ContrAuction: An Integrated Contract and Auction Design for Dynamic Spectrum Sharing

Lin Gao, Jianwei Huang, Ying-Ju Chen, and Biying Shou

Abstract—**Designing mechanisms with proper economic incentives is essential for the success of dynamic spectrum sharing, and market-driven secondary spectrum trading is one effective way to achieve this goal. In this paper, we consider secondary spectrum trading between one seller (i.e., the primary spectrum owner, PO) and multiple buyers (i.e., the secondary users, SUs) in a *hybrid* spectrum market with both guaranteed contract buyers (*future market*) and spot purchasing buyers (*spot market*). We focus on the *PO's profit maximization* with *stochastic* network information, and formulate the PO's expected profit maximization problem, where the optimal solution serves as a *policy* guiding the allocation of every spectrum under every possible information realization. We study systematically the optimal solutions under both *information symmetry* and *information asymmetry*, depending on whether the PO can observe the SUs' realized information. Under information symmetry, we show that the optimal solution (benchmark) maximizes both the PO's expected profit (optimality) and the social welfare (efficiency). Under information asymmetry, an incentive-compatible mechanism is necessary for eliciting the SUs' realized information. We propose the *ContrAuction*, an integrated contract and auction design, where the PO acts as virtual bidders (in addition to the role of an auctioneer) on behalf of the guaranteed contracts. We derive the optimal ContrAuction under the constraint of efficiency, and characterize the PO's expected profit loss (compared to that under information symmetry) induced by the information rent for the SUs.**

I. Introduction

With the explosive development of wireless services and networks, spectrum is becoming more congested and scarce. Dynamic spectrum sharing has been recently viewed as a novel approach to increase spectrum efficiency and alleviate spectrum scarcity. The key idea is to enable unlicensed secondary users (SUs) to access the spectrum licensed to primary spectrum owners (POs) opportunistically [1], [2]. The long-term successful implementation of dynamic spectrum sharing requires many innovations in technology, economics, and policy. In particular, it is essential to design a sharing mechanism that offers enough *incentives* for POs to open their licensed spectrum for secondary sharing.

Market-driven secondary spectrum trading is a promising paradigm to address the incentive issue in dynamic spectrum sharing, where POs temporarily *lease* the un-utilized (idle) spectrums to unlicensed SUs to obtain additional profit. In this paper, we consider the *short-term* secondary spectrum trading (e.g., on a slot-by-slot basis) between a single PO (monopoly seller) and multiple SUs (buyers) in a *hybrid* spectrum market with both the *future market* and the *spot market*. In the future market, each SU reaches an agreement, called a *guaranteed contract*, directly with the PO. The contract pre-specifies some key elements in trading, e.g., the price and the demand in a future period. In the spot market, the SUs compete openly with each other for spectrums (e.g., by an auction), and each spectrum is traded in a real-time and open manner. Figure 1 illustrates a hybrid spectrum market with contract users {1,2,3,4} and spot market users {a,b,c,d}, where two idle spectrums at time slot t are allocated to the contract user SU-3 and the spot market user SU-a, respectively.

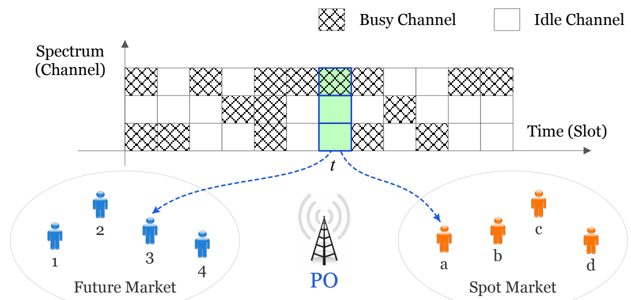

Fig. 1. An illustration of the hybrid spectrum market.

One significant advantage of such a hybrid spectrum market is its flexibility in achieving desirable Quality of Service (QoS) differentiations. Specifically, the future market insures SUs against uncertainties of the future supply, and insures POs against uncertainties of the future demand; the spot market allows SUs to compete for specific spectrums when they need resource, and permits fine grained resource allocation based on the SU's real-time demand and preference. Thus, an SU with elastic services (e.g., file transferring that is delay tolerant) may be more interested in spot trading to achieve the best resource-price tradeoff, while an SU with inelastic services (e.g., Netflix video streaming) requiring a minimum data rate may prefer the certainty of contract in the future market.

In this paper, we focus on the PO's profit maximization: *how should the PO allocate his idle spectrums in a future period among the guaranteed contract users and the spot market users (and charge) to maximize his overall profit?* Since the PO usually cannot obtain the complete and deterministic network information in advance, we formulate the PO's *expected* profit maximization problem based on the stochastic information. The optimal solution defines a *policy*, which determines the allocation of every spectrum with every possible information

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realization. We study the optimal solutions under *information symmetry* and *information asymmetry* systematically.

