Profit Maximization of Cognitive Virtual Network Operator in A Dynamic Wireless Network

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***Abstract*—In this paper, we study a cognitive virtual network operator's profit maximization problem in a dynamic network environment. We consider various network dynamics, including dynamic user demands, unstable sensing spectrum resources, dynamic spectrum prices, and time-varying channel conditions. We develop a low-complexity on-line control policy that determines pricing and resource scheduling without knowing the distribution of dynamic network parameters. We show that the proposed algorithm can achieve arbitrarily close to the optimal profit with a proper trade-off of the queuing delay.**

I. Introduction

Dynamic spectrum sharing has the potential to solve the spectrum shortage problem by enabling unlicensed radio devices to intelligently and dynamically discover and access the "spectrum holes" (*i.e.,* underutilized licensed spectrum) without disturbing the licensed users. Such opportunistic sharing can happen at both user and operator levels. Recently, Duan *et al.* in [1] introduced a sharing architecture of Cognitive Virtual Network Operator (CVNO) at the operator level. A CVNO obtains spectrum from a spectrum owner (through spectrum sensing and leasing) to serve its own customers. The concept of CVNO is a natural generalization of MVNO (Mobile Virtual Network Operator), which has been enjoying world-wide success during the last decade [2]. Departing from the static and stylized network model in [1], in this paper *we analyze the performance of the CVNO in a realistic dynamic network environment*.

There are two approaches for a CVNO to obtain spectrum resource: spectrum sensing and spectrum leasing [1]. Spectrum sensing is based on the cognitive radio technology, and allows the CVNO to identify and utilize spectrum holes without paying fees to the spectrum owner.[1] Spectrum sensing will not consume much time or energy if the sensing technology is efficient. The availability of spectrum holes, however, is often stochastic, and thus it is hard to predict the amount of usable spectrum until sensing finishes. Spectrum leasing, on the other hand, guarantees the availability of spectrum for a predefined time period by allowing the CVNO to pay a (often high) fee to the spectrum owner. To maximize the investment flexibility, spectrum leasing may happen at a short time scale comparable to spectrum sensing. This requires an efficient near real-time spectrum market, which is an active research area itself.

Compared with the static model in [1], this paper studies a more realistic dynamic network environment as follows.

- *Exogenous network dynamics:* We model sensing channel availability, leasing market price, and channel conditions as exogenous stochastic processes. These characterize the time-varying nature of wireless networks.
- *Dynamic user demands:* We allow users to dynamically join the network with random demands (file sizes). The demand is affected by both the transmission price (decision variable) and market state (exogenous stochastics).
- *Realistic cognitive radio model:* We incorporate various practical issues such as imperfect spectrum sensing, primary users' collision tolerance, and sensing technology selection. The CVNO needs to choose a sensing technology to optimize the tradeoff between cost and performance.

The literature on evaluating cognitive radio technology from *an operator's point of view* only started to emerge recently, *e.g.,* [4]–[10]. Similar to [1], [4]–[6] studied the monopoly pricing based on a Stackelbeg game model. References [7]–[10] studied the competition among multiple operators: either among two operators ([7], [8]) or many operators ([9], [10]). These previous results all adopted a static network model.

The main contributions of this paper include:

- *A dynamic network decision model:* Our model incorporates various key dynamic aspects of a cognitive radio network and the dynamic decision process of a CVNO.
- *A low-complexity on-line control policy:* We design a low-complexity on-line pricing and resource allocation policy, which can achieve arbitrarily close to the CVNO's optimal profit. The policy does not require precise information of the dynamic network parameters, has a low system overhead, and is easy to implement.
- *Performance guarantee for users:* Though the algorithm is designed for the operator, it can also provide performance guarantee (delay bound) to end users.

II. System Model

We consider the profit maximization problem of a CVNO, which transmits data to its users (which are *unlicensed secondary users*) from a base station using the OFDM technology,[2] as shown in Fig. 1. Time is divided into equal length slots, indexed by $t = 1, 2, \ldots$. The system contains a set $\mathcal{I}$ of multiple orthogonal channels licensed to a spectrum owner, which serves its *primary licensed users*. The channels are

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[1]This is what FCC has decided on how the TV white space should be used by unlicensed devices in the US [3].

[2]The discussions of multi-cell transmissions with inter-cell interferences will be part of our future work. Performance optimization for multi-cell networks is challenging even for today's cellular networks.

