Physical Interference Model Based Spectrum Sharing with Generalized Spatial Congestion Games

(Invited Paper)

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***Abstract*— With the rapid development of heterogeneous wireless technologies, the issue of how selfish wireless users can share spectrum is becoming increasingly relevant. In this paper we introduce the generalized spatial congestion game (GSCG), and use it to model wireless spectrum sharing over a large area. The idea behind the GSCG is to think of the players as vertices in a weighted graph. The amount of congestion two players cause each other (when they use the same resource) is determined by the weight of the edge linking them. The GSCG is more suitable for modeling spectrum sharing than many previously considered models, because one can select the edge weights and payoff functions to correspond with several practical inte rference models (such as the physical interference model). We focus on determining which GSCGs possess pure Nash equilibria (i.e., mutually acceptable resource allocations), and how selfish players can organize themselves into pure Nash equilibria.**

I. Introduction

As the number of wireless devices grows rapidly, it is becoming increasingly important to understand how wireless users can share the spectrum. Spectrum allocation has been studied extensively from a centralized point of view (*e.g.,* [1]). The complexity of the problem and the selfish nature of wireless users, however, often make it desirable to achieve efficient and fair spectrum sharing in a distributed fashion (*i.e.,* allowing users to select channels for themselves). This is especially true in cognitive radio networks [2], where the users are empowered with the ability to scan the spectrum and adapt their channels rapidly. Game theory is ideal for modeling distributed spectrum sharing.

The spectrum sharing problem has an intrinsic spatial element, because the mutual interferences generated among users on the same channel heavily depends on the locations of the corresponding transmitters and receivers. The generalized spatial congestion game (GSCG) model proposed in this paper allows us to use "weights" to reflect how much interference one user may cause to an other user. An illustrative example is shown in Fig. 1, where vertices represents wireless users (transmitter-receiver pairs) and directed edges represent interference relationships. The edge weights represent the level of interference. Notice that two vertices (users) might generate different interferences to each other (*e.g.,* the two edges with weights 4 and 9 between two black nodes 2 and 3), possibly representing different locations of transmitters/receivers and different transmission power levels. By selecting edge weights appropriately, we can account for the spatial aspects of spec-

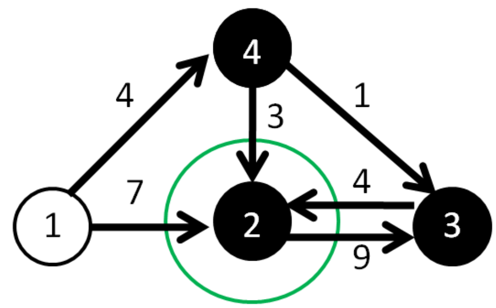

Fig. 1. A GSCG. The users are represented by vertices in a weighted graph. Users suffer interference from other users of the same channel which are connected to them. For example, user 2 suffers an interference level of 3 from user 4 (which is also using the *black* channel), but user 2 suffers no interference from user 1 (which is using *white* channel instead).

trum sharing in an accurate way. This is quite satisfying, as the spatial aspect of spectrum sharing is less understood than many other aspects [3].

Many game theoretic models have been used to study spectrum sharing (*e.g.,* [4]–[9]), but these generally assume that the users have complete knowledge about the network parameters and each other's information. Congestion game based models [10] (including the GSCG model) have the advantage of modeling network scenarios where the users have limited information, which is the case in many networks (cognitive radio networks in particular). More precisely, many congestion game models have the finite improvement property, which means is that if the players keep performing selfish better response updates based on local information, then the system will eventually reach a pure Nash equilibrium.

In [12] and [13] we introduced spatial congestion games and applied them to spectrum sharing in wireless networks. The idea behind these games is to model the users as vertices upon an undirected unweighted graph, which can only cause interference to their neighbors. This corresponds to the protocol interference model [14], within which a pair of users are either considered *linked* (in which case they can cause one another some fixed amount of interference) or *not linked* (in which case they are considered to be too distant to interfere with one another). Spatial congestion games serve as a more realistic model for spectrum sharing than classical congestion games because they allow for *spectrum reuse*-

where distantly spaced individuals can use the same channel without interfering. However the protocol inference model (upon which these games are based) is too basic to capture the real interference relationships between the users. In reality, inference decrease continuously with separation distance, and depends on transmission power levels. Interference effects can be captured much more accurately using the SINR based physical interference model [14], which is widely used in wireless network modeling.