The main contributions of this paper are as follows:

- *New modeling and solution technique*: As far as we know, this is the first paper tackling secondary spectrum trading with the coexistence of future and spot markets.
- *Multiple information scenarios*: We study the optimal allocations in two stochastic information scenarios: information symmetry and asymmetry, depending on whether the PO can obtain the SUs' realized information.
- *Optimal Mechanism under information symmetry*: We derive the optimal solution analytically, where each spectrum is allocated to the most "profitable" SU directly (called *perfect price discrimination*).
- *Optimal Mechanism under information asymmetry*: We propose the *ContrAuction*, an integrated contract and auction design, where the PO acts as virtual bidders (in addition to the role of an auctioneer) on behalf of the guaranteed contracts. We derive the optimal ContrAuction under the constraint of efficiency.
- *Performance analysis*: Under information symmetry, the optimal solution (benchmark) maximizes both the PO's expected profit (optimality) and the social welfare (efficiency). Under information asymmetry, the optimal ContrAuction suffers certain profit loss (15% on average in our simulations) under the constraint of efficiency.

The rest of this paper is organized as follows. In Section II, we review the related literature. In Sections III and IV, we provide the system model and problem formulation. In Sections V and VI, we propose the optimal solutions under information symmetry and asymmetry, respectively. We present the simulations in Section VII and conclude in Section VIII.

II. Related Work

Recent years have witnessed a growing body of literature on the economic analysis (in particular the incentive issue) of dynamic spectrum sharing [1], [2]. Market-driven secondary spectrum trading is an effective way to address the incentive issue [3], [4]. The literature on secondary spectrum trading often considers *pricing*, *contract* and *auction*.

Pricing and contract are generally adopted by a *future market*, wherein the buyers enter certain agreements with the sellers beforehand, specifying the price, quality and demand. *Pricing* is often used when the seller knows precisely the value of the resource being sold (information symmetry) [5]–[7]. *Contract* is effective when the seller only knows limited information about the buyers' valuation (information asymmetry) [8]–[10]. In contrast to pricing and contract, *auction* is generally adopted by a *spot market*, where the buyers compete openly with each other for spectrums. Auction is particularly suitable for the information asymmetry scenario, where the seller does not know the value of the resource being sold whereas the buyers know [11]–[16].

However, the above works on secondary spectrum trading consider only the pure spectrum market (i.e., either in the spot market or in the future market), while we consider a *hybrid* spectrum market with both the spot market and the future market. As far as we know, this is the first work tackling the secondary spectrum trading with the coexistence of the spot market and the future market. For clarity, we summarize the key literature of secondary spectrum trading in Table I.

TABLE I
A SUMMARY OF SPECTRUM TRADING LITERATURE

Market Type	Related Work
Future Market	Pricing: [5]–[7], Contract: [8]–[10]
Spot Market	Auction: [11]–[16]
Hybrid Market	**This paper**

III. System Model and Problem Formulation

A. System Description

We consider a network with one PO and multiple SUs. The PO owns a set of licensed spectrums, and serves the licensed holders with a slotted transmission protocol (e.g., GSM, WCDMA, and LTE). That is, the total transmission time is divided into fixed-time intervals, called *time slots*, and the licensed holders access the PO's spectrums according to a synchronous slot structure. Depending on the activities of licensed holders, there may be some idle spectrums at each time slot, which can be temporally used by the SUs.

We consider *short-term* secondary spectrum trading, where the PO leases the idle spectrums to the SUs on a slot-by-slot basis. Thus, the basic unit of resource for trading is "a particular spectrum at a particular time slot", called a *spectrum* for short. The total number of idle spectrums in a certain period (say T time slots) is defined as the PO's *supply*, denoted by $\mathcal{S} = \{1, 2, ..., S\}$, where $S = \sum_{t=1}^{T} S_t$ and S_t is the number of idle spectrums at time slot t. In Figure 1, we have $T = 12$, $S = 20$, $S_1 = 1$, $S_2 = 2$ and so on.

We assume that each idle spectrum can only be used by one SU, that is, spectrum spatial reuse is not considered in this work. We further assume $S_t = 1$ for each time slot t, and thus a spectrum is equivalent to a slot. Note that this assumption does not affect the generality of our derivation, as we can view the multiple idle spectrums in the same time slot as a set of sequentially emerging spectrums.

B. Future Market and Spot Market

We consider a *hybrid* spectrum market consisting of the future market and the spot market.

1) Future Market: In the future market, each SUs enters into an agreement, called a future contract or *guaranteed contract*, directly with the PO. A contract pre-specifies not only the SU's payment and demand for spectrums in a given period, but also the PO's *penalty* when violating the contract.

2) Spot Market: In the spot market, each idle spectrum is traded in a real-time and on-demand manner. That is, an SU initiates a purchasing request only when needed, and all SUs requesting the same spectrum compete openly (e.g., through an auction) with each other for the spectrum. The spectrum is delivered immediately to the winner at a real-time market price, which not only depends on the SUs' valuations, but also on the market's supply-demand relationship.