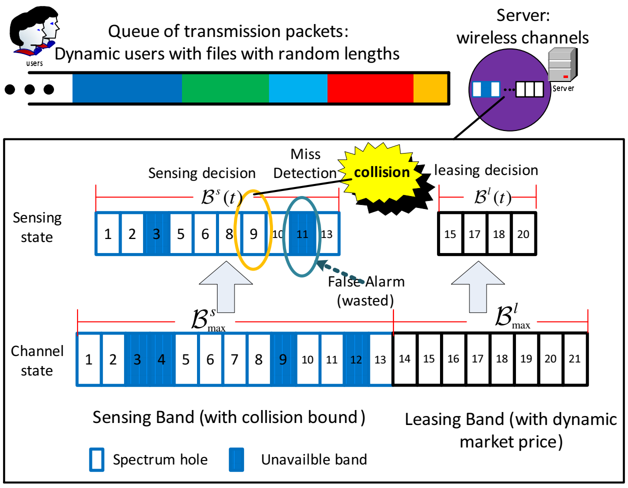

Fig. 1. Business model of the CVNO (Cognitive Virtual Network Operator).

divided into two sets: the *sensing band* $\mathcal{B}^s_{\max} = \{1, \ldots, B^s_{\max}\}$ and the *leasing band* $\mathcal{B}^l_{\max} = \{1, \ldots, B^l_{\max}\}$.

A. Imperfect Spectrum Sensing

The sensing band $\mathcal{B}^s_{\max}$ includes channels that the spectrum owner allows to be sensed by the CVNO.[3] We will define **the state of a channel** $i \in \mathcal{B}^s_{\max}$ **in time slot** t as $S_i(t)$, which equals 1 if channel i is busy (used by a primary user), and equals 0 if channel i is idle.

For each channel $i \in \mathcal{B}^s_{\max}$, we assume that the channel state is an i.i.d. Bernoulli random variable, with an idle probability $p_0 \in (0,1)$ and a busy probability $1 - p_0$. This approximates the reality well if the time slots for secondary transmissions are sufficiently long or the primary transmissions are highly bursty [13]. We will define the **sensing state of a channel** $i \in \mathcal{B}^s_{\max}$ **in time slot** t as $W_i(t)$, which equals to 1 if channel i is *sensed* busy, and 0 if sensed idle.

As there are no direct communications between the CVNO and spectrum owner in the sensing band, imperfect spectrum sensing can lead to false alarms and missed detections. The accuracy of spectrum sensing depends on the particular choice of technology [14]. If we denote C^s as the sensing cost (per channel)[4] which depends on the sensing technology, then we can write the false alarm probability as $\alpha(C^s) \triangleq Pr\{W_i = 1 | S_i = 0\}$ (same for each channel i) and missed detection probability as $\beta(C^s) \triangleq Pr\{W_i = 0 | S_i = 1\}$ (same for each channel i). Both functions are decreasing in C^s. Intuitively, a better technology leads to a higher cost C^s, a lower false alarm probability $\alpha(C^s)$, and a lower missed detection probability $\beta(C^s)$. We denote all possible choices of cost C^s by a finite set $\mathcal{C}^s$.

As different channels have different channel conditions (to be explained in details in Sec. II-C), the CVNO needs to decide which channels to sense at the beginning of each time slot. We use $\mathcal{B}^s(t)$ to denote **the set of channels sensed by the CVNO at time** t, which satisfies

$$\mathcal{B}^s(t) \subseteq \mathcal{B}^s_{\max}, \forall\, t. \tag{1}$$

B. Spectrum Leasing with Dynamic Market Price

A spectrum owner may have some channels that do not want to be sensed, for either privacy reasons or the fear of collisions due to sensing errors. However, these channels may not be always fully utilized. The spectrum owner can lease the unused part of these channels to the CVNO dynamically over time to earn more revenue.

Recall that we denote the set of these channels as the leasing band $\mathcal{B}^l_{max}$. We use $\mathcal{B}^l_i(t)$ to denote **the set of channels leased by the CVNO at time** t, which satisfies

$$\mathcal{B}^l(t) \subseteq \mathcal{B}^l_{max}, \forall\, t, \tag{2}$$

at the market price $C^l(t)$ per channel. These channels will be exclusively used by the CVNO in the current time slot. We assume that the market price $C^l(t)$ stochastically changes according to the supply and demand relationship in the spectrum market (which might involve many spectrum owners and CVNOs). It can be modeled by an exogenous (not affected by this particular CVNO's decisions) random process with countable discrete states and the stationary distribution (not necessarily known by the CVNO).

