On-Demand Spectrum Sharing By Flexible Time-Slotted Cognitive Radio Networks

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***Abstract*—In this paper, we present a novel framework for spectrum sharing in cognitive radio networks. The secondary users (SUs) can share the spectrum resource with primary users (PUs) in a cooperative manner, where PUs trade their information and surplus resource, and SUs access the primary spectrum intelligently based on SUs' heterogeneous demands and PUs' resource prices. After paying PUs a subscription fee for the spectrum information, SUs become spectrum-aware and avoid the overhead on spectrum sensing. During SUs' channel access, PUs further charge SUs based on the amount of resource taken by SUs. We model this sharing problem in a flexible time-slotted structure, where SUs' decisions include the selection of proper transmission channel and slot length to meet their demands. This joint decision problem is studied as a spectral temporal allocation game. We prove the existence of a Nash equilibrium and design a strategy update process which can converge to an equilibrium.**

***Index Terms*—Cognitive radio, spectrum sharing, potential game**

I. Introduction

Spectrum-awareness in cognitive radio networks is the basic requirement for harmonic spectrum sharing between secondary users (SUs) and primary users (PUs). Due to the hardware constraint or the lack of an efficient sensing algorithm, spectrum-awareness through independent spectrum sensing is rather difficult to implement especially in dynamic environment. Another alternative is for PUs or third parties to maintain a spectrum database containing PUs' future activities over space and time, and thus relieve SUs from the burden of spectrum sensing. Compared with independent spectrum sensing, the database-assisted spectrum-awareness can be more economic for scenarios where the PUs' activities do not change frequently. One example is the TV bands, where the FCC has made spectrum database the default approach and spectrum sensing optional [1]. As the spectrum database predictively details PUs' activities in near future (e.g., TV programs in 24 hour are scheduled in advance and reported to the spectrum database), SUs can query the database and obtain necessary information to make intelligent decisions on channel access [2], [3].

In this paper, we assume the existence of a spectrum database, and try to *design an efficient resource sharing mechanism that resolves the competition among different SUs*. We assume that SUs have different demand requirements, and show how they compete and negotiate with each other to achieve a balance among their selfish pursuits. The key challenge is to design a spectrum sharing mechanism that is *distributed* and requires only *limited information exchange*. As game theory is inherently strong at analyzing user interactions, it becomes an important method of studying the resource sharing problem in cognitive radio networks.

A key difference between this work and various results in literature is how the spectrum resource is characterized. Bandwidth sharing among SUs was studied in [4] as an auction-based game, where each SU makes a bid for the bandwidth and the PU decides how to allocate its spectrum based on all SUs' bids. In the most recent work [5], the authors further studied the auction-based spectrum allocation in both time and spectral dimensions. Considering SUs' flexible requirements, each SU is allowed to submit a bid consisting of a bundle of time-frequency slots. Power consumption was also considered in spectrum sharing. The authors in [6], [7], [8] considered joint spectrum allocation and power control in a potential game with interference mitigation as the major target. Pricing scheme was introduced to regulate SUs' behavior in [9], where bandwidth trading is modeled as a secondary market. A similar spectrum leasing model is proposed in [10], where the trading resource is viewed as the total tolerable interference at primary receiver.

In this paper, spectrum resource is considered in terms of channels and time slots of variable lengths, or more precisely, in spectral temporal blocks. Each SU selfishly maximizes its satisfaction by choosing the proper primary resource (i.e., a transmission slot of particular length on a particular primary channel), while considering the costs incurred which contains two parts: the first part is due to information service at the spectrum database, and the other is related to channel usage. An SU's channel usage is characterized by the length of time slot used for its own transmission. As all SUs and PUs share the primary channels, an SU's channel usage cost is related to the length of its own time slot, and also depends on other SUs' requests of time slots on the same channel (i.e., congestion cost). We formulate the distributed joint channel and slot length selection problem among all SUs as a *spectral temporal allocation game* with the following features:

- *Flexible Slotted Access*: We consider a time-slotted frame-

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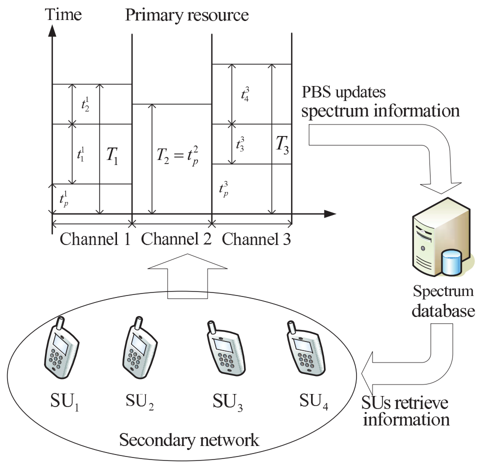


Fig. 1: System Model

work in which SUs' channel access is based on the joint selection of channel and time slot. Each SU chooses a proper time slot length to meet its own transmission demand.
- *Potential Function and Nash Equilibrium*: We show that the spectral temporal allocation game is a potential game. According to the finite improvement property of potential game, we design a distributed strategy update process that can converge to a pure Nash equilibrium.

The remainder of this paper is as follows. Section II introduces the system model. Section III studies the game-theoretic resource sharing problem and proposes the protocol design. Numerical results and conclusions are presented in Section IV and V, respectively.

II. System Model

We consider a cognitive radio network with N SUs and M primary channels in equal bandwidth (e.g., the bandwidth of whites-pace channels equals 6 MHz in U.S. and 8 MHz in EU). The set of SUs and primary channels are denoted as $\mathcal{N} = \{1, 2, \dots, N\}$ and $\mathcal{M} = \{1, 2, \dots, M\}$, respectively. Each SU is a dedicated pair of secondary transmitter and receiver, and can only access one PU's channel at one time. With a lower priority, SUs' channel access is subject to the availability of primary channels. To assist SUs' channel access, PUs predict the spectrum usage and update this information in a database, which then provides information service for all SUs and frees the SUs from the overhead of spectrum sensing. Fig. 1 gives an illustration of spectrum sharing model. Some notations are left for further explanation. We assume the information exchange among PUs, spectrum database, and SUs are through dedicated control channels [11].

Due to the stochastic natural of PUs' traffic, statically allocated spectrum resource is usually not fully utilized when PUs experience the off-peak hour. To this end, PUs consider selling the unused spectrum resource to SUs and to obtain some additional revenue. We consider the spectrum resource as transmission opportunities characterized by time slots, and propose the slotted structure in Fig. 2. The first sub-slot is left for negotiation: PUs negotiate with SUs through updating the spectrum database and offering the channel prices. Then, SUs compete for the transmission slots. When the negotiation process stabilizes, PUs' and SUs' transmissions are scheduled sequentially. We assume that the negotiation process takes much less time compared to the transmission time of PUs and SUs.

A. Flexible Time Slot

The major difference between our model and previously proposed models is the flexibility of choosing transmission time slot lengths. The higher priority PUs can set flexible primary slot t_p^k on each channel k for their exclusive usage in the first place. This setting of t_p^k should adapt according to the fluctuations of PUs' traffic, such that the probability of spectrum shortage is less than an acceptable level. The detailed procedure for PUs to adjust t_p^k requires further investigation and is beyond scope of this paper. Here we will assume a fixed primary slot length t_p^k on each channel k. For the ease of understanding, we assume that all PUs are served by the same primary base station (PBS)[1], which sets the prices and reserves the primary slots on all channels.

Given the channel prices on channel k, each SU i chooses a time slot with length $t_i^k \in \boldsymbol{T}_i \triangleq [t_{i,\min}, t_{i,\max}]$, which is also flexible according to its transmission demand (see Section II-C for detailed discussions). Therefore the total length of the time slots on channel k is the summation of two parts, i.e., $T_s^k = t_p^k + t_s^k$ where T_s^k is defined as a *super slot* and $t_s^k = \sum_{i \in \mathcal{N}^k} t_i^k$ denotes the total transmission time purchased by SUs on channel k. In practice, the PBS can set an additional upper bound T_k for the super slot on each channel k. If there are some SUs with very high demand and requesting large time slots that exceed the length of a super slot, their requests cannot be fulfilled and they get nothing. In this case, the PBS may change the channel price and restart the negotiation. We will discuss this problem further in the protocol design in Section III.