To summarize, the future market insures SUs (PO) against uncertainties in the future supply (demand), while the spot market allows SUs to compete for specific spectrums based on their real-time requirements. Thus, a hybrid market is more flexible in *achieving desirable QoS differentiations*. We assume each SU is only associated with one trading scheme.[1] Let $\mathcal{N} = \{1, ..., N\}$ and $\mathcal{M} = \{1, ..., M\}$ denote the sets of SUs in the future market and spot market, respectively.

A guaranteed contract, in principle, could be quite complicated. In order to provide an intuitive expression and gain meaningful insights, we focus on a basic contract form, which consists of (i) the SU's payment and demand in one period (T time slots), and (ii) the PO's penalty when violating the contract. Thus, we can write the contract of SU $n \in \mathcal{N}$ as

$$\mathbb{C}_n \triangleq \{B_n, D_n, \widehat{P}_n\}, \quad (1)$$

where B_n and D_n are the SU's payment and demand, and $\widehat{P}_n$ is the unit punishing price for every undelivered spectrum.

C. Valuation

The *valuation* of an SU for a particular spectrum represents the SU's benefit from using the spectrum. It depends on both the *spectrum quality*, reflecting how efficiently an SU can utilize the spectrum, and *user preference*, reflecting how eagerly an SU needs a spectrum.

We consider a general scenario where the SUs' valuations randomly and independently vary with time. For convenience, we introduce the following random variables:

- $\boldsymbol{v}_m$: the valuation of spot market user m;
- $\boldsymbol{u}_n$: the valuation of contract user n;
- $\boldsymbol{\Theta} \triangleq (\boldsymbol{v}_1, ..., \boldsymbol{v}_M; \boldsymbol{u}_1, ..., \boldsymbol{u}_N)$: valuation vector of SUs.

We denote the realization of $\boldsymbol{v}_m$, $\boldsymbol{u}_n$ and $\boldsymbol{\Theta}$ at time slot t by $v_m(t)$, $u_n(t)$ and $\theta(t) = \big(v_1(t), ..., v_M(t); u_1(t), ..., u_N(t)\big)$, respectively. Since each spectrum t is fully characterized by $\theta(t)$, we also refer to $\theta(t)$ as the **information of spectrum** t.

IV. Problem Formulation with Stochastic Information

We focus on the *PO's profit maximization*: how should the PO allocate the idle spectrums among the contract users and the spot market users (and charge) to maximize his profit?

Due to the randomly varying of information, the PO usually cannot obtain the complete and deterministic network information (i.e., $\theta(t)$ for all t) in advance. Therefore, we focus on the PO's profit maximization under *stochastic* network information, where the PO knows only the stochastic distribution of $\boldsymbol{\Theta}$ in advance. To tackle this problem, we formulate the PO's *expected* profit maximization problem, and the optimal solution specifies the allocation of every spectrum under every possible information realization. Formally, the allocation strategy is defined as follows.

Definition 1 (Allocation Strategy)**.** *An allocation strategy $\boldsymbol{A}(\theta)$ is a mapping from every θ to a probability vector*

$$\boldsymbol{A}(\theta) \triangleq \big(a_0(\theta), a_1(\theta), ..., a_N(\theta)\big), \quad \forall \theta \in \boldsymbol{\Theta},^{2}$$

where $a_0(\theta)$ denotes the allocation probability of a spectrum with information θ (also called spectrum θ for short) to the spot market,[3] *and $a_n(\theta)$ to the contract user n, $\forall n \in \mathcal{N}$.*

The PO's expected revenue from the spot market is:

$$\mathbb{E}[R_0] = S \cdot \int_{\theta} a_0(\theta) r_0(\theta) \cdot f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta, \quad (2)$$

where $f_{\boldsymbol{\Theta}}(\theta) \triangleq \prod_{m=1}^{M} f_{\boldsymbol{v}_m}(v_m) \prod_{n=1}^{N} f_{\boldsymbol{u}_n}(u_n)$ is the joint PDF of $\boldsymbol{\Theta}$, and $r_0(\theta)$ is the maximum revenue that the PO can achieve from selling a spectrum θ to the spot market (which depends on whether the PO can observe the SUs' realized information at each time slot, i.e., information *symmetry* or *asymmetry*). Note that we write $\mathrm{d}v_1 ... \mathrm{d}v_M \mathrm{d}u_1 ... \mathrm{d}u_N$ as $\mathrm{d}\theta$ and $\int_{v_1} ... \int_{v_M} \int_{u_1} ... \int_{u_N}$ as $\int_{\theta}$ for convenience.

The expected number of spectrums for a contract user n is:

$$\mathbb{E}[d_n] = S \cdot \int_{\theta} a_n(\theta) \cdot f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta. \quad (3)$$

The PO's expected revenue from a contract user n is:

$$\mathbb{E}[R_n] = B_n - \mathbb{P}(\mathbb{E}[d_n], D_n), \quad (4)$$

where $\mathbb{P}(x, y)$ is the PO's penalty given by

$$\mathbb{P}(\mathbb{E}[d_n], D_n) \triangleq [D_n - \mathbb{E}[d_n]]^+ \cdot \widehat{P}_n \quad (5)$$

Note that the penalty is based on the expected number (rather than the actual number) of spectrums for a contract user n.