C. Power Allocation

Proper power allocation is important to combat channel fading and achieve satisfactory data rates. For each channel $i \in \mathcal{I}$, $h_i(t)$ represents its channel gain in time slot t and follows an i.i.d. distribution over time.[5] Different channels have independent and possibly different channel gain distributions. We assume that secondary users are homogeneous and experience the same channel conditions.[6] The CVNO can measure $h_i(t)$ for each i at the beginning of each slot t, but may not know the distributions. Let $P_i(t)$ denote the **power allocated to channel** i **at time** t. Furthermore, for a channel $i \in \mathcal{B}^s_{\max}$ in the sensing band, we use the binary variable $I_i(t)$ to denote the **transmission result**, *i.e.*, $I_i(t) = 1$ if successful (*i.e.*, $S_i(t) = 0$ and $W_i(t) = 0$) and $I_i(t) = 0$ otherwise (either not sensed, or sensed busy, or sensed idle but actually busy). Then the **total transmission rate** obtained by the CVNO in time slot t is (based on the Shannon formula)

$$r(t) \triangleq \sum_{i \in \mathcal{B}^s(t)} I_i(t) \log(1 + h_i(t) P_i(t)) + \sum_{i \in \mathcal{B}^l(t)} \log(1 + h_i(t) P_i(t)). \tag{3}$$

The CVNO needs to satisfy the **total power constraint** $P_{\max}$ at its base station,

$$\sum_{i \in \mathcal{I}} P_i(t) \le P_{\max}, \forall t. \tag{4}$$

Further, we assume that the CVNO has a finite maximum transmission rate, *i.e.*, $r(t) \le r^{\max}, \forall t$.

[3]The CVNO will collect the sensing information from a sensor network and provide it to its users, *i.e.*, providing "sensing as service" [11]. This means that users do not need have cognitive radio capable mobile devices, and thus the network can accommodate legacy mobile devices. For detailed discussions on how this can be done in practice, see [12].

[4]For example, power or time used for sensing.

[5]This assumption can be generalized to the case of ergodic Markov process without changing the main results.

[6]This is the case where the users are located close by, and thus the downlink channel condition from the base station to the users is user independent.

D. Collision Constraint

Missed detections in spectrum sensing will lead to transmission collisions with the primary users. We represent the **collision in channel** $i \in \mathcal{B}^s_{\max}(t)$ as a binary random variable $X_i(t) \in \{0, 1\}$, *i.e.,* $X_i(t) = 1$ if $S_i(t) = 1$ and $W_i(t) = 0$.

To protect primary users' transmissions, the CVNO needs to ensure that the collisions in each channel i do not exceed a tolerable level η_i (measured in terms of time-average maximum number of collisions) specified by the spectrum owner. We define the time-average number of collision in channel i as $\overline{X_i} \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\left[X_i(\tau)\right]$. The collision constraints are

$$\overline{X_i} \leq \eta_i, \forall i \in \mathcal{B}^s_{\max}. \tag{5}$$

E. Demand Model

We will focus on elastic data traffic in this paper. Secondary users randomly arrive at the network to request files with random and finite file sizes (measured in the number of packets) from the CVNO. A user will leave the network once it has completely downloaded the file. The CVNO can price the packet transmission dynamically over time, which will affect the users' arrival rate. For example, a higher price at peak time can refrain users from downloading files, as they can wait until a later time with a lower price. To model this, we use $M(t)$ to denote the random market state, which can be measured precisely at the beginning of each time slot t and can help estimate the users demand[7]. The random variable is drawn from a finite set $\mathcal{M}$ over time in an i.i.d. fashion. The distribution of $M(t)$ may not be known by the CVNO.

At a time t, the CVNO can decide whether to accept any additional demand from secondary users. We define the **binary transmission control variable as** $O(t)$, where $O(t) = 1$ means that the CVNO allows new packet transmissions, and $O(t) = 0$ otherwise. When the CVNO decides to accept new requests of packet transmissions, it will announce **a price** $q(t)$ **for transmitting one packet (to any user)**. Users will respond to this price by adjusting their requests. More precisely, **the number of incoming users at time** t **is a discrete random variable** $N(t) \triangleq N(M(t), q(t)) \in \mathbb{N} \triangleq \{0, 1, 2 \ldots\}$, which is a function of the transmission price $q(t)$ and market state $M(t)$. Moreover, a user n's requested file size is $L_n(t)$, with $n \in \{1, 2, \ldots, N(q(t))\}$. All users's file sizes follow the same discrete distribution, which have K possible values $\{l_k, k = 1, \ldots, K\}$ with the corresponding probabilities $\{\theta_k, k = 1, \ldots, K\}$ and $\sum_{k=1}^K \theta_k = 1$. The file sizes of different users are independent of each other and do not depend on $q(t)$ or $M(t)$.

To summarize, **users' instantaneous demand at time** t is

$$A(t) \triangleq \sum_{n=1}^{N(M(t),q(t))} L_n(t), \tag{6}$$

which is a random variable due to random file sizes and the random number of incoming users (even given $q(t)$ and $M(t)$). We define the **users' (expected) demand function** as $D(t) \triangleq D(M(t), q(t)) \triangleq \mathbb{E}\left[A(M(t), q(t))\right]$, and its value is completely determined by $M(t)$ and $q(t)$. It is then reasonable to assume the CVNO can rather accurately characterize the demand function $D(M(t), q(t))$ through long-term observations. We further assume that the instantaneous demand is upper-bounded as $A(t) \leq A_{\max}$ for all t, and that the demand function $D(t)$ is non-negative and non-increasing function of the price $q(t)$. When the price is higher than some upper-bound, *i.e.,* $q(t) \geq q_{\max}$, the demand function $D(t)$ will be zero.