However, as selfish players, the SUs' may not accept the channel selling price. To motivate SUs' payment for their channel access, the PBS needs to provide more than unguaranteed channel access. In this paper, we consider nonpreemptive PUs, which will not preempt SUs' transmissions in the middle of a super slot. With this assumption, SUs are also provided with exclusive spectrum usage. Nevertheless, SUs' privilege is limited, since they are still required to suspend transmissions at the end of a super slot T_s^k whenever PUs return during the super slot. If the traffic demands of PUs are high, the PBS can set a small upper bound T_k on the super slot to improve PUs' througput.

B. PUs' Pricing Policy

As the license holder, the PBS would like to charge a price on SUs' channel access. We adopt a linear price scheme which

[1]Actually the spectrum database can serve as the PBS.

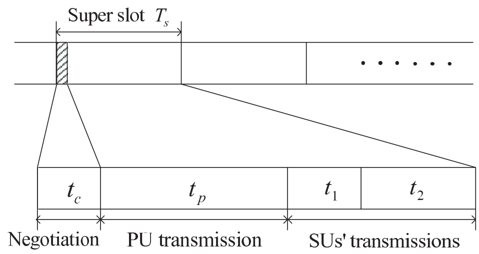


Fig. 2: Slotted structure

is commonly assumed in literature [12]:

$$c_i(k_i, t_i) = t_i\beta_{k_i} + \beta_d \tag{1}$$

where k_i and t_i are the operating channel and slot length of SU i, respectively. Here β_{k_i} denotes the unit usage price on channel k_i, and β_d is a constant subscription fee for the information service which is charged by the PBS for the spectrum database installation and maintenance, and for the information retrieve for SUs' strategy update in negotiation stage (please refer to Section III-B).

As we consider nonpreemptive PUs in this model, we need to have a mechanism that holds PUs' transmission and caches the incoming traffic. Therefore, PUs' performance will be very sensitive to the size of super slot. A larger super slot means a higher probability of holding PUs' traffic and a longer waiting time. Although the cost of cache (buffer) is not high today, PUs' delay performance may be significantly affected by a larger super slot. This consideration urges us to redesign the pricing function in (1). Besides the unit usage price β_{k_i} and constant fee β_d, we add a new pricing term that is proportional to the size of super slot, i.e.,

$$c_i(k_i, t_i) = \alpha_{k_i}T_s^{k_i}t_i + \beta_{k_i}t_i + \beta_d. \tag{2}$$

Here $\alpha_{k_i}T_s^{k_i}$ is the unit price that reflects PUs' sensitivity to transmission delay on channel k_i, and reflects the delay cost charged to SU i based on its contribution t_i. For simplicity, we define the new unit price as $\theta_{k_i}(T_s^{k_i}) = \alpha_{k_i}T_s^{k_i} + \beta_{k_i}$, and denote $\gamma_{k_i} = (\alpha_{k_i}, \beta_{k_i})$ as the pricing parameters on channel k_i. Such parameters may be different on different channels. Then we can write the new pricing model as $c_i(k_i, t_i) = \theta_{k_i}(T_s^{k_i})t_i + \beta_d$. When PUs' traffic are in the peak, the PBS sets a higher α_{k_i} in order to preserve the primary channels for PUs' transmissions. During the off-peak time, the PBS will choose a smaller coefficient α_{k_i}, and thus will reduce the impact of the super slot size on SUs' activities.

C. SUs' Strategy

An SU i's problem is to choose a channel $k_i \in \boldsymbol{A}_i$ and a slot length $t_i \in \boldsymbol{T}_i$ to maximize its payoff, where subset $\boldsymbol{A}_i \subset \mathcal{M}$ lists all primary channels available for SU i. This set represents the local spectrum map retrieved from the spectrum database. We define SUs' payoff as the difference between SUs' utility and channel access cost as follows:

$$\pi_i(\boldsymbol{k}, \boldsymbol{t}) = u_i(t_i, \bar{d}_i) - \alpha_{k_i}t_i\left(\sum_{j\in\mathcal{N}^{k_i}} t_j\right) - \beta_{k_i}t_i - \beta_d, \tag{3}$$