The PO's total expected revenue in one period is:

$$\mathbb{E}[R] = \mathbb{E}[R_0] + \sum_{n=1}^{N} \mathbb{E}[R_n]. \quad (6)$$

Although there is no explicit relation between a contract user's valuation and his actual payment, allocating a contract user the spectrums with low valuation will decrease the SU's long-term satisfaction, which may potentially affect the SU's future payment (or the PO's future revenue). In other words, the PO runs the risk of losing the contract user in future. We capture this potential loss by introducing an elastic *cost* for the contract user's long-term satisfaction (loss).

Let $c_n(\theta)$ denote the cost for a contract user n's long-term satisfaction over an (allocated) spectrum θ. The overall cost for contract user n's long-term satisfactions is

$$\mathbb{E}[C_n] = S \cdot \int_{\theta} a_n(\theta) c_n(\theta) \cdot f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta. \quad (7)$$

The PO's expected *profit* is a weighted sum of the current revenue (6) and the future cost (7):

$$\mathbb{E}[U] = \mathbb{E}[R_0] + \sum_{n=1}^{N} \mathbb{E}[R_n] - \sum_{n=1}^{N} w_n \cdot \mathbb{E}[C_n], \quad (8)$$

where w_n is the weight for evaluating the cost for contract user n's long-term satisfaction. A larger w_n means that the PO cares more about the satisfaction of contract user n.

The PO's objective is to find the optimal allocation $\boldsymbol{A}^*(\theta)$ for any possible realization θ to maximize his profit:

$$\boldsymbol{A}^*(\theta) = \arg\max_{\boldsymbol{A}(\cdot)} \left(\mathbb{E}[R_0] + \sum_{n=1}^{N} \mathbb{E}[R_n] - \sum_{n=1}^{N} w_n \mathbb{E}[C_n] \right),$$
$$s.t. \quad a_n(\theta) \in [0, 1], \ \forall n \in \{0\} \cup \mathcal{N}, \forall \theta \in \boldsymbol{\Theta}; \quad (9)$$
$$\textstyle\sum_{n=0}^{N} a_n(\theta) \leq 1, \ \forall \theta \in \boldsymbol{\Theta}.$$

Note that solving (9) does not require the deterministic information of all spectrums, and thus the PO is able to derive the optimal solution (i.e., the optimal policy) in advance.

[1]For the SU running multiple services, we can simply divide the SU into multiple virtual SUs, each associated with one trading scheme.

[2]Here we omit the time index t of $\theta(t)$, as we are not talking about a particular spectrum t. Furthermore, we use the same notation $\boldsymbol{\Theta}$ to denote the information space, i.e., the set of all possible information realization θ.

[3]Once allocated to the spot market, we will show that it is always optimal to allocate the spectrum to the user with the highest valuation.

V. Optimal Solution under Information Symmetry

Under information symmetry, the PO is able to observe the realized information $\theta(t)$ at each time slot t (but not before time slot t). Thus, when selling a spectrum on the spot market, the PO can extract all the surplus by allocating the spectrum to the highest valuation SU and charging exactly that SU's valuation. Such a selling mechanism is referred to as the *perfect price discrimination*.

Obviously, the maximum revenue $r_0(\theta)$ is given by

$$r_0(\theta) = Y_M^1(\theta) \triangleq \max_{m\in\mathcal{M}} \ v_m \ ,$$

where $Y_M^1(\theta)$ (or Y_M^1 for short) denote the highest valuation of all spot market users for a spectrum θ. In what follows, we will derive the optimal policy, i.e., the optimal solution of (9). Due to space limitations, we present all of the detailed proofs in our technical report [20].

A. Necessary Conditions

Suppose $\boldsymbol{A}^*(\theta) = (a_0^*(\theta), a_1^*(\theta), ..., a_N^*(\theta))$ is the optimal solution to (9). The following necessary conditions hold.

Lemma 1 (Necessary Condition I)**.** *For any information $\theta \in \boldsymbol{\Theta}$, we have: $\sum_{n=0}^N a_n^*(\theta) = 1$.*

Lemma 2 (Necessary Condition II)**.** *For any contract user $n \in \mathcal{N}$, we have: $\mathbb{E}[d_n] \le D_n$.*

Intuitively, Lemma 1 states that every idle spectrum will be sold to an SU. This is because the PO can always gain a positive revenue by selling a spectrum on the spot market. Lemma 2 states that none of contract users will get spectrums (on average) more than his demand. This is because there is no bonus for the PO from allocating extra spectrums to an SU beyond what is specified in the contract.

