is easy to show the following proposition.

Proposition 1. *There exists an equilibrium pricing solution* $\boldsymbol{h}^*$ *that satisfies*

$$h^*_{i\leftarrow j} = h^*_{i\leftarrow k} = p_i^*, \quad \forall i, j, k \in \mathcal{I}, \tag{15}$$

where p_i^* *is the equilibrium data price.*

Proposition 1 suggests that one of the equilibrium price solutions for the MNOs is *gateway independent*. In other words, to derive the equilibrium hybrid pricing matrix $\boldsymbol{h}^*$, it is enough to optimize the data price vector $\boldsymbol{p} \triangleq \{p_i\}_{i\in\mathcal{I}}$ only, and then set the hybrid price identical for different gateways. Hence, in the rest of this section, we will focus on this gateway independent pricing strategy. After obtaining the optimal data price vector $\boldsymbol{p}^*$, we can compute the optimal tethering prices as

$$d^*_{i\leftarrow j} = p_i^* - p_j^*, \ \forall i, j \in \mathcal{I}. \tag{16}$$

By the gateway independence of the price for each MU and (7), we can derive MU i's *demand function* $q_i(p_i^*)$, defined as

$$q_i(p_i^*) \triangleq \sum_{j\in\mathcal{I}} x^*_{i\leftarrow j}(\boldsymbol{h}^*) = \left(\frac{\theta_i}{p_i^* + c}\right)^{1/\alpha}, \ \forall i \in \mathcal{I}. \tag{17}$$

Note that it is necessary to define the demand functions because $\boldsymbol{x}^*(\boldsymbol{h}^*)$ is not unique but $\boldsymbol{q}(\boldsymbol{p}^*)$ is unique, since the objective of (UOP) is strictly concave in $\boldsymbol{q}$ but not strictly concave in $\boldsymbol{x}$. We will show in the next subsection that deriving the unique $\boldsymbol{q}(\boldsymbol{p}^*)$ is enough for designing the optimal pricing scheme.

By (17), we define the *utilized capacity for downlink* j as

$$z_j(\boldsymbol{p}^*) \triangleq \sum_{i\in\mathcal{I}} x^*_{i\leftarrow j}(\boldsymbol{p}^*), \ \forall j \in \mathcal{I},$$
$$\overset{(a)}{=} \begin{cases} C_j & \text{if} \quad j < k_{\text{act}} \\ \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{p_i^*+c}\right)^{1/\alpha} - \sum_{i=0}^{k_{\text{act}}-1} C_i > 0 & \text{if} \quad j = k_{\text{act}} \\ 0 & \text{if} \quad j > k_{\text{act}} \end{cases}, \tag{18}$$

where $k_{\text{act}} \in \mathcal{I}$ is the *largest active downlink index* which satisfies $\sum_{i=0}^{k_{\text{act}}-1} C_i < \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{p_i^*+c}\right)^{1/\alpha} \leq \sum_{i=0}^{k_{\text{act}}} C_i$, and (a) is due to our earlier assumption. Here we define $C_0 \triangleq 0$ for notational completeness.

Recall that we have ordered the downlinks such that a smaller k_{act} corresponds to a smaller operational cost $e_{k_{\text{act}}}$. Hence, (18) implies that the MNOs set the price matrix $\boldsymbol{h}$ so that MUs will prioritize the utilization of downlinks with lower operational costs.

B. MNOs' Cooperative Pricing in Stage I

In this subsection, we will design the cooperative pricing scheme to maximize MNOs' total profit by solving (OPP). Note that the objective of (OPP) is not easy to deal with directly, as we do not have a close-form expression for $\boldsymbol{x}^*(\boldsymbol{h})$. However, given the equilibrium $\boldsymbol{h}^*$, we can use Proposition 1, $q_i(p_i^*)$ in (17), and $z_j(\boldsymbol{p}^*)$ in (18) to transform (OPP) into the following equivalent Problem (OPP-T),

$$(\text{OPP} - \text{T})$$
$$\max_{\boldsymbol{p}, k_{\text{act}}\in\mathcal{I}} \quad \sum_{i\in\mathcal{I}} (p_i - e_{k_{\text{act}}}) q_i(p_i) + \sum_{i=0}^{k_{\text{act}}-1} (e_{k_{\text{act}}} - e_i) C_i$$
$$\text{s.t.} \qquad \sum_{i=0}^{k_{\text{act}}-1} C_i < \sum_{i\in\mathcal{I}} q_i(p_i) \leq \sum_{i=0}^{k_{\text{act}}} C_i \tag{19}$$

for the choice of k_{act} such that $\sum_{i=0}^{k_{\text{act}}-1} C_i < \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{p_i^*+c}\right)^{1/\alpha} \leq \sum_{i=0}^{k_{\text{act}}} C_i$.