In this paper we extend the spatial congestion game model, from undirected unweighted graphs to directed weighted graphs. This non-trivial generalization greatly increases the flexibility of the model. Rather than simply assuming a pair of players n and m are either *linked* (i.e., they can cause a fixed amount of congestion to one another) or *not linked* (i.e., are too distant to cause congestion), we allocate each pair of players with a weight $S_{n,m} \geq 0$ which represents how much congestion n can cause to m in a continuous way. The amount of congestion a player n incurs is given by the sum of the weights $S_{m,n}$ of the directed edges which are incoming upon n from other players m who are using the same resource. The payoff a player receives is given by a decreasing function of their congestion level. Our key results and contributions can be summarized as follows:

- *A Powerful New Model of Spectrum Sharing:* After introducing the GSCG formally (Section II-A), we explain how GSCGs can be used to model spectrum sharing based on the physical interference model (Section II-B).
- *Properties of Resource-Homogeneous GSCGs:* We characterize several properties of resource-homogeneous GSCGs, which are very useful for modeling spectrum sharing (Section III).
- *Realistic Simulations of Spectrum Sharing:* We perform realistic simulations of spectrum sharing (Section IV), and illustrate the practical implication of our theoretical results.

II. The Model

A. GSCG Game Formulation

Let us define a **generalized spatial congestion game (GSCG)** as a 5-tuple

$$(\mathcal{N}, \mathcal{R}, (\mathcal{R}_n)_{n\in\mathcal{N}}, (f_n^r)_{n\in\mathcal{N}, r\in\mathcal{R}_n}, S),$$

where

- $\mathcal{N} = \{1, 2, ..., N\}$ is a finite set of players.
- $\mathcal{R} = \{1, 2, ..., R\}$ is a finite set of resources.
- $\mathcal{R}_n \subseteq \mathcal{R}$ is the set of resources available to player $n \in \mathcal{N}$.
- f_n^r is a strictly decreasing continuous function that describes the payoff that player $n \in \mathcal{N}$ gets when it uses resource $r \in \mathcal{R}_n$. The payoff player n gets for using resource r is $f_n^r(x)$ where x is the current congestion level of resource r at player n, (as we shall describe).
- S is an $n \times n$ matrix of non-negative entries. Here $S_{n,m}$ measures the amount of congestion that player n causes to player m when it uses the same resource. We assume[1] $S_{n,n} = 0, \forall n \in \mathcal{N}$.

Matrix S captures the *spatial* information in the game. Connectivity data from S can be visualized with a **directed graph** $D(S) = (\mathcal{N}, E)$. The vertex set of this graph is the player set $\mathcal{N}$. An edge (n, m) belongs to the directed edge set E if and only if the corresponding edge weight $S_{n,m} > 0$. The graph $D(S)$ describes which players can cause congestion to one another. Sometimes we refer to a GSCG with a spatial matrix S as "a GSCG on graph $D(S)$". The spatial matrix associated with the graph shown in Fig. 1 is

$$S = \begin{pmatrix} 0 & 7 & 0 & 4 \\ 0 & 0 & 9 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 3 & 1 & 0 \end{pmatrix}.$$

A **state** $\boldsymbol{X} = (X_1, X_2, ..., X_N) \in \Pi_{n\in\mathcal{N}}\mathcal{R}_n$ is an assignment of one[2] resource $X_n \in \mathcal{R}_n$ to each player $n \in \mathcal{N}$. The **congestion level** of player n (when the system is in state $\boldsymbol{X}$) is $\sum_{m\in\mathcal{N}:X_m=X_n} S_{m,n}$ or simply $\sum_{m:X_m=X_n} S_{m,n}$. Graphically, we can think of the congestion level as the sum of the weights $S_{m,n}$ of all of the edges of $D(S)$ that are incoming upon n, from players m which are using the same resource. In Fig. 1, the congestion level of the player 2 (which is using the *black* resource) is $S_{3,2} + S_{4,2} = 4 + 3 = 7$.