[7] For example, $M(t)$ can be users' willingness to pays, or whether the system is in peak time or off-peak time.

F. Queuing dynamics

Since we focus on the profit maximization problem in this paper, we will take a simple view of the network and model users' dynamic arrivals and departures as a one server queue. When a user accesses the network, the corresponding file will be queued at the base station, waiting to be transmitted to the user based on the First Come First Serve (FCFS) discipline. Shama and Lin in [15] showed that the one server queue model is a good approximation for an OFDM system, especially when the number of users and channels are large.

We denote the **queue length** (*i.e.,* the backlog, or the number of all packets from all queued files) at time t as $Q_i(t)$. Thus the queuing dynamic can be written as

$$Q(t+1) = (Q(t) - r(t))^+ + O(t)A(t), \tag{7}$$

where the notation $(a)^+ \triangleq \max(a, 0)$. Throughout the paper, we adopt the following notion of *queue stability*:

$$\overline{Q} \triangleq \limsup_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}\left[Q(\tau)\right] < \infty. \tag{8}$$

In addition, to satisfy primary users' collision constraint (12), we introduce a **virtual control queue** $Z_i(t)$, $i \in \mathcal{B}^s_{\max}$ with $Z_i(0) = 0$ and the dynamics as:

$$Z_i(t+1) = (Z_i(t) - \eta_i)^+ + X_i(t). \tag{9}$$

III. Problem Formulation

For notation convenience, we introduce several condensed notations and use them together with the original notations.

We define $\phi(t) \triangleq (M(t), \boldsymbol{h}(t), C^l(t))$ as **all observable parameters**, including the market state $M(t)$, channel conditions (vector) $\boldsymbol{h}(t)$, and the leasing price $C^l(t)$ in the spectrum market. Based on previous assumptions, $\phi(t)$'s are i.i.d over time and take values from a finite set Φ. More precisely, $\beta(\phi)$ denotes the stationary probability density function, *i.e.,*

$$\beta(\phi) \triangleq Pr(\phi(t) = \phi), \quad \forall t, \quad \forall \phi \in \Phi.$$

We define $\gamma(t) \triangleq (O(t), q(t), C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t), \boldsymbol{P}(t))$ as **all decision variables**, including the market control variable $O(t)$, the transmission price for users $q(t)$, the sensing cost (with corresponding sensing technologies) $C^s(t)$, the sensing channels $\mathcal{B}^s(t)$, leasing channels $\mathcal{B}^l(t)$, and power allocations (vector) $\boldsymbol{P}(t)$ of the CVNO. We assume that $\gamma(t)$ takes values form a countable (finite or infinite) set $\Gamma_\phi(t)$, which is a Cartesian product of the feasible regions of all variables, *i.e.,* non-negative values satisfying constraints (1), (2), and (4). With the condensed notations, functions in this paper can be simply represented as functions of $\gamma(t)$ with parameter $\phi(t)$.

We further define the **instantaneous profit in time** t

$$\begin{aligned}R(t) &\triangleq R(\gamma(t);\phi(t))\\ &\triangleq q(t)O(t)A(t) - C^s(t)|\mathcal{B}^s(t)| - C^l(t)|\mathcal{B}^l(t)|. \quad (10)\end{aligned}$$

The **time average profit** is denoted as

$$\overline{R} \triangleq \limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}\left[R(t)\right].$$

All expectations in this paper are taken with respect to system parameters $\phi(t)$ unless stated otherwise.

We look at the profit maximization problem with pricing and resource allocation. At the beginning of each time slot t, the CVNO observes the value of $\phi(t)$ and makes a decision $\gamma(t)$, in order to maximize the time average profit, subject to the system stability constraint (11) and the collision upper-bound requirement (12). The Profit Maximization (PM) problem is

$$\begin{aligned} PM: \quad &\text{Maximize} \quad \overline{R}\\ &\text{Subject to} \quad \overline{Q} < \infty, \quad (11)\\ &\qquad \overline{X_i} \leq \eta_i, i \in \mathcal{B}^s_{max}, \quad (12)\\ &\text{Variables} \quad \gamma(t) \geq 0, \forall t,\\ &\text{Parameters} \quad \phi(t), \forall t. \end{aligned}$$

It is easy to see that this PM problem has a nonempty feasible set, since 0 is a feasible solution. We denote $\overline{R^*}$ as the **optimal time average profit**.