Note that (OPP-T) is not a convex problem due to the integer decision variable k_{act}. However, if we fix k_{act}, (OPP-T) is

[9] In terms of practical implementation, MNOs can always set the hybrid price for the larger operational cost downlink ϵ larger than that of the smaller operational cost downlink, so that MUs will first utilize the lower cost downlink according to Lemma 1, where $\epsilon > 0$ is an arbitrarily small number.

a convex problem in $\boldsymbol{p}$. We are now ready to introduce the following proposition to obtain the optimal solution of the above (OPP-T).

Proposition 2. *Given k^*_{act}, the optimal $\boldsymbol{p}^*$ is given by*

- *If* $\sum_{i\in\mathcal{I}}\left(\frac{\theta_i(1-\alpha)}{e_{k^*_{\mathrm{act}}}+c}\right)^{1/\alpha} \le \sum_{j=0}^{k^*_{\mathrm{act}}} C_j$, *then*

$$p_i^* = \frac{e_{k_{\mathrm{act}}{}^*}+c}{1-\alpha} - c, \quad \forall i\in\mathcal{I}; \tag{20}$$

- *Otherwise,*

$$p_i^* = \left(\frac{\sum_{i\in\mathcal{I}}\theta_i^{1/\alpha}}{\sum_{i=1}^{k_{\mathrm{act}}{}^*} C_i}\right)^{\alpha} - c, \quad \forall i\in\mathcal{I}. \tag{21}$$

Hence, we can obtain all solution candidates $\boldsymbol{p}^*(k_{\mathrm{act}})$ for all k_{act} by Proposition 2, and select $(\boldsymbol{p}^*(k^*_{\mathrm{act}}), k^*_{\mathrm{act}})$ that leads to the maximal objective value among all candidates as the optimal solution.

As is shown in Proposition 2, the optimal data prices $\boldsymbol{p}^*$ for all MUs are identical, even if the MUs are heterogeneous (having different values of θ_i and thus different demands q_i). According to (16), the optimal tethering price is therefore zero, i.e., $d^*_{i\leftarrow j}=0, \forall i,j\in\mathcal{I}$.[10]

VI. Competitive MNOs

We now analyze the MNOs and MUs' decisions in the competitive MNOs model. In this model, MNOs fail to cooperate, and hence each of them participates in a *competitive pricing game* and aims to selfishly maximize its own profit, which corresponds to a competitive market.

Here we use $\boldsymbol{h}=(\boldsymbol{h}^n,\boldsymbol{h}^{-n})$, where $\boldsymbol{h}^n \triangleq \{h_{i\leftarrow j}\}_{i\in\mathcal{I}, j\in\mathcal{I}_n}$ is the hybrid price matrix of MNO n, and $\boldsymbol{h}^{-n}$ refers to the hybrid price matrix of MNOs other than MNO n. The competitive pricing game and the SPE $(\boldsymbol{h}^\star, \boldsymbol{x}^\star(\boldsymbol{h}^\star))$ for the competitive MNOs model are thus defined as

$$\boldsymbol{h}^{n\star} \triangleq \arg\max_{\boldsymbol{h}^n} V_n(\boldsymbol{h}^n, \boldsymbol{h}^{-n\star}; \boldsymbol{x}^\star(\boldsymbol{h}^n, \boldsymbol{h}^{-n\star})), \ \forall n\in\mathcal{N}. \tag{22}$$

$$\boldsymbol{x}^\star(\boldsymbol{h}^\star) \triangleq \arg\max_{\boldsymbol{x}} J(\boldsymbol{x}; \boldsymbol{h}^\star). \tag{23}$$

Similar to the cooperative MNOs model, we will derive the SPE by backward induction, i.e., given the hybrid pricing matrix $\boldsymbol{h}$, we characterize the MUs' traffic decision $\boldsymbol{x}^\star(\boldsymbol{h})$ in Stage II, and then we characterize the equilibria of the pricing and tethering scheme.

