Wireless Access Pricing

LEUNG, Kwan Fong

A Thesis Submitted in Partial Fulfillment
of the Requirements for the Degree of
Master of Philosophy
in
Information Engineering

The Chinese University of Hong Kong
August 2011

Abstract

This thesis studies the access pricing issues in wireless network competition. Access price is a critical issue in network competition as the operator is selling her own resources (profit-making) to her competitor (profit-losing). Mobile-to-mobile (MTM) access pricing has been adopted by many countries such as Japan, Italy, and Kenya. In this thesis, we study the interaction between user price and access price in both regulated and deregulated market. In a regulated market, the regulator set the social optimal user price to achieve social optimality. On the other hand, in a deregulated market, we model the market by a Stackelberg game assuming each operators sets a user price and access price to maximize her own profit.

We adopt the model proposed by [8] and enrich it with wireless properties. Three market models are considered - fixed coverage wireless market model where user-network association is fixed, full coverage wireless market model where each networks has full coverage, and three base stations full coverage wireless market model where one of the operator builds a new base station. We compare the social welfare obtained in the regulated market and the operator's profit in the deregulated market. We show that in full coverage model (both in two base stations model and three base stations model), when difference between the two operators' per unit bandwidth costs is large, the society will be better off if only the lower cost operator serves the market. Also, no matter in two base stations or three base stations case, providing full coverage does not always improve the social welfare. In building one more base station, the

overall social welfare is improved only when the operator's per bandwidth cost is lower compared to the rival networks. In a deregulated market, full coverage does not always result in higher profits. Building one more base station does not always result in higher operator's profit either. In the full coverage (both in two base stations model and three base stations model), a higher per unit bandwidth cost operator will lose market when the difference between the per unit bandwidth cost of the two operators is large.

摘要

在本論文中，我們探討在無線網絡中的網絡互連費（access pricing）的定價問題。網絡互連費比使用者的收費的定價更為複雜，這是因為網絡互連本身是賣自己的資源（增加利潤）給她的競爭對手（虧損利潤）。移動網絡與移動網絡（MTM）之間的網絡互連費存在在多個國家，例如日本、意大利和肯尼亞等。在本篇論文，我們將研究網絡互連費及使用者收費之間的關係。我們會研究在一個規範的市場，監管機構如何利用使用者收費及網絡互連費以最優化社會福利。另一方面，在放鬆管制的市場裡，我們假設網絡營運者利用使用者收費和網絡互連費以最大化自己的利潤，我們使用斯塔克爾伯格模型（Stackelberg Game）去模擬網絡營運者如何定價。

我們採用[8]所提出的數學模型，並加入無線網絡原素。我們研究三個市場模型 - 固定無線網絡覆蓋模式（即用戶及網絡連接是固定的）、全覆蓋無線市場模型（即每個網絡已全面覆蓋，由使用者自行決定選擇的網絡）、三基站全覆蓋無線市場模型（即其中一個網絡營運者建立一個新的基站）。我們比較在這三個市場模型之下，規管市場內的社會福利，以及在放鬆管制的市場內網絡營運者的利潤。我們發現，在全覆蓋模型（無論在兩個基站模型或三個基站模型之下），當兩網絡營運者的每單位帶寬成本差異較大的時候，如果只有較低成本的網絡營運者服務市場，社會福利會更佳。在全覆蓋模型（無論在兩個基站模型或三個基站模型），放鬆管制的市場下，當網絡營運者的每單位帶寬成本相差較大的時候，每單位帶寬成本較高的網絡營運者將失去市場。

Acknowledgments

In completing this thesis, I am most grateful to my thesis advisor, Professor Jianwei Huang, for his patience in advising me in the past two years. I am grateful to have his guidance so that I can finish this project. Professor Huang is such an enthusiastic and encouraging person that helps me improve a lot.

I would also like to thank the lab members in the Network Communications and Economics Laboratory especially Dr. Liqun Fu, Dr. Fen Hou, Dr. Lin Gao, Dr. Richard Southwell, Lok Man (Janice) Law, Shuqin (Helen) Li, Lingjie Duan, Xu Chen, Scotty Leung, Guosen (Sam) Feng. Many good people I met in these two years including Pui Ho (Adonis) Fung, Michael Hui, Scottie Tse, Ka Chun Lam and Kam Hon Tse. They are there to support me in my hardship and I treasure the time we spent together.

Finally, I would like to thank my parents, Ka Ho Poon, and my brother, for their unconditional love and support.

Contents

List of Figures

Chapter 1

Introduction

1.1 Motivation and Overview

Due to the rapid growth of wireless services development, it is important to understand how to fairly allocate valuable wireless resources. Researchers have been analysing the problem using prices. This line of work tries to set a proper price by using techniques developed from optimization(e.g.,[1] and [2]), game theory (e.g.,[3] and [4]) or auction theory (e.g.,[5]). Our work falls into the second category. Several issues have been considered in this line of studies: Network Externalities, Network Effect, and Interconnection. For this thesis, we look at the interconnection issues in wireless competition.

Interconnection happens when an operator needs access to resources owned by the other network. Inter-network communications bring benefits to all networks involves, as a network does not necessarily need to build an infrastructure covering the entire market. In telecommunications, interconnection could happen between fixed-to-fixed, fixed-to-mobile, and mobile-to-mobile network. Serving users from other networks brings negative impacts on a network's own available resource. This raises the notion of interconnection fee or access pricing, which is the settlement between networks in consuming resources provided by the other networks. Access pricing is more complex in nature than user pricing since it is selling the resources (profit-gaining) to a competitor (profit-losing).

Access pricing has long been a practice in long-distance calls between countries. It has also been the practice between mobile operators such as in Japan, Germany, Italy [6] and in Africa. As the wireless traffic is becoming more data-drive, it is expected that access pricing would become a more common issues between mobile operators.

In real life practices, operators usually are given the flexibility to negotiate with each other to set the access pricing. Regulators will take intervention in cases of disputes or inefficiency. This happens in long distance call in US [7] and recently between mobile operators in South Africa in 2010.

Access pricing does not only apply to telecommunication networks. In fact, it was first studied in electricity and transportation as well. Access pricing has been considered in two network structures - one way or two way networks. One way network involves vertically related firms at which the downstream firm needs access to the upstream firm while two way networks involves vertically unbundled firms at which each firm needs access to each other.

Traditionally, the literature analyzes on how to regulate access pricing. The need for regulation in a one-way network is particularly crucial. In a one-way network, there is a monopoly upstream firm (the incumbent) that upholds the resource needed by the downstream firm (the new entrant). The incumbent will set access charges as high as possible to make further entry difficult, but not vice versa. The new entry concern in one-way access is not so severe in two-way access since both networks need access from each other. However, this does not mean regulation is unnecessary in two-way access. Problems such as operators collude to set high access pricing still arise. Approaches such as Ramsey access pricing (e.g.,[8],[9]), the efficient component-pricing rule (ECPR) (e.g.,[10],[11],[12]), cost-based access prices (e.g.,[13]) has been largely studied in the literature.

Our work fills in the gap between the existing studies on wireless network competitions and network access pricing. Existing studies on network access prices considered either a traditional wireline network [9] or a hybrid market of a wireline telephone network and a wireless cellular network [14]. References [15], [16] studied how to determine the access pricing among hierarchical Internet Service Providers with a specific traffic model. None of the above results captured the impacts of channel conditions and transmission power. Existing work on wireless network competition (e.g., [17],[18],[19],[20]) did not consider access prices between operators.

Also, traditional access pricing models did not take into account the fact that resources demand from downstream and upstream traffic is different in a wireless environment. Existing literature on telecommunication access pricing, even to a cellular network (e.g.,[14])), considers the cost in a per-call basis. Yet, the resources demanded from traffic terminating at users close to his own base-station could be very different to the traffic ending at user far away from his base-station. The location of the originating and terminating users do matter in this aspect.

This thesis focuses on the interaction between access pricing and user pricing, and how they are set in both a regulated and deregulated scenario. We are interested in understanding if a fixed coverage scenario could provide a better social welfare and profits than a full coverage scenario. Also, does investing in setting up a new base station help increasing the social welfare and profit?

In a regulated scenario, regulators such as the government could control the social welfare by means of user pricing. We showed also that regulators do not have to always control the user pricing directly. Setting the right access pricing could also achieve the same goal. On the other hand, in a deregulated scenario, the selfish operators will set the user pricing and access pricing to maximize his/her own profits.

This thesis considers three different market models - fixed coverage model, full coverage model, and full coverage three base stations model. Fixed coverage model considers the scenario where the network-user association is fixed. This happens in real life such as in country side where certain area is covered by only one operator but not the other. Full coverage model considers the scenario where operators provide full coverage and each user choose his/her own service providers. The full coverage three base stations model models the situation where an operator invest in building up a new base station under the full coverage scenario.

In each of these three market models, we formulate the social welfare and each operator's profit in regulated and deregulated scenario respectively. Then, we compare the social welfare and profits achieved in each of these cases. By numerical study, we showed that:

Full coverage market model could result in a better social welfare and profits than in full coverage market model. However, it is not always the case. Investing in setting up a new base station could improve social welfare when the per bandwidth cost is low. As the cost increases, social welfare is even worse by setting up a new base station.

Profit is not always improved in the full coverage scenario compared to the fixed coverage. Investing in setting up a new base station could improve profits. Also, it helps relieving the effect brought about when the per bandwidth cost increases so that the operator will lose market at a larger cost.

1.2 Thesis Outline

This thesis is organized in six chapters. In Chapter 2, we describe the models that will be used in the analysis. Specifically, we present the wireless market model, network's cost structure and users' payoff function. In Chapter 3, we propose the fixed market model at which network-users association is fixed. Also, we formulate the social welfare and profit of each operators. We show

how the regulator should control the user pricing or access pricing to achieve social optimality. We also formulate a deregulated market by a Stackelberg game. We show how each operator will set the user pricing and access pricing accordingly to maximize her own profitin the deregulated environment.

In Chapter 4, we propose the full market model at which each user chooses his/her own service provider. We will study how each user chooses his/her own operator. Similar as in Chapter 3, we formulate the social welfare and each operator's profit function so as to solve for the social optimal user pricing, profit-maximizing user pricing and profit-maximizing access pricing. The social optimal case, or the equilibrium case could be one of the three cases: Only network 1 in the market, only network 2 in the market, or both network 1 and network 2 in the market. We show the corresponding social optimal case or equilibrium case under different cost structure, and compare the social welfare obtained in regulated market and operator's profit in deregulated market to the fixed wireless market.

In Chapter 5, we present the three base stations model which models one of the operators builds up a new base station. Similar as before, we will formulate the social welfare and each operators' profit and solve for the social optimal user pricing, equilibrium user pricing and access pricing. We will compare the social welfare obtained in the regulated market, and operator's profits in a deregulated market to that of the two base stations model.

Finally, we conclude and suggest some future direction for this thesis in Chapter 6.

Chapter 2

Problem Formulation

In this section, we will describe the basic wireless market model that we use, followed by users' utility function, network costs and the access price.

2.1 Basic Wireless Market Model

Our model is the extension to the model first proposed by [8] in 1996. [8] considers the problem in linear cost and linear access price. It illustrates how regulator could control the access price using methods such as Efficient Component Pricing Rule (ECPR) in a one-way access problem. Since then, several work is built upon this framework (e.g., [21], [22]). We took similar model as in this line of work that a one-dimensional market model as in Fig. 2.1 is considered. It consists of two network operators. The number of users are fixed. Users are uniformly located as in [18] along the segment [0,1]. This means that users do not move, or users move without changing the user distribution. Each network companies could have one base station only as in the figure, or more than one in the three base stations market model as described later on. As in the Hotelling model, the base stations are placed at the two ends. The users' distance to the base stations affects the experienced channel gains of users. An operator could build more than one base-stations to serve users.

This one-dimensional model is a simplification of a three-dimensional or

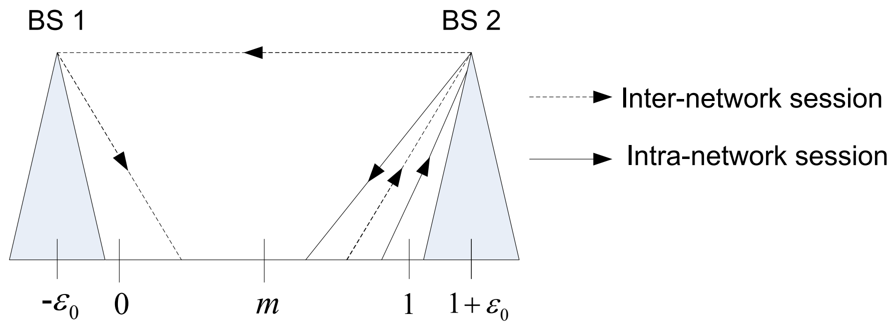

Figure 2.1: An illustration of the wireless market.

two dimensional scenario [4], which are complicated to analyse. Also, the model could be extended to involve multiple networks. In the previous work in [23], we talked about the result that could be applied to the case involving three networks.

From the model described in [8], we enrich the model with specific wireless components to better model a wireless scenario. The users' different location would mean a different channel gain between a user and a base-station. We adopt the two-ray wireless model such that the channel gain follows the large-scale distance based attenuation. Small scale fading does exist but in the long term the average attenuation by small scale fading does not matter. To avoid having an infinitely large channel gain, the base-stations are ϵ_0 away from their closest users.

In Fig 2.1, the market is split between operator 1 and operator 2 such that users located between $[0, m]$ are served by operator 1 and users located between $[m, 1]$ are served by operator 2. The way how the market is split between operators is described by the specific wireless market model and will be discussed further in the subsequent chapters.