Now we consider a special class of stationary randomized policies, called *ϕ-only policy*, which makes the decision $\gamma(t)$ in slot t only depending on the observation of system parameters $\phi(t)$. In this policy, the CVNO chooses $\gamma(t)$ from the finite set of $\Gamma_\phi(t) = \{\gamma^1_\phi, \gamma^2_\phi, \dots\}$ with probabilities $\{\rho^1_\phi, \rho^2_\phi \dots\}$, where $\sum_{u=1}^{\infty} \rho^u_\phi = 1$. Notice that the decision is independent of time t, and thus is stationary. We can prove that the optimal profit R^* of the PM problem can be reached by some ϕ-only policy as the following theorem.

Theorem 1: There exists a stationary ϕ-only policy $\gamma^*(t) = \big(O^*(t), q^*(t), C^{s*}(t), B^s(t), B^l(t), \boldsymbol{P}(t)\big)$ that stabilizes the network, *i.e.,*

$$\mathbb{E}\left[A(\gamma^*(t);\phi(t))\right] \leq \mathbb{E}\left[r\left(\gamma^*(t);\phi(t)\right)\right] \quad (13)$$

and has the following properties:

$$\overline{R} = \mathbb{E}\left[R(\gamma^*(t);\phi(t))\right] = \overline{R^*} \quad (14)$$

$$\overline{Y_i} = \mathbb{E}\left[Y_i(\gamma^*(t);\phi(t))\right] \leq \eta_i, \ \forall i. \quad (15)$$

Theorem 1 is a special case of Theorem 4.5 in [16]. The proof is omitted here due to space limitations.

Though we can restrict ourselves to ϕ-only policies, computing the optimal ϕ-only policy (*i.e.,* determining the optimal values of $\rho^1_\phi, \rho^2_\phi, \dots$) is very difficult in practice, since it requires precise information of the distribution of the parameter $\phi(t)$. However, many elements in $\phi(t)$ are hard to predict or estimate, *e.g.,* the leasing cost in the spectrum market and the activities of the primary users.

In next section, we will design a low complexity **Profit Maximization Control (PMC) Policy**, which requires no statistical information of the parameter $\gamma(t)$ and can reach the optimal profit R^* arbitrarily close. We will further provide its performance bounds in Section V.

IV. Profit Maximization Control Policy

We next describe various components of the PMC Policy. In this policy, we will have a system design parameter V that trades off system performance and users' experienced delay. The queue lengths $Q(t)$ and $Z_i(t)$, $i \in \mathcal{B}^s_{\max}$, are updated by (7) and (9).

A. Transmission Pricing and Market Control

We compute the optimal transmission price $q^*(t)$ by solving the following problem.

$$\begin{aligned}&\text{Maximize} \quad \left(q(t) - \frac{Q(t)}{V}\right) D\left(q(t), M(t)\right) \quad (16)\\ &\text{Variables} \quad q(t) \geq 0\end{aligned}$$

If the maximum objective in (16) is positive, the CVNO sets market control $O^*(t) = 1$ and accepts users' new file download requests at the price $q^*(t)$. Otherwise, the CVNO sets $O^*(t) = 0$ and rejects any new requests.

B. Sensing Cost Selection And Resource Allocation

We solve the following optimization problem to determine sensing cost and resource allocation:

$$\begin{aligned}\text{Minimize } & V\big(C^s(t)|\mathcal{B}^s(t)|+C^l(t)|\mathcal{B}^l(t)|\big)+\sum_{i\in\mathcal{B}^s(t)} Z_i(t)\beta(C^s(t))(1-p_0)\\ & - Q(t)\left(\sum_{i\in\mathcal{B}^s(t)\cup\mathcal{B}^l(t)} f_i(t)\log\left(1+h_i(t)P_i(t)\right)\right) \quad (17)\\ \text{Subject to } & (1), (2), (4)\\ \text{Variables } & C^s(t), \mathcal{B}^s(t), \mathcal{B}^l(t), P_i(t) \geq 0\end{aligned}$$

where we use the notation

$$f_i(t) \triangleq \begin{cases}(1-\alpha(C^s(t))\, p_0, & \text{if } i \in \mathcal{B}^s(t),\\ 1, & \text{if } i \in \mathcal{B}^l(t).\end{cases}$$

Through several equivalent transformations (details in [17]), we obtain the optimal solution of (17) as follows.