A. MUs' Traffic and Demand in Stage II

Similar to the interaction in Stage II for the cooperative MNOs model, the MUs work cooperatively to optimize $\boldsymbol{x}$ by solving (UOP) given $\boldsymbol{h}$.

Note that the sufficient and necessary KKT conditions to (UOP) given in (7)-(11) and Lemma 1 also hold for the competitive model, as these results apply to an arbitrary $\boldsymbol{h}$. In addition, similar to Theorem 1 and Proposition 1, there exists a best response $\boldsymbol{h}^{n\star}$ such that $h^\star_{i\leftarrow j}=h^\star_{i\leftarrow k}, \forall i\in\mathcal{I}, \forall j,k\in\mathcal{I}_n$ for each MNO n, which implies that for each MNO n, it is enough to focus on a gateway independent pricing scheme. This allows us to focus on each MNO n's strategy $\boldsymbol{p}^n=\{p_i^n\}_{i\in\mathcal{I}}$, where

$$p_i^n \triangleq h_{i\leftarrow j}, \ \forall i\in\mathcal{I}, \ \forall j\in\mathcal{I}_n. \tag{24}$$

B. MNOs' Pricing Competition in Stage I

It turns out that analytically deriving the SPE is difficult.[11] However, in this subsection, we derive the properties of the *potential equilibria*, in the sense that either they are the SPE or there does not exist any SPE. To characterize the potential equilibria, we introduce the following definitions.

Let p_s^π denote the downlink-clearing price for the first s downlinks, that is,

$$p_s^\pi \triangleq \left(\frac{\sum_{i\in\mathcal{I}}\theta_i^{1/\alpha}}{\sum_{i=1}^{s} C_i}\right)^{\alpha} - c, \ \forall s\in\mathcal{I}, \tag{25}$$

which is decreasing in s and e_s is increasing in s. As we will show, $\{p_i^n=p_s^\pi\}_{i\in\mathcal{I}}$ for some s corresponds to a pricing strategy that MNO n may choose at an equilibrium.

Let $g\triangleq \arg\min_{i\in\mathcal{I}/\mathcal{I}_{\sigma(1)}} e_i$ denote the index of the smallest operational cost downlink of all MNOs other than MNO $\sigma(1)$.

We start with the following lemma:

Lemma 2. *Any equilibrium strategy profile $\{\boldsymbol{p}^{n\star}\}$ should satisfy $p_i^{\sigma(1)\star}\le e_g, \ \forall i\in\mathcal{I}$, if and only if $p^\pi_{g-1}\le e_g$.*

Note that if $p_i^{\sigma(1)\star}\le e_g, \ \forall i\in\mathcal{I}$, then no MNO except MNO $\sigma(1)$ has the incentive to set prices lower than e_g. This provides MNO $\sigma(1)$ chances to monopolize the market. On the other hand, if $p_i^{\sigma(1)\star}>e_g$ for some i, then at least MNO $\sigma(g)$ will compete with MNO $\sigma(1)$. Thus, we can categorize the potential equilibria into the following two cases, depending on the relationship between p^π_{g-1} and e_g.

1) Case I (Monopolistic Equilibria): $p^\pi_{g-1}\le e_g$

According to Lemma 2, any MNO except $\sigma(1)$ does not have the incentive to set the price p_i^n lower than the lowest downlink cost $p_i^{\sigma(1)\star}$. Otherwise, it will obtain a negative profit. Thus, at an equilibrium, we must have that only the MNO $\sigma(1)$ is a *traffic-supporting* MNO.

In other words, MNO $\sigma(1)$ can act as a monopolist in this case. Let $\tilde{\boldsymbol{p}}^\star=\{\tilde{p}_i^\star\}_{i\in\mathcal{I}}$ be the monopoly price vector obtained by solving (OPP-T). We thus have the following theorem to characterize the equilibria in this case.