We consider three wireless market models - fixed coverage model, full coverage model and three base-stations full coverage model. We begin our analysis with a fixed coverage model at which the base-stations of each operators will cover a fixed amount of users. Such coverage could be a result of geographical constraints. After that, we will continue our analysis in a full coverage model at which both operators could cover all the users and users will have the right to choose their service providers. Also, based on the full coverage market model, we look three base stations model where one of the operator builds one more base-station at the other end to attract users who are not currently served by her.

2.2 User's Utility, Payment, Payoff, and Demand

A one-way data communication session involves a source user and destination user. The session is *intra-network* when both users belong to the same network, or *inter-network* when the two users belong to different networks. We adopt the α-fair utility function [24] to represent the quality of service (QoS) of a session in terms of its data rate y,

$$u(y) = \frac{y^{1-\alpha}}{1-\alpha},$$

where the utility parameter $\alpha \in (0, 1)$ [9].

If a source user i belonging to network j starts a session with the data rate y_{ij}, then it pays network j at the *user pricing* π_j and hence the user payment is $\pi_j y_{ij}$. The source user i's payoff (which is also the payoff of the communication session) is

$$r_i(y_{ij}, \pi_j) = \frac{y_{ij}^{1-\alpha}}{1-\alpha} - \pi_j y_{ij}.$$

We assume the Calling Party Pays charging scheme [25] such that the destination node does not need to pay the user pricing.

The optimal *demand* of data rate that maximizes utility is

$$y_{ij}^*(\pi_j) = \pi_j^{-1/\alpha}, \tag{2.1}$$

which has a constant elasticity of $1/\alpha$. A small α denotes an elastic application and a large α denotes an inelastic application. Source user i's optimal payoff is

$$r_i(y_{ij}^*(\pi_j), \pi_j) = \frac{\pi_j^{1-1/\alpha}}{1-\alpha} - \pi_j^{1-\frac{1}{\alpha}},$$

which only depends on the network price π_j and is independent of the user's location. This is desirable in practice, as the user only needs to keep track of the total data usage instead of where he conducts the communications.

2.3 Network Costs

A communication session involves both a uplink transmission (from the source user to its network's base station) and a downlink transmission (from the destination user's base station to the destination user). Each part of the transmissions involves a cost proportional to the bandwidth consumed. For a source user i belonging to network j, the relationship between the transmission rate y_{ij} and the consumed bandwidth B_{ij} depends on the distance between the user and the base station d_{ij}, the uplink transmission power per unit bandwidth P^u, and the background noise density n_0. We assume an Orthogonal Frequency Division Multiple Access scheme with equal power allocation, and no two users (either in a same or different networks) interfere with each other. Thus

$$y_{ij} = B_{ij} \log\left(1 + \frac{P^u h_{ij}\left(d_{ij}\right)}{n_0}\right),$$

where $h_{ij}(d_{ij})$ is the channel gain depending on the distance d_{ij}. One possible choice is $h_{ij}(d_{ij}) = \frac{1}{{d_{ij}}^{\beta}}$, where β is the channel attenuation factor (usually between 2 to 4). The analysis in this paper can be generalized to other distance based channel models as we keep the function $h_{ij}(d_{ij})$ abstract most of the time.

The total bandwidth cost for supporting this uplink transmission is $c_j B_{ij}$. The cost for the downlink transmission can be computed similarly, except that P^u will be replaced by P^d and c_j will be replaced by the cost of the corresponding network. For notation simplicities, we denote

$$g^d(d_{ij}) \equiv \log\left(1+\frac{P^d h_{ij}\left(d_{ij}\right)}{n_0}\right),$$
$$g^u(d_{ij}) \equiv \log\left(1+\frac{P^u h_{ij}\left(d_{ij}\right)}{n_0}\right).$$

Let us compute the total cost of serving one communication session. If source user i subscribes network 1 and is at location d_i, then network 1's cost of the uplink transmission is:

$$c_1 B_{i1} = c_1 \frac{y_{i1}}{\log\left(1+\frac{P^u h_{ij}(d_i)}{n_0}\right)} = c_1 \frac{y_{i1}}{g^u(d_i)}.$$

Serving a user close-by (with a small value of d_i and thus a larger $g^u(d_i)$) costs less compared with serving a user far away. For the downlink traffic, since the destination may be with network 1 or network 2, we need to compute the *expected* downlink cost. In our model, users are uniformly distributed along the segment [0,1], and each of them will have equal probability of receiving data. The expected cost is $c_1 \int_0^m \frac{y_{i1}}{g^d(r)} dr$ to network 1 and $c_2 \int_0^{1-m} \frac{y_{i1}}{g^d(r)} dr$ to network 2. Hence, the total expected cost to support a session initiated by a user i located at d_i in network 1 with data rate y_{i1} is:

$$c_1 \frac{y_{i1}}{g^u(d_i)} + c_1 \int\limits_0^m \frac{y_{i1}}{g^d(r)} dr + c_2 \int\limits_0^{1-m} \frac{y_{i1}}{g^d(r)} dr.$$

The first two terms are related to network 1 and the third term is related to network 2.

In the rest of the paper, we denote

$$B^u(m) \equiv \int_0^m \frac{1}{g^u(r)} dr \text{ and } B^d(m) \equiv \int_0^m \frac{1}{g^d(r)} dr,$$

which represent the average bandwidth needed to serve one unit of uplink and a downlink transmission of a network with a market share m, respectively.

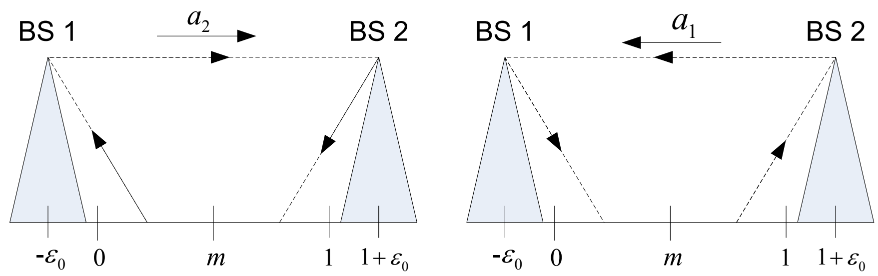

Figure 2.2: Illustration of the access pricing. Left: To complete an inter-network traffic initiated from network 1, network 2 charges network 1 a_2 per unit of bandwidth. Right: the access price a_1 is defined similarly.

2.4 Access Price

For an inter-network session, the network with the destination user cannot charge the source user, but bears the cost of the downlink transmission. To compensate this additional cost, the destination network charges the source network an *access pricing*. As illustrated in Fig. 2.2, to complete an inter-network session initiated from network 1, network 1 pays network 2 a_2 per unit of bandwidth consumed in network 2. The access price a_1 is defined similarly.

Chapter 3

Fixed Coverage Two Base Stations Model

In this chapter, we study the regulated and deregulated market in the fixed coverage two base stations model. We first formulate the social welfare and solve for the social optimal user pricing in the regulated market. Then, we showed that regulators can also achieve social optimality by means of access pricing. Finally, we study the deregulated market and solve for the profit-maximizing access pricing and user pricing.

3.1 Social Optimal User Pricing

In a fixed coverage model, we assume that the user-network associations are fixed, such that users in $[0, m]$ subscribe to network 1 and users in $[m, 1]$ subscribe to network 2. This means that network 1 has a market share of m, and network 2 has a market share of $1 - m$. Reference [26] illustrates one such example, where some part of California is covered by AT&T but not Verizon (and vice versa). The analysis based on this restrictive assumption of fixed market share will help us understand the more general scenario of flexible market share.

A regulator cares about the social welfare, which is the total payoffs of all network entities in the market, i.e., the total user utility minus the total network cost. The payments (either from users to networks or between networks) are internal transfers and do not affect the social welfare. However, a regulator typically cannot directly control how much resources that users consume. In this section, we will look at the case where the regulator maximizes the social welfare by controlling the user pricing π_1 and π_2.[1]

The social welfare $SW(\pi_1, \pi_2, m, c_1, c_2)$ is:

$$\begin{aligned} SW(\pi_1, \pi_2, m, c_1, c_2) =& m\frac{{\pi_1}^{1-\frac{1}{\alpha}}}{1-\alpha} + (1-m)\frac{{\pi_2}^{1-\frac{1}{\alpha}}}{1-\alpha} \\ & - {\pi_1}^{-\frac{1}{\alpha}} f(m, c_1, c_2) - {\pi_2}^{-\frac{1}{\alpha}} f(1-m, c_2, c_1), \end{aligned} \tag{3.1}$$

where $f(m, c_j, c_{-j})$ represents the total cost in serving sessions originated from network j of a market share m,

$$f(m, c_j, c_{-j}) = c_j B^u(m) + m c_j B^d(m) + m c_{-j} B^d(1-m).$$

Here c_{-j} denotes the per unit bandwidth cost of the network other than network j. For example, if $j = 1$, then $c_{-j} = c_2$.

The regulator's objective is to choose π_1 and π_2 to maximize the social welfare. From (3.1), it is clear that the social welfare is decoupled between π_1 and π_2. We can also show that the social welfare is quasi-concave in both π_1 and π_2, and thus the optimal user prices can be obtained through solving the first order conditions.

Proposition 1 Socially optimal user prices are $\pi_1^S = f(m, c_1, c_2)/m$ and $\pi_2^S = f(1-m, c_2, c_1)/(1-m)$.

Observation 1 The social optimal user prices do not depend on the utility parameter α.

Proposition 1 works for any channel function. In other words, the social optimal user price changes accordingly in different channel gain functions.

[1]The access prices a_1 and a_2 cancel out and do not affect social welfare.

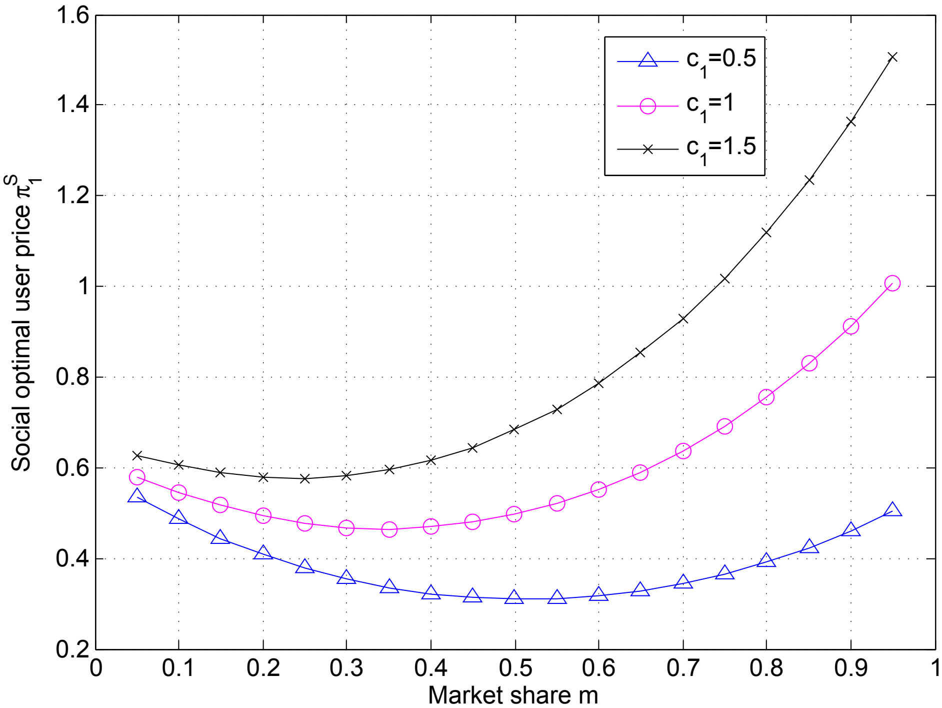

Figure 3.1: Social optimal user price π_1^S of network 1 with different market share m and cost c_1. Here $h_{ij}(d_{ij}) = d_{ij}^{-3}$, $P^d = P^u = 1$, and $c_2 = 1$.

In fact, since we are abstracting the average bandwidth function $B^u(m)$ and $B^d(m)$, the social optimal user price could be applied to wireless model other than the two-way propagation model with slight modification to the average bandwidth function.

Also, Proposition 1 suggests that the social optimal user price increases as the bandwidth costs for any channel gains. This is intuitive that since higher the operating cost, the higher the user prices as a compensation to the operators. Figure 3.1 shows a plot of social optimal user price of operator 1 with respect to a change in the operator 1's market share under different operating cost c_1.

3.2 Social Optimal Access Pricing

Very often the regulator cannot even control the user pricing in practice. This may be due to the complexity of the regulation, or due to the fact that the government just does not want to micro-manage the telecommunication industry. However, the regulator can still achieve the social optimality by setting the access pricing, which induce the right user pricing set by the profit-maximizing operators.

We can model the system as a three-stage decision process as in Fig 3.2. In stage one, the regulator determines the access pricing a_1 and a_2 between two network operators. In stage two, each network j chooses the user pricing π_j to maximize its profit, given a_1 and a_2. Access price is determined first before user price since user price changes more frequently than access price. Finally, each users determine their rate demand according to the user price announced by their own operators.

We will use backward induction to analyze the three-stage dynamic game [27]. We start with Stage III and analyze the users' optimal rate demand given the operators' user prices and access prices. Then we look at Stage II and analyze how operators make the user pricing decision taking users' rate demand into consideration. Finally, at Stage I, we look at how the operators set the access price assuming the user price and users' rate demand are determined. At Stage III, the users' rate demand is $\pi_1^{-\frac{1}{\alpha}}$ as determined in 2.1. Next, we will analyze Stage II of this dynamic game.