B. Optimal Solution

By Lemma 1, we can eliminate the decision variable $a_0(\theta)$ by substituting $a_0(\theta) = 1 - \sum_{n=1}^N a_n(\theta)$. By Lemma 2, we have $\mathbb{P}(\mathbb{E}[d_n], D_n) = \widehat{P}_n\left(D_n - \mathbb{E}[d_n]\right)$. Thus, the optimization problem (9) can be rewritten as:

$$\begin{aligned}
\boldsymbol{A}_0^*(\theta) &= \arg\max_{\boldsymbol{A}_0(\theta)} \Big(S \textstyle\int_\theta \big(1 - \sum_{n=1}^N a_n(\theta)\big) r_0(\theta) f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta \\
&\quad + \textstyle\sum_{n=1}^N \Big(B_n - \widehat{P}_n(D_n - \mathbb{E}[d_n]) \Big) - \sum_{n=1}^N w_n \cdot \mathbb{E}[C_n] \Big) \\
&= \arg\max_{\boldsymbol{A_0}(\theta)} \ \Big(F + S \cdot \textstyle\sum_{n=1}^N \int_\theta H_n(\theta) a_n(\theta) f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta \Big) \\
s.t. \quad & \text{(i)} \quad a_n(\theta) \ge 0, \ \forall n \in \mathcal{N}, \forall \theta \in \boldsymbol{\Theta}; \\
& \text{(ii)} \quad \textstyle\sum_{n=1}^N a_n(\theta) \le 1, \ \forall \theta \in \boldsymbol{\Theta}; \\
& \text{(iii)} \quad \mathbb{E}[d_n] \le D_n, \ \forall n \in \mathcal{N};
\end{aligned} \tag{10}$$

where

- $\boldsymbol{A}_0(\theta) = \boldsymbol{A}(\theta)/\{a_0(\theta)\} = \big(a_1(\theta), ..., a_N(\theta)\big)$;
- $F = S \cdot \int_\theta r_0(\theta) f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta + \sum_{n=1}^N \big(B_n - \widehat{P}_n D_n\big)$;
- $H_n(\theta) = -r_0(\theta) + \widehat{P}_n - w_n c_n(\theta)$.

In what follows, we first study the dual problem of (10) using the primal-dual method [19]. Then we determine the optimal dual variables and optimal primal solution.

1) Primal-Dual Method: We introduce Lagrange multipliers (also called *dual variables* or *shadow prices*) $\mu_n(\theta)$ for constraint (i), $\eta(\theta)$ for constraint (ii), and λ_n for constraint (iii). The Lagrangian can be written as $\mathbb{L} \triangleq \int_\theta \ \mathcal{L}(\theta) \cdot f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta$, where $\mathcal{L}(\theta)$, called Sub-Lagrangian, is given by

$$\begin{aligned}
\mathcal{L}(\theta) &= F + S \cdot \textstyle\sum_{n=1}^N H_n(\theta) a_n(\theta) + \sum_{n=1}^N \mu_n(\theta) a_n(\theta) \\
&\quad + \eta(\theta)\big(1 - \textstyle\sum_{n=1}^N a_n(\theta)\big) + \sum_{n=1}^N \lambda_n \big(D_n - S \cdot a_n(\theta)\big).
\end{aligned}$$

We define the PO's *marginal profit* (with respect to the allocation probability of contract user n) as the first partial derivative of $\mathcal{L}(\theta)$ over allocation $a_n(\theta)$. Formally,

Definition 2 (Marginal Profit)**.**
$\mathcal{L}^{(n)}(\theta) \triangleq \frac{\partial \mathcal{L}(\theta)}{\partial a_n(\theta)} = S \cdot H_n(\theta) + \mu_n(\theta) - \eta(\theta) - S \cdot \lambda_n.$

From Definition 2, we can see that the marginal utility $\mathcal{L}^{(n)}(\theta)$ is independent of $\boldsymbol{A}_0(\theta)$. By Euler-Lagrange conditions for optimality [19], the optimal solution $\boldsymbol{A}_0^*(\theta)$ must occur at the boundaries, i.e.,

$$a_n^*(\theta) = \begin{cases} 0, & \mathcal{L}^{(n)}(\theta) < 0 \\ 1, & \mathcal{L}^{(n)}(\theta) > 0 \\ \delta \in [0,1], & \mathcal{L}^{(n)}(\theta) = 0 \end{cases} \tag{11}$$

From (11), the optimal solution is determined by the dual variables $\mu_n(\theta), \eta(\theta)$, and λ_n. By the primal-dual method, every dual variable must be nonnegative and can be non-zero only when the associated constraint is tight, i.e., the following dual constraints must hold [19]:

(D.1) $\mu_n^*(\theta) \ge 0, \ a_n^*(\theta) \ge 0,$
$\mu_n^*(\theta) a_n^*(\theta) = 0, \quad \forall n \in \mathcal{N}, \theta \in \boldsymbol{\Theta};$

(D.2) $\eta^*(\theta) \ge 0, \ 1 - \sum_{n=1}^N a_n^*(\theta) \ge 0,$
$\eta^*(\theta)\big(1 - \sum_{n=1}^N a_n^*(\theta)\big) = 0, \quad \forall \theta \in \boldsymbol{\Theta};$

(D.3) $\lambda_n^* \ge 0, \ D_n - S \int_\theta a_n^*(\theta) f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta \ge 0,$
$\lambda_n^*\big(D_n - S \int_\theta a_n^*(\theta) f_{\boldsymbol{\Theta}}(\theta) \mathrm{d}\theta\big) = 0, \quad \forall n \in \mathcal{N}.$

By the duality principle, the primal problem (10) is equivalent to the problem of finding a set dual variables $\mu_n^*(\theta), \eta^*(\theta)$, and λ_n^*, such that all dual constraints are satisfied.

