Economics of Mobile Data Trading Market

Junlin Yu, Man Hon Cheung, and Jianwei Huang

***Abstract*—To exploit users' heterogeneous data demands, several mobile network operators worldwide have launched the mobile data trading markets, where users can trade mobile data quota with each other. In this work, we aim to understand the users' optimal trading decisions and the operator's revenue maximizing strategy. We model the interactions between the mobile operator and the users as a two-stage Stackelberg game. In Stage I, the operator chooses the operation fee imposed on sellers to maximize its revenue. In Stage II, each user decides whether to be a seller or a buyer and optimizes the corresponding trading price and quantity. We derive the closed-form expression of the unique Nash equilibrium (NE) in Stage II in closed-form, and prove that the users' decisions can converge to the NE through distributed best response updates. We show that at the NE, different types of sellers and buyers should propose the same price such that the total demand matches the total supply. We further show that the Stage I operation fee optimization problem is convex, and derive the optimal operation fee in closed-form. Our analysis and numerical results show that the users who have less uncertainty of their data usages can benefit more from data trading. We also show that an operation fee that is too high hurts both the users' payoffs and the operator's revenue.**

I. Introduction

A. Background and Motivation

Due to the significant increase of video traffic on smartphones and tablets, global mobile data traffic has been growing tremendously in the past few years [1]. To alleviate the tension between the mobile data demand and mobile network capacity, mobile network operators have been experimenting with several innovative pricing schemes, such as time and location dependent pricing, shared data plans, and sponsored data pricing [2]–[5]. However, these pricing schemes do not fully take advantage of the *heterogeneous demands* across all mobile users, as in all these schemes a user's unused portion of his month data quota will be wasted even if another user is in need of additional data. Seizing this opportunity, China Mobile Hong Kong (CMHK) in 2014 launched the 2nd exChange Market (2CM) [6], which is a mobile data trading platform that allows its users to trade their 4G mobile data quota with each other. In this platform, a seller can list his desirable selling price on the platform, together with the amount of data to be sold (up to his monthly data quota). If there is a buyer who is willing to buy the data at the listed price, the platform will clear the transaction and transfer the corresponding amount of the data to the buyer's monthly quota limit.

However, the current 2CM market mechanism is not efficient, as only a seller can list his trading price and quantity. This means that a buyer needs to frequently check the platform to see whether he is willing to buy according to the current (lowest) selling price. This motivates us to consider a first-price multi-unit double auction mechanism based on the one adopted in stock markets [9], [10]. With such a mechanism, in every time slot, a user can choose his role (seller or buyer) and submit his (selling or buying) price and quantity to the platform.[1] The platform clears the market at the users' proposed prices, when the buying price of some buyers is no smaller than the selling price of some sellers. The sellers with very high selling prices and the buyers with very low buying prices may not get all their proposed quantities transacted.

B. Contributions

In the mobile data trading market, the mobile network operator can obtain revenue in two ways. First, the operator charges the sellers an *operation fee* for each unit of sold data in the form of a "transaction tax" [6]. Second, the operator profits from the difference between the proposed prices of matched buyers and sellers. For example, if the data quota of a seller proposing \$2/GB is transferred to a buyer proposing \$3/GB, then the gap of \$1 goes to the operator for each transacted GB. We would like to understand two important questions in such a market: *(i) How should the operator set the operation fee to maximize its revenue? (ii) Given a fixed operation fee, what are the equilibrium trading behaviors among the users?*

To answer the above questions in a coherent framework, we model the interactions between the mobile operator and the users as a two-stage *Stackelberg game*. In Stage I, the operator optimizes its operation fee imposed on the sellers to maximize its revenue. In Stage II, the users decide their roles as sellers or buyers and the corresponding trading prices and quantities given the operation fee. Solving the Stage II problem in such a two-sided market is very challenging, because it is difficult to guarantee the existence of the Nash equilibrium (NE) due to the discontinuity of users' utility functions (to be discussed in details in Section II) [7], [8]. Nevertheless, we are able to characterize the unique NE for our problem in closed-form, and show that different types of buyers and sellers propose the *same price* and the total supply matches the total demand in the market at the NE.

In summary, our key results and contributions are as follows.

- *Two-sided data trading market formulation*: We propose a two-sided data trading market model, in which users decide their trading prices and quantities, without knowing in advance how much data they can sell or buy at the proposed prices. We also consider the optimization of the operation fee and analyzes its impact on the market.

J. Yu, M. H. Cheung, and J. Huang are with the Department of Information Engineering, the Chinese University of Hong Kong, Hong Kong, China; Emails: {yj112, mhcheung, jwhuang}@ie.cuhk.edu.hk. This work is supported by the General Research Funds (Project Number CUHK 14219016) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

[1] If a user is not willing to trade in a time slot, he can choose to be either a buyer with a very high buying price or a seller with a very low selling price.

- *Closed-form solution of the two-stage problem*: Despite the discontinuity in the user's utility functions, we characterize the unique NE in closed-from for the Stage II game. In addition, we show that the users' decisions will converge to the unique NE through distributed best response dynamics. We further derive the optimal operation fee in closed-form for the Stage I problem.
- *Engineering insights*: Our analysis indicates that at the market equilibrium in Stage II, all the users who want to trade should propose the same price and the total demand matches the total supply. Our numerical results show that the users who have less uncertainty about their data usage can benefit more from the trading. We also show that an operation fee that is too large hurts both the users' payoffs and the operator's revenue.

C. Related Literature on Mobile Data Trading

The research on mobile data trading market only emerged recently [11]–[13]. In [11], Zheng *et al.* studied the users' optimal bids in the market and proposed an algorithm for mobile operator to match the buyers and sellers. In [12], Yu *et al.* studied a single user's optimal mobile data trading problem under the future demand uncertainty from a behavioral economics perspective. However, the authors in [11] and [12] assumed that the sellers and buyers in the mobile data trading market can always bid prices that ensure that all their demand are satisfied and all their supply are cleared. In [13], Andrews presented a dynamic programming problem to characterize the trading behavior of mobile users without considering the interactions between the operator and the users. To the best of our knowledge, this work is the first paper that studies the mobile data trading market involving the active decisions of both the operator and the users, where users make trading decisions without knowing in advance how much data they can sell or buy at the proposed prices.

The rest of the paper is organized as follows. In Section II, we introduce the system model. In Section III, we analyze the users' best responses and compute the equilibrium. In Section IV, we analyze the mobile operator's operation fee optimization problem. We present the numerical results in Section V, and conclude in Section VI.