3.2.1 Networks' Profit-Maximizing User Pricing Given Fixed Access Prices: $\pi_1^*(a_1, a_2)$ and $\pi_2^*(a_1, a_2)$

We begin by deriving the network profits. Consider a session originated from a user i located at d_i in network 1. Network 1's profit from this session equals the payment received from user i minus the total expected cost considering

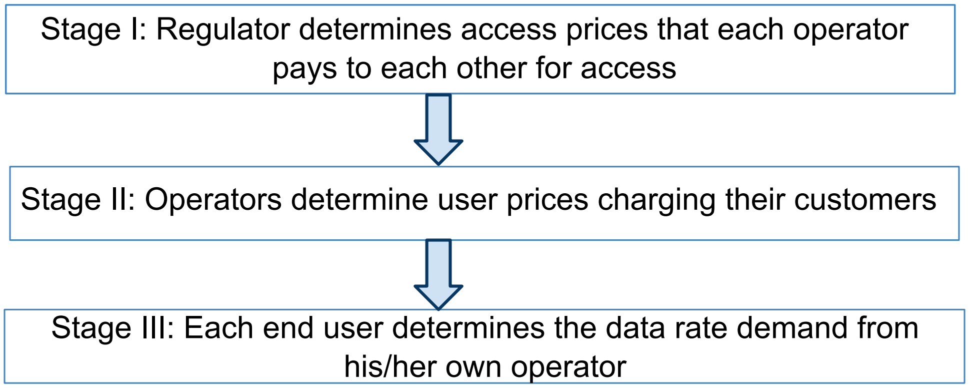

Figure 3.2: Three-stage dynamic game in fixed coverage market model: Regulator determines the access price. Each operators then determines the user price, and finally each user determines his/her data rate demand.

both the possibility of the intra-network and inter-network sessions, i.e.,

$$\pi_1^{1-\frac{1}{\alpha}} - c_1 \frac{\pi_1^{-\frac{1}{\alpha}}}{g^u(d_i)} - c_1 \int_0^m \frac{\pi_1^{-\frac{1}{\alpha}}}{g^d(r)} dr - a_2 \int_0^{1-m} \frac{\pi_1^{-\frac{1}{\alpha}}}{g^d(r)} dr.$$

For an inter-network session originating from a user i located at d_i in network 2, network 1's profit equals the access price payment received from network 2 minus the cost in supporting the downlink communication, i.e.,

$$(a_1 - c_1) \int_0^m \frac{\pi_2^{\frac{-1}{\alpha}}}{g^d(r)} dr.$$

Combining the above analysis, the profits of network 1 (R_1) and of network 2 (R_2) are

$$\begin{aligned} R_1(\pi_1, \pi_2) = m\pi_1^{1-\frac{1}{\alpha}} - c_1 \pi_1^{-\frac{1}{\alpha}} B^u(m) - m c_1 \pi_1^{-\frac{1}{\alpha}} B^d(m) \\ - m a_2 \pi_1^{-\frac{1}{\alpha}} B^d(1-m) + (1-m)(a_1 - c_1) \pi_2^{-\frac{1}{\alpha}} B^d(m) \end{aligned}$$

and

$$\begin{aligned} R_2(\pi_1, \pi_2) =& (1-m)\pi_2^{1-\frac{1}{\alpha}} - c_2 \pi_2^{-\frac{1}{\alpha}} B^u(1-m) \\ & - (1-m) c_2 \pi_2^{-\frac{1}{\alpha}} B^d(1-m) - (1-m) a_1 \pi_2^{-\frac{1}{\alpha}} B^d(m) \\ & + m(a_2 - c_2) \pi_1^{-\frac{1}{\alpha}} B^d(1-m). \end{aligned}$$

By optimizing R_1 over π_1 and optimizing R_2 over π_2, we have the following results.

Proposition 2 For any given access prices a_1 and a_2 set by the regulator, networks 1 and 2 set the following user prices to maximize their individual profits,

$$\pi_1^*(a_2) = \frac{c_1 B^u(m) + mc_1 B^d(m) + a_2 B^d(1-m)}{m(1-\alpha)}, \tag{3.2}$$

and

$$\pi_2^*(a_1) = \frac{c_2 B^u(1-m) + (1-m)c_2 B^d(1-m) + a_1 B^d(m)}{(1-m)(1-\alpha)}. \tag{3.3}$$

Observation 2 A network j's profit-maximizing user pricing π_j^* depends on the rival network's access pricing a_{-j} and is independent of its own access pricing a_j.

Before studying the regulator's optimal choice of access pricing, let us consider the case where no access pricing is set, i.e., $a_1 = a_2 = 0$. We can compare networks' profit maximizing user pricing (π_1^* and π_2^*) with the social optimal user pricing (π_1^S and π_2^S) computed in Section 3.1.

Observation 3 With $a_1 = a_2 = 0$, the profit-maximizing user prices π_1^* and π_2^* always lead to a smaller social welfare comparing with the one achieved under π_1^S and π_2^S, except for one value of the market share.

Figure 3.3 shows the social optimal user price π_1^S (which is independent of the utility parameter α as shown in Observation 1) and the profit-maximizing user price π_1^* for three different values of α under different market shares. For each choice of α, π_1^* only intersects with π_1^S once, and the prices are different for all other values of m. For example, when $\alpha = 0.2$ and market share $m < 0.55$, we have $\pi_1^* < \pi_1^S$ and users' demand is larger in the profit-maximizing case than in the social optimal case. It is the other way around when $m > 0.55$. Neither case is desirable from the regulator's point of view.

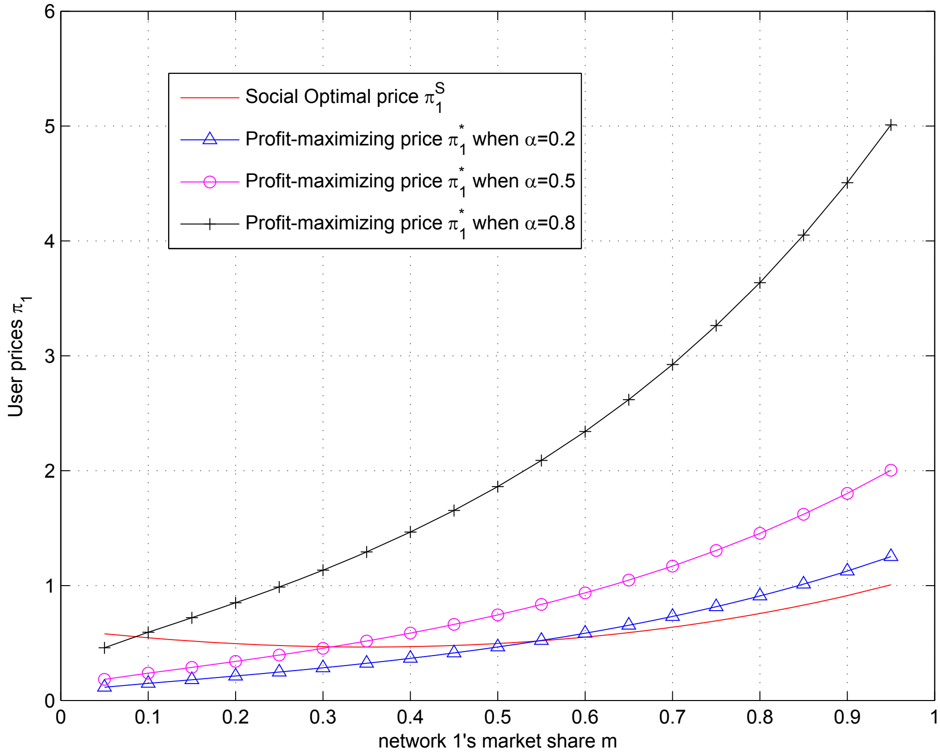

Figure 3.3: Social optimal user price π_1^S and profit-maximizing user price π_1^* with different values of market share m and utility parameter α. Here $h_{ij}(d_{ij}) = d_{ij}^{-3}$, $P^d = P^u = 1$, and $c_1 = c_2 = 1$.

3.2.2 Social Optimal Access Pricing: a_1^S and a_2^S

Now consider the stage 1 problem. The regulator can set the proper access pricing a_1^S and a_2^S so that the networks' profit-maximizing behavior is aligned with the social optimality objective, i.e., $\pi_1^*(a_2^S) = \pi_1^S$ and $\pi_2^*(a_1^S) = \pi_2^S$.

By comparing the values of π_1^S and π_2^S in Proposition 1 and the values of π_1^* and π_2^* in (3.2) and (3.3), we have the following.

Proposition 3 The social optimal access prices are

$$a_1^S = \frac{(1-m)(1-\alpha)\pi_2^S - c_2 B^d(1-m)}{(1-m)B^d(1-m)} - \frac{c_2 B^d(1-m)}{B^d(m)},$$
$$a_2^S = \frac{m(1-\alpha)\pi_1^S - c_1 B^d(m)}{mB^d(m)} - \frac{c_1 B^d(m)}{B^d(1-m)},$$

where π_1^S and π_2^S are defined in Proposition 1.

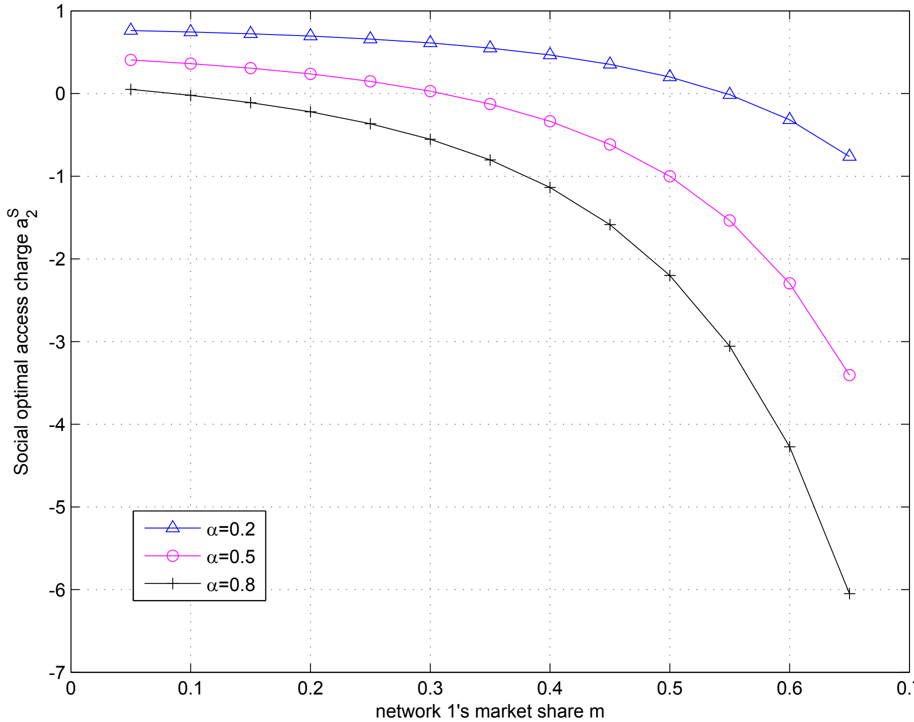

Figure 3.4: Social optimal access price a_2^S with different market share m and utility parameter α. Here $h_{ij}(d_{ij}) = d_{ij}^{-3}$, $P^d = P^u = 1$, and $c_1 = c_2 = 1$.

Figure 3.4 shows the social optimal access price a_2^S with different market share of network 1 m and utility parameter α. We have the following observation.

Observation 4 The social optimal access price of a network decreases in its rival network's market share.

Observation 4 can be explained as follows. When an operator's market share (e.g., m of operator 1) increases, more customers are being connected to and more farther away users are being served. As a result, the average bandwidth cost of serving a user also increases. Due to the profit-maximzing nature of the network, it tends to increase the user pricing (π_1^*) significantly to compensate such cost increase (see Fig. 3.3). Though the social optimal user pricing increases as the market share of operator 1 increases, the profit-maximizing user pricing increases more than the social optimal user pricing for an increase in market share as observed in Fig. 3.3. Thus the access price charged by the other network (network 2) should decrease to provide incentive for the network to maintain serving at the social optimal user pricing.

In fact, in the extreme case where the average cost has increased so much due to a very large market share, the access price from its rival network needs to be negative (i.e., network 2 pays to network 1 for using network 2's resource, as the case of $\alpha = 0.8$ and $m = 0.6$ in Fig. 3.4) to reach the social optimality. For example, assume that network 1 is a cellular network that has a large coverage area, a large market share, and an average of not very high channel condition to the users. Network 2 is a commercial Wi-Fi service provider that has a small coverage area, a small market share, and an average of excellent channel condition to the users. Then in order to maintain the social optimal user pricing, i.e., keeping the user price of the cellular network π_1^* low enough, the Wi-Fi service provider needs to pay the cellular provider for a file transfer from a cellular user to a Wi-Fi user, as the cellular network is bearing most of the network costs in supporting the communication session. Notice that here we consider the case where the regulator determines the access pricing; it will be a quite different story if the networks themselves optimize the access pricing to maximize their profits.

Observation 5 The social optimal access price increases in the elasticity of the users' utilities.

We also note from Fig. 3.4 that the utility parameter α has a significant impact on the social optimal access pricing. A smaller α (e.g., a higher elasticity) means a higher optimal access price (under the same market share). Since today's wireless networks are becoming more data-centric, we can expect that the overall users' utility functions (determined by the applications) will become more elastic and thus the optimal access price will become higher.