The **payoff** that a player n gets from using a resource $X_n = r$ (when the game is in state $\boldsymbol{X}$) is a strictly decreasing continuous function of their congestion level, *i.e.,*

$$f_n^r\left(\sum_{m\in\mathcal{N}:X_m=r} S_{m,n}\right).$$

In Fig. 1, the payoff of the circled node is $f_2^{black}(7)$.

B. Modeling Wireless Networks Using GSCGs

We start by showing that the GSCG can be used to model the SINR-based physical interference model with fixed transmission power levels. Later we discuss how the GSCG can be used to model more general scenarios.

Consider a wireless network where each user is a fixed transmitter-receiver pair. In the physical interference model, the interference received by a user is the summation of the power received from all other users in the network [14]. The maximum achievable transmission rate (according to the Shannon capacity) that a user n gets by using channel r is $B_r \log_2(1 + \text{SINR})$, where B_r is the bandwidth of channel r and SINR is the signal-to-interference plus noise ratio,

$$\text{SINR} = \frac{h_{n,n}P_n}{\tau_0 B_i + \sum_{m:m\neq n, X_m=r} h_{m,n}P_m}.$$

Here τ_0 is the thermal noise density, P_m is the transmission power of m's transmitter, and $h_{m,n}$ is the channel gain from m's transmitter to n's receiver.

[1] We make the assumption for notational convenience. We could relax this assumption to take into account the congestion a player causes to itself, but it seems to be easier just to modify the payoff functions to achieve this.

[2] This means our generalized spatial congestion games belongs to the class of singleton congestion games. This is well motivated in practice, for example, a wireless user only has one transceiver and can only access one cellular channel.

If each user has a fixed transmission power (which is the default operation mode in today's Wi-Fi networks), then the spectrum sharing scenario can be modeled as a GSCG $(\mathcal{N}, \mathcal{R}, (\mathcal{R}_n)_{n\in\mathcal{N}}, (f_n^r)_{n\in\mathcal{N}, r\in\mathcal{R}_n}, S)$ as explained below:

- Each player $n \in \mathcal{N}$ corresponds to a fixed transmitter-receiver pair.
- Each resource $r \in \mathcal{R}$ corresponds to an orthogonal channel. When channels have an equal bandwidth (which is true in Wi-Fi , WiMax, and LTE networks) and the channels are interleaved (and thus have the same channel conditions for the same user), the system corresponds to **a resource-homogenous GSCG**.
- Each user n has a *user-dependent* available channel set $\mathcal{R}_n \subseteq \mathcal{R}$. This flexibility is especially useful for modeling cognitive radio networks, where the channels available to a secondary user depend on the activities of the licensed users within its vicinity.
- Each player uses exactly one resource/channel at a given time. This is the case in networks using FDMA technologies.
- $S_{m,n}$ measures the amount of interference that m causes n when both users are on the same channel. More precisely, $S_{n,n} = 0$ and $S_{m,n} = h_{m,n}P_m$ for $n \neq m$.
- Player n's payoff of using resource r depends on the interference level $x = \sum_{m:X_m=X_n} S_{m,n}$, and is equal to

$$f_n^r(x) = B_r \log_2 \left(1 + \frac{h_{n,n}P_n}{\tau_0 B_r + x}\right). \quad (1)$$

When the power levels of the users are equal, and distinct transmitter-receiver pairs are distantly spaced (relative to the distance between individual transmitters and their receivers), the assumption that the interference relationship between users is symmetric (*i.e.,* $h_{n,m} = h_{m,n}$) is at least approximately valid. Such cases correspond to GSCGs with **symmetric spatial matrices**, or "on undirected graphs".

In Section IV, we will simulate spectrum sharing in real wireless networks using GSCG models. Note that other aspects of wireless communication can also be modeled by the GSCG. For example, users may have different payoffs and different channel preferences because of different transmission technologies. We can choose player-specific payoff functions (instead of the same Shannon capacity) to model this. Also, we can also use edge weights to reflect different user priorities in cognitive radio networks. For example, if n is a primary license holder and m is an secondary unlicensed user, then we could set $S_{n,m}$ to be very large to reflect the price that m must pay (or the punishment which m may receive) for causing interference to the license holder n.