II. System Model

In this paper, we consider the mobile operator's operation fee optimization and the corresponding data trading decisions of all the users in the mobile data trading platform. We first discuss a user's profile in Section II-A. Then we introduce the mobile data trading market in Section II-B and the user's payoff in Section II-C. Finally, we formulate the two-stage Stackelberg game model in Section II-D.

A. Users' Profiles and Decisions

1) Quota and Future Demand: Let $\mathcal{I} = \{1, ..., I\}$ be the set of users. We assume that the number of users in the data trading market is very large (e.g., $I \to \infty$), hence the impact of a single user's action on the whole population can be ignored [14]. We assume that all users have the same *monthly quota* Q, but different different users have values and probabilities of future data demands.[2] For the ease of exposition, we assume that there are two possible realizations of a user i's future data demand (or simply called *demand*): $d_i \in \{d_{i,h}, d_{i,l}\}$, with $d_{i,h} > Q > d_{i,l} > 0$.[3] The probability for user i to observe a high demand $d_{i,h}$ is p_i, and the probability of observing a low demand $d_{i,l}$ is $1 - p_i$. For the analytical convenience, we assume p_i, $d_{i,l}$, and $d_{i,h}$ are uniformly and independently distributed in $[0, 1]$, $[\underline{d_l}, \overline{d_l}]$, and $[\underline{d_h}, \overline{d_h}]$, respectively.[4] We assume that the distribution of p_i, $d_{i,l}$, and $d_{i,h}$ in the entire market remains unchanged everyday, although a single user's p_i, $d_{i,l}$, and $d_{i,h}$ changes over time.[5]

2) Satisfaction Loss: Each user i will incur a *satisfaction loss* when his demand $d_i \in \{d_{i,h}, d_{i,l}\}$ exceeds his monthly data quota Q. We consider a linear satisfaction loss function

$$L(Q - d_i) = -\kappa[d_i - Q]^+, \tag{1}$$

where $[z]^+ = \max\{0, z\}$. Here $[d_i - Q]^+$ is the amount of insufficient data. When $d_i - Q$ is positive, it means that the quota is exceeded. The linear coefficient κ represents the usage-based pricing imposed by the mobile operator.[6] By selling or buying data in the market, a user can change his *effective remaining data quota* (for the current month only), and hence will change his expected satisfaction loss. Next, we will show the users' trading decisions and the corresponding payoff functions.

3) Trading Decisions: The decision of user i is defined as $\boldsymbol{x}_i = (a_i, \pi_i, q_i)$, which consists of three components. First, user i needs to decide his *role* $a_i \in \{s, b\}$, i.e., whether to be a seller ($a_i = s$) or a buyer ($a_i = b$), or not participate in the market.[7] Correspondingly, he has to determine his *trading price* π_i and his *trading quantity* q_i. Specifically, if user i chooses to be a seller, the price π_i and the quantity q_i refer to his selling price and selling quantity, respectively; if user i chooses to be a buyer, the price π_i and the quantity q_i refer

[2]We make this assumption for analytical convenience. The consideration of heterogeneous monthly quota adds another dimension in the user type (which we will introduce later in this section). It will lead to a more complicated user type distribution, without changing the main conclusions in this paper.

[3]The analysis for the case where both $d_{i,h}$ and $d_{i,l}$ are higher (or lower) than the monthly quota Q is relatively trivial, and hence is omitted here due to space limitations.

[4]The uniform distribution has been widely used to model the users' demands [15], [16]. The consideration of other distributions will not change the main conclusions in this paper.

[5]Each user's parameters p_i, $d_{i,l}$, and $d_{i,h}$ will change based on the number of remaining days of his billing cycle. According to [6], every user has a different billing date (e.g., the billing date of a monthly data plan can be on the 5th day or the last day of every month). If there are many users in the market and the users' billing dates are spread out in the entire calendar month, then the distribution of the number of remaining days in the current billing cycle will not change.

[6]We have assumed a two-part pricing tariff, where the user pays a fixed fee for the data consumption up to a monthly quota, and a linear usage-based cost for any extra data consumption. Such a pricing model is widely used by major mobile operators. For example, for a 4G CMHK user, $\kappa = \$60$ with a monthly data quota of 1 GB.

[7]Note that if a user is not willing to trade in a time slot, he can choose to be either a buyer with a very high price or a seller with a very low price.

to his buying price and buying quantity, respectively. In the CMHK market, the proposed prices must be smaller than the usage-based price κ, otherwise no transaction will happen in the market. Hence, we have $\pi_i \in \Pi = [0, \kappa]$ and $q_i \in [0, \infty)$. We define the set of feasible strategies of user i as $\mathcal{X}_i = \{(a_i, \pi_i, q_i) : a_i \in \{s, b\}, \pi_i \in [0, \kappa], q_i \in [0, \infty)\}$.

B. Mobile Data Trading

1) Sellers' and Buyers' Markets: We consider a two-sided mobile data trading platform based on the first-price multi-unit double auction mechanism, which consists of four main steps:

- Step 1 (Bidding): At the beginning of every time slot (e.g., one day), all users make their trading decisions and submit their *bids*, which include their roles, trading prices, and trading quantities, simultaneously to the platform.
- Step 2 (Prioritization): The platform sorts the bids of the users in the sellers' and buyers' markets, respectively, according to their proposed prices. For the example in Fig. 1, in the first row of the table, the total demand in the buyers' market at the highest buying price of \$15 is 5 GB, which will be satisfied with the highest priority. The total supply in the sellers' market with the lowest selling price of \$13 is 10 GB, which will be sold with the highest priority.
- Step 3 (Allocation): The platform clears the markets by allocating the bids through an auction mechanism with the priority orders. Notice that the selling bids are only allocated to the buying bids whose buying price is no smaller than the corresponding selling price. For example, the platform first allocates the first row's bids (5 GB data quota supply with the selling price of \$13 to 5 GB data quota demand with the buying price of \$15), and then allocates the remaining 5 GB supply with the selling price of \$13 on the first row to 5 GB demand with the buying price of \$14 on the second row. The remaining 10 GB demand with the buying price of \$14, however, is not satisfied, because the remaining supply on the second row is with the selling price of \$15.[8]
- Step 4 (Payment): Finally, the platform decides the payment of users. If there is a gap between the selling and buying prices of the allocated data, then the price gap leads to the revenue of the operator. For example, the operator gains \$2 for allocating each GB with the selling price of \$13 to that with the buying price of \$15. The platform will also charge a seller θ dollars of operation fee (tax) for each GB sold.

After each trading, all users can access the aggregate information on other users' decisions of the last time slot, by checking the updated market information on the platform.[9]

[8]If the 15 GB of demand with the buying price of \$14 is from multiple buyers, we will discuss how to decide the allocation among multiple buyers later in Section II-B2.