3.3 Deregulated User Pricing and Access Pricing

In the real world, very often, regulators do not always control the market. Instead, the market is deregulated most of the time. Operators have full control over the user prices and access prices. Only when undesirable outcome such as disputes and litigation (e.g., litigation between Telecom Corporation of New Zealand and Clear Communications in 1994) appears will then the regulator mitigates the situation. A deregulated scenario is similar to the scenario we described in the section 3.2 except that here we consider no regulators setting social optimal access prices. Similarly, we model this as a three-stage decision process as before such that operators determine their access prices at the first stage. Then, operators determine user prices charging their users at the second stage. Finally, each users determines his/her data rate demand based on the user pricing announced by the operator. Once again, we will analyse the problem by Backward Induction. We begin with Stage III.

3.3.1 User's Optimal Data Rate Demand in Stage III

Assuming the user pricing and access pricing is fixed at Stage I and II, the optimal data rate demand by an operator j's subscriber is given as $\pi_j^{-1/\alpha}$.

3.3.2 Operators' User Pricing in Stage II

Assume the operators already fix their access prices at the first stage, they will then determine the best response user prices at the second stage with the access price being fixed in Stage I. With the same profits formulated in the previous section, the best response user price would be given the same as in Proposition 2, i.e.:

$$\pi_1^*(a_2) = \frac{c_1 B^u(m) + m c_1 B^d(m) + a_2 B^d(1-m)}{m(1-\alpha)}, \tag{3.4}$$

and

$$\pi_2^*(a_1) = \frac{c_2 B^u(1-m) + (1-m) c_2 B^d(1-m) + a_1 B^d(m)}{(1-m)(1-\alpha)}. \tag{3.5}$$

3.3.3 Operators' Access Pricing in Stage I

By the best response user prices formulated in Equation 3.4 and 3.5, we substitute them back to the profits R_1 and R_2 as defined in Section 3.2. Again, it is easy to check that R_1 and R_2 are quasi-concave in access pricing a_1 and a_2. By applying first order condition, we could obtain the best response access prices ${a_1}^*$ and ${a_2}^*$ as:

$$ {a_1}^* = \frac{\alpha c_2 B^u(1-m^*) + \alpha(1-m^*) c_2 B^d(1-m^*) + (1-m^*) c_1 B^d(m^*)}{(1-m^*)\,(1-\alpha)\, B^d(m^*)} \tag{3.6}$$

and

$${a_2}^* = \frac{\alpha c_1 B^u(m) + \alpha m c_1 B^d(m) + m c_2 B^d(1-m)}{m(1-\alpha) B^d(1-m)}. \tag{3.7}$$

We first look at the interaction between the profit maximizing access price that operator 1 has to pay to operator 2 with operator 1's market share. As observed from Fig. 3.5, the higher the market share of operator 1, the larger the access price that operator 1 will have to pay to operator 2. This suggests that the increase in market share of a network not only increases the number of users and hence the total internal cost in serving the customers, but also the access price charged by the other network.

Fig. 3.6 shows a plot of operator 1's profit maximizing user price against her market share. We could observe that the user price increases as the cost c_1 increases. Higher user price is needed as a compensation to an increase in service charge.

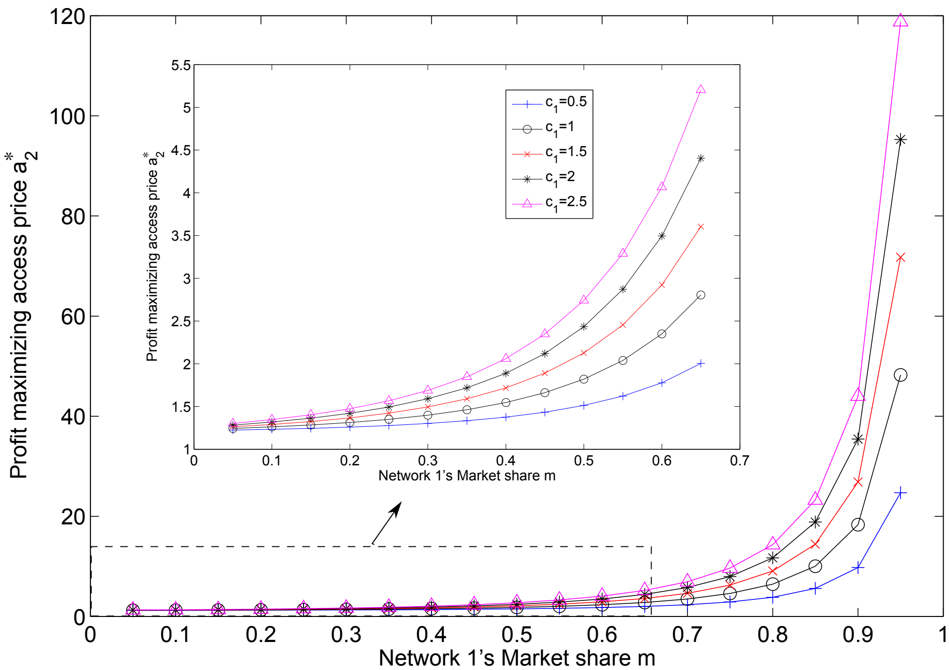

Figure 3.5: Plot of profit maximizing access price that operator 1 will pay to operator 2 against operator 1's market share under different cost c_1.

Also, we can observe that the user price first increases with the market share and then decreases. As the cost c_1 increases, the turning point at which user price changes from decreasing with market share to increasing with market share appear at a smaller market share. For example, at $c_1 = 1$, the turning point appears at $m = 0.6$. At $c_2 = 2.5$, the turning point appears at $m = 0.5$. This is because when the market share is relatively large, the increase in market share means the more users an operator has to serve. The new users are much further away and hence the average bandwidth cost per users increases. Also, the access price charged by the other operator increases as the market share increases. User prices increases as a result.

When the market share is small, an increase in market share means that more users to be served. Even though the access price and average cost in serving a user increases, the increase in user price is offset by the fact that less access payment that needs to be paid to the other network.

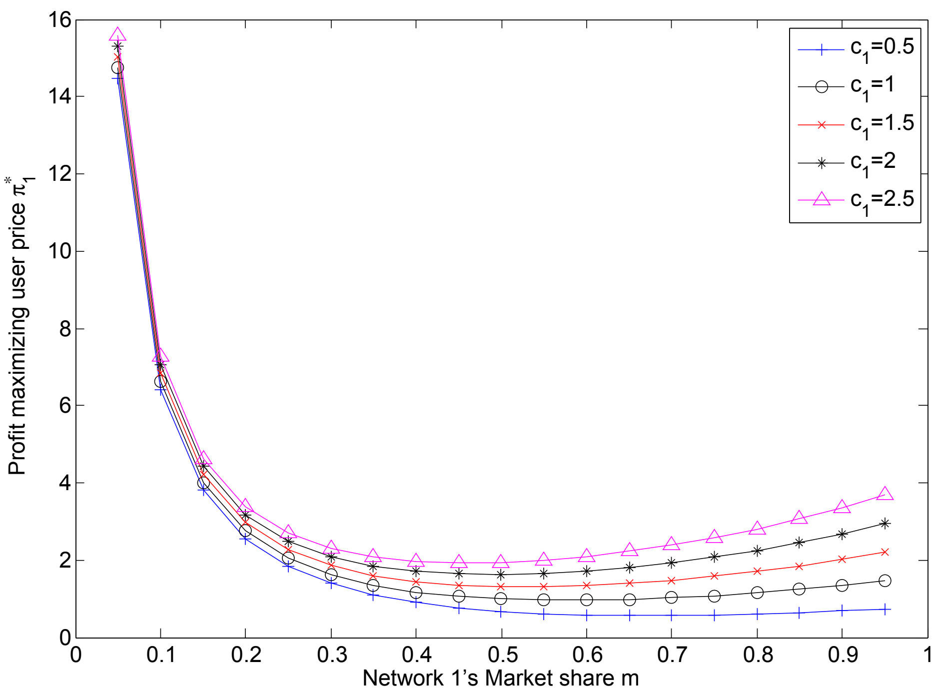

Figure 3.6: Plot of Operator 1's profit maximizing user price against the operator's market share under different cost c_1.

Chapter 4

Full Coverage Two Base Stations Model

In this chapter, we consider a full coverage scenario that both operators provide full coverage. We first look at the regulated market and formulate the social welfare. We study the social optimal scenario under different costs. Then we look at the deregulated market and discuss the equilibrium scenario resulted. Finally, we will compare the social welfare and profits obtained compared to that in the fixed coverage market model.

4.1 Full Coverage Wireless Market Model

In real life, it is likely that base stations from both operators cover the entire area. Consider the cellular service nowadays. In many cases, signals from more than one networks could be received in the office or in the lecture room. In this case, it is the users' choice over which network providers to choose. It happens especially in a populated region such as the city of a country. We propose the full coverage market model to model this situation.

Unlike to the fixed coverage model, each of the network covers all the users located in $[0, 1]$. Initially, operators will send out pilot signals from their base-stations telling users about the existence of the network service as mentioned by [28] and [29]. The strength of the signal as received by users depends on the distance to the operator's base station and the power of the pilot signal. For a pilot signal sent out by an operator, the closer to the base-station of this operator, the stronger the pilot signal strength is. Operators set the strength of each of their pilot signal. For any possible pilot signal strength that each operators sets, the boundary user located at m, where m is between $[0, 1]$, receives the same pilot signal from both operators. Hence, the users located between $[0, m]$ will receive better pilot signal from operator 1 while users between $[m, 1]$ will receive better pilot signal from operator 2. Note that users in $[m, 1]$ could receive the pilot signal from operator 2 in this model.

Prior to the users' transmission, each operators announces the user price that charges her end users via the control channel. Since each of the users could only be connected to either one of the networks, users will decide which operators to connect to based on the user price charged by each of the operators as well as the strength of the pilot signal received. This selection process is described in the next section.

4.2 Users' choice of service providers

For any user i, he/she will prefer operator j to operator $-j$ if and only if:

$$r_i(y_{ij}^*(\pi_j), \pi_j) > r_i(y_{i-j}^*(\pi_{-j}), \pi_{-j}),$$

which happens when $\pi_j < \pi_{-j}$. Since the user price charged by an operator is the same for all users, all users will prefer network j to network $-j$ when $\pi_j < \pi_{-j}$. We assume the operators do have enough resources to serve the entire market. This could be ensured by buying enough spectrum together with the enhanced technology to improve the spectral efficiency. As a result, operators could handle the service demand even if it is from the entire market.

On the other hand, a user will be indifferent between operator j and $-j$ if and only if:

$$r_i(y_{ij}^*(\pi_j), \pi_j) = r_i(y_{i-j}^*(\pi_{-j}), \pi_{-j}),$$

which happens when $\pi_j = \pi_{-j}$. In this case, the user will connect to the base station that with the strongest pilot signal strength as received by the user.

This wireless operator selection method happens in everyday life. Consider the case where there are two WiFi services available. One of them charges and the other one is free. Users will choose the one that provides free access. If both services do not charge, users will connect to the one with the stronger pilot signal.

4.3 Social Optimal User Pricing

The users' choice of service providers suggests that there could be three possible cases: i) Operator 1 only (ii) Operator 1 and Operator 2 (iii) Operator 2 only, the regulator could obtain the optimal social welfare. To a regulator, their concern is to maximize the overall social welfare regardless of monopoly market or both networks serving the market. By comparing the social welfare achieved in each of these three cases, the regulator could obtain the optimal social welfare. Note that in case (ii), in addition to fixing the user price at social optimal value, the regulator will have to fix the social optimal coverage which results in the highest social welfare.

Case (i) - Operator 1 only

Only operator 1 serving the market implies that user price of operator 1 is smaller than that of operator 2. In a fully regulated environment, this means regulator fixes operator 1's user price at a social optimal level and that of operator 2 at any value above that of operator 1. The social optimal price of network 1 is the solution to the following optimization problem:

$$\begin{aligned} \max_{\pi_1} \quad & SW = \frac{\pi_1^{1-1/\alpha}}{1-\alpha} - \pi_1^{-1/\alpha}\left(c_1 B^u(1) + c_1 B^d(1)\right) \\ \text{s.t.} \quad & \pi_1 > \pi_2. \end{aligned}$$

It can be easily checked that the objective function is concave in π_1. The social optimal user price in this case would be:

$$\pi_1{}^S = c_1 B^u(1) + c_1 B^d(1); \pi_1^S < \pi_2^S.$$

Case (ii) - Operator 1 and Operator 2

When both operators serving the users, the user prices must be the same. To realize such a case, operators would set the two user prices equal and at the same social optimal level.

$$\begin{aligned} \max_{\pi_1,\pi_2} \quad & SW = m\frac{\pi_1{}^{1-1/\alpha}}{1-\alpha} + (1-m)\frac{\pi_2{}^{1-1/\alpha}}{1-\alpha} \\ & \quad - \pi_1{}^{-1/\alpha} f(m, c_1, c_2) - \pi_2{}^{-1/\alpha} f(1-m, c_1, c_2) \\ \text{s.t.} \quad & \pi_1 = \pi_2. \end{aligned}$$

The function $f(m, c_1, c_2)$ has been defined previously in 3.1. The social optimal user price in this case would be given as:

$$\pi_1{}^S = \pi_2{}^S = f(m, c_1, c_2) + f(1-m, c_2, c_1)$$

Case (iii) - Operator 2 only

$$\begin{aligned} \max_{\pi_2} \quad & SW = \frac{\pi_2^{1-1/\alpha}}{1-\alpha} - \pi_2^{-1/\alpha}\left(c_2 B^u(1) + c_2 B^d(1)\right) \\ \text{s.t.} \quad & \pi_1 > \pi_2. \end{aligned}$$