- *Power allocation $P_i(t)$:*

  $$P_i(t) = \frac{1}{h_i(t)}\left(\frac{Q(t) f_i(t) h_i(t)}{\lambda} - 1\right), \forall i \in \mathcal{B}^s(t)\cup\mathcal{B}^l(t), \quad (18)$$

  where the parameter λ is calculated as

  $$\lambda = \frac{Q(t)\sum_{i\in\mathcal{B}^s(t)\cup\mathcal{B}^l(t)} f_i(t)}{P_{max} + \sum_{i\in\mathcal{B}^s(t)\cup\mathcal{B}^l(t)} \frac{1}{h_i(t)}}. \quad (19)$$

- *Leasing channel $\mathcal{B}^l(t)$:* choose each leasing band channel $i \in \mathcal{B}^l_{\max}$ with a channel gain $h_i(t)$ satisfying

  $$Q(t)G(Q(t)h_i(t),\lambda) \geq VC^l(t), \quad (20)$$

  where function $G(\cdot,\cdot)$ is defined as

  $$G(a,b) \triangleq \begin{cases}\log\left(\frac{a}{b}\right) & \text{if } a > b,\\ 0 & \text{if } a \leq b;\end{cases}$$

- *Sensing channel set $\mathcal{B}^s(t)$:* choose each leasing band channel $i \in \mathcal{B}^s_{\max}$ with a channel gain $h_i(t)$ satisfying

  $$\begin{aligned}&Q(t)f_i(t)G(Q(t)f_i(t)h_i(t),\lambda) \geq\\ &\qquad VC^s(t) + Z_i(t)(1-p_0)\beta(C^s(t)). \quad (21)\end{aligned}$$

Algorithm 1 Sensing Cost Selection And Resource Allocation

$C^{s*} \leftarrow 0$, % initialize sensing cost
$U^* \Leftarrow U(C^{s*})\{1\}$ % see Algorithm 2
for $C^s \in \mathcal{C}^s$ **do**
  if $U^* > U(C^s)$ **then**
    $U^* \leftarrow U(C^s)$,
    $C^{s*} \leftarrow C^s$,
  end if
end for
$\mathcal{B}^{s*} \Leftarrow U(C^{s*})\{2\}, \mathcal{B}^{l*} \Leftarrow U(C^{s*})\{3\}, P_i^* \Leftarrow U(C^{s*})\{4\}$
return $C^{s*}, \mathcal{B}^{s*}, \mathcal{B}^{l*}, P_i^*$

- *Sensing cost $C^s(t)$ (thus the sensing technique):* Choosing the sensing cost to minimize the following objective:

$$\text{Minimize} \sum_{i\in\mathcal{B}^s(t)} [VC^s(t) - f_i(t)G(Q(t)f_i(t)h_i(t),\lambda)] + \sum_{i\in\mathcal{B}^s(t)} Z_i(t)(1-p_0)\beta(C^s(t)). \quad (22)$$

Intuitively, we want to choose channels with large enough channel gains for sensing and leasing, and allocate higher power for better channels. In practice, when the parameter $\phi(t) \triangleq (M(t), \boldsymbol{h}(t), C^l(t))$ is given, we can run Algorithm 1 to numerically calculate the **exact** optimal solution and profit. Here we ignore the time index t in the algorithm. We label all channels in a decreasing order of the virtual channel gain $g_i \triangleq h_i f_i$, *i.e.,* $g_1 \geq g_2 \geq \cdots, \geq g_I$.

The key idea of Algorithm 1 is to check which cost in set $\mathcal{C}^s$ leads to the optimal solution of (17). We invoke the function $U(C^s)$ defined in Algorithm 2, which searches the optimal resource allocation $(\mathcal{B}^{s*}, \mathcal{B}^{l*}, P_i^*)$ for a given sensing cost. $U(C^{s*})$ returns a vector of four elements, and we use the notation $U(C^{s*})\{k\}$, $k = 1,2,3,4$, to denote the k-th element.

In the PMC policy, the two parts are nicely decoupled into two subproblems, each of which is easy to solve. The pricing and market control is a simple one-variable optimization problem. For the sensing cost selection and resource allocation part, we generate the simple searching algorithm as shown in Algorithm 1 with a complexity $O(|\mathcal{C}^s| \times |\mathcal{I}|)$. In addition, we find that the dynamic pricing $q(t)$ performs the functionality of congestion control, *i.e.,* the price $q(t)$ will be higher when the network congestion level is higher. More detailed discussions can be found in the technical report [17].

V. Performance of the PMC Policy

Next we outline the performance bounds of the PMC Policy. The proof details are in our online technical report [17].

Theorem 2: For any V, the PMC Policy guarantees that

- The queue stability (11) and collision constraints (12) are satisfied. The queue length is upper bounded by

$$Q(t) \leq Vq_{\max} + A_{\max}, \quad \forall t. \quad (23)$$

- The average profit $\overline{R}$ satisfies

$$\inf \overline{R} \geq \overline{R^*} - O(1/V), \quad (24)$$

  where $\overline{R^*}$ is the optimal value of the PM problem.