III. Analytic Results

A. Better Responses, Nash Equilibria, and the Finite Improvement Property

Often we assume our system evolves over discrete time slots, with no more than one player updating its resource choice during any given time slot. This assumption is often used in congestion games, it is realistic when the time scale is so small that simultaneous updating becomes unlikely.

When we have a system in state $\boldsymbol{X}$, and then a single player $n \in \mathcal{N}$ changes their resource choice to $r \in \mathcal{R}$ the system changes to a new state $(X_1, \cdots, X_{n-1}, r, X_{n+1}, \cdots, X_N)$. We say that such a update is a better response update when it increases player n's payoff.

Definition 3.1: The event where a player $n \in \mathcal{N}$ changes its resource choice from X_n to $r \in \mathcal{R}_n$ is a **better response update** if and only if

$$f_n^r \left(\sum_{m:X_m=i} S_{m,n}\right) > f_n^{X_n} \left(\sum_{m:X_m=X_n} S_{m,n}\right).$$

A Nash equilibrium is a stable resource allocation, from which no player has any incentive to deviate unilaterally.

Definition 3.2: A state $\boldsymbol{X}$ is a **Nash equilibrium**[3] if and only if no player can perform a better response update, *i.e.,* $f_n^r\left(\sum_{m:X_m=r} S_{m,n}\right) \leq f_n^{X_n}\left(\sum_{m:X_m=X_n} S_{m,n}\right)$, $\forall n \in \mathcal{N}$, $\forall r \in \mathcal{R}_n$.

Definition 3.3: A GSCG has the **finite improvement property** if every sufficiently long sequence of better response updates leads to a Nash equilibrium.

When a game with the finite improvement property evolves via asynchronous better response updates, it is guaranteed to reach a Nash equilibrium within a finite number of time slots. Loosely speaking, this means that greedy behavior always leads to a mutually acceptable resource allocation.

B. Properties of GSCGs

Firstly we should note that there are GSCGs which do not possess pure Nash equilibria. This result was effectively proved in [12], where we exhibited a spatial congestion game on an undirected unweighted graph, which has no pure Nash equilibria. The game has player-specific and resource-specific payoff functions. One can construct many other GSCGs with no pure Nash equilibria which are played on directed graphs. For example, the three player GSCG played on a directed triangle, with two resources, and a payoff function $f_n^r(x) = -x$, for each player n and each resource r.

In the following we focus on GSCGs with homogenous resources.

Definition 3.4: A GSCG is **resource-homogenous** if and only if $f_n^1(x) = f_n^2(x) = ... = f_n^R(x)$, $\forall n \in \mathcal{N}$ and $\forall x$ (i.e., all resources appear identical from any particular player n's perspective).

Resource-homogenous GSCGs can model wireless networks where each channel provides the same data rate. In most wireless standards (*e.g.,* Wi-Fi, WiMax, LTE, and bluetooth), channels have equal bandwidth. Moreover, interleaving techniques can be used to homogenize the quality of different channels (e.g., the interleaved channelization in IEEE 802.16d/e [16]).

We show that no player in a Nash equilibrium, of a resource-homogenous GCGWE, will have a congestion level that is above their maximum possible congestion level divided by the number of resources they have available to them. This is good news, because it means that if enough resources are

[3]In this paper we only focus on *pure* Nash equilibria.

available then all Nash equilibria will have low levels of congestion.

Theorem 3.1: Suppose we have a resource-homogenous GSCG at a Nash equilibrium $\boldsymbol{X}$. Now the congestion level $\sum_{m:X_m=X_n} S_{m,n}$ of any player $n \in \mathcal{N}$ is no larger than $\left(\sum_{m=1}^{N} S_{m,n}\right)/|\mathcal{R}_n|$.

Our next result states that when the resources are homogenous and the spatial relationships between the players are symmetric, the population will be eventually organize itself into a Nash equilibrium. This combines well with Theorem 3.1, which implies that the resulting equilibria will involve relatively low levels of congestion. In many wireless networks the assumptions that channels *are* homogenous and interference *is* symmetric between users are often quite accurate. In such cases, Theorem 3.2 implies that selfish radio users can find a way to share the spectrum for themselves efficiently.