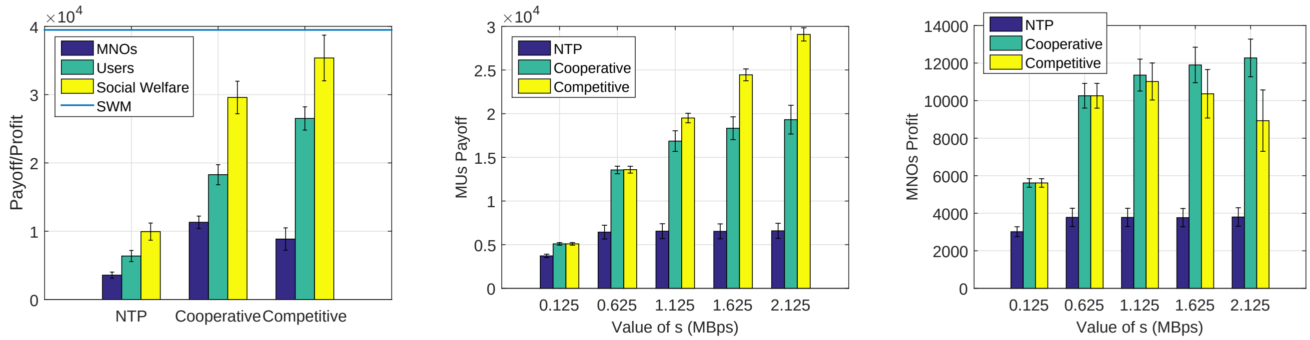
Theorem 2. *When $p^\pi_{g-1}\le e_g$, the potential equilibria are characterized as:*

- *(perfect monopoly) if $\tilde{p}_i^\star<e_g$, then*

$$\boldsymbol{p}^{n\star}\begin{cases} =\tilde{\boldsymbol{p}}^\star & \text{if} \quad n=\sigma(1) \\ >\tilde{\boldsymbol{p}}^\star & \text{otherwise}\end{cases}; \tag{26}$$

[10]This result is partially due to the choice of the α-fair utility function. We will study the impact of the general utility functions in an extended version of the paper.

[11]In fact, sometimes the equilibrium may not exist due to a reason similar as that of the well-known Edgeworth paradox [31], where two producers with limited capacities engage in price competitions in the same market. Our model, however, is much more complicated than the standard model leading to Edgeworth paradox.



(a) Experiment 1 (b) Experiment 2: MNOs' Profit (c) Experiment 2: MUs' Payoff

Fig. 2. Comparison of the NTP scheme, the cooperative scheme, the competitive scheme, and SWM.

- *(partial monopoly) if $\tilde{p}_i^\star \geq e_g$, then*

$$p_i^{n\star} \begin{cases} = e_g - \epsilon, & \text{if } n = \sigma(1) \\ \geq e_g, & \text{if } n \in \mathcal{N} \setminus \{\sigma(1)\} \end{cases}, \quad \forall i \in \mathcal{I}, \qquad (27)$$

with, for each $i \in \mathcal{I}$, at least one of $n \in \mathcal{N} \setminus \{\sigma(1)\}$ satisfying $p_i^{n\star} = e_g$.

Intuitively, if $\tilde{p}_i^\star < e_g$ for any $i \in \mathcal{I}$, then MNO $\sigma(1)$'s monopoly price $\tilde{\boldsymbol{p}}^\star$ is the equilibrium strategy, since no other MNOs will set a lower price than e_g. However, if $\tilde{p}_i^\star \geq e_g$ for some MU i, then MNOs $\sigma(1)$ and $\sigma(g)$ will compete down their prices until $p_i^{\sigma(1)\star} = e_g - \epsilon$ for each i.

2) Case II (Competitive Equilibria): $p_{g-1}^\pi > e_g$

We can characterize the potential equilibria in the following theorem.

Theorem 3. *When $p_{g-1}^\pi > e_g$, the potential equilibria are characterized by*

$$p_i^{n\star} \begin{cases} = p_{s^\star}^\pi, & \text{if } \; n \in \{\sigma(i) | i \in [1, 2, ..., s^\star]\} \\ \in (p_{s^\star}^\pi, +\infty), & \text{otherwise} \end{cases}, \quad \forall i \in \mathcal{I};$$

with $s^\star$ being the solution to the following system:

$$\begin{cases} e_{s^\star} \leq p_{s^\star}^\pi \leq e_{s^\star+1}, & \text{if } s < |\mathcal{I}| \\ e_{s^\star} \leq p_{s^\star}^\pi, & \text{if } s = |\mathcal{I}| \end{cases}. \qquad (28)$$

Combining the preceding discussions for two cases ($p_{g-1}^\pi \leq e_g$ and $p_{g-1}^\pi > e_g$), we conclude that in all potential equilibria, all traffic-supporting MNOs choose the same data prices and zero tethering prices for all MUs. For the remaining MNOs, they can set the hybrid prices for MUs arbitrarily higher than those imposed by traffic-supporting MNOs.