2) Optimal Shadow Prices and Optimal Solution: Now we study the optimal shadow prices (dual variables) and optimal primal solution. For convenience, we introduce the *mantle marginal profit* and *core marginal profit* as variations of the marginal profit in Definition 2. Formally,

Definition 3 (Mantle Marginal Profit)**.**
$\mathcal{J}_1^{(n)}(\theta) \triangleq S \cdot H_n(\theta) - \eta(\theta) - S \cdot \lambda_n = \mathcal{L}^{(n)}(\theta) - \mu_n(\theta).$

Definition 4 (Core Marginal Profit)**.**
$\mathcal{J}_2^{(n)}(\theta) \triangleq S \cdot H_n(\theta) - S \cdot \lambda_n = \mathcal{L}^{(n)}(\theta) - \mu_n(\theta) + \eta(\theta).$

We start by showing the necessary conditions for $\mu_n^*(\theta)$.

Lemma 3 (Feasible Range of $\mu_n^*(\theta)$)**.** *The optimal shadow prices $\mu_n^*(\theta)$ must satisfy the following conditions:*
(a) $\mu_n^*(\theta) = 0$, *if* $\mathcal{J}_1^{(n)}(\theta) \ge 0$*;*
(b) $\mu_n^*(\theta) \in [0, \ |\mathcal{J}_1^{(n)}(\theta)|]$, *if* $\mathcal{J}_1^{(n)}(\theta) < 0$.

It is easy to see that $\mathcal{L}^{(n)}(\theta) = \mathcal{J}_1^{(n)}(\theta) + \mu_n(\theta)$. Lemma 3 implies that (a) $\mathcal{J}_1^{(n)}(\theta) \ge 0$ if and only if (iff) $\mathcal{L}^{(n)}(\theta) \ge 0$, and (b) $\mathcal{J}_1^{(n)}(\theta) < 0$ iff $\mathcal{L}^{(n)}(\theta) \le 0$. That is, $\mu_n^*(\theta)$ never changes the sign of marginal profit $\mathcal{L}^{(n)}(\theta)$, which implies that $\mu_n^*(\theta)$ has no impact on the optimal solution by (11).

Intuitively, this is because the optimal solution given by (11) never violates the first constraint of primal problem (10).

Let $K_1(\theta) \triangleq \max_{n\in\mathcal{N}} \mathcal{J}_2^{(n)}(\theta)$ denote the highest core marginal profit, and $K_2(\theta) \triangleq \max_{n\in\mathcal{N}/n_1} \mathcal{J}_2^{(n)}(\theta)$ the second highest, where $n_1 \triangleq \arg\max_{n\in\mathcal{N}} \mathcal{J}_2^{(n)}(\theta)$. Next we show the necessary conditions for the optimal $\eta^*(\theta)$.

Lemma 4 (Feasible Range of $\eta^*(\theta)$)**.** *The optimal shadow prices $\eta^*(\theta)$ must satisfy the conditions:*

(a) $\eta^*(\theta) \in [\max(0, K_2(\theta)),\ K_1(\theta)]$, *if* $K_1(\theta) > 0$,
(b) $\eta^*(\theta) = 0$, *if* $K_1(\theta) \leq 0$.

It is easy to see that $\mathcal{J}_1^{(n)}(\theta) = \mathcal{J}_2^{(n)}(\theta) - \eta^*(\theta), \forall n$. That is, the shadow price $\eta^*(\theta)$ can be viewed as an *identical* reduction of each contract user's marginal profit at point θ. Lemma 4 suggests that a feasible $\eta^*(\theta)$ lies between $K_1(\theta)$ and $\max(0, K_2(\theta))$, with which there is *at most one* SU (say n) with positive $\mathcal{J}_1^{(n)}(\theta)$, and therefore $\mathcal{L}^{(n)}(\theta) \geq 0$ by Lemma 3 and $a_n^*(\theta) = 1$ by (11). Thus the user coupling constraint (ii) in the primal problem (10) will be satisfied.

By Lemmas 3 and 4, the optimal solution is to allocate each spectrum θ to the contract user with the highest and positive core marginal profit $\mathcal{J}_2^{(n)}(\theta)$. Let $\boldsymbol{n}^* \triangleq \{n \mid \mathcal{J}_2^{(n)}(\theta) = K_1(\theta)\}$ denote the set of contract users with the highest core marginal profit. Formally, the optimal solution is given by the following lemma.