[9]We assume that a user knows the distribution of user types in the market, which can be learnt from the historical market information.

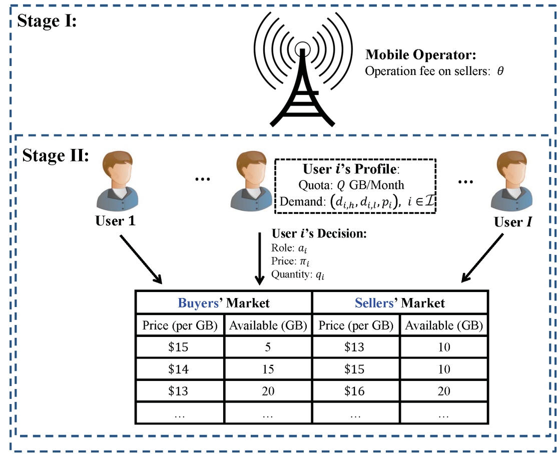

Fig. 1. Mobile data trading market.

2) Realized Transaction: Note that a user may not be able to get all his proposed buying or selling quantity transacted. For example, if a user chooses to be a buyer ($a_i = b$) with a proposed buying price π_i lower than other buyers, then he may get nothing transacted (e.g., the buyer who propose \$13 in Fig. 1). In other words, user i's realized transaction quantity depends on other users' decisions $\boldsymbol{x}_{-\boldsymbol{i}} = (\boldsymbol{x}_j, \forall j \in \mathcal{I}, j \neq i)$. This leads to the game theoretical interactions among users in Stage II.

To help explain the realization rules, we first define several set notations for user i as follows. First, let us define

$$\mathcal{LS}_i = \begin{cases} \{(a_j, \pi_j, q_j) \colon a_j = s \text{ and } \pi_j < \pi_i, \forall j \neq i, j \in \mathcal{I}\}, & \text{if } a_i = s, \\ \{(a_j, \pi_j, q_j) \colon a_j = s \text{ and } \pi_j \leq \pi_i, \forall j \neq i, j \in \mathcal{I}\}, & \text{if } a_i = b. \end{cases} \tag{2}$$

If user i is a seller, then set $\mathcal{LS}_i$ refers to the set of sellers, who have higher priorities than user i. If user i is a buyer, then set $\mathcal{LS}_i$ refers to the set of sellers who are feasible to be matched with user i. Next, we define the set

$$\mathcal{E}_i = \{(a_j, \pi_j, q_j) \colon a_j = a_i \text{ and } \pi_j = \pi_i, \forall j \neq i, j \in \mathcal{I}\}. \tag{3}$$

Set $\mathcal{E}_i$ refers to the set of users who have the same role and the same priority as user i. Finally, we define

$$\mathcal{HB}_i = \begin{cases} \{(a_j, \pi_j, q_j) \colon a_j = b \text{ and } \pi_j \geq \pi_i, \forall j \neq i, j \in \mathcal{I}\}, & \text{if } a_i = s, \\ \{(a_j, \pi_j, q_j) \colon a_j = b \text{ and } \pi_j > \pi_i, \forall j \neq i, j \in \mathcal{I}\}, & \text{if } a_i = b. \end{cases} \tag{4}$$

If user i is a buyer, then set $\mathcal{HB}_i$ refers to the set of buyers who have higher priorities than user i. If user i is a seller, then set $\mathcal{LS}_i$ refers to the set of buyers who are feasible to be matched with user i.

If the accumulated buying quantity proposed by the buyers within the set $\mathcal{HB}_i$ is smaller than the accumulated selling quantity proposed by the sellers within the set $\mathcal{LS}_i$, then a seller i's supply cannot be cleared. On the other hand, if $\sum_{\boldsymbol{x}_j \in \mathcal{HB}_i} q_j \geq \sum_{\boldsymbol{x}_j \in \mathcal{LS}_i} q_j$, then the seller i will *equally share* the demands with the sellers of the same priority (i.e., those in set $\mathcal{E}_i$). The same rule applies to the buyer's case.

Specifically, user i's transacted trading quantity under strategy profile $(\boldsymbol{x}_i, \boldsymbol{x}_{-i})$ is[10]

$$r_i(\boldsymbol{x_i}, \boldsymbol{x_{-i}}) = \begin{cases} \left[\dfrac{\sum_{\boldsymbol{x_j} \in \mathcal{HB}_i} q_j - \sum_{\boldsymbol{x_j} \in \mathcal{LS}_i} q_j}{|\mathcal{E}_i|}\right]^+, & \text{if } a_i = s, \\ \left[\dfrac{\sum_{\boldsymbol{x_j} \in \mathcal{LS}_i} q_j - \sum_{\boldsymbol{x_j} \in \mathcal{HB}_i} q_j}{|\mathcal{E}_i|}\right]^+, & \text{if } a_i = b. \end{cases} \tag{5}$$

Based on the realization rule, among the sellers or buyers proposing the same price, the users who propose a smaller quantity will get all their quantity transacted before the users who propose a higher quantity. As an example in Fig. 1, consider three buyers, 1, 2, and 3, proposing the same buying price of \$14, where $q_1 = 3$ GB, $q_2 = 4$ GB, and $q_2 = 8$ GB. Hence their total demand is 15 GB as shown on the second row, but there are only 5 GB left in seller's market (with the selling price of \$13) that can be allocated to them. According to the realization rule in (5), they will equally divide the 5 GB, i.e., $r_1(\boldsymbol{x}) = r_2(\boldsymbol{x}) = r_3(\boldsymbol{x}) = 5/3$ GB. Consider a different scenario where the three buyers' demands are $q_1 = 1$ GB, $q_2 = 6$ GB, and $q_3 = 8$ GB, then the allocations are $r_1(\boldsymbol{x}) = 1$ GB and $r_2(\boldsymbol{x}) = r_3(\boldsymbol{x}) = 2$ GB. This is because with the result under the equal division (5/3) exceeds user 1's demand, hence the exceeded part is equally shared by the remaining two buyers.

C. User's Payoff Function

We define user i's payoff in (6) on the top of page 5, which is the difference of his expected satisfaction loss and the net payment of the trade. For the seller case ($a_i = s$), the first term $(\pi_i - \theta) r_i((s, \pi_i, q_i), \boldsymbol{x_{-i}})$ is the revenue from selling data. The second and third terms correspond to the seller's expected satisfaction loss after selling q_i data quota under the high and the low demand realization, respectively. Notice that each seller has to pay the operation fee of θ to the mobile operator for each unit of transacted data. The buyer's payoff function on the last line of (6) is similar, except that the operator does not charge the buyer an operation fee. Notice that the payoff function in (6) is discontinuous. For example, if a seller's supply is only partially cleared, he can clear all his supply by decreasing his selling price by a unit ϵ, and hence makes a discontinuous increase in his payoff.