where $\boldsymbol{k} = [k_1, ..., k_N]$ and $\boldsymbol{t} = [t_1, ..., t_N]$ denote the channel and slot length selection of all SUs, respectively. Subset $\mathcal{N}^{k_i}$ denotes the SUs choosing the same channel k_i. The utility $u_i(t_i, \bar{d}_i)$ describes how user's demand $\bar{d}_i$ is satisfied by the chosen time slot t_i. A simple illustration is the quadratic demand function, i.e., $u_i(t_i, \bar{d}_i) = -(t_i - \bar{d}_i)^2$. In this demand model, SUs are rational since they only pursuit the transmission opportunities that exactly meet their demands. A more aggressive demand model is based on the α-fairness utility function [12]:

$$u_i(t_i, \bar{d}_i) = \begin{cases} \frac{\bar{d}_i}{1-\omega}t_i^{1-\omega} & 0 \le \omega < 1, \\ \bar{d}_i\log(t_i) & \omega = 1. \end{cases} \tag{4}$$

When SUs are charged with the same unit price $\theta_k(T_s^k)$ on channel k, their optimal slot lengths are proportional to their demands, i.e., $t_i^*(\theta_k(T_s^k)) = \arg\max_{t_i} u_i(t_i, \bar{d}_k) - \theta_k(T_s^k)t_i - \beta_d = (\bar{d}_i)^{\frac{1}{\omega}}/\theta_k(T_s^k)^{\frac{1}{\omega}}$. When primary resource is sufficient, each SU will pursuit the slot length as longer as possible. However, besides SUs' selfish pursuits, this demand model also considers proportional fairness between competing SUs, which is an appealing property in congestion control.

As illustrated in Fig. 1, the PBS can set different prices for 3 primary channels, resulting in different strategies of 4 SUs. Due to the heavy primary traffic load on channel 2, the PBS sets a high price on that channel and drives SUs to channels 1 and 3 with light primary traffic load. As a result, PUs' transmissions take up the complete super slot on channel 2, i.e., $t_p^2 = T_2$ (here T_2 can be viewed as the maximum length of the super slot on channel 2), while SUs 1 and 2 transmit on channel 1 with a slot length t_1^1 and t_2^1, respectively.

III. Game Analysis and Protocol Design

In this paper, we focus on the analysis of SUs' activities given PBS' decisions on channel price (α_k, β_k) and primary slot t_p^k. Then, each SU makes independent decision on channel and slot length selection to maximize the individual payoff, by taking into account SUs' competition, PUs' channel availabilities, and channel prices. We express the conflicting goal of all SUs into a *spectral temporal allocation game* $\mathcal{G} = (\mathcal{N}, \mathcal{M}, \mathcal{J}, \Pi)$. Specifically, each player (i.e., SU) $i \in \mathcal{N}$ chooses one available channel k_i from its local spectrum repository $\boldsymbol{A}_i \subseteq \mathcal{M}$, and purchases a transmission slot $t_i \in \boldsymbol{T}_i$ on the chosen channel. We denote the joint strategy space as $\mathcal{J} = \prod_{i\in\mathcal{N}}\boldsymbol{J}_i \triangleq \mathcal{A} \times \mathcal{T}$, where $\mathcal{A} \triangleq \prod_{i\in\mathcal{N}}\boldsymbol{A}_i$ and $\mathcal{T} \triangleq \prod_{i\in\mathcal{N}}\boldsymbol{T}_i$. SUs' payoffs are defined as $\Pi \triangleq \{\pi_i\}_{i\in\mathcal{N}} : \mathcal{J} \to \mathcal{R}^N$ where π_i is given in (3). We are interested in characterizing the Nash equilibrium (NE) of this game, where SUs' independent and selfish behaviors may reach a stable point where no SU has an incentive to deviate from its current strategy unilaterally.

A. Existence of Nash Equilibrium

Definition 1: A strategy profile $\boldsymbol{q} \in \mathcal{J}$ is a *Nash equilibrium* for the game, if and only if $\pi_i(q_i^{'}, \boldsymbol{q}_{-i}) \leq \pi_i(q_i, \boldsymbol{q}_{-i}), \quad \forall q_i^{'} \in \boldsymbol{J}_i, \quad i \in \mathcal{N}$, where $(q_i^{'}, \boldsymbol{q}_{-i})$ denotes the strategy profile when player i changes its strategy to $q_i^{'}$, while other players keep their previous strategies $(q_j, j \in \mathcal{N}, j \neq i)$ unchanged.