Theorem 3.2: Every resource-homogenous GSCG with a symmetric spatial matrix S has the finite improvement property.

The full proofs of both theorems can be found in the online technical report [15].

IV. Simulating wireless networks

In this section we will use GSCGs to simulate how $N = 20$ selfish radio users (scattered across a square region of length L) will share $R = 5$ homogenous channels. We study how the users' ability to share the spectrum is influenced by L. We suppose that each player corresponds to a fixed transmitter-receiver pair that wishes to maximize its transmission rate by selecting the best channel. We shall also make the following assumptions:

- Each of user n transmits at a fixed power level of $P_n = 100\,\mathrm{mW}$.
- Each channel r has a bandwidth of $B_r = 20\,\mathrm{MHz}$, and is available to every user.
- The payoff a user n gets for using a channel $X_n = r$ is equal to its transmission rate, as given by Equation (1).
- We shall use the distance-based physical interference model [14], by writing the channel gain $h_{m,n}$, from user m's transmitter to n's receiver (see Equation (1)) as $h_{m,n} = 1/d_{m,n}^{\alpha}$, where α is the attenuation factor and $d_{m,n}$ is the distance from m's transmitter to n's receiver.
- We will suppose that the attenuation factor $\alpha = 4$ and the spectral noise density $\tau_0 = -174\,\mathrm{dBm}/\mathrm{Hz} = 10^{-17.4}\,\mathrm{mW}/\mathrm{Hz}$.
- We place each transmitter at a point (chosen uniformly at random) from our $L \times L$ square. Each receiver is uniformly randomly located within $100\,\mathrm{m}$ of its transmitter. We insure that no receiver is with $1\,\mathrm{m}$ of a transmitter (since our distance based SINR model breaks down at such close ranges).

For each simulation run, we randomly generate a network and randomly allocate one of the five channels to each user. Then the network evolves by having one random unsatisfied[4] user update its channel to a better response at every time slot.

[4]An *unsatisfied* user is a user which can increase its payoff by switching to a different channel.

The simulation run stops when a Nash equilibrium has been reached (or when some pre-specified large number of time slots have elapsed). We show the choices of channels in a Nash equilibrium of a network under our parameters in Fig. 2. Figures 3 and 4 depict the results of further simulations.

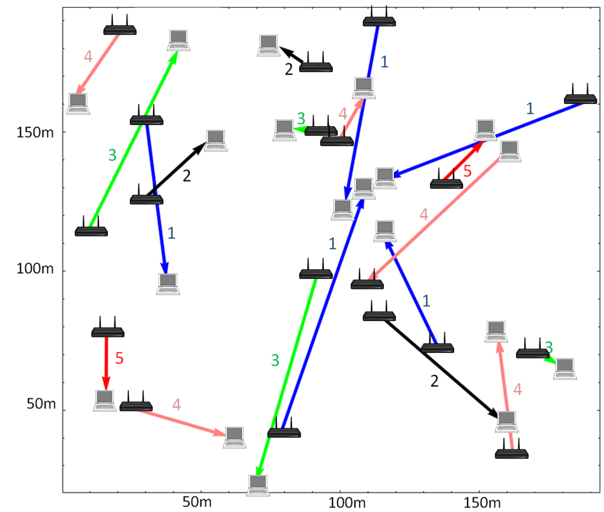

Fig. 2. A Nash equilibrium in a network where the $N = 20$ users are scattered across a square of length $L = 200\,\mathrm{m}$. Each arrow points from a transmitter (represented by a black router) to its receiver (represented by a gray computer). The link colors represent the channels that users choose at this Nash equilibrium. The average transmission rate of users at this Nash equilibrium is $101\,\mathrm{Mbps}$. Notice how links (users) of the same color naturally spread out to avoid strong mutual interferences.