Here we only characterize the potential equilibria due to the space limit. In an extended version of the paper, we will analytically derive the conditions for the uniqueness of the SPE. Nevertheless, we can still check whether the potential equilibria are the exact SPE numerically by (22) and (23).

VII. Numerical Results

We perform numerical study in this section. First, we introduce two benchmark schemes for comparison: the *no-tethering pricing* (NTP) scheme (described in Section III) and the *social welfare maximizing* pricing (SWM) scheme (maximizing the sum of MUs' payoff and MNOs' profit). Then, we provide the numerical results to evaluate the performance gain of the cooperative and the competitive pricing schemes.

In our numerical studies, we consider an MCA model involving $|\mathcal{N}| = 2$ MNOs and $|\mathcal{I}| = 10$ MUs, and study their interactions for a time period of $T = 10$ minutes. We randomly assign 5 MUs as the subscribers of MNO 1 and the remaining 5 MUs as the subscribers of MNO 2. The model parameters are configured as follows: (i) $\alpha = 0.4$ and θ_i is selected from $[300, 600]$ randomly and uniformly for all MUs; (ii) MU 1 does not have any Internet access, MUs 2-4 have LTE connections, and MUs 5-10 have 3G connections; (iii) the operational cost e_i is randomly and uniformly selected from $[64, 96]$ Joule/MB for LTE connection and from $[400, 800]$ Joule/MB for 3G connection [32]; and (iv) the energy cost is $c = 7.5$ Joule/MB, which consists of an average downloading cost of 4.65 Joule/MB and a Wi-Fi connection cost for 2.85 Joule/MB [33]. In what follows, we run two experiments, each 10,000 times, to illustrate the performance gain of our proposed pricing schemes as well as the impact of network capacity on the achieved performance gain.

In the first experiment, we set the capacity C_i to be uniformly distributed in $[0.94, 2.5]$ MBps for LTE downlink and in $[0.06, 0.19]$ MBps for 3G downlink, according to the results from the real-world measurements [34], [35]. In Fig. 2 (a), we compare the NTP scheme, the cooperative scheme, and the competitive scheme, in terms of MUs' payoff, MNOs' profit, and social welfare. We observe that the tethering scheme with our proposed cooperative and competitive schemes can improve both the MUs' payoffs and the MNOs' profits, comparing with the NTP scheme. The cooperative pricing scheme can increase the MNOs' profit up to 216% and the MUs' payoff up to 186%, and the competitive pricing scheme can increase the MNOs' profit up to 147% and the MUs' payoff up to 315%.

In the second experiment, we study how the network performance depends on the capacities of low energy cost (LTE) downlinks. We set the LTE downlinks (with low operational cost) of MUs 2-4 to be s MBps and fix the 3G downlinks (with high operational cost) of MU 5-10 to be 1 MBps. The results are shown in Fig. 2 (b) and (c). We observe that as the capacities of the low-cost LTE downlinks increase, the advantages of the cooperative scheme over NTP scheme becomes more significant. As we have discussed before, in the cooperative scheme, MUs will download the data from the downlinks of low operational costs first. Hence the capacity increase of these downlinks will reduce the MNOs' cost and thus the prices to MUs. In addition, as the capacities of the low-cost LTE downlinks increase, MNOs' profit achieved by the competitive scheme first increases and then decreases. This is because the capacity increase of low-cost downlinks first reduces the MNOs' cost. However, the further capacity increase in these downlinks increases the competition among the MNOs, which leads to prices decrease, the MNOs' profits decrease, and the MUs' payoff increase.