After applying the first order condition, the social optimal user price in this case would be:

$$\pi_2{}^S = c_2 B^u(1) + c_2 B^d(1); \pi_2^S < \pi_1^S.$$

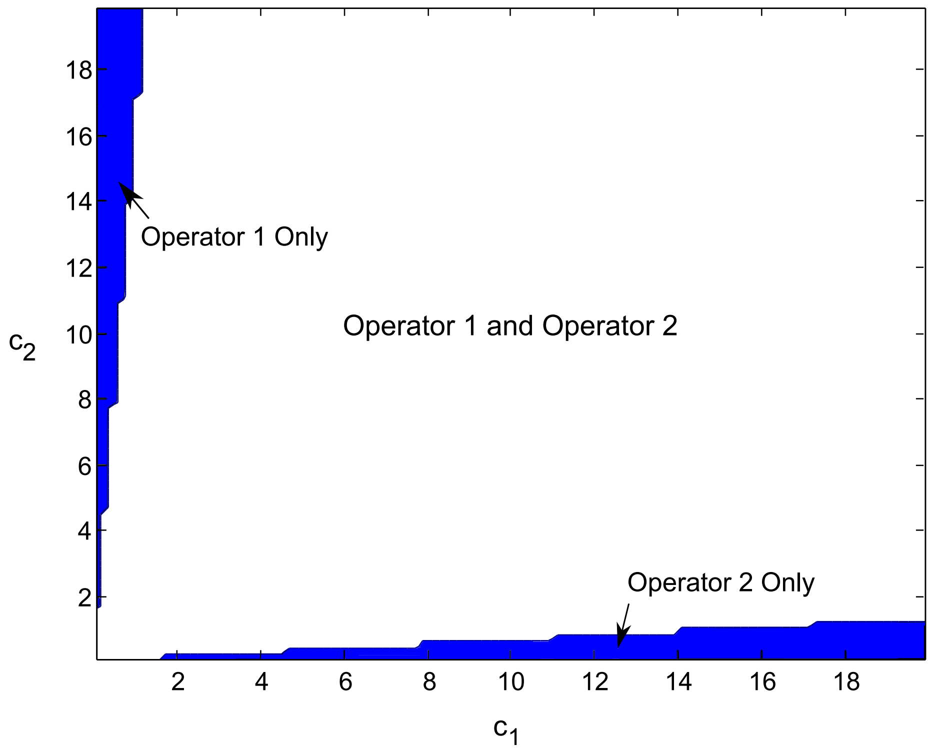

Figure 4.1: Plot of different resulted cases (i.e. only network 1 serving, or only network 2 serving, or both networks serving) under different cost c_1 and c_2 in the two base-stations full coverage model.

4.3.1 Numerical study

We will perform numerical study to find the market split between the operators that result in the optimal social welfare under different costs. Then we compare the social welfare between the fixed coverage wireless model and full coverage wireless model.

Market split that result in optimal social welfare under different costs

By comparing the social welfare obtained in these three cases, a regulator can figure out the optimal market share and how to regulate the market accordingly. In other words, we will compare the social welfare obtained in case (i),

(iii) and the maximum social welfare that obtained in case (ii).

Fig. 4.1 shows the resulted cases under different cost c_1 and c_2. It can be observed that when the cost of a network is significantly lower compared to the other one's, the market is better off to have only the lower-cost operator serving the customers. Most of the time, it is better to have set the situation such that both networks are serving. This is because in the case where the two per unit bandwidth cost are significantly different, the lower-cost operator can serve the market with a much lower cost even for customers further away from her base station. When the two per unit bandwidth costs are similar, each operators serve the users who are more efficiently and less costly served by her.

Social welfare comparison between fixed coverage model and full coverage model

Figure 4.2 shows the plot of Network 1's market share at fixed coverage that results in a better social welfare than the optimal social welfare in full coverage case under different cost c_1. The region bounded by the two lines are the region of interest. Note that in the full coverage case, there is only one optimal market share m that leads to the optimal social welfare and it is not shown in this figure. The figure compares the social welfare of different network 1's coverage to the optimal social welfare obtained in the full coverage case. The optimal market share m in the full coverage is obtained by comparing the social welfare obtained in each of the market share [0,1].

It can be seen that having full coverage may not always lead to a better social welfare. It is because in full coverage case, the user prices are constrained to be the same, which may not be the best in terms of social optimality. By carefully designing the coverage in fixed coverage case, a higher social welfare could be obtained. However, the fixed coverage is constrained by the geographical factors and is not easy to change. In most of the time, full

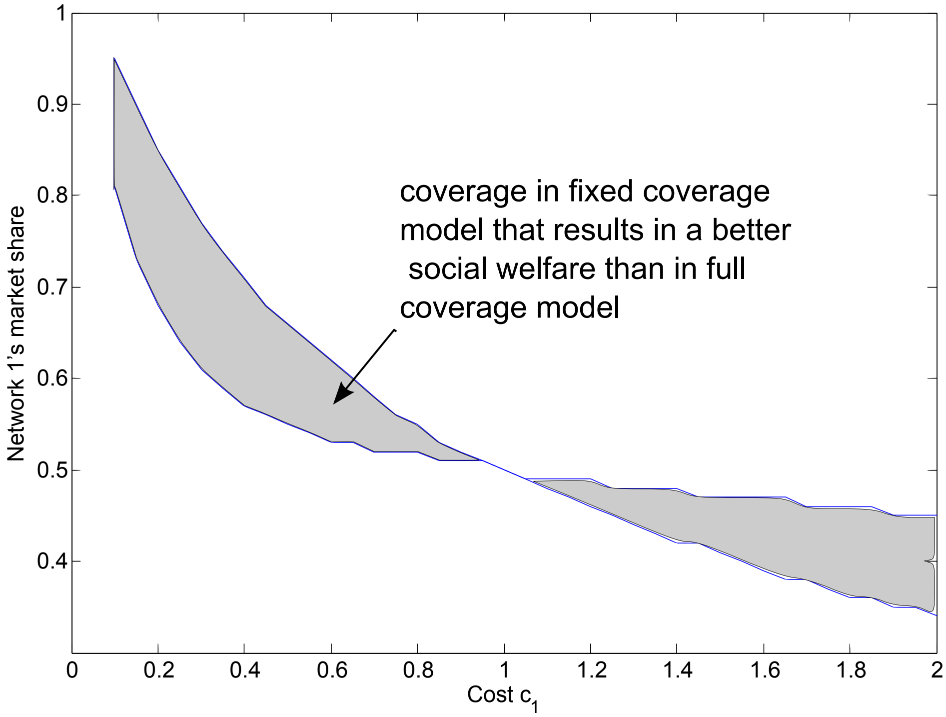

Figure 4.2: Network 1's market share which results in a better social welfare in fixed coverage case than in full coverage. The comparison is between the social welfare obtained in the fixed coverage to the optimal social welfare in the full coverage. The optimal market share in full coverage is not shown in the figure.

coverage would lead to a social welfare improvement.

4.4 Deregulated case - Profit-maximizing access price and user price

In deregulated case under full coverage scenario, each operators has full control over access price, user price, and the strength of her pilot signal. Based on the user price announced by each operator and the received pilot signal strength, each user selects which operator to connect to and the demand rate from the network operator. At each cost c_1 and c_2, there is an equilibrium. There are three possible equilibria:

1. $\pi_1^* < \pi_2^*$. Users will all choose operator 1.

2. $\pi_1^* > \pi_2^*$. Users will all choose operator 2.

3. $\pi_1^* = \pi_2^*$. Some users (i.e. users located within $[0, m)$) will choose operator 1 and some (i.e. users located within $[m, 1]$ will choose operator 2.

In case (1) and (2), the operator who announces a higher user price, say operator A, will have incentives to lower its user price to a value smaller than that of another operator, operator B, if she could by this mean capture the market and attain positive profit. Operator B will in turn set her user price smaller than the new user price set by operator A to attain positive profit and capture the market. The user price reduces until either one of the operator do not have incentive to further lower her user price. This happens when the user price sets the operator's profit to be zero.

In case (1), the equilibrium user price will be slightly smaller than operator 2's user price that results in zero profit (which means operator 2 has no incentive to further reduce its user price to gain non-positive profit). On the other hand, in case (2), the equilibrium user price will be slightly smaller than the operator 1's user price. Operator 1's profit becomes zero in this case.

Case (3) could be an equilibrium since both operators are charging at the corresponding profit-maximizing user price. Any change in the user price will force one operator losing market while the other operator captures the whole market.

The lower-cost operator has more control over the market split. If the lower cost operator sets her user price at her monopoly value, the higher cost operator cannot further under-cut her user price. The higher cost operator will lose market in the price-undercutting process described earlier in case (1) and (2). However, being the monopoly does not necessarily result in maximum profit. The lower-cost operator may gain better profit in case (3). As a result, she will compare the profits obtained in the duopoly case and in the monopoly case and decide which market split is the best in terms of her profit.

Next, we will formulate the profits of each operator in each of the three equilibrium cases. From the profits formulated, we will study the access price and user price in each of the cases.

Case (i) - Operator 1 Only

Operator 1 will set the user price to at which operator 2 has no further incentive to undercut operator 1's user price. In this case, operator 1's profit becomes:

$$\pi_1^{1-1/\alpha} - \pi_1^{-1/\alpha} c_1 B^u(1) - \pi_1^{-1/\alpha} c_1 B^d(1).$$

The monopoly user price of operator 2 that operator 2 has zero profit: $\pi_2 = c_2 B^u(1) + c_2 B^d(1)$.

Operator 1 will set a user price that is smaller than this value, i.e.: $\pi_1^* = c_2 B^u(1) + c_2 B^d(1) - \delta_1$, where δ_1 is small and determined by the individual operator.

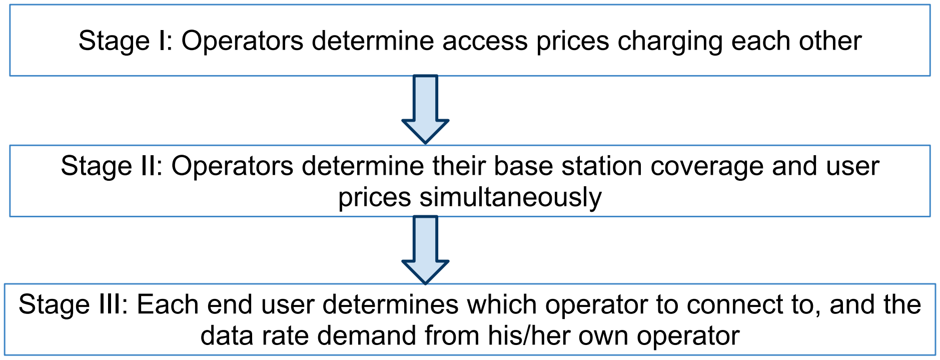

Figure 4.3: Three-stage dynamic game in full coverage market model: the operators determine access price, then user price, and the strength of pilot signal; each users determines which operators to connect to and the corresponding data rate demand.

Case (ii) - Operator 2 Only

Similarly, operator 2 will set the user price to at which operator 1 has no further incentive to undercut operator 2's user price. In this case, operator 2's profit becomes:

$$\pi_2^{1-1/\alpha} - \pi_2^{-1/\alpha} c_2 B^u(1) - \pi_2^{-1/\alpha} c_2 B^d(1).$$

The monopoly user price of operator 1 that she has zero profit: $\pi_1 = c_1 B^u(1) + c_1 B^d(1)$.

Hence, Operator 2 will set a user price that is smaller than this value, i.e.: $\pi_2^* = c_1 B^u(1) + c_1 B^d(1) - \delta_2$, where δ_2 is small and determined by the individual operator.

Case (ii) - Operator 1 and Operator 2

This could be modelled as a three stage game as in Fig. 4.3. The game setting is similar as in the fixed market model except operators could determine both the user prices and each one's coverage in Stage II and users will choose which operator to connect to and their corresponding data rate demand in Stage III.

Users' choice of operators and rate demand in Stage III

In the equilibrium, the two user prices are equal and for the boundary set at m, users located within $[0, m]$ will connect to operator 1 while users located within $[m, 1]$ will connect to operator 2.

Similar as before, an operator 1's user will demand the data rate $\pi_1^{-1/\alpha}$. An operator 2's user will demand data rate $\pi_2^{-1/\alpha}$.