D. Two-Stage Stackelberg Game Formulation

We model the interactions between the mobile operator and the users as a two-stage Stackelberg game as follows.

- Stage I: The operator optimizes its operation fee to maximize his revenue.
- Stage II: The users decide their roles as sellers or buyers and the corresponding trading prices and quantities in the non-cooperative mobile data trading game.

A general technique for solving a Stackelberg game is to use the *backward induction*. Thus in Section III, we first analyze the existence and uniqueness of the NE in Stage II. Then, in Section IV, we compute the optimal operation fee for revenue maximization in Stage I.

III. Stage II: Mobile Data Trading Game

In this section, we first formulate the mobile data trading game in Section III-A and define the market indicators in Section III-B. Next we analyse the best response in Section III-C. Finally, we characterize the game equilibrium in Section III-D and study how to reach the equilibrium in Section III-E.

A. Non-cooperative Game Formulation

We model the interactions among users as the following non-cooperative game[11]:

Definition 1. *A mobile data trading game is a tuple $\Omega = (\mathcal{I}, \mathcal{X}, \boldsymbol{U})$ defined by*

- Players*: The set $\mathcal{I}$ of users, where user $i \in \mathcal{I}$ is associated with a type $(p_i, d_{i,h}, d_{i,l})$.*
- Strategies*: Each player chooses an action (pure strategy) $\boldsymbol{x_i} = (a_i, \pi_i, q_i) \in \mathcal{X}_i$, which is his bid to the platform. The strategy profile of all the players is $\boldsymbol{x} = (\boldsymbol{x_i}, \forall i \in \mathcal{I})$ and the set of feasible strategy profile of all the players is $\mathcal{X} = \mathcal{X}_1 \times \ldots \times \mathcal{X}_I$.*
- Payoffs*: The vector $\boldsymbol{U} = (U_i, \forall i \in \mathcal{I})$ contains all users' payoffs as defined in (6).*

B. Transacted Price

Given a strategy profile $\boldsymbol{x} = (\boldsymbol{x_i}, \forall i \in \mathcal{I})$, we first define the *transaction selling price* $\hat{\pi}_s$ and the *transaction buying price* $\hat{\pi}_b$ in Definitions 2 and 3, respectively. Here we use $\epsilon > 0$ to denote the smallest price unit.[12] When a user makes a decision, he does not need to know the choice of each of the other users, but only needs to know the accumulated bids (e.g., the total demands and supplies in terms of GBs at each price). This allows us to analyze the users' best responses based on $\hat{\pi}_s$ and $\hat{\pi}_b$ instead of $\boldsymbol{x}$.

Definition 2. *The* transaction selling price[13] $\hat{\pi}_s$ *corresponds to the minimum price such that a seller, whose proposed selling price is one unit larger than the transaction selling price, cannot get any of his selling quantity transacted:*

$$\hat{\pi}_s = \min\{\pi_i : r_i((s, \pi_i + \epsilon, q_i), \boldsymbol{x_{-i}}) = 0, \forall i\}. \tag{7}$$

Definition 3. *The* transaction buying price $\hat{\pi}_b$ *corresponds to the maximum price such that a buyer, whose proposed buying*

[10] If $(\sum_{\boldsymbol{x_j} \in \mathcal{HB}_i} q_j - \sum_{\boldsymbol{x_j} \in \mathcal{LS}_i} q_j)/|\mathcal{E}_i| > q_i$ for some seller i ($a_i = s$), then his $r_i(\boldsymbol{x_i}, \boldsymbol{x_{-i}}) = q_i$, and the "burden left" is averaged over the other sellers with the same price. We will continue with this procedure until each seller j has an $r_j(\boldsymbol{x_j}, \boldsymbol{x_{-j}}) \le q_j$. Details of the procedure can be found in [8], [17], and a similar procedure also applies to the buyers.

[11] As we have assumed that distribution of user types do not change over time, we model the users' interactions as a one-shot game. We will discuss the dynamics of the users' decisions in Section III-E.

[12] We assume that ϵ is the smallest price unit used in the mobile data trading platform. For example, $\epsilon = 1$ HKD in 2CM.

[13] Based on the definition, the sellers who propose this price can get *some* or *all* of their selling quantities transacted. The sellers who propose selling prices lower than this price will get *all* their selling quantities transacted, because they have higher priorities. Notice that lower priority bids can only be cleared when all the higher priority bids are cleared.

$$U_i(r_i(\boldsymbol{x_i},\boldsymbol{x_{-i}}))=U_i\left(r_i\left((a_i,\pi_i,q_i),\boldsymbol{x_{-i}}\right)\right)$$
$$=\begin{cases}(\pi_i-\theta)r_i((s,\pi_i,q_i),\boldsymbol{x_{-i}})+p_iL(Q-r_i((s,\pi_i,q_i),\boldsymbol{x_{-i}})-d_{i,h})+(1-p_i)L(Q-r_i((s,\pi_i,q_i),\boldsymbol{x_{-i}})-d_{i,l}), & \text{if } a_i=s,\\ -\pi_i r_i((b,\pi_i,q_i),\boldsymbol{x_{-i}})+p_iL(Q+r_i((b,\pi_i,q_i),\boldsymbol{x_{-i}}))-d_{i,h})+(1-p_i)L(Q+r_i((b,\pi_i,q_i),\boldsymbol{x_{-i}}))-d_{i,l}), & \text{if } a_i=b.\end{cases} \quad (6)$$

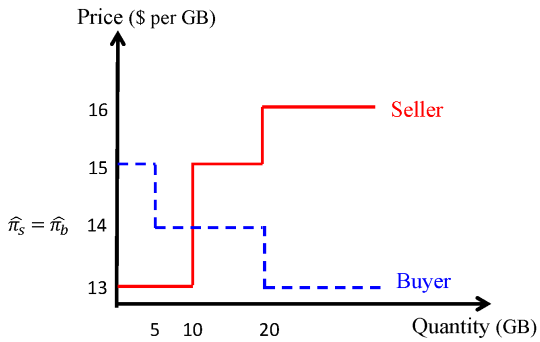

Fig. 2. An example of $\hat{\pi}_s$ and $\hat{\pi}_b$.

price is one unit smaller than the transaction buying price $\hat{\pi}_b$, cannot get any of his buying quantity transacted:

$$\hat{\pi}_b=\min\{\pi_i : r_i((b,\pi_i-\epsilon,q_i),\boldsymbol{x_{-i}})=0,\forall i\}. \quad (8)$$

An example of the definitions is shown in Fig. 2, based on the market supply and demand shown in Fig. 1. The platform sorts user's bids according to their prices, then clears the bids with the highest priorities first. In this example, the transaction selling price $\hat{\pi}_s=14$ and the transaction buying price $\hat{\pi}_b=14$, because the users who propose a price of \$14 can get some quantity transacted, and the sellers who propose one unit higher (\$15) and the buyers who propose one unit lower (\$13) cannot get any quantity transacted.