Lemma 5 (**Optimal Solution**)**.** *For $\forall\theta \in \boldsymbol{\Theta}$, we have $a_n^*(\theta) = 0$ if $n \notin \boldsymbol{n}^*(\theta)$; and if $n \in \boldsymbol{n}^*(\theta)$, then*

(a) $a_n^*(\theta) \in [0,1]$ *such that* $\sum_{n\in\boldsymbol{n}^*} a_n^*(\theta) = 1$, *if* $K_1(\theta) > 0$;
(b) $a_n^*(\theta) \in [0,1]$ *such that* $\sum_{n\in\boldsymbol{n}^*} a_n^*(\theta) \leq 1$, *if* $K_1(\theta) = 0$;
(c) $a_n^*(\theta) = 0$, *if* $K_1(\theta) < 0$;

Lemma 5 states that only the contract users with the highest positive $\mathcal{J}_2^{(n)}(\theta)$ may win a spectrum θ. If $\mathcal{J}_2^{(n)}(\theta)$ is smaller than 0 for all contract users (i.e., $K_1(\theta) < 0$), the spectrum is allocated to the spot market (or more strictly, the highest valuation user in the spot market).

When SUs have continuous and heterogeneous valuations, the range of θ such that $\mathcal{J}_2^{(n)}(\theta) = 0$ (which may lead to case (b) in Lemma 5) or multiple contract users having the same $\mathcal{J}_2^{(n)}(\theta)$ (which may lead to $|\boldsymbol{n}^*| > 1$) has a zero size support, and therefore can be ignored. Thus, the optimal allocation given by Lemma 5 is equivalent to:

$$a_n^*(\theta) = 1 \text{ iff } \mathcal{J}_2^{(n)}(\theta) > 0 \ \&\ \mathcal{J}_2^{(n)}(\theta) > \max_{i\neq n} \mathcal{J}_2^{(i)}(\theta). \quad (12)$$

Let $\boldsymbol{\Theta}_n^+ \triangleq \left\{\theta | \mathcal{J}_2^{(n)}(\theta) > 0 \& \mathcal{J}_2^{(n)}(\theta) > \max_{i\neq n} \mathcal{J}_2^{(i)}(\theta)\right\}$ denote the set of spectrums allocated to a contract user n. Obviously, $\boldsymbol{\Theta}_n^+$ is determined by the shadow price vector $\boldsymbol{\Lambda}^* \triangleq (\lambda_1^*, ..., \lambda_N^*)$, and thus we can write $\boldsymbol{\Theta}_n^+$ as a function of $\boldsymbol{\Lambda}^*$, denoted by $\boldsymbol{\Theta}_n^+(\boldsymbol{\Lambda}^*)$. The optimal $a_n^*(\theta)$ can be equivalently written as: $a_n^*(\theta) = 1$ for all $\theta \in \boldsymbol{\Theta}_n^+(\boldsymbol{\Lambda}^*)$, and $a_n^*(\theta) = 0$ for all $\theta \notin \boldsymbol{\Theta}_n^+(\boldsymbol{\Lambda}^*)$. Thus, the expected number of spectrums to contract user n can be written as $\mathbb{E}[d_n] = S\int_{\theta\in\boldsymbol{\Theta}_n^+(\boldsymbol{\Lambda}^*)} f_{\boldsymbol{\Theta}}(\theta)\mathrm{d}\theta$.

According to (D.3), the optimal shadow price λ_n^* is either 0 if the associated constraint is not tight, or otherwise a non-negative value makes the associated constraint tight. Formally,

Lemma 6 (Optimal Shadow Price λ_n^*)**.** *The optimal shadow price $\lambda_n^*, \forall n \in \mathcal{N}$, is given by:*

$$\lambda_n^* = \max\left\{0, \arg_{\lambda_n} S \cdot \int_{\theta\in\boldsymbol{\Theta}_n^+(\boldsymbol{\Lambda}_{-n}^*, \lambda_n)} f_{\boldsymbol{\Theta}}(\theta)\mathrm{d}\theta = D_n\right\}.$$

Substituting the optimal shadow price λ_n^* into the core marginal profit $\mathcal{J}_2^{(n)}(\theta)$, we can derive the optimal primal allocation $a_n(\theta)^*$ of any spectrum θ according to Lemma 5 or (12). Intuitively, the shadow price λ_n^* can be viewed as a *vertical* shift for the core marginal profit $\mathcal{J}_2^{(n)}(\theta)$ of contract user n, such that $\mathbb{E}[d_n]$ meets the demand D_n.

C. Efficiency

An allocation is *efficient* if it maximizes the *social welfare*, which is defined as the overall welfare of all SUs.

1) Spot market user's welfare: For each SU m in the spot market, the welfare is directly defined by his maximum willingness-to-pay (i.e., his valuation v_m).

2) Future market user's welfare: For each SU n in the future market, the welfare consists of three parts: (i) a fixed value $\widetilde{B}_n$ related to his willingness to pay, (ii) an elastic cost $\widetilde{w}_n \cdot \mathbb{E}[C_n]$ used to evaluate his satisfaction in the long run, and (iii) a potential welfare loss $\widetilde{P}_n \cdot (D_n - \mathbb{E}[d_n])$ if the demanded number of spectrums is not satisfied.