Operator' User Pricing and pilot signal strength in Stage II

The best response user prices would be the same as before and are dependent of the strength of the pilot signal in terms of m^*. The best response user prices are given as:

$$\pi_1^*(a_2, m^*) = \frac{c_1 B^u(m^*) + m^* c_1 B^d(m^*) + a_2 B^d(1-m^*)}{m^*(1-\alpha)}, \tag{4.1}$$

and

$$\begin{aligned}\pi_2^*(a_1, m^*) = & \frac{c_2 B^u(1-m^*) + (1-m^*) c_2 B^d(1-m^*)}{(1-m^*)(1-\alpha)} \\ & + \frac{a_1 B^d(m^*)}{(1-m^*)(1-\alpha)}.\end{aligned} \tag{4.2}$$

The access prices are assumed to be fixed at Stage I. The pilot signals of each operator is set such that the boundary m is set such that $m = m^*$ where m^* equates the two user prices. That is, m^* is the solution to the following equation:

$$\begin{aligned}& \frac{c_1 B^u(m^*) + m^* c_1 B^d(m^*) + a_2 B^d(1-m^*)}{m^*} \\ = & \frac{c_2 B^u(1-m^*) + (1-m^*) c_2 B^d(1-m^*) + a_1 B^d(m^*)}{(1-m^*)}.\end{aligned} \tag{4.3}$$

Operator' Access Pricing in Stage I

The access price set at stage I is computed from the same objective function as in fixed coverage market model and hence is the same as before, i.e.:

$$a_1{}^* = \frac{\alpha c_2 B^u(1-m^*) + \alpha(1-m^*) c_2 B^d(1-m^*) + (1-m^*) c_1 B^d(m^*)}{(1-m^*)\,(1-\alpha)\,B^d(m^*)} \tag{4.4}$$

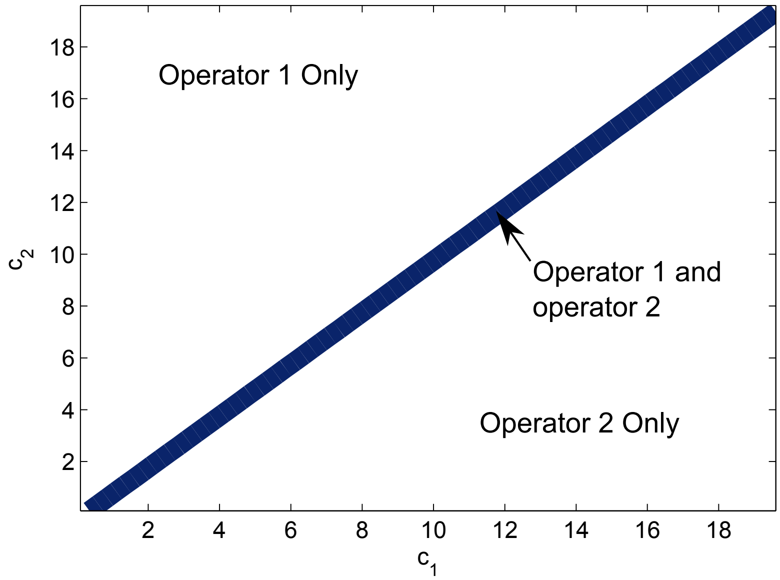

Figure 4.4: Equilibrium market split under different cost c_1 and c_2. The shaded region represents the duopoly scenario while the non-shaded area represent a monopoly scenario. This shows that when c_1 and c_2 are the same, the two operators serve the market together.

and

$$a_2{}^* = \frac{\alpha c_1 B^u(m) + \alpha m c_1 B^d(m) + m c_2 B^d(1-m)}{m(1-\alpha)B^d(1-m)}. \tag{4.5}$$

4.4.1 Numerical Study

We will perform numerical study to investigate the equilibrium market split under different per unit bandwidth cost c_1 and c_2.

Equilibrium market split

Fig. 4.4 shows the equilibrium cases under different cost c_1 and c_2. It can be observed that when the two costs are relatively the same, the equilibrium would be when the two operators operating together and the user prices are the same. Otherwise, the operator with a lower per unit bandwidth cost could capture the market by setting a user price that is smaller than the user price that makes the other operator obtains zero profit.

Chapter 5

Full Coverage Three Base Stations Model

In this chapter, we consider a three base stations full coverage scenario that one of the operators set up a new base station. Similar as before, we first look at the regulated market and formulate the social optimal scenario. We study the social optimal market split under different costs. Then we look at the deregulated market and discuss the equilibrium scenario resulted. Finally, we will compare the social welfare and profits obtained in the three base stations full coverage market model compared to that in the two base stations full coverage market model.

5.1 Three base-stations Full Coverage Market Model

In practice, operators often have many base stations. Investing in building new base stations incur extra initial investment cost, but may be gaining benefits later on. For instance, the new base-station could be built in a region that could barely receive the service so as to fight for more subscribers. Also, the new base station could lower the average bandwidth cost per customers since some further away users could now be served by the closer new base station.

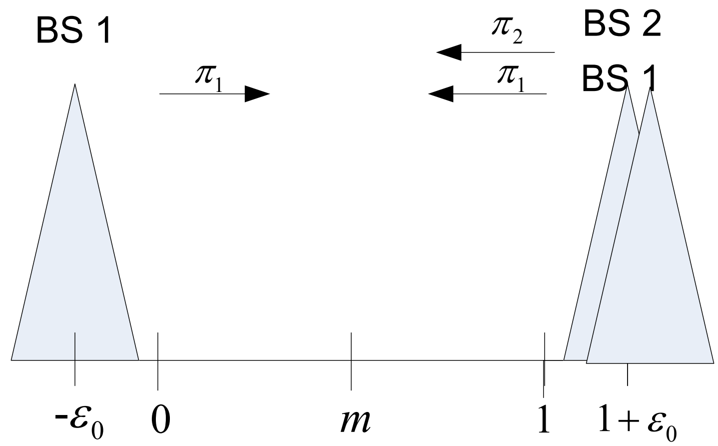

Figure 5.1: An illustration of the three base-stations market model. All base-stations have full coverage. Each user connects to his/her preferred base-station based on the user prices π_1 and π_2 announced by the base-stations, and the received pilot signal strength.

In this section, we consider a three base stations model at which one of the operators invest in setting up a new base station and place it at the opposite end. We consider operator 1 builds one more base station. The new base station is set up and co-located with the rival's base station at the location $1 + \epsilon_0$. Fig. 5.1 shows an illustration of the three base-stations setting.

We will compare the social welfare and profits obtained in the case of a two base stations against that in three base stations. To facilitate the comparison, we first of all analyse without considering the fixed cost incurred in building base stations. We will leave the analysis involving the cost of base stations as an extension to this work.

The three base stations case is similar to the two base stations full coverage market model at which each users could choose their operators and data rate demand. If the user price π_1 is lower than π_2, all users prefer Operator 1 to Operator 2. Operator 1 will serve the users in an efficient way such that half of the users will be connected operator 1's left hand side base station while another half of the users will be connected to operator 1's right hand side base station. Similarly, if the user price π_2 is lower than π_1, all users will choose operator 2.

As before, only when the two user prices are equal will the users be indifferent between the two networks. In due case, both operators will be serving the customers. Operators will choose their best pilot signal strength such that user at the boundary m will be indifferent between the three base-stations. Users located at $[0, m]$ will receive better pilot signal from operator 1's left hand side base-station while Users located at $[m, 1]$ will receive better pilot signal from the right hand side base-stations and are indifferent between operator 1 and operator 2. In this case, each of the users will have half probability of being served by operator 1 and half probability of being served by operator 2.

5.2 Social Optimal User Prices

There are three possible scenario as in the two base-stations market model - (i) Operator 1 Only (ii) Both Operator 1 and Operator 2 (iii) Operator 2 Only. We will formulate the social welfare in all these three cases. By comparing the social welfare in these three cases, regulator can obtain the optimal social welfare. We will formulate the social welfare obtained in each of these cases.

Operator 1 Only

When only operator 1 serving the whole market, the service load will be shared between the left and the right base-station. In other words, half of the users will be served by the left-hand-side base-station while another half of the users will be served by the right-hand-side base-station. The social welfare in this case equals:

$$\frac{{\pi_1}^{1-1/\alpha}}{1-\alpha} - {\pi_1}^{-1/\alpha}\left(2c_1 B^u(\frac{1}{2}) + 2c_1 B^d(\frac{1}{2})\right), \tag{5.1}$$

while π_2^S is set at any value larger than π_1^S

As a result, the social optimal user price π_1^S would be: $2c_1B^u(\frac{1}{2})+2c_1B^d(\frac{1}{2})$

Operator 1 and Operator 2

To operator 1, the cost in realizing a session originated from user located d_i away from her left-hand-side base-station is the same as a session originated from user located d_i away from the right-hand-side base-station:

$$\pi_1{}^{-1/\alpha}\left(\frac{c_1}{g^u(d_i)}+\int_0^m \frac{c_1}{g^d(r)}dr+\frac{1}{2}\int_0^{1-m}\frac{c_1}{g^d(r)}dr\right)$$
$$+\,\pi_1{}^{-1/\alpha}\frac{1}{2}\int_0^{1-m}\frac{c_2}{g^d(r)}dr$$

To realizing a session originated from user i from d_i away from operator 2's base-station, operator 2 incurred a cost of:

$$\pi_2{}^{-1/\alpha}\left(\frac{c_2}{g^u(d_i)}+\frac{1}{2}\int_0^{1-m}\frac{c_2}{g^d(r)}dr+\frac{1}{2}\int_0^{1-m}\frac{c_1}{g^d(r)}dr\right)$$
$$+\,\pi_2{}^{-1/\alpha}\left(\int_0^m\frac{c_1}{g^d(r)}dr\right)$$

As a result, the social welfare when both operators are serving would be:

$$\begin{aligned}
SW = &\left(\frac{1+m}{2}\right)\frac{\pi_1{}^{1-1/\alpha}}{1-\alpha}+\left(\frac{1-m}{2}\right)\frac{\pi_2{}^{1-1/\alpha}}{1-\alpha}\\
&-\pi_1{}^{-1/\alpha}\left(c_1B^u(m)+mc_1B^d(m)\right)\\
&-\pi_1{}^{-1/\alpha}\left(\frac{m}{2}c_1B^d(m)+\frac{m}{2}c_2B^d(1-m)\right)\\
&-\pi_1{}^{-1/\alpha}\left(\frac{1}{2}c_1B^u(1-m)+\frac{1-m}{4}c_1B^d(1-m)\right)\\
&-\pi_1{}^{-1/\alpha}\left(\frac{1-m}{4}c_1B^d(m)+\frac{1-m}{4}c_2B^d(1-m)\right)\\
&-\pi_2{}^{-1/\alpha}\left(\frac{1}{2}c_2B^u(1-m)+\frac{1-m}{4}c_2B^d(1-m)\right)\\
&-\pi_2{}^{-1/\alpha}\left(\frac{1-m}{4}c_1B^d(1-m)+\frac{1-m}{4}c_1B^d(m)\right)
\end{aligned}\tag{5.2}$$

By maximizing social welfare subject to the constraint that the two user prices are the same, we obtain the social optimal prices to be:

$$\begin{aligned}
{\pi_1}^S = {\pi_2}^S &= c_1 B^u(m) + m c_1 B^d(m) \\
&+ \frac{m}{2} c_1 B^d(m) + \frac{m}{2} c_2 B^d(1-m) \\
&+ \frac{1}{2} c_1 B^u(1-m) + \frac{1-m}{4} c_1 B^d(1-m) \\
&+ \frac{1-m}{4} c_1 B^d(m) + \frac{1-m}{4} c_2 B^d(1-m) \\
&+ \frac{1}{2} c_2 B^u(1-m) + \frac{1-m}{4} c_2 B^d(1-m) \\
&+ \frac{1-m}{4} c_1 B^d(1-m) + \frac{1-m}{4} c_1 B^d(1-m)
\end{aligned} \tag{5.3}$$

Operator 2 Only

In this case, π_1^S is larger than π_2^S. The social welfare formulation is as same as in the two base-stations case. Hence, the social optimal user price π_2^S is given as:

$$ {\pi_2}^S = \int_0^1 \frac{c_2}{g^u(r)} dr + \int_0^1 \frac{c_2}{g^d(r)} dr; \pi_2^S < \pi_1^S. \tag{5.4}$$

5.2.1 Numerical Study

We will perform numerical study to find the market split that results in the optimal social welfare under different costs. Then we compare the social welfare between the fixed coverage wireless model and full coverage wireless model.

Market split results in optimal social welfare under different costs

By computing and comparing the resulted social welfare in each of these cases, regulator knows which case result in a higher social welfare and hence set the user prices at the social optimal value. Fig 5.2 shows the resulted cases under different per unit bandwidth cost c_1 and c_2 in the three base-stations case. It can be noticed that when c_1 is significantly larger than c_2, the market is

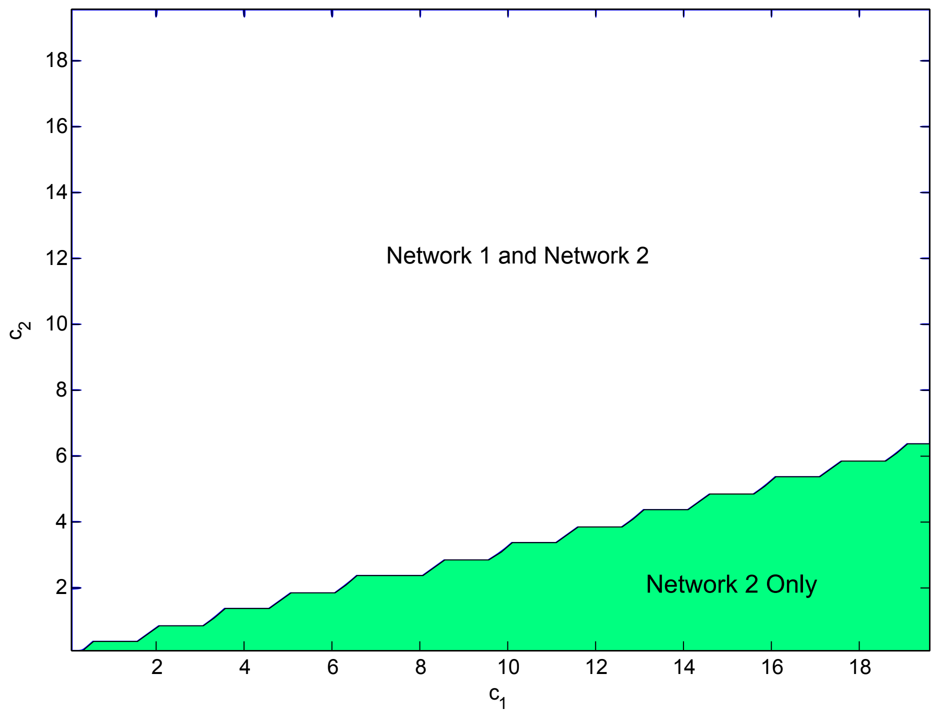

Figure 5.2: Plot of different resulted cases (i.e. only network 1 serving, or only network 2 serving, or both networks serving) under different cost c_1 and c_2 in three base-stations market model. Unlike to two base stations model, the market is not better off by only letting network 1 to serve as c_2 increases.

better off to have only network 2 serving. This observation is similar as before. However, this will be more easily triggered by an increase in cost c_1 than in the two base-stations case. For example, if $c_1 = 8$ and $c_2 = 2$, the market is better off if only operator 2 serves under the three base stations case. However, in the two base stations case, the market is better off if operator 1 and operator 2 co-exist. This is because in the three base stations market model, operator 1 is serving more users than in two base stations case. An increase in operator 1's cost increases operator 1's service burden more in three base stations case.