C. Best Response Analysis

Next, we investigate a user's best response, which is the strategy that maximizes his payoff given the fixed strategies of other users.

Definition 4. *User i's best response is*

$$\begin{aligned}\boldsymbol{x}_i^{BR}(\boldsymbol{x}_{-i}) &\triangleq (a_i^{BR}(\boldsymbol{x_{-i}}),\pi_i^{BR}(\boldsymbol{x_{-i}}),q_i^{BR}(\boldsymbol{x_{-i}}))\\ &=\arg\max_{\boldsymbol{x_i}\in\mathcal{X}_i} U_i(r_i((a_i,\pi_i,q_i),\boldsymbol{x_{-i}})).\end{aligned} \quad (9)$$

In Proposition 1, we characterize user i's best response based on the market information indicators $\hat{\pi}_s$ and $\hat{\pi}_b$ defined in Section III-B.

We find the user's best response of role $a_i^{BR}(\boldsymbol{x_{-i}})$, price $\pi_i^{BR}(\boldsymbol{x_{-i}})$, and quantity $q_i^{BR}(\boldsymbol{x_{-i}})$ given other users' strategies $\boldsymbol{x_{-i}}$ in the following proposition.

Proposition 1. *The best response $\boldsymbol{x}_i^{BR}(\boldsymbol{x}_{-i})$ of user i with type $(p_i,d_{i,h},d_{i,l})$ is given in (10) on the top of page 6.*

The proof of Proposition 1 is given in our online technical report [18].

First, Proposition 1 states that every user who wants to trade will either choose to sell the quota at the quantity $Q-d_{i,l}$ or choose to buy at the quantity $d_{i,h}-Q$, due to the piecewise linearity of (6) in quantity q_i.

Proposition 1 further states that every seller who wants to trade will sell at the price $\hat{\pi}_s$ or $\hat{\pi}_s-\epsilon$, and every buyer who wants to trade will buy at the price $\hat{\pi}_b$ or $\hat{\pi}_b+\epsilon$, due to the special structure in the realization function in (5).

Next, we discuss the five lines of equation (10) in details:

- The first and third lines: According to the realization function in (5), if the supply is not enough, the users with the same price will equally share the supply. In this case, the bids of users with a low q_i will be fully satisfied[14], while the bids of users with a high q_i will only be partially satisfied. Hence, the users with a low p_i and a low $Q-d_{i,l}$ will choose to sell with a lower price $\hat{\pi}_s$ (as shown in the first line of (10)), and the users with a high p_i and a low $d_{i,h}-Q$ will choose to buy with a higher price $\hat{\pi}_b$ (as shown in the third line of (10)).
- The second and fourth lines: To make their bids fully satisfied, the users with a higher quantity can propose a price with a slightly higher priority. Hence, the users with a low p_i and a high $Q-d_{i,l}$ will choose to sell with a lower price $\hat{\pi}_s-\epsilon$ (as shown in the second line of (10)), and the users with a high p_i and a high $d_{i,h}-Q$ will choose to buy with a higher price $\hat{\pi}_b+\epsilon$ (as shown in the fourth line of (10)).
- The fifth line: The users who are not willing to participate will randomly propose a high price as a seller or a low price as a buyer, and randomly propose a quantity.

D. Nash Equilibrium Analysis

Next, we define the Nash equilibrium as the intersection of all users' best response correspondences.

Definition 5. *(Nash Equilibrium): A strategy profile $\boldsymbol{x}^*$ is a Nash Equilibrium (NE) if and only if*

$$U_i(r_i(\boldsymbol{x}_i^*,\boldsymbol{x}_{-i}^*))\geq U_i(r_i(\boldsymbol{x}_i',\boldsymbol{x}_{-i}^*)),\forall \boldsymbol{x}_i'\in\mathcal{X}_i, i\in\mathcal{I}. \quad (11)$$

If a strategy profile is an NE, none of the users has the incentive to change his strategy, and the transacted prices will not change. To obtain the NE, we first show the conditions that an NE should satisfy in Lemma 1.

Lemma 1. *At any NE $\boldsymbol{x}^*$, there exists a unique price $\hat{\pi}^*$ such that the following two conditions are satisfied.*

$$\sum_{i\in\{j:a_j^*=s,\pi_j^*=\hat{\pi}^*\}} q_i=\sum_{i\in\{j:a_j^*=b,\pi_j^*=\hat{\pi}^*\}} q_i, \quad (12)$$

$$\pi_i^{BR}(\boldsymbol{x}_i^*)=\hat{\pi}^*,\forall i\in\{i:p_i<\frac{\hat{\pi}^*-\theta}{\kappa} \text{ or } p_i>\frac{\hat{\pi}^*}{\kappa}\}. \quad (13)$$

[14]By saying "the bid of a user is satisfied", we mean that "his demand is satisfied" if the user is a buyer, or "his supply is cleared" if the user is a seller.