VIII. Conclusions

In this paper, we proposed a novel hybrid pricing framework for MCA, and studied the optimal data and tethering pricing schemes for cooperative and competitive MNOs. Our analysis shows that the equilibrium data and tethering prices have the following structure with the α-fair MU utility functions: the tethering price is zero and the data price is identical for all MUs, for both cooperative MNOs and competitive MNOs. This provides the insight that as long as MNOs set the data prices properly, there is no need for charging additional fees for tethering. This result is encouraging, as it allows the MNOs to increase their profits (compared to the no tethering case) even if they are prohibited from charging for tethering [9]. Furthermore, such pricing schemes can also increase the MUs' payoff and the social welfare.

For the future work, we will study how the model and results can be further generalized. Specifically, we will study the equilibrium pricing schemes under more general assumptions of the MUs' utility functions, energy cost, and the MNOs' operational cost. In addition, we are interested in studying the impact of other pricing schemes, e.g., the shared data scheme, instead of the usage-based pricing scheme considered here.

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Appendix

A. Proof of Lemma 1

When $h_{i\leftarrow j} < h_{i\leftarrow k}$, by (7), we have:

$$\lambda_j^* - \beta_{i\leftarrow j}^* > \lambda_k^* - \beta_{i\leftarrow k}^*. \tag{29}$$

When $x_{i\leftarrow k}^* > 0$, combining (9)-(11) and (29), we have:

$$\lambda_j^* > \lambda_k^* + \beta_{i\leftarrow j}^* \geq 0, \tag{30}$$

which indicates that $\sum_{i\in\mathcal{I}} x_{i\leftarrow j}^* - C_j = 0$ according to (8).

B. Proof of Theorem 1

Suppose that, given $\boldsymbol{p}^*$, there exists an optimal solution $\{\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\lambda}^*\}$ (that satisfies the KKT conditions) with $\lambda_j^* > 0$ for some downlink $j \in \mathcal{I}$. In this case we can see that if MNOs increase $h_{i\leftarrow j}^*$ to $h'_{i\leftarrow j} \triangleq h_{i\leftarrow j}^* + \lambda_j^*$ for all $i \in \mathcal{I}$, the solution $\{\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\lambda}' \triangleq \boldsymbol{0}\}$ will still satisfy the KKT conditions under the updated price, implying that it is an optimal solution under the updated price. Hence, the MNOs' profit increases from $\sum_{j\in\mathcal{I}}\sum_{i\in\mathcal{I}}(h_{i\leftarrow j}^* - e_j)x_{i\leftarrow j}^*$ to $\sum_{j\in\mathcal{I}}\sum_{i\in\mathcal{I}}(h'_{i\leftarrow j} - e_j)x_{i\leftarrow j}^*$, which implies that $\boldsymbol{h}^*$ is not the equilibrium (best) price matrix for the MNOs. Hence, we can see that under the equilibrium price matrix $\boldsymbol{h}^*$, the optimal solution $\{\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\lambda}^*\}$ must satisfy $\lambda_i^* = 0,\ \forall i \in \mathcal{I}$. Substitute $\boldsymbol{\lambda}^* = \mathbf{0}$ into (7), we have that

$$\begin{cases}\theta_i \left(\sum_{l\in\mathcal{I}} x^*_{i\leftarrow l}\right)^{-\alpha} = h^*_{i\leftarrow j} + c, & \text{if} \quad x^*_{i\leftarrow j} > 0 \\ \theta_i \left(\sum_{l\in\mathcal{I}} x^*_{i\leftarrow l}\right)^{-\alpha} \leq h^*_{i\leftarrow j} + c, & \text{if} \quad x^*_{i\leftarrow j} = 0\end{cases}.$$

C. Proof of Lemma 2

Suppose when $p^{\pi}_{g-1} \leq e_g$, there exists an equilibrium with $p_i^{\sigma(1)\star} = p_j^{\sigma(1)\star} > e_g,\ \forall i, j \in \mathcal{I}$, then the total demand of all MUs satisfies

$$\sum_{i\in\mathcal{I}} q_i \overset{(b)}{=} \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{p_i^{\sigma(1)\star} + c}\right)^{1/\alpha} < \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{e_g + c}\right)^{1/\alpha} \leq \sum_{i=1}^{g-1} C_i, \tag{31}$$

where (b) is due to (24). We observe that the first $g-1$ downlinks are not fully utilized. In this case, MNO $\sigma(g)$ can always reduce its price $p_i^{\sigma(g)\star} \leftarrow p_i^{\sigma(1)\star} - \epsilon$ to increase its profit, where $\epsilon > 0$ is an arbitrarily small value, which contradicts with the fact that $\boldsymbol{p}^{\sigma(1)\star}$ is the equilibrium price.