Definition 2: Given the current strategy profile $\boldsymbol{q}$, strategy update $q_i^{'}$ is a *better response* of SU i if $\pi_i(q_i^{'}, \boldsymbol{q}_{-i}) > \pi_i(q_i, \boldsymbol{q}_{-i})$.

Next, we show that our game falls into the class of *potential game* [13], which always admits a Nash equilibrium due to some appealing mathematical properties in users' payoff functions. In a potential game, any change in player's payoff function can be reflected by the change of a global *potential function* $\Phi(\cdot)$. Moreover, we will have an *exact potential game (EPG)* if the unilateral payoff change is the same as the change in the potential function, i.e.,

$$\Phi(q_i^{'}, \boldsymbol{q}_{-i}) - \Phi(q_i, \boldsymbol{q}_{-i}) = \pi_i(q_i^{'}, \boldsymbol{q}_{-i}) - \pi_i(q_i, \boldsymbol{q}_{-i}). \quad (5)$$

For our spectral temporal allocation game, the change of strategy profile from $(q_i, \boldsymbol{q}_{-i})$ to $(q_i^{'}, \boldsymbol{q}_{-i})$ implies that user i unilaterally changes its strategy to $q_i^{'} = (k_i^{'}, t_i^{'})$.

Proposition 1: The spectral temporal allocation game $\mathcal{G}$ possesses a potential function in the form of $\Phi(\boldsymbol{k}, \boldsymbol{t}) = \sum_{i \in \mathcal{N}} \Phi_i(\boldsymbol{k}, \boldsymbol{t})$, where

$$\Phi_i(\boldsymbol{k}, \boldsymbol{t}) = u_i(t_i, \bar{d}_i) - t_i \left(\alpha_{k_i} \sum_{j \leq i, k_j = k_i} t_j + \beta_{k_i} \right).$$

Proof: Given other users' actions $(\boldsymbol{k}_{-s}, \boldsymbol{t}_{-s})$ unchanged, the potential function before user s deviates from its current strategy (k_s, t_s) to $(k_s^{'}, t_s^{'})$ is represented in (6) on the next page. Since $Q(\boldsymbol{k}_{-s}, \boldsymbol{t}_{-s})$ is irrelevant with the strategy change and will not effect the potential, we can prove that $\Phi(\boldsymbol{k}, \boldsymbol{t})$ is an exact potential function as follows:

$$\begin{aligned}
& \Phi(\boldsymbol{k}^{'}, \boldsymbol{t}^{'}) - \Phi(\boldsymbol{k}, \boldsymbol{t}) \\
= \quad & u_s(t_s^{'}, \bar{d}_s) - t_s^{'} \left(\sum_{j \leq s, k_j = k_s^{'}} \alpha_{k_s^{'}} t_j + \beta_{k_s^{'}} \right) \\
& - u_s(t_s, \bar{d}_s) + t_s \left(\sum_{j \leq s, k_j = k_s} \alpha_{k_s} t_j + \beta_{k_s} \right) \\
& - \sum_{i > s, k_i = k_s^{'}} \alpha_{k_s^{'}} t_s^{'} t_i + \sum_{i > s, k_i = k_s} \alpha_{k_s} t_s t_i \\
= \quad & u_s(t_s^{'}, \bar{d}_s) - t_s^{'} \left(\sum_{j \in \mathcal{N}, k_j = k_s^{'}} \alpha_{k_s^{'}} t_j + \beta_{k_s^{'}} \right) \\
& - u_s(t_s, \bar{d}_s) + t_s \left(\sum_{j \in \mathcal{N}, k_j = k_s} \alpha_{k_s} t_j + \beta_{k_s} \right) \\
= \quad & \pi_s(k_s^{'}, t_s^{'}) - \pi_s(k_s, t_s).
\end{aligned}$$

■

The nice property in (5) implies that the potential function will be monotonically increasing if users sequentially play their better responses. Therefore, a deterministic strategy profile $\boldsymbol{q} \in \mathcal{J}$ is a pure Nash equilibrium for the potential game if and only if $\Phi(\boldsymbol{q}) \geq \Phi(q_i, \boldsymbol{q}_{-i})$, $\forall q_i \in \boldsymbol{J}_i$ for every player i. This implies that finding the Nash equilibrium in a potential game is equivalent to maximizing its potential function.