$$\boldsymbol{x}_i^{BR}(\boldsymbol{x}_{-i}) \begin{cases} =(s,\hat{\pi}_s,Q-d_{i,l}), & \text{if } p_i \le \frac{\hat{\pi}_s-\theta}{\kappa} \quad \text{and} \quad \frac{\sum_{j\in\mathcal{HB}_i} q_j - \sum_{j\in\mathcal{LS}_i} q_j - \sum_{j\in\{k:k\in\mathcal{E}_i, q_k<Q-d_{i,l}\}} q_j}{|\{k: k\in\mathcal{E}_i, q_k \ge Q-d_{i,l}\}|} \ge Q-d_{i,l}, \\ =(s,\hat{\pi}_s-\epsilon,Q-d_{i,l}), & \text{if } p_i \le \frac{\hat{\pi}_s-\epsilon-\theta}{\kappa} \quad \text{and} \quad \frac{\sum_{j\in\mathcal{HB}_i} q_j - \sum_{j\in\mathcal{LS}_i} q_j - \sum_{j\in\{k:k\in\mathcal{E}_i, q_k<Q-d_{i,l}\}} q_j}{|\{k: k\in\mathcal{E}_i, q_k \ge Q-d_{i,l}\}|} < Q-d_{i,l}, \\ =(b,\hat{\pi}_b,d_{i,h}-Q), & \text{if } p_i \ge \frac{\hat{\pi}_b}{\kappa} \quad \text{and} \quad \frac{\sum_{j\in\mathcal{LS}_i} q_j - \sum_{j\in\mathcal{HB}_i} q_j - \sum_{j\in\{k:k\in\mathcal{E}_i, q_k<d_{i,h}-Q\}} q_j}{|\{k: k\in\mathcal{E}_i, q_k \ge d_{i,h}-Q\}|} < d_{i,h}-Q, \\ =(b,\hat{\pi}_b+\epsilon,d_{i,h}-Q), & \text{if } p_i \ge \frac{\hat{\pi}_b+\epsilon}{\kappa} \quad \text{and} \quad \frac{\sum_{j\in\mathcal{LS}_i} q_j - \sum_{j\in\mathcal{HB}_i} q_j - \sum_{j\in\{k:k\in\mathcal{E}_i, q_k<d_{i,h}-Q\}} q_j}{|\{k: k\in\mathcal{E}_i, q_k \ge d_{i,h}-Q\}|} \ge d_{i,h}-Q, \\ \in\{(a_i,\pi_i,q_i): a_i=b, \pi_i\in[0,\hat{\pi}_b), q_i\in[0,\infty)\} \cup \{(a_i,\pi_i,q_i): a_i=s, \pi_i\in(\hat{\pi}_s,\kappa], q_i\in[0,\infty)\}, & \text{otherwise.} \end{cases} \quad (10)$$

The Proof of Lemma 1 is in Appendix A.

Equation (12) implies that there exists an equilibrium price $\hat{\pi}^*$ such that the market is *cleared*, i.e., the total supply matches the total demand. From (13), we observe that those who want to make a transaction in the market will propose the *same price* in their best responses.[15]

By jointly solving equations (12) and (13) in Lemma 1, we obtain the unique NE in Theorem 1. For notation convenience, we first define

$$P_L \triangleq \frac{(\overline{d_h}+\underline{d_h}-2Q)(\kappa-\theta)}{(\overline{d_h}+\underline{d_h}-\overline{d_l}-\underline{d_l})\kappa} \quad (14)$$

and

$$P_H \triangleq \frac{(\overline{d_h}+\underline{d_h}-2Q)\kappa+(2Q-\overline{d_l}-\underline{d_l})\theta}{(\overline{d_h}+\underline{d_h}-\overline{d_l}-\underline{d_l})\kappa}, \quad (15)$$

which are the boundary indicators of user types. With these indicators, we can divide the users into different groups according to their user types.

Theorem 1. *The unique NE $\boldsymbol{x}^*$ of Game Ω is shown in Table I, where the equilibrium price*

$$\hat{\pi}^* = \frac{(\overline{d_h}+\underline{d_h}-2Q)\kappa+(2Q-\overline{d_l}-\underline{d_l})\theta}{\overline{d_h}+\underline{d_h}-\overline{d_l}-\underline{d_l}}. \quad (16)$$

The proof of Theorem 1 is given in the online technical report [18].

Theorem 1 states that at the NE, all the users who participate in the market will propose the same price, and all their proposed quantities will be transacted. This is because the sellers proposing higher prices cannot get their selling quantities transacted, and the sellers proposing lower prices receive lower payoffs. The same intuition applies to the buyers. The users who are not willing to participate will randomly propose a high selling price or a low buying price with random quantities, and will not affect the market trading results.

In the next subsection, we will further study how to reach the NE explicitly.

[15]The users with type $p_i < \frac{\hat{\pi}^*-\theta}{\kappa}$ are sellers and the users with type $p_i > \frac{\hat{\pi}^*}{\kappa}$ are buyers. The sellers' transaction selling price equals to the buyers' transaction buying price.

TABLE I
THE EQUILIBRIUM USERS' DECISIONS ON ROLES, QUANTITIES, AND PRICES

User Type p_i	Role a_i^*	Quantity q_i^*	Price π_i^*
$0 < p_i \le P_L$	s	$Q-d_{i,l}$	$\hat{\pi}^*$
$P_L < p_i < P_H$	s or b	$q_i^*\in[0,\infty)$	$\pi_i^*\in(p_i\kappa,\kappa]$ if $a_i^*=s$ $\pi_i^*\in[0,p_i\kappa)$ if $a_i^*=b$
$P_H \le p_i < 1$	b	$d_{i,h}-Q$	$\hat{\pi}^*$

E. Distributed Convergence to the Unique NE

In this section, we propose a best response dynamics algorithm with a diminishing step size, and show that the algorithm converges to the unique NE. To describe the best response dynamics, we first define a time-slotted system with slots (e.g., one day per time slot) $t=0,1,2,\ldots$, and allow users to change their decisions in every time slot based on the newly derived market prices. Let $\hat{\pi}_s(t)$ and $\hat{\pi}_b(t)$ in (17) and (18) be the transaction selling price and transaction buying price at time slot t, respectively, which can be derived from the users' strategies $\boldsymbol{x}(t)$. According to (10), we can compute the best response of user i in next time slot $t+1$ denoted as $\boldsymbol{x}_i(t+1)=\boldsymbol{x}_i^{BR}(\boldsymbol{x}_{-i}(t))$ (Line 4). Then, by $\boldsymbol{x}_i(t+1)$, we can derive the new transacted selling and buying price $\hat{\pi}_s(t+1)$ and $\hat{\pi}_b(t+1)$ (Line 5).

Due to the choice of the step size ϵ in Line 4, if any user i is going to change his price, the new price has to be at least $1/t$ different from the price in the last time slot. That is,

$$|\pi_i(t+1)-\pi_i(t)| \ge \frac{1}{t} \quad \text{or} \quad |\pi_i(t+1)-\pi_i(t)|=0, \forall i\in\mathcal{I}. \quad (19)$$

We have Theorem 2 on the convergence of the strategy profile in Algorithm 1 to the unique NE in Stage II.