On the other hand, unlike to the two base stations scenario, the market is not better off by only letting network 1 to serve as c_2 increases. For example, in two base stations case, the market will be better off by allowing only Operator 1 to serve the market when $c_1 = 1$ and $c_2 = 10$. But in three base stations

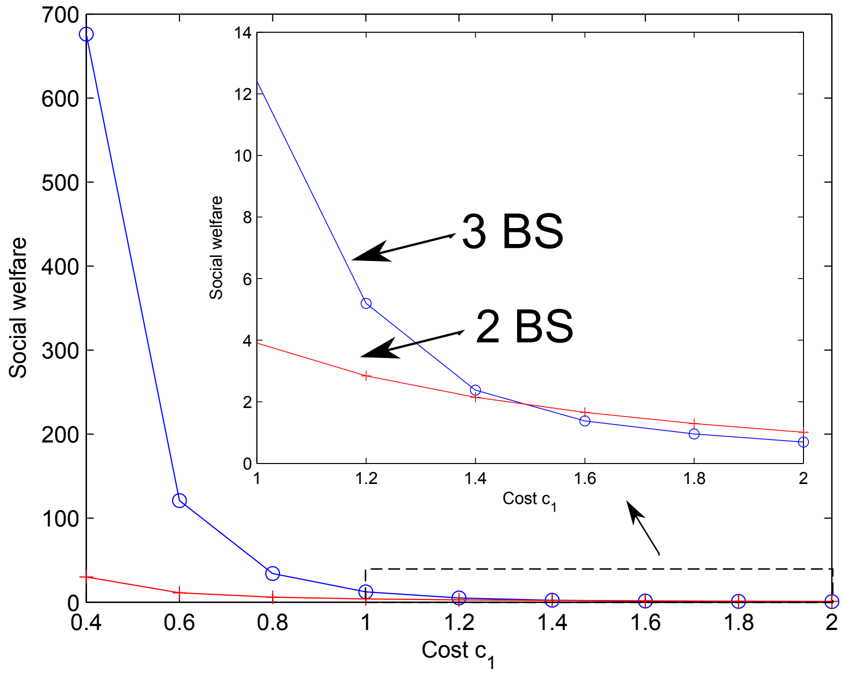

Figure 5.3: Plot of social welfare in regulated case in the three base-stations market model and in the two base-stations market model. Three base-stations case results in a better social welfare when the cost c_1 is low and worse than the two base-stations case when c_1 gets larger. Note that c_2 is fixed at 1 and the market is split between operator 1 and operator 2 (i.e. not a monopoly case).

case, the market is better off by allowing both operators to serve the market. This is because in three base-stations market model, operator 2 is serving less customers since some of the users are served by both base stations of operator 1. The increase in c_2 hence impact less on the service burden of operator 2.

Comparison between the maximum social welfare obtained in two base stations market and three base stations market

Figure 5.3 plots the highest possible social welfare obtained in the two base-stations full coverage case and that in the three base-sttaions full coverage case. Note that the cost c_1 varies while c_2 is fixed at 1.

It could be observed that when the cost c_1 is low, building one more base-stations leads to a higher social welfare. On the other hand, when the cost c_1 is large, building one more base-stations leads to a smaller social welfare. This is because running one more base-stations means the new base-stations serving some of customers who are previously from the other network, assuming both operators are servicing the customers. When network 1's cost is small, this means that more customers are being served by a network operating at the lower cost. As a result, higher social welfare is obtained. If network 1's cost is large, more customers are being served by the more costly network. This lowers the overall social welfare.

5.3 Deregulated scenario

In the deregulated scenario where each of the operators wants to control and maximize her own profit, operators set access prices, user prices and pilot signal strength. Similar as before, there could be three possible equilibrium:

1. $\pi_1^* < \pi_2^*$. Users will all choose operator 1.
2. $\pi_1^* > \pi_2^*$. Users will all choose operator 2.
3. $\pi_1^* = \pi_2^*$. Some users (i.e. users located within $[0, m)$) will choose operator 1 and some (i.e. users located within $[m, 1]$ will choose operator 2.

We will look at the profit, user prices, access prices, boundary m set in each of the equilibrium.

Operator 1 Only

Operator 1 will set the user price smaller than the smallest user price that operator 2 can accept for a non-negative profit. Such user price is such that operator 2 has zero profit, i.e. $c_2B^u(1)+c_2B^u(1)$. The profit that operator 1 will get is:

$$\pi_1^{1-1/\alpha}-\frac{1}{2}\pi_1^{-1/\alpha}c_1B^u(\frac{1}{2})-\frac{1}{2}\pi_1^{-1/\alpha}c_1B^d(\frac{1}{2}). \tag{5.5}$$

Operator 2 Only

Operator 2 will set the user price smaller than the user price that operator 2 receives zero profit, i.e. $\frac{1}{2}c_1B^u(\frac{1}{2})+\frac{1}{2}c_1B^d(\frac{1}{2})$. The profit that operator 2 will get is as in the two base stations case, i.e.:

$$\pi_2^{1-1/\alpha}-\pi_2^{-1/\alpha}c_2B^u(1)-\pi_2^{-1/\alpha}c_2B^d(1).$$

Operator 1 and operator 2

Again, we will analyse the problem by a three-stage Stackelberg game. As before, we will firstly compute the best response access prices, pilot signal strength in terms of m and user prices.

The profit of network 1 made from an intra-network traffic originated from the left-hand-side base-station would be the same as the intra-network traffic originated from the right-hand-side base-station and is given as:

$$\begin{aligned}&{\pi_1}^{1\text{-}\frac{1}{\alpha}}-c_1{\pi_1}^{\text{-}\frac{1}{\alpha}}\frac{1}{g^u(d_i)}-c_1{\pi_1}^{\text{-}\frac{1}{\alpha}}\int\limits_0^m\frac{1}{g^d\left(r\right)}dr\\&-c_1{\pi_1}^{\text{-}\frac{1}{\alpha}}\int\limits_0^{1-m}\frac{1}{g^d\left(r\right)}dr\times\frac{1}{2}-a_2{\pi_1}^{\text{-}\frac{1}{\alpha}}\int\limits_0^{1-m}\frac{1}{g^d\left(r\right)}dr\times\frac{1}{2}\end{aligned}$$

For an inter-network traffic originated from network 2 and terminated at network 1's left-hand-side base-station, the profit gained by operator 1 would be:

$$\left(a_1-c_1\right){\pi_2}^{\text{-}\frac{1}{\alpha}}\int\limits_0^m\frac{1}{g^d\left(r\right)}dr$$

For an inter-network traffic originated from network 2 and terminated at network 2's right-hand-side base-station, the profit gained by operator 2 would be:

$$\frac{1}{2}\left(a_1-c_1\right)\pi_2{}^{-\frac{1}{\alpha}}\int\limits_0^{1-m}\frac{1}{g^d\left(r\right)}dr$$

As a result, the total profit of operator 1 is given as:

$$\begin{aligned}&\left(\frac{1+m}{2}\right)\pi_1{}^{1-\frac{1}{\alpha}}-\left(B^u(m)+\frac{1}{2}B^u(1-m)\right)c_1\pi_1{}^{-\frac{1}{\alpha}}\\&-\left(\frac{1+m}{4}B^d(m)+\left(\frac{1+m}{4}\right)B^d(1-m)\right)c_1\pi_1{}^{-\frac{1}{\alpha}}\\&-\left(\frac{1+m}{4}\right)a_2\pi_1{}^{-\frac{1}{\alpha}}B^d(1-m)\\&+\left(a_1-c_1\right)\left(\left(\frac{1-m}{2}\right)\left(\frac{1+3m}{4}\right)\right)\pi_2{}^{-\frac{1}{\alpha}}B^d(m)\end{aligned}\tag{5.6}$$

To operator 2, the profit made from an intra-network traffic is:

$$\begin{aligned}&\pi_2{}^{1-\frac{1}{\alpha}}-c_2\pi_2{}^{-\frac{1}{\alpha}}\frac{1}{g^u\left(d_i\right)}-c_2\pi_2{}^{-\frac{1}{\alpha}}\int\limits_0^{1-m}\frac{1}{g^d\left(r\right)}dr\frac{1}{2}\\&-a_1\pi_2{}^{-\frac{1}{\alpha}}\left(\int\limits_0^{1-m}\frac{1}{g^d\left(r\right)}dr\frac{1}{2}+\int\limits_0^{m}\frac{1}{g^d\left(r\right)}dr\right)\end{aligned}$$

The profit made from an inter-network traffic is:

$$\frac{1}{2}(a_2-c_2)\pi_1{}^{-\frac{1}{\alpha}}\int\limits_0^{1-m}\frac{1}{g^d\left(r\right)}dr$$

Combining these, the total profit of operator 2 would be:

$$\begin{aligned}&\frac{1-m}{2}\pi_2{}^{1-\frac{1}{\alpha}}-\frac{1}{2}c_2\pi_2{}^{-\frac{1}{\alpha}}B^u(1-m)\\&-\frac{1-m}{2}c_2\pi_2{}^{-\frac{1}{\alpha}}B^d(1-m)\frac{1}{2}\\&-\frac{1-m}{2}a_1\pi_2{}^{-\frac{1}{\alpha}}\left(B^d(1-m)\frac{1}{2}+B^d(m)\right)\\&+\left(m+\frac{1-m}{2}\right)\left(\frac{1-m}{2}\right)\frac{1}{2}(a_2-c_2)\pi_1{}^{-\frac{1}{\alpha}}B^d(1-m)\end{aligned}\tag{5.7}$$

By backward induction, we obtain the best-response user prices as:

$$\begin{aligned}\pi_1{}^*\left(m^*, a_2^*\right) =& \frac{c_1 B^u(m) + \frac{1}{2}c_1 B^u(1-m)}{(1-\alpha)\left(\frac{1+m}{2}\right)} \\ &+ \frac{\left(\frac{1+m}{2}\right) c_1 B^d(m) + \left(\frac{1+m}{4}\right) c_1 B^d(1-m)}{(1-\alpha)\left(\frac{1+m}{2}\right)} \\ &+ \frac{\left(\frac{1+m}{4}\right) a_2 B^d(1-m)}{(1-\alpha)\left(\frac{1+m}{2}\right)},\end{aligned} \tag{5.8}$$

and

$$\begin{aligned}\pi_2{}^*\left(m^*, a_1^*\right) =& \frac{c_2\left(\frac{1}{2}B^u(1-m) + \frac{1-m}{4}B^d(1-m)\right)}{(1-\alpha)\left(\frac{1-m}{2}\right)} \\ &+ \frac{\left(\frac{1-m}{2}\left(\frac{1}{2}B_d(1-m) + B_d(m)\right)\right) a_1}{(1-\alpha)\left(\frac{1-m}{2}\right)}.\end{aligned} \tag{5.9}$$

At stage 2, operators set their market share such that the two user prices are equal. That is:

$$\begin{aligned}&\frac{c_1 B^u(m) + \frac{1}{2}c_1 B^u(1-m) + \left(\frac{1+m}{2}\right) c_1 B^d(m)}{\left(\frac{1+m}{2}\right)} \\ &+ \frac{\left(\frac{1+m}{4}\right) c_1 B^d(1-m) + \left(\frac{1+m}{4}\right) a_2 B^d(1-m)}{\left(\frac{1+m}{2}\right)} \\ =& \frac{c_2\left(\frac{1}{2}B^u(1-m) + \frac{1-m}{4}B^d(1-m)\right)}{\left(\frac{1-m}{2}\right)} \\ &+ \frac{\left(\frac{1-m}{2}\left(\frac{1}{2}B_d(1-m) + B_d(m)\right)\right) a_1}{\left(\frac{1-m}{2}\right)}\end{aligned} \tag{5.10}$$

At stage 1, operators set their access prices in order to maximize her own profit. By applying first order condition, we obtain the best-response access prices to be:

$$a_1{}^* = \frac{\left(c_2\left(\frac{1}{2}B^u(1-m) + \frac{1-m}{4}B^d(1-m)\right)\right)\alpha}{(\left(\frac{1-m}{2}\left(\frac{1}{2}B_d(1-m) + B_d(m)\right)\right)(1-\alpha)} + \frac{c_1}{(1-\alpha)} \tag{5.11}$$

and

$$\begin{aligned}a_2{}^* =& \frac{\left(c_1 B^u(m^*) + \frac{1}{2}c_1 B^u(1-m^*) + \left(\frac{1+m^*}{2}\right) c_1 B^d(m^*)\right)\alpha}{\left(\frac{1+m^*}{4}\right) B^d(1-m^*)(1-\alpha)} \\ &+ \frac{\alpha\left(\frac{1+m^*}{4}\right) c_1 B^d(1-m^*)}{\left(\frac{1+m^*}{4}\right) B^d(1-m^*)(1-\alpha)} + \frac{c_2}{(1-\alpha)}.\end{aligned} \tag{5.12}$$

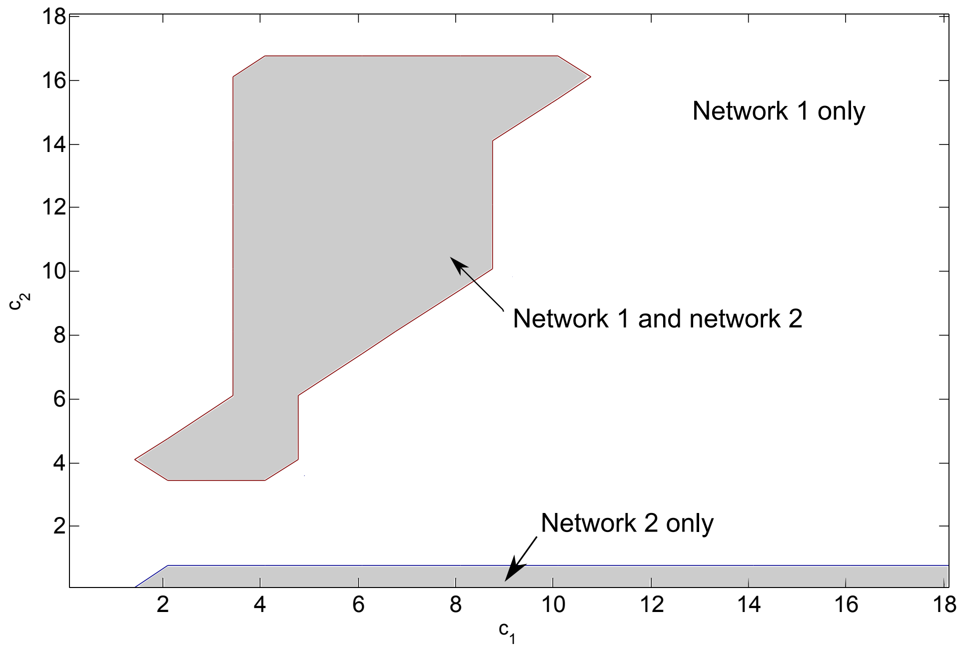

Figure 5.4: Plot of equilibrium market split in deregulated market against different c_1 and c_2 under three base stations full coverage market model.