According to Little's law, the average queuing delay is proportional to the queue length. Thus users experience bounded queuing delays under the PMC algorithm by (23). By (24), we find that the profit obtained by the PMC Policy can be made closer to the optimal profit by increasing V. However, as V increases, the queuing delay will also increase as shown in (23). The best choice of V depends on the desirable trade-off between queuing delay and profit optimality.

Algorithm 2 Optimal Resource Allocation for a Given C^s

function $U(C^s)$
  $J = I$, $P_i \leftarrow 0, \forall\, i \in \mathcal{I}$
  $\lambda^* \leftarrow \frac{Q\sum_{i=1}^J f_i}{P_{\max}+\sum_{i=1}^J \frac{1}{h_i}}$
  if $Q \neq 0$ **then**
    while $Qf_J h_J \leq \lambda^*$ **do**
      $J \leftarrow J-1$, $\lambda^* \leftarrow \frac{Q\sum_{i=1}^J f_i}{P_{\max}+\sum_{i=1}^J \frac{1}{h_i}}$
    end while
    $\mathcal{B}^s \leftarrow \{i : i \leq J, i \in \mathcal{B}^s_{max}\}$, $B^s \leftarrow \max_{i\in\mathcal{B}^s} i$
    $\mathcal{B}^l \leftarrow \{i : i \leq J, i \in \mathcal{B}^l_{max}\}$, $B^l \leftarrow \max_{i\in\mathcal{B}^l} i$
    SELECT($\mathcal{B}^s, \mathcal{B}^l$) % see Algorithm 3
    if B^l OR B^s **then**
      $\lambda^* \leftarrow \frac{Q(\sum_{i\in\mathcal{B}^s} f_i + \sum_{i\in\mathcal{B}^l} f_i)}{P_{\max}+\sum_{i\in\mathcal{B}^s}\frac{1}{h_i}+\sum_{i\in\mathcal{B}^l}\frac{1}{h_i}}$
      if B^s **then**
        $P_i \leftarrow \frac{1}{h_i}\left(\frac{Qf_ih_i}{\lambda^*}-1\right), \forall i \in \mathcal{B}^s$
      end if
      if B^l **then**
        $P_i \leftarrow \frac{1}{h_i}\left(\frac{Qf_ih_i}{\lambda^*}-1\right), \forall i \in \mathcal{B}^l$
      end if
    end if
    $U \leftarrow V(C^s|\mathcal{B}^s|+C^l|\mathcal{B}^l|) - Q\sum_{i\in\mathcal{B}^s\cup\mathcal{B}^l} f_i \log(1+h_iP_i) + \sum_{i\in\mathcal{B}^s} Z_i\beta(C^s)(1-p_0)$
  else
    $U \leftarrow 0$ $\mathcal{B}^s \leftarrow$Null $\mathcal{B}^l \leftarrow$Null
  end if
  return $U, \mathcal{B}^s, \mathcal{B}^l, P_i$
end function

VI. Simulation

We conduct simulations with the following parameters. The number of incoming users in each slot satisfies a Poisson distribution with a rate $D(q(t), M(t)) = \frac{1}{M(t)}(q(t)-5)^2$, where $M(t) = 1$ with probability 0.5, and $M(t) = 2$ with probability 0.5. The file length of each user satisfies the i.i.d. (discrete) uniform distribution between 1 and 10. There are 32 channels in total, and 20 of them belongs to the sensing band $\mathcal{B}^s_{\max}$, and the rest 12 channels belongs to the leasing band $\mathcal{B}^l_{\max}$. The channel gain h_i of each channel satisfies i.i.d. (continuous) Rayleigh distribution with parameter $\sigma = 4.5$ (such that virtual SNRs of most channels lie in range of 10 – 30dB). The total power is upper bounded by $P^{\max} = 8$. The idle time probability of sensing band p_0 is 0.6. The primary collision probability bound is $\eta = 0.01$. There are 6 different sensing techniques with costs $\mathcal{C}^s = \{0\,(\text{not sensing at all}), 0.5, 1, 1.5, 2, 2.5\}$. The corresponding false alarm probabilities are $\alpha = \{0.5, 0.3, 0.1, 0.06, 0.03, 0.008\}$, and the missed detection probabilities are $\beta = \{0.5, 0.2, 0.08, 0.03, 0.01, 0.005\}$.