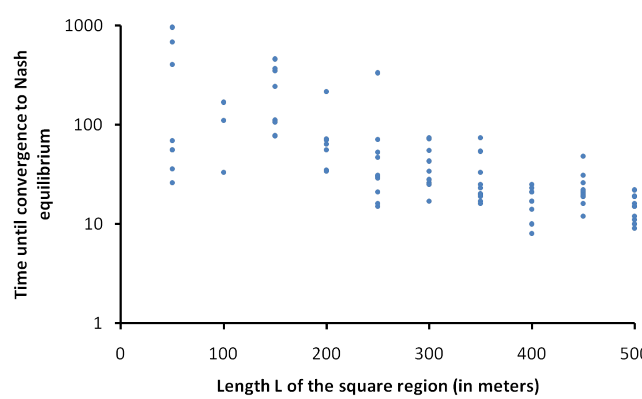

Fig. 3. The x-axis gives the length L (in meters) of the square region which the users are scattered over. We performed 10 simulations for each $L \in \{50\,\mathrm{m}, 100\,\mathrm{m}, ..., 500\,\mathrm{m}\}$. We ran each simulation for up to 2000 time slots. The y-axis shows how many time slots it takes a given simulation to converge. We do not show the points corresponding to simulations which did not converge. Note that the scale of the y axis is logarithmic.

The majority of our simulation runs converged to Nash equilibria. As Fig. 3 shows, however, convergence time is highly variable in simulation runs. Figure 3 shows that 20 users normally reach a Nash equilibrium within 200 time slots. In rare cases, however, it can take over 1000 time slots for a simulation to converge. This illustrates an inherent difficulty in studying the convergence dynamics of our systems. When a simulation does not converge at all, it will not have the finite improvement property. It is also possible that the system

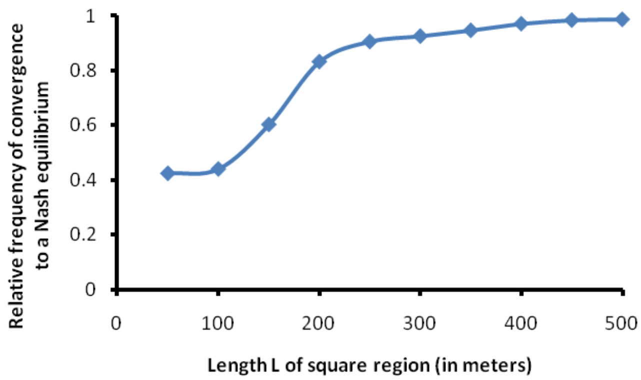

Fig. 4. The x-axis gives the length L, in meters, of the square region which the users are scattered across. We performed 1000 simulations for each value of L. The y-axis gives the fraction of these simulations which converged to a Nash equilibrium within 500 time slots.

possesses no Nash equilibria. The problem is, each of the systems we are simulating has 5^{20} states. This make it impractical to use a computer to rigorously verify whether a given system has the finite improvement property or Nash equilibria.

For this reason, we studied how the relative frequency that a given system will converge within 500 time slots, depends upon the geometry of the network. Figure 4 shows that it is easier for the users to organize themselves into a Nash equilibrium when the area they are spread across is larger. When the users are scattered across a large area, the distance between distinct transmitter-receiver pairs will often be much larger than the distance between a particular transmitter and its receiver. This will cause the interference relationship between the users to be approximately symmetric (in that the distance from n's transmitter to m's receiver will be approximately equal to the distance from m's transmitter to n's receiver). This means the system will (approximately) correspond to a resource-homogenous GSCG with a symmetric spatial matrix. In this case, Theorem 3.2 states that the system will have the finite improvement property, and therefor will eventually converge to a Nash equilibrium.

V. Discussions

Our simulation results reveal that the ability of wireless users to find a mutually acceptable spectrum allocation depends sensitively upon the network topology. GSCGs on directed graphs with no pure Nash equilibria can easily be constructed, although many GSCGs on undirected graphs have the finite improvement property. This suggests that spectrum allocation is easier when the interference relationships between the users are symmetric.

Acknowledgment

This work is supported by the General Research Funds (Project Number CUHK412710, CUHK412511, and CityU144209) established under the University Grant Committee of the Hong Kong Special Administrative Region, and CityU SRG grants No. 7002517 and No. 7008116.

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