5.3.1 Numerical Study

We will perform numerical study to investigate the equilibrium market split under different cost c_1 and c_2. Then we compare the profit obtained by operator 1 between two base stations model and three base station model under different cost c_1 while c_2 is fixed at 1.

Equilibrium market split

Fig. 5.4 shows the equilibrium cases under different c_1 and c_2. Comparing to Fig. 4.4, in the three base stations case, operator 1 captures the market in more combinations of c_1 and c_2. It is because the new base station built by operator 1 shares some of the service workload. The lowest user price of operator 1 that results in zero profits is lower than in the two base stations model. As a result, operator 1 is able to offer lower user price and remains in the market in more combinations of c_1 and c_2.

Operator 1's profit

Fig. 5.5 shows the plot of operator 1's profit in 2 base stations scenario and 3 base stations scenario under different cost c_1. Similar as before, the cost of operator 2 c_2 is fixed at 1 and c_1 increases. In both scenario, operator 1's profit decreases as the operator's operating cost c_1 increases. Eventually, the profit approaches zero since operator 2 reduces its user price so much that operator 1 no longer has an incentive to undercut its user price to compete for the market. Operator 1 leaves the market in these cases.

In the 3 base stations scenario, operator 1's profit is higher in the 3 base stations case than in the 2 base stations case. This is because the service load is shared between the left hand side base station and the right hand side base station in the 3 base stations scenario.

Though operator 1 will eventually lose the market as its operating costs increases, it happens at a smaller cost c_1 for 2 base stations scenario than for 3 base stations scenario. The increase in per unit bandwidth cost c_1 is mitigated by the fact that the service load is shared between the two base stations in the 3 base stations scenario. As a result, operator 1 leaves market at a higher cost c_1.

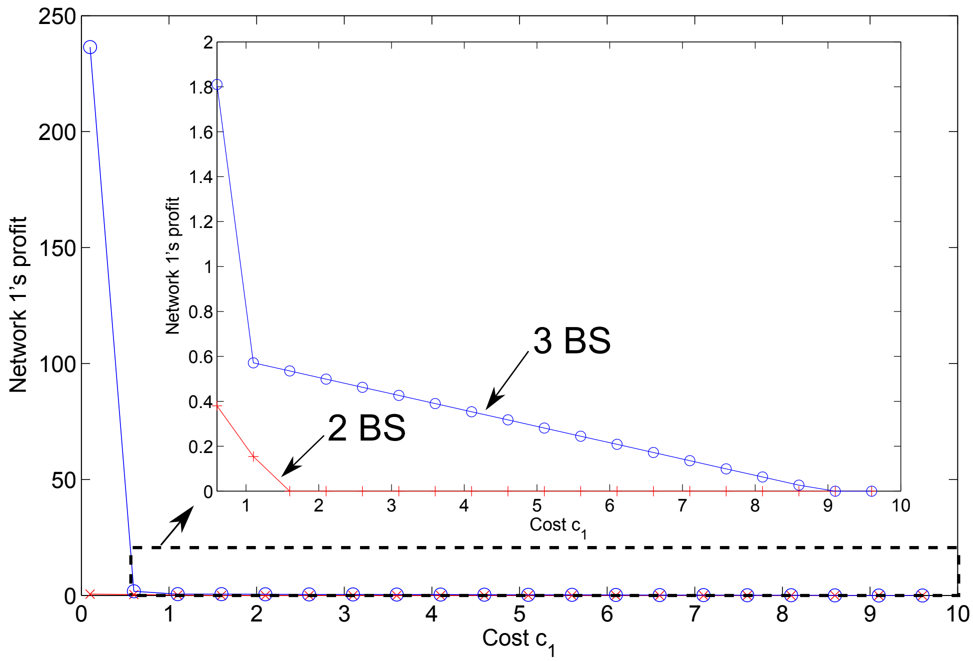

Figure 5.5: Plot of operator 1's profits against different cost c_1 in the 2 base stations full coverage case ('o' marker) and in the 3 base stations full coverage case ('x' marker). Cost of operator 2 remains fixed at 1.

Chapter 6

Conclusions and Future Work

Wireless network competition is a hot research area in wireless communication. The goal of the study is to understand how to allocate valuable wireless resources fairly. Most work in this area considers setting the right prices to achieve this goal. Approaches such as optimization, game theory, and auction theory have been applied in the study. The existing studies in wireless competition consider either the interaction between users and operators, or operator and operator, but not both. In this thesis, we fill in this gap by considering also the interconnection issue.

The difficulty in analysing interconnection issues lies in the complex cost structure of wireless channel condition. Unlike to fixed-line network, resource demand (e.g., bandwidth demand) is different for different users' location due to different channel gains. A connection between a source-destination pair close to a base station would require less resource than a source-destination pair far away from a base station. Also, the complexity in the cost function hardens the interpretation of a closed-form solution. As a result, we performed numerical study instead.

The focus of this thesis is to provide relevant suggestion to both regulator and profit-maximizing operators. We consider three real life scenarios and compare the social welfare in the regulated scenario and operator's profit in the deregulated scenario. Other than suggesting which scenario and under which condition is the best to regulator or operator, we would also like to shed the light for future researchers by providing a general framework in analysing wireless competition in the future.

The scenarios being considered are - fixed coverage at which user-network association is fixed (Chapter 3), full coverage at which each user choose his/her own operator (Chapter 4), and three base stations scenario at which a new base station is set up to provide services to users (Chapter 5). In each of these scenarios, we formulated the social welfare and each operator's profit. Then, we solved for the social optimal user pricing in regulated market, profit-maximizing user pricing and access pricing in deregulated market.

We are interested in comparing the social welfare in the regulated market and operator's profit in the deregulated market between the aforementioned scenarios. We compare the two metrics between fixed coverage model and full coverage model, and between 2 base stations full model and 3 base stations full model. We showed that social welfare and operator's profit is not always improved in the full coverage case. On the other hand, by building one more base station, the overall social welfare and the operator's profit are both improved only when the operator's per bandwidth cost is lower compared to the rival networks.

In this thesis, the model that we adopted is similar to the one used by [8]. Our results depend heavily on the total cost incurred by a network, which is affected by the users' distribution. A change in the assumption about uniform distribution of users might have impact on the results and that needs a further study.

Also, the investment cost and fixed cost incurred in a base station is not considered in our model. In the three base stations market model, the investment cost could be one of the factors that determine if the investment is profitable. For a particular per bandwidth cost, there might be an upper bound over the investment cost such that the investment is profitable.

There are other related areas which might be interesting for the future research. One possibility is to consider network effects as well. Price discrimination could happen such that intra-network traffic could be charged lower than the inter-network traffic. In this way, users are encouraged to stay in the same network. Operators will require less resources from its rival network and can better control the total cost incurred.

Bibliography

[1] J. Laffont, P. Rey, and J. Tirole, "Access pricing and competition," *European Economic Review*, vol. 38, no. 9, pp. 1673–1710, 1994.

[2] M. Chiang and J. Bell, "Balancing supply and demand of bandwidth in wireless cellular networks: utility maximization over powers and rates," in *INFOCOM 2004. Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies*, vol. 4, pp. 2800 – 2811, 2004.

[3] P. Marbach and R. Berry, "Downlink resource allocation and pricing for wireless networks," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, pp. 1470 – 1479, 2002.

[4] S. Adlakha, R. Johari, and A. J. Goldsmith, "Competition in wireless systems via bayesian interference games," *Computing Research Repository*, 2007.

[5] V. Gajic, J. Huang, and B. Romoldi, "Competition of wireless providers for atomic users: Equilibrium and social optimality," in *Communication, Control, and Computing, 2009. Allerton 2009. 47th Annual Allerton Conference on*, pp. 1203 –1210, 2009.

[6] X. Zhou, S. Gandhi, S. Suri, and H. Zheng, "ebay in the sky: strategy-proof wireless spectrum auctions," in *Mobile Computing and Networking*, pp. 2–13, 2008.

[7] N. DoCoMo, "International comparison of mobile phone interconnection fees @ONLINE."

[8] M. Cave, "The relationship between access pricing regulating and infrastructure competition," *Report to OPTA and DG Telecommunications and Post*, 2001.

[9] J. Laffont, P. Rey, and J. Tirole, "Network competition: I. overview and nondiscriminatory pricing," *RAND Journal of Economics*, vol. 29, no. 1, pp. 1–37, 1998.

[10] R. Willig, "The theory of network access pricing," tech. rep., Michigan State University Public Utilities Papers, 1979.

[11] W. Baumol, "Some subtle pricing issues in railroad regulation," *International Journal of Transport Economics 10*, 1983.

[12] M. Armstrong, C. Doyle, and J. Vickers, "The access pricing problem: A synthesis," *Journal of Industrial Economics*, vol. 44, no. 2, pp. 131–50, 1996.

[13] M. Salinger, "Regulating prices to equal forward-looking costs: Cost-based prices or price-based costs?," *Journal of Regulatory Economics*, vol. 14, pp. 149–164, 1998.

[14] J. Wright, "Access pricing under competition: An application to cellular networks," *The Journal of Industrial Economics*, vol. 50, no. 3, pp. 289–315, 2002.

[15] Y. Cheung, D. Chiu, and J. Huang, "Can bilateral isp peering lead to network-wide cooperative settlement," in *Computer Communications and Networks, 2008. ICCCN '08. Proceedings of 17th International Conference on*, pp. 1 –6, 2008.

[16] R. Ma, D. Chiu, J. Lui, V. Misra, and D. Rubenstein, “Interconnecting eyeballs to content: a shapley value perspective on isp peering and settlement,” in *ACM SIGCOMM Conference*, pp. 61–66, 2008.

[17] J. Jia and Q. Zhang, “Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market,” in *Proceedings of the 9th ACM international symposium on Mobile ad hoc networking and computing*, MobiHoc ’08, pp. 313–322, 2008.

[18] E. Altman, A. Kumar, C. Singh, and R. Sundaresan, “Spatial sinr games combining base station placement and mobile association,” in *INFOCOM 2009, IEEE*, pp. 1629 –1637, 2009.

[19] G. S. Kasbekar, E. Altman, and S. Sarkar, “A hierarchical spatial game over licenced resources,” in *Game Theory for Networks, 2009. GameNets ’09. International Conference on*, pp. 70 –79, 2009.

[20] D. Niyato and E. Hossain, “A game theoretic analysis of service competition and pricing in heterogeneous wireless access networks,” *Wireless Communications, IEEE Transactions on*, no. 12, pp. 5150 –5155, 2008.

[21] T. Doganoglu and Y. Tauman, “Network competition with reciprocal proportional access charge rules,” Tech. Rep. 9611001, EconWPA, 1996.

[22] M. Carter and J. Wright, “Interconnection in network industries,” *Review of Industrial Organization*, vol. 14, pp. 1–25, 1999.

[23] K. Leung and J. Huang, “Regulating wireless access pricing,” *IEEE ICC*, June 2011.

[24] J. Mo and J. Walrand, “Fair end-to-end window-based congestion control,” *IEEE/ACM Trans. Netw.*, vol. 8, pp. 556–567, 2000.

[25] H. Gruber, *The Economics of Mobile Telecommnications.* Cambridge University Press, 2005.

[26] J. Cohn, "Cell phone coverage maps - at&t, verizon, sprint, t-mobile @ONLINE."

[27] R. B. Myerson, *Game Theory: Analysis of Conflict.* Cambridge, MA: Harvard University Press, 2002.

[28] M. Felegyhazi and J.-P. Hubaux, "Wireless operators in a shared spectrum," in *INFOCOM 2006. 25th IEEE International Conference on Computer Communications. Proceedings*, pp. 1 –11, 2006.

[29] M. Felegyhazi, M. Cagalj, D. Dufour, and J.-P. Hubaux, "Border games in cellular networks," in *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*, pp. 812 –820, 2007.