Theorem 2. *Algorithm 1 asymptotically converges to the unique NE of Game Ω, in the sense that there exists a time threshold $\hat{t}$ such that for any $t>\hat{t}$, we have*

$$|\hat{\pi}_s(t)-\hat{\pi}^*| \le \frac{1}{t} \quad (20)$$

and

$$|\hat{\pi}_b(t)-\hat{\pi}^*| \le \frac{1}{t}. \quad (21)$$

According to Theorem 2, both the transacted selling and buying prices $\hat{\pi}_s(t)$ and $\hat{\pi}_b(t)$ converge to the equilibrium price

Algorithm 1: Mobile Data Trading Algorithm

Input: Initialize the time slot $t := 0$, strategy profile $\boldsymbol{x}(0) := (\boldsymbol{x}_i(0), \forall i \in \mathcal{I})$, the equilibrium price $\hat{\pi}^*$ from (16), and the corresponding transacted selling and buying prices $\hat{\pi}_s(0)$ and $\hat{\pi}_b(0)$ from the following two equations:

$$\hat{\pi}_s(t) := \min\{\pi_i : a_i(t) = s \text{ and } r_i((a_i(t), \pi_i + \epsilon, q_i(t)), \boldsymbol{x}_{-i}(t)) = 0, \forall i\}. \quad (17)$$

$$\hat{\pi}_b(t) := \min\{\pi_i : a_i(t) = b \text{ and } r_i((a_i(t), \pi_i - \epsilon, q_i(t)), \boldsymbol{x}_{-i}(t)) = 0, \forall i\}. \quad (18)$$

while $\hat{\pi}_s(t) \neq \hat{\pi}^*$ *or* $\hat{\pi}_b(t) \neq \hat{\pi}^*$ **do**
  for $i = 1$ *to* I **do**
    $x_i(t+1) := \boldsymbol{x}_i^{BR}(\boldsymbol{x}_{-i}(t))$ in (10), where $\epsilon := 1/t$.
  Update the transacted selling and buying prices $\hat{\pi}_s(t+1)$ and $\hat{\pi}_b(t+1)$ from (17) and (18), respectively.
  Update time slot $t := t + 1$.

Output: Strategy Profile $\boldsymbol{x}^* := \boldsymbol{x}(t)$.

$\hat{\pi}^*$ when $t \to \infty$. This shows that our proposed best response dynamics converges to the unique NE. The proof of Theorem 2 is given in the online technical report [18].

IV. Stage I: Operation Fee Optimization

In this section, we discuss the mobile operator's revenue maximization problem in deciding the optimal operation fee, given the NE of Stage II.

The operator's revenue comes from two parts: (a) The differences between the prices of matched sellers and buyers, and (b) the operation fee charged on *sellers* for the transacted data quota. Based on Theorem 1, the sellers and buyers propose the same price (i.e., the revenue from (a) is zero), hence the operator's revenue only comes from the operation fee in (b). Notice that the unit operation fee θ should be less than the usage-based unit price κ, otherwise no transaction will happen in the market.

The operator's revenue can be calculated as the product of the unit operation fee θ, the total number of sellers, and the average trading quantity per seller. First, according to a_i^* in Table I, the total number of sellers equals to IP_L, where P_L is the fraction of users who can successfully sell some data[16] as defined in (14). Second, from q_i^* in Table I, every seller i with a type $0 \leq p_i \leq P_L$ will propose a quantity $Q - d_{i,l}$. Since $d_{i,l}$ is uniformly distributed in $[\underline{d_l}, \overline{d_l}]$, the average quantity per seller equals to $(2Q - \overline{d_l} - \underline{d_l})/2$. Hence we can write the operator's revenue maximization as

$$\max_{0 \leq \theta \leq \kappa} \quad P(\theta) = I\theta \frac{(2Q - \overline{d_l} - \underline{d_l})(\overline{d_h} + \underline{d_h} - 2Q)(\kappa - \theta)}{2(\overline{d_h} + \underline{d_h} - \overline{d_l} - \underline{d_l})\kappa}. \quad (22)$$

[16] Notice that some users with type $P_L < p_i < P_H$ may choose to be a seller but cannot sell any data.

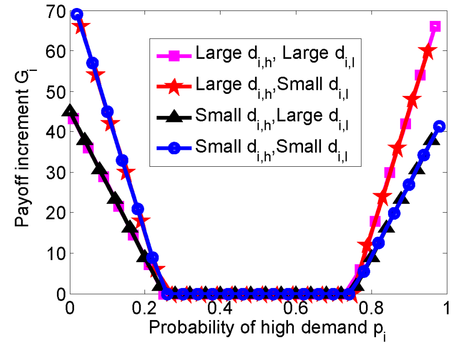

Fig. 3. The gap G_i between user i's utilities with and without data trading market.

We can verify that $P(\theta)$ is a concave function of θ, so problem (22) is a *convex* optimization problem. As a result, by the first order condition, we can obtain the unique optimal operation fee in *closed-form*.

Theorem 3. *The operator's optimal seller's operation fee is*

$$\theta^* = \frac{\kappa}{2}. \quad (23)$$

When θ is too small, the mobile operator gains a low operation fee from each trade. On the other hand, when θ is too large, there are fewer transactions, which also hurts the mobile operator's revenue. Theorem 3 shows that the optimal operation fee is in the middle of the feasible region $[0, \kappa]$.[17]

V. Performance Evaluation

In this section, we first provide simulation results to illustrate how users benefit from this mobile data trading market. We then evaluate the equilibrium social welfare and mobile operator's payoff under different operation fees.

A. Mobile Data Trading Market's Benefit to Users

We evaluate the users' benefits by calculating the gap G_i between user i's payoffs with and without data trading market.

In Fig. 3, we plot the payoff increment G_i against the probability of high demand p_i, high demand $d_{i,h}$, and low demand $d_{i,l}$. We show that the users with small p_i and large p_i will benefit from trading in the market, while those with medium p_i will not benefit. This is because the users with medium p_i are the most uncertain about whether their usage will exceed quota or not, and hence will not trade data in the market. On the other hand, the users who have less uncertainty about their usage (i.e., the users whose p_i are close to zero or one) will benefit a lot through trading.

In addition, the high p_i (i.e., $p_i > 0.75$) users will benefit more if they have a larger $d_{i,h}$, and the low p_i (i.e., $p_i < 0.25$) users will benefit more if they have a smaller $d_{i,l}$. This is because the users will benefit more when they trade a larger quantity. Based on Theorem 1, a high p_i user will be a buyer and propose a demand of $d_{i,h} - Q$, while a low p_i user will be a seller and propose a supply of $Q - d_{i,l}$.

[17] This result is based on the assumption that users' demand ($d_{i,l}$ and $d_{i,h}$, $i \in \mathcal{I}$) and probability (p_i, $i \in \mathcal{I}$) are uniformly distributed. The consideration of other distributions will not change the main insight stated here.

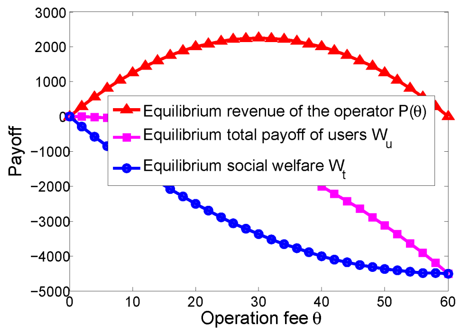

Fig. 4. The mobile operator's equilibrium revenue $P(\theta)$, the total payoff of users W_u, and the equilibrium social welfare W_t versus the operation fee θ.