The simulation is conducted with control parameters $V \in \{5, 10, 50, 100, 500\}$. Figure 2 (a) shows that the average queue length grows linearly in V, and is always less than the

Algorithm 3 Select the Sensing And Leasing Channels

function SELECT($\mathcal{B}^s, \mathcal{B}^l$)
  flag=1
  while B^s AND B^l AND flag **do**
    $\lambda^* \leftarrow \frac{Q\sum_{i\in\mathcal{B}^s\cup\mathcal{B}^l} f_i}{P_{\max}+\sum_{i\in\mathcal{B}^s\cup\mathcal{B}^l}\frac{1}{h_i}}$, $L \leftarrow QG(Qh_{B^l}, \lambda^*)$
    $S_i \leftarrow Qf_i G(Qf_i h_i, \lambda^*)$, $i \in B^s$
    $CS_i \leftarrow Z_i(1-p_0)\beta(C^s)$, $i \in B^s$
    if $S_{B^s} > VC^s + CS_{B^s}$ AND $L > VC^l$ **then**
      flag$\leftarrow 0$
    else if $S_{B^s} \le VC^s + CS_{B^s}$ AND $L \le VC^l$ **then**
      $J \leftarrow J - 1$
      $\mathcal{B}^s \leftarrow \{i : i \le J, i \in \mathcal{B}^s_{\max}\}$, $B^s \leftarrow \max_{i\in\mathcal{B}^s} i$
      $\mathcal{B}^l \leftarrow \{i : i \le J, i \in \mathcal{B}^l_{\max}\}$, $B^l \leftarrow \max_{i\in\mathcal{B}^l} i$
    else if $S_{B^s} > VC^s + CS_{B^s}$ AND $L \le VC^l$ **then**
      $\mathcal{B}^l \leftarrow \mathcal{B}^l - \{B^l\}$, $B^l \leftarrow \max_{i\in\mathcal{B}^l} i$
    else if $S_{B^s} \le VC^s + CS_{B^s}$ AND $L > VC^l$ **then**
      $\mathcal{B}^s \leftarrow \mathcal{B}^s - \{B^s\}$, $B^s \leftarrow \max_{i\in\mathcal{B}^s} i$
    end if
  end while
  flag=1
  while B^s AND $B^l == 0$ AND flag **do**
    $\lambda^* \leftarrow \frac{Q\sum_{i\in\mathcal{B}^s} f_i}{P_{\max}+\sum_{i\in\mathcal{B}^s}\frac{1}{h_i}}$
    $S_i \leftarrow Qf_i G(Qf_i h_i, \lambda^*)$, $i \in \mathcal{B}^s$
    if $S_{B^s} > VC^s + CS_{B^s}$ **then**
      flag$\leftarrow 0$
    else
      $\mathcal{B}^s \leftarrow \mathcal{B}^s - \{B^s\}$,$B^s \leftarrow \max_{i\in\mathcal{B}^s} i$
    end if
  end while
  flag=1
  while $B^s == 0$ AND B^l AND flag **do**
    $\lambda^* \leftarrow \frac{Q\sum_{i\in\mathcal{B}^l} f_i}{P_{\max}+\sum_{i\in\mathcal{B}^l}\frac{1}{h_i}}$, $L \leftarrow QG(Qh_{B^l}, \lambda^*)$
    if $L > VC^l$ **then**
      flag$\leftarrow 0$
    else
      $\mathcal{B}^l \leftarrow \mathcal{B}^l - \{B^l\}$, $B^l \leftarrow \max_{i\in\mathcal{B}^l} i$
    end if
  end while
  $\mathcal{B}^s \leftarrow \mathcal{B}^s - \{i : S_i \le VC^s + CS_i, i \in \mathcal{B}^s\}$
end function

worst case bound $Vq^{max} + A^{max}$. Figure 2 (b) shows that the average profit achieved by PMC policy converges quickly as V grows, and is close to the maximum profit when $V \ge 50$.

VII. Conclusion

In this paper, we study the profit maximization problem of a CVNO in a dynamic network environment. We propose a low-complexity control algorithm that performs pricing and resource scheduling without knowing the distribution of dynamic network parameters. We show that our algorithm can achieve arbitrarily close to the optimal profit, and has a flexible trade-off between profit optimality and queuing delay.

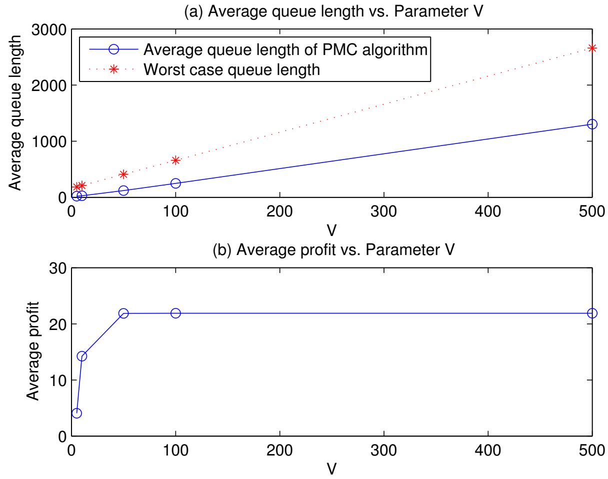

Fig. 2. (a) Average queue length vs. Parameter V, (b) Average profit vs. Parameter V

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