Given any allocation $\boldsymbol{A}(\theta)$, the expected social welfare can be formally written as:

$$\mathbb{E}[W] = \mathbb{E}[W_0] + \sum_{n=1}^{N} \mathbb{E}[W_n], \quad (13)$$

where $\mathbb{E}[W_0] = S \cdot \int_\theta a_0(\theta) Y_M^1(\theta) \cdot f_{\boldsymbol{\Theta}}(\theta)\mathrm{d}\theta$ is the total welfare from the spot market, and $\mathbb{E}[W_n] = \widetilde{B}_n - \widetilde{w}_n \cdot \mathbb{E}[C_n] - \widetilde{P}_n \cdot (D_n - \mathbb{E}[d_n])$ is the welfare from contract user n.

To concentrate on the difference induced by different information scenarios, we introduce the following assumptions:

(a) $\widetilde{w}_n = w_n, \forall n$, i.e., the PO and the contract user n have the same evaluating weight for the contract user n's long-term satisfaction loss (cost);
(b) $\widetilde{P}_n = \widehat{P}_n, \forall n$, i.e., the PO's penalty is totally used to compensate the contract user's welfare loss induced by the violation of the contract.

With above assumptions, the social welfare (13) is equivalent to the PO's expected profit (8) up to a constant, which implies that the PO's profit maximizing solution maximizes the social welfare as well.

Lemma 7 (Efficiency)**.** *Suppose $\widetilde{w}_n = w_n$ and $\widetilde{P}_n = \widehat{P}_n$. The optimal solution for PO's profit maximization, i.e., Lemmas 3-6, also maximizes the social welfare.*

VI. Optimal Solution Under Information Asymmetry

Under information asymmetry, the PO cannot observe the realized SUs' private information at each time slot, and thus an incentive-compatible mechanism is necessary to elicit the SUs' private information. Without loss of generality, we apply an *VCG-based auction* (e.g., a second-price auction), which is provably optimal [18], as the underlying mechanism.

A. ContrAuction Mechanism

The first question is: *how to involve the contract users into the auction mechanism?*[4] To address this, we propose an integrated contract and auction design, called *ContrAuction*, wherein the PO acts as *virtual bidders* on behalf of the guaranteed contracts. That is, the PO now plays two types of roles: a seller by offering spectrums on the market, and N bidding agents by bidding on behalf of the contracts.

The second question for the PO is: *how to determine the optimal bid for each contract?* In what follows, we will derive the optimal bidding rules under the constraint of efficiency.

Note that although we consider information asymmetry, we assume that the PO can obtain the realized information of contract users, while cannot obtain those of spot market users. This assumption is motivated by the fact that the contract users have no incentive to hide their private information, since their payments are independent of their valuations.

B. Optimal ContrAuction under Efficiency

By Lemma 7, an efficient mechanism achieves the same allocation as with information symmetry. Inspired by Lemma 5, we propose the following bidding rule for each contract user n (the PO bids on behalf of the contract user n):

$$b_n(\theta) \triangleq \widehat{P}_n - w_n c_n(\theta) - \lambda_n^* = x_n(\theta) - \lambda_n^*, \qquad (14)$$

where λ_n^* is given by Lemma 6, i.e., the optimal shadow price under information symmetry.

As the overall mechanism is a second-price auction, the *truthfulness* for spot market users is obvious in ContrAuction. The following two Lemmas show the efficiency and optimality of the ContrAuction with the bidding rule (14).

Lemma 8 (Efficiency)**.** *With the bidding rule (14), the ContrAuction maximizes the social welfare.*

The efficiency can be easily proved by showing that the ContrAuction with (14) achieves the same allocation as the optimal allocation under information symmetry.

Lemma 9 (Optimality)**.** *The ContrAuction with (14) maximizes the PO's profit among all* ***efficient*** *mechanisms.*

Any efficient mechanism must achieve the same allocation as that under information symmetry. Thus, we can divide the whole spectrums (under an efficient mechanism) into two parts: those allocated to the future market (part I) and those to the spot market (part II). The optimality can be easily proved by the facts that (i) the profit from part I is the maximum profit, since it is same as that in information symmetry, and (ii) the profit from part II is also the maximum profit that the PO can achieve *in information asymmetry*; this can be shown by the revenue equivalent principle [18].

VII. Simulation Results

Our simulations show that under the constraint of efficiency, the optimal ContrAuction suffers certain profit loss (15% on average in our simulations) induced by the information rent for the spot market users. We skip the detailed simulations due to space limitations. For details, please refer to [20].

[4]This question is not trivial, because a contract user usually cares only about whether the contract can be satisfied. Thus, a contract user is usually not willing to (or not able to) be involved in the competition of an auction.

VIII. Conclusion

We study the short-term secondary spectrum trading in a hybrid spectrum market with the future market and the spot market, and focus on the PO's expected profit maximization problem with stochastic information. We derive the optimal solutions under both information symmetry and asymmetry. Under information symmetry, the optimal solution (benchmark) maximizes both the PO's expected profit and the social welfare. Under information asymmetry, We propose the *ContrAuction*, an integrated contract and auction design, and derive the optimal ContrAuction under the constraint of efficiency. The optimal ContrAuction suffers certain profit loss (15% on average in our simulations) under the constraint of efficiency.

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