If there exists i such that $p_i^{\sigma(1)\star} > e_g$ and $p_i^{\sigma(1)\star} > p_j^{\sigma(1)\star}$ for some j, then:

- if $\sum_{i\in\mathcal{I}} x^{\star}_{i\leftarrow g} < C_g$, then MNO $\sigma(g)$ is always willing to set the price $p_i^{\sigma(g)\star} \leftarrow p_i^{\sigma(1)\star} - \epsilon$ which can attract more traffic demand according to Lemma 1, where $\epsilon > 0$ is arbitrarily small;
- if $\sum_{i\in\mathcal{I}} x^{\star}_{i\leftarrow g} = C_g$ and $\sum_{j=1}^{g-1}\sum_{i\in\mathcal{I}} x^{\star}_{i\leftarrow j} \leq \sum_{j=1}^{g-1} C_j$, then MNO $\sigma(1)$ can always set the price $p_i^{\sigma(g)} \leftarrow p_i^{\sigma(g)} - \epsilon$ to attract more traffic demand, which will increase her profit.

Thus, when $p^{\pi}_{g-1} \leq e_g$, any equilibrium must satisfy $p_i^{\sigma(1)\star} \leq e_g, \forall i$.

On the other hand, suppose when $p^{\pi}_{g-1} > e_g$, there exists an equilibrium with $p_i^{\sigma(1)\star} \leq e_g,\ \forall i, j \in \mathcal{I}$, then the total demand of all MUs satisfies

$$\sum_{i\in\mathcal{I}} q_i = \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{p_i^{\sigma(1)\star} + c}\right)^{1/\alpha} \geq \sum_{i\in\mathcal{I}} \left(\frac{\theta_i}{e_g + c}\right)^{1/\alpha} > \sum_{i=1}^{g-1} C_i. \tag{32}$$

However, the MUs' total demand exceeds MNO $\sigma(1)$'s first $g-1$ downlinks. Thus, in this case, MNO $\sigma(1)$ having a profit of $p_i^{\sigma(1)\star}\sum_{i=1}^{g-1} C_i$ always has the incentive to increase $p_i^{\sigma(1)}, \forall i$, which is a contradiction to the fact that $\boldsymbol{p}^{\sigma(1)\star}$ is the equilibrium price. Thus, when $p^{\pi}_{g-1} > e_g$, any equilibrium must satisfy $p_i^{\sigma(1)\star} > e_g$ for each MU i.

D. Proof of Theorem 2

As it is stated in Lemma 2, when $p^{\pi}_{g-1} \leq e_g$, any equilibrium $\{\boldsymbol{p}^{n\star}\}$ satisfies $p_i^{\sigma(1)\star} \leq e_g$ for all $i \in \mathcal{I}$ and MNO $\sigma(1)$ is the only traffic-supporting MNO in this case.

Hence, if the monopoly price $\tilde{p}_i < e_g$ for each MU i, then MNO $\sigma(1)$'s best strategy is to set $p_i^{\sigma(1)} = \tilde{p}_i$, since the remaining MNOs do not have any incentive to set lower prices due to their high operational cost.

When $\tilde{p}_i \geq e_g,\ \forall i$, if MNO $\sigma(1)$ sets the price $p_i^{\sigma(1)} = \tilde{p}_i$ for each MU i, then MNO $\sigma(g)$ will set a lower price to attract traffic, leading MNO $\sigma(1)$ to set an even lower price. If MNO $\sigma(1)$ sets the price $p_i^{\sigma(1)}$ below e_g for some i, then it has an incentive to increase $p_i^{\sigma(1)}$ since there is no competition from the remaining MNOs and the monopoly price $\tilde{p}_i$ is higher than e_g. Hence, at any equilibrium, the only possible equilibrium price for MNO $\sigma(1)$ is $p_i^{\sigma(1)\star} = e_g - \epsilon$ for all MU i. Moreover, although the remaining MNOs do not receive any profit in this monopolistic case, some of them must set $p_i^n = e_g$ at any equilibrium, for each MU i, in order to preclude the case where MNO $\sigma(1)$ still has the incentive to increase its price.