Theorem 1: Spectral temporal allocation game $\mathcal{G}$ possesses a pure Nash equilibrium.

Proof: Since the potential function $\Phi(\boldsymbol{k}, \boldsymbol{t})$ is finite, there is always a maximal point $\boldsymbol{q}$ for the potential function. If $\boldsymbol{q}$ is not a NE point and an SU i has incentive to deviate, then $\Phi(q_i^{'}, \boldsymbol{q}_{-i}) - \Phi(\boldsymbol{q}) = \pi_i(q_i^{'}, \boldsymbol{q}_{-i}) - \pi_i(\boldsymbol{q}) > 0$, which contradicts with the assumption. ■

In order to achieve a Nash equilibrium, we design a sequence of strategy updates $\boldsymbol{I}_s \triangleq \{\boldsymbol{q}^{i_1}, \ldots, \boldsymbol{q}^{i_n}, \ldots\}$. Here $\boldsymbol{q}^{i_n}$ indicates that, in the n-th strategy update, player i_n updates its strategy. If each strategy update in the sequence $\boldsymbol{I}_s$ strictly increases the payoff function of the corresponding player, we call $\boldsymbol{I}_s$ an *improving sequence*.

Proposition 2: In a potential game, any improving sequence $\boldsymbol{I}_s$ converges to a Nash equilibrium within finite steps.

Proposition 2 presents the *finite improvement property* of a potential game [13]. Since our potential function $\Phi(\boldsymbol{k}, \boldsymbol{t})$ is finite, any improving sequence will maximize it within finite steps and achieve a Nash equilibrium.

B. Protocol Design

Based on the finite improvement property, we now design a distributed protocol for the channel negotiation in Fig. 2. Suppose that an SU is currently choosing strategy (k, t). Then we can rewrite its cost function as $c_i(k, t) = \alpha_k t^2 + \beta_k t + \alpha_k p_k^i t + \beta_d$, where $p_k^i = \sum_{j \in \mathcal{N}^k, j \neq i} t_j$ represents the congestion level SU i experienced on channel k. The first two terms in the cost function relate to PBS' pricing parameters on channel k. The third term relates to user-specific channel experience. And the last term is the constant subscription fee for all SUs. Therefore, we define an information set $\{p_k^i, \gamma_k\}_{k \in \boldsymbol{A}_i}$ to characterize the channel quality perceived by SU i. Note that the pricing parameter $\gamma_k = (\alpha_k, \beta_k)$ is channel specific, and can be broadcasted by the PBS when SUs enter the network. While the congestion level needs to be informed by PBS during the negotiation. The cost of information broadcast by PBS can be integrated into the information service fee. We denote SU's better response as a set $\text{BR}_i(\boldsymbol{k}, \boldsymbol{t}) \triangleq \{(k^{'}, t^{'}) \in \boldsymbol{A}_i \times \boldsymbol{T}_i | \pi_i(\boldsymbol{k}_{-i}, k^{'}, \boldsymbol{t}_{-i}, t^{'}) > \pi_i(\boldsymbol{k}, \boldsymbol{t})\}$ given the current strategy profile $(\boldsymbol{k}, \boldsymbol{t})$.

Finite improvement property requires asynchronous strategy updates, which can be achieved through the PBS' coordination. Before the competition among SUs reaching a Nash equilibrium, the PBS sequentially polls SUs to update their strategies and passes them the information set. After receiving SU's strategy update, the PBS then updates the information set and passes it to the next SU. This process iterates until no SU is willing to update its strategy any more. In the transmission stage, all SUs will be scheduled to transmit within