B. Impact of Operation Fee on Social Welfare

We define the total equilibrium user payoff W_u as the sum of all users' payoffs $W_u = \sum_{i=1}^{I} U_i(\boldsymbol{x}^*)$, and the equilibrium social welfare W_t as the sum of all users' payoffs plus the operator's maximal revenue $W_t = \sum_{i=1}^{I} U_i(\boldsymbol{x}^*) + P(\theta^*)$.

In Fig. 4, we show that both the total equilibrium user payoff W_u and the equilibrium social welfare W_t decrease in the operation fee θ. This is because when θ is larger, users need to pay more to the operator, and as a result, there are fewer tradings happening. Hence, an operation fee that is too large hurts both the users' payoffs and the operator's revenue.

VI. Conclusion

In this paper, we studied the users' trading behavior in a two-sided market, in which users decide their trading prices and quantities without knowing in advance how much data they can sell or buy at their proposed prices. Our analysis indicated that all the users who want to trade should propose the same price, such that the total demand matches the total supply. Our numerical results showed that the users who have less uncertainty about their usages can benefit more from data trading. We also showed that an operation fee that is too high hurts both the users' payoffs and the operator's revenue.

In the future, we would like to understand how the distribution of user type impacts the market equilibrium. Apart from theoretical analysis, we would also conduct a market survey to understand the users' realistic responses to market dynamics.

References

[1] Ericsson, "Ericsson mobility report." White Paper, Jun. 2016.
[2] S. Ha, S. Sen, C. Joe-Wong, Y. Im, and M. Chiang, "TUBE: Time-dependent pricing for mobile data," *ACM SIGCOMM Computer Communication Review*, vol. 42, no. 4, pp. 247-258, Nov. 2012.
[3] T. Yu, Z. Zhou, D. Zhang, X. Wang, Y. Liu, and S. Lu, "Indapson: An incentive data plan sharing system based on self-organizing network," in *Proc. of IEEE INFOCOM*, Toronto, Canada, Apr. 2014.
[4] M. Andrews, U. Ozen, M. Reiman, and Q. Wang, "Economic models of sponsored content in wireless networks with uncertain demand," in *Proc. of IEEE INFOCOM*, Turin, Italy, Apr. 2013.
[5] Q. Ma, Y. F. Liu, and J. Huang, "Time and location aware mobile data pricing," *IEEE Trans. on Mobile Computing*, vol. 15, no. 10, pp. 2599-2613, Oct. 2016.
[6] China Mobile Hong Kong, "2cm (2nd Exchange Market)," *online at: http://www.hk.chinamobile.com/tc/2cm_intro.html*, 2016
[7] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*. Philadelphia, PA: SIAM Series in Classics in Applied Mathematics, 1999.
[8] Y. Wang, W. Saad, Z. Han, H. V. Poor, and T. Basar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1439-1450, May 2014.
[9] K. D. Garbade and W. L. Silber, "Structural organization of secondary markets: Clearing frequency, dealer activity and liquidity risk," *The Journal of Finance*, vol. 34, no. 3, pp. 577-593, Jun. 1979.
[10] H. R. Stoll and R. E. Whaley, "Stock market structure and volatility," *Review of Financial studies*, vol. 3, no. 1, pp. 37-71, 1990.
[11] L. Zheng, C. Joe-Wong, C. W. Tan, S. Ha and M. Chiangs, "Secondary markets for mobile data: Feasibility and benefits of traded data plans," in *Proc. of IEEE INFOCOM*, Kowloon, Hong Kong, Apr. 2015.
[12] J. Yu, M. H. Cheung, J. Huang and H. V. Poor, "Mobile data trading: A behavioral economics perspective," in *Prof. of IEEE WiOpt*, Mumbai, India, May 2015.
[13] M. Andrews, "Understanding the effects of quota trading on mobile usage dynamics," in *Proc. of IEEE WiOpt*, Tempe, AZ, May 2016.
[14] A. Ozdaglar, R. Srikant, "Incentives and Pricing in Communication Networks," *Algorithmic Game Theory*, Cambridge University Press, 2007.
[15] L. Zhang, W. Wu, and D. Wang, "Time-dependent pricing in wireless data networks: Flat-rate vs. usage-based schemes," *in Proc. of IEEE INFOCOM*, Toronto, Canada, Apr. 2014.
[16] Y. Jin and Z. Pang, "Smart data pricing: To share or not to share?" in *Proc. of IEEE INFOCOM Workshop Smart Data Pricing (SDP)*, Turin, Italy, Apr. 2014.
[17] P. Huang, A. Scheller-Wolf, and K. Sycara, "Design of a multi-unit double auction e-market," *Comput. Intell.*, vol. 18, no. 4, pp. 596-617, Feb. 2002.
[18] J. Yu, M. H. Cheung, and J. Huang, "Economics of Mobile Data Trading Market," Tech. Rep., [Online]. Available: http://jianwei.ie.cuhk.edu.hk/publication/AppendixWiOpt17DataTrading.pdf.

Appendix

A. Proof of Lemma 1

In this proof, we do not consider the users who are not willing to trade. In other words, we only consider the user $i \in \tilde{\mathcal{I}} = \{j : j \in \mathcal{I}, r_j(\boldsymbol{x}) > 0\}$. First, we show that the following lemma holds at the equilibrium.

Lemma 2. *For any two sellers or two buyers proposing the same price, both of them can get all their quantity transacted. That is, if $a_j^* = a_k^*$ and $\pi_j^* = \pi_k^*$, we have $r_j(\boldsymbol{x}^*) = q_j^*$ and $r_k(\boldsymbol{x}^*) = q_k^*$.*

The proof of Lemma 2 is in our online technical report [18].

Next we prove that Lemma 3 also holds at the equilibrium.

Lemma 3. *Any two users j and k who want to trade will propose the same price, i.e.,*

$$\pi_j^* = \pi_k^*, \ \forall j, k \in \tilde{\mathcal{I}}. \tag{24}$$

The proof of Lemma 3 is in our online technical report [18].

By Lemma 2, we know that all users can get their proposed quantity fully transacted, which will only happen when the total proposed buying quantity equals the total proposed selling quantity. By (10), we know that the users with type $p_i < (\hat{\pi}^* - \theta)/\kappa$ will trade as a seller, and the users with type $p_i > \hat{\pi}^*/\kappa$ will trade as a buyer. By Lemma 3, we know that all the buyers and sellers will propose the same price. Combining the above analysis, we obtain the result in (12) and (13). ■