E. Proof of Theorem 3

To study the potential equilibria in this case, we first define $\mathcal{N}^e(\{p^{n\star}\}) \triangleq \{\sigma(i) | i \in [1, 2, ..., s^\star]\}$, where $s^\star$ is defined in (28).

Similar to the proof for Theorem 1 in Appendix B, we can show that the competitive MNOs set equilibrium price so that the optimal solution $(\boldsymbol{x}^\star, \boldsymbol{\lambda}^\star)$ satisfying KKT conditions must also satisfy $\boldsymbol{\lambda}^\star = \mathbf{0}$. Thus, it follows that $p_i^{n\star} = p_i^{m\star},\ \forall i \in \mathcal{I}, m, n \in \mathcal{N}^e(\{p^{n\star}\})$. Suppose that at some equilibrium, $p_i^{m\star} = p_i^{n\star} < p_j^{n\star} = p_j^{m\star},\ \forall i, j \in \mathcal{I},\ \forall m, n \in \mathcal{N}^e(\{p^{n\star}\})$, without loss of generality. By setting $p_j^{n\star} \leftarrow p_j^{n\star} - \epsilon$, MU j will prioritize the utilization of MNO n's downlinks according to Lemma 1, which can increase MNO n's profit and contradicts with the fact that $\boldsymbol{p}^\star$ is the equilibrium price. Thus, we can see that, if exists, an equilibrium satisfies, $\forall i, j \in \mathcal{I}, \forall m, n \in \mathcal{N}^e(\{p^{n\star}\})$,

$$p_i^{m\star} = p_i^{n\star} = p_j^{n\star} = p_j^{m\star},$$

while any remaining MNO $n \notin \mathcal{N}^e(\{p^{n\star}\})$ set prices arbitrarily higher.

Next, we will show that for any equilibrium $\{p^{n\star}\}$ with $p^{\pi}_{g-1} > e_g$, it satisfies $\sum_i q_i(\{\boldsymbol{p}^{n\star}\}) = \sum_{i=1}^{s} C_i$, for some s. Suppose that the equilibrium price satisfies $\sum_{i=1}^{s-1} C_i < \sum_i q_i(\boldsymbol{p}^\star) < \sum_{i=1}^{s} C_i$, for some s. As we have discussed, $p_i^{n\star} = p_i^{m\star}, \forall i \in \mathcal{I},\ \ \forall n, m \in \mathcal{N}^e(\{\boldsymbol{p}^{n\star}\})$. Note that MNO $\sigma(s)$'s downlink s is not fully utilized. Suppose MU i who does not receive the data all from MNO $\sigma(s)$ without loss of generality. In this case, MNO $\sigma(s)$ can set $p_i^{\sigma(s)} \leftarrow p_i^{n\star} - \epsilon, \forall n \in \mathcal{N}^e(\{\boldsymbol{p}^{n\star}\})$. According to Lemma 1, MU i will first utilize downlink s, and thus MNO $\sigma(s)$ can use the initially unutilized downlinks to support more traffic and increase its profit, which contradicts with the fact that $\{p^{n\star}\}$ is at the equilibrium. Hence, for any equilibrium $\{p^{n\star}\}$ with $p^{\pi}_{g-1} > e_g$, it satisfies $\sum_i q_i(\{\boldsymbol{p}^{n\star}\}) = \sum_{i=1}^{s} C_i$, i.e.,

$$p_i^{m\star} = p_i^{n\star} = p_j^{n\star} = p_j^{m\star} = p^{\pi}_s,\ \forall i, j \in \mathcal{I}, \forall m, n \in \mathcal{N}^e(\{\boldsymbol{p}^{n\star}\}).$$

Finally, we also observe that i) $p_i^{m\star} = p_i^{n\star} = p_j^{n\star} = p_j^{m\star} \geq e_s^\star$ and ii) $p_i^{m\star} = p_i^{n\star} = p_j^{n\star} = p_j^{m\star} \leq e_{s^\star+1}$, if $s^\star < |I|$. If i) is not satisfied, MNO $\sigma(s^\star)$ having a negative profit would set higher prices. If ii) is not satisfied, MNO $\sigma(s^\star+1)$ has incentive to set a lower price to participate in the price competition.