$$
\begin{aligned}
\Phi(\boldsymbol{k},\boldsymbol{t}) &= u_s(t_s,\bar{d}_s) + \sum_{i\neq s} u_i(t_i,\bar{d}_i) - t_s\left(\alpha_{k_s}\sum_{j\leq s,k_j=k_s} t_j + \beta_{k_s}\right) - \sum_{\substack{i\neq s,\\ k_i\neq k_s,k_s^{'}}} t_i\left(\alpha_{k_i}\sum_{j\leq i,k_j=k_i} t_j + \beta_{k_i}\right) \\
&\quad - \sum_{i\neq s,k_i=k_s} t_i\left(\alpha_{k_i}\sum_{j\leq i,k_j=k_s} t_j + \beta_{k_i}\right) - \sum_{i\neq s,k_i=k_s'} t_i\left(\alpha_{k_i}\sum_{j\leq i,k_j=k_s'} t_j + \beta_{k_i}\right) \\
&= u_s(t_s,\bar{d}_s) - t_s\left(\alpha_{k_s}\sum_{j\leq s,k_j=k_s} t_j + \beta_{k_s}\right) + Q(\boldsymbol{k}_{-s},\boldsymbol{t}_{-s}) \\
&\quad - \sum_{i>s,k_i=k_s} t_i\left(\alpha_{k_s}\sum_{\substack{j\leq i,j\neq s,\\ k_j=k_s}} t_j + \alpha_{k_s}t_s + \beta_{k_s}\right) - \sum_{i>s,k_i=k_s'} t_i\left(\alpha_{k_s'}\sum_{\substack{j\leq i,j\neq s,\\ k_j=k_s^{'}}} t_j + \beta_{k_s'}\right). \qquad (6)
\end{aligned}
$$

their own time slots. We propose the asynchronous strategy update process in Algorithm 1.

However, there is still a possibility that the sum of SUs' required time opportunities at the equilibrium exceeds PUs' acceptable slot length (i.e., the length of super slot on a channel k is greater than the pre-defined upper bound T_k). We leave this to the PBS' discretion as shown in Fig. 3. More specifically, if the converging equilibrium by Algorithm 1 does not satisfy the total time length constraints, the PBS rejects this equilibrium. Based on the time slots purchased by different SUs on different channels, the PBS predicts the demands of SUs. Then the PBS updates the channel price accordingly, and initiates another round of negotiation process. When the negotiation reaches a feasible equilibrium, all PUs and SUs are scheduled to transmit according to the converging spectral temporal allocations.

Algorithm 1 Spectral Temporal Allocation

initialization:
SUs choose initial strategy $(\boldsymbol{k},\boldsymbol{t})$
PBS broadcasts the pricing information
end initialization
loop until convergence
PBS polls an SU to update strategy
PBS passes information set $\{p_k^i,\gamma_k\}_{k\in\mathcal{M}}$ to SU
SU i **updates** strategy $(k^{'},t^{'})$ from set $\mathrm{BR}_i(\boldsymbol{k},\boldsymbol{t})$
end poll
end loop

Once the context (i.e., spectrum environment) has been changed, the PBS will suspend the periodic transmission and start another around of negotiation at the end of the previous super slot. For TV bands, the change of spectrum environment is usually due to prearranged program shifting in a larger time scale. In this case, this protocol can provide long time stable operations for SUs with small negotiation overhead at a Nash equilibrium.

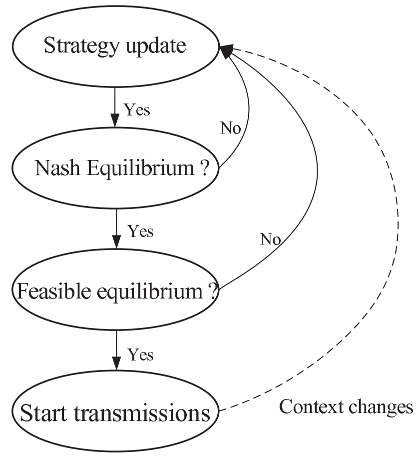
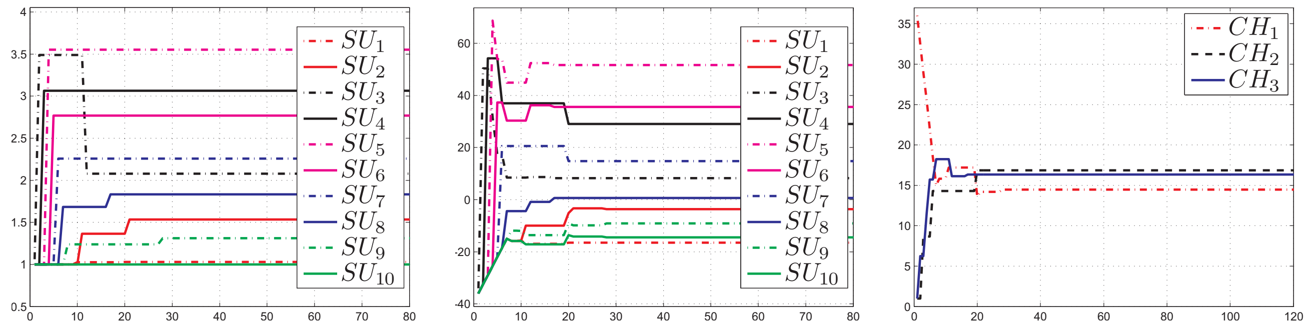


Fig. 3: Flowchart of the competition and negotiation process.

IV. Simulation Results

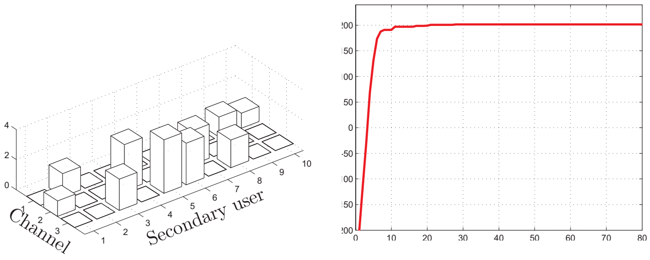
In the simulation, we consider $N = 10$ SUs sharing $M = 3$ primary channels. We use the α fairness utility in (4) and set $\omega = 1$ for all SUs. SUs' demand requirements are characterized by different parameters $\bar{\boldsymbol{d}} = [20, 30, 40, 50, 60, 55, 45, 35, 25, 15]$. Each SU's choice of slot length is limited to the interval set $\boldsymbol{T} = [0.5, 5]$. We consider the PUs in the peak hour, thus the unit price β_k is small compared to $\alpha_k T_s^k$. According to PUs' traffic demands, we assume that the PBS sets the pricing parameters for 3 channels as $\boldsymbol{\alpha} = [3.5, 2.5, 1.5]$ and $\beta_k = 0$ for $k = 1, 2, 3$. We also assume that the subscription fee for the spectrum database is negligible and hence set $\boldsymbol{\beta}_d = 0$.

Fig. 4a shows the dynamics of SUs' selection on their time slots during the algorithm iterations. Initially, we set all SUs transmit on channel 1 with the unit slot length. Due to severe congestion and the high price on channel 1 ($\alpha_1 = 3.5$), SUs obtain small payoffs initially as shown in Fig. 4b. After some iterations (around 25), SUs' choices of slot length coincide with their demands. Note that SUs 4, 5, and 6 are high



(a) Slot allocation **(b)** User payoff **(c)** Channel price

Fig. 4: Dynamics of slot length selection, user payoff, and channel price



(a) Nash equilibrium **(b)** Potential function

Fig. 5: Convergence to Nash equilibrium

demanding users (higher values in vector $\bar{d}$), therefore they choose longer time slots and their final payoffs stabilize on the top of Fig. 4b. Meanwhile, the slot lengths and payoffs of low demanding SUs 1, 9, and 10 converge to smaller values.

The initial high price on channel 1 urges competitive SUs to deviate from severe congestion, therefore raises the prices on other channels. When prices on different channels become close enough to each other as shown in Fig. 4c, there will be no incentive for any SU to deviate. Fig. 5a plots the channel access map at an NE point, where every SU's channel access strategy is the best response of other SUs' strategies. The Nash equilibrium is further verified by the maximum point of its potential function as shown in Fig. 5b, which gradually increases as different SUs asynchronously update their strategies.

V. Conclusion

In this paper, we study the joint channel and slot selection problem for competitive SUs with primary pricing. This problem is modeled as spectral temporal allocation game which is shown to be a potential game. We prove the existence of a pure Nash equilibrium, and design a strategy update process that converges to a Nash equilibrium. As an extension, we may consider the interactions between PBS' pricing and SUs' strategy changes, and thus model the whole system using a multi-stage dynamic game.

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