Bargaining and Peering between Network Content/Coverage Providers

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Abstract

In content distribution market, the content popularity and market coverage have significant impacts on a network content provider's revenue. One way for a network content provider to obtain popular content is to peer with other providers and share contents. But the providers need to bargain and reach a win-win peering strategy. Thus, the aim of the present study is to characterize the popularity as the content quality through the analysis of how providers' peering can affect content quality they offer and change their market coverage. The findings identify the peering conditions and optimal peering strategies for sharing contents.

Firstly, we consider a static baseline model, whereby network content providers have static content and do not peer. We derive the coverage of the providers based on the quality of the contents and user subscription fees. Then we consider how peering and content sharing can help providers to improve their revenues. The key insight is that peering will be desirable when the providers' total revenue is increased and properly shared by inter-provider financial transfers. In the case of linear advertisement functions, peering will take place when providers have different abilities in generating advertisement revenue and set subscription fees properly.

Secondly, we consider the dynamic content model, whereby a provider can introduce some high quality special content for a short period of time in order to attract users. Two cases of the special content budget sources are discussed: additional investment and finite budget. The findings indicate that the budget

source, the special content timing, the switching cost, the valuation of content, and time discount factor all play important roles in deciding the benefit of special content.

Finally, we consider the peering incentive in dynamic content model and compare the optimal peering strategy with the the one in static content case. Two peering agreements for additional investment case are first analyzed: Peering over T time slots and Peering over One time slot. We discuss different advertisement revenue functions and present providers' peering incentive. In the case of linear advertisement revenue function, the findings show that the difference between static model and the two peering cases in dynamic model is for the scenario that the special content right holder has a stronger ability in generating advertisement revenue. For peering over T time slots case, the peering condition is stricter if the special content quality increases. Then, we identify that the peering incentive in finite budget case is higher than in additional case. While for peering over one time slot case, we show that when providers consider the future revenues more in both special content budget cases, their peering incentive is lower.

摘要

在內容傳播的市場里，內容的受歡迎程度和市場覆蓋率對網絡內容運營商的收入有非常重要的影響。網絡內容運營商獲取優質內容的一種方式是與其他內容運營商合作并共享受歡迎的內容。但這些合作者需要互相協商，從而達成雙贏的合作策略。在本研究中，我們將內容的受歡迎程度定義為內容的質量。我們將分析內容運營商之間的合作是如何影響他們提供的內容質量和市場覆蓋率，并提出他們合作并共享內容所需要的條件及其相應的最優合作策略。

我們首先考慮的是基準靜態內容模型，網絡內容運營商提供靜態的內容質量。在運營商不合作的基礎上，我們推導出基於內容質量和用戶服務費用的市場覆蓋率。然後我們研究運營商是如何通過合作及內容共享來提高他們的用戶覆蓋率及收入。產生合作的關鍵是運營商的總收入增加及在合作者間有互惠互利的收入分配策略。在廣告收入函數是線性函數時，合作發生於運營商擁有不同的廣告收入能力及恰當的用戶服務費作用的情況下。

其次，我們進一步考慮動態的內容模型。其中一個運營商可以引入一個很短時間的高質量特殊內容來吸引用戶。我們將研究兩種引用特殊內容的預算方式：額外投資方式和限額預算方式。研究結果表明，預算方式，用戶轉換運營商成本，用戶對內容的估值以及時間銀子等在決定特殊內容的收益時都發揮重要的作用。

最後，我們研究了網絡內容運營商在動態內容模型中的合作意愿，并與之前的靜態模型中的合作意愿相比較。我們首先考慮了在額外投資特殊內容時兩種合作的例子：長時間合作和一次性合作。我们讨论了在不同广告收入函数情況下运营商的合作意愿。研究結果表明，在广告收入函数为线性的例子中，特殊內容擁有者具有較強廣告收入能力的情況下，動態模型的合作條件不同於靜態模型中的合作條件。在長時間合作中，當特殊內容的受歡迎程度提高时，合作條件会更加嚴格。而在限額預算時的合作條件要比額外投資方式時要寬鬆。另一方面，在一次性合作中，在兩種預算方式中，當特殊內容擁有者看重未來收入的情況下，運營商的合作意愿會降低。

Acknowledgement

I would like to take this opportunity to give my sincere thanks to Professor Dah Ming Chiu and Professor Jianwei Huang. I have received plenty of precious guidance from them not only on doing research, but also on how to form the habit of critical thinking. It is difficult to find words powerful enough to convey the extent of his help and my gratitude. I am so lucky to have such great scholars as my supervisors and work closely with them to learn their aspiration for innovation. They are knowledgable, conscientious, responsible and full of passionate. This study would not have been completed without their support and help.

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Chapter 1

Introduction

1.1 Background for Network Content Providers' Peering

The digital revolution has prompted the development of the content distribution service. At present, users can access their chosen contents through a variety of advanced technological distribution methods, such as satellite TV, cellular network and internet, as well as through traditional methods, including radio, broadcasting television, cable system and so on. This growing variety of content distribution methods has resulted in increased number of *network content providers*, who endeavor to attract users from other providers. To do this, providers should produce or purchase popular contents continuously, so as to draw as many users as possible to subscribe to their network. Popular content, such as live sports and Hollywood Movies, has a significant value in helping the network content providers to build up a substantial subscriber base. These competitive and contemporary contents can attract great interest from users, and can significantly increase the market coverage of the network content providers. This, in turn, will increase the providers' advertisement and subscription revenues. One way for a network content provider to obtain popular contents is to peer with other providers in order to share contents.

However, peering agreements among the network content providers are not always easy to reach. For example, Google TV, a new Internet-connected television platform, aims at providing users with new experiences of enjoying both traditional TV and web contents [1]. But some content providers (*e.g.*, NBC and ABS in the U.S.) choose to block Google TV from accessing their TV programs [2]. These content providers are afraid that this new technology may influence advertisers' choice of advertising platforms (*e.g.*, Google TV vs. NBC's website) and reduce their advertisement incomes. A proper financial agreement between Google and the content providers may resolve this issue. Besides regular contents, there are some special contents (*e.g.*, world cup programs) that can attract many users during a certain period of time. Such attractive contents can be used as a powerful tool for network content providers to gain additional market share. The 2010 world cup broadcasting right issue in Hong Kong showed how fiercely content providers bargain over the special content delivering right [3]. The official broadcast right holder (iCable in Hong Kong) has limited subscribers and wanted the content to reach a bigger coverage together with its own advertisement, while other content providers (*e.g.*, TVB and ATV in Hong Kong) had a large audience and wished to purchase the broadcasting right without the advertisement from iCable. A final agreement was reached which led to a win-win situation of both sides. Otherwise, users who have not subscribed to iCable can not access the contents and enjoy the amazing football matches. This is definitely not beneficial from the social welfare's point of view. In a third example, mobile TV has been developed for a long time and has gone into the daily life of people. Users can access the contents and television programs through handhold devices while traveling. The worldwide mobile TV market has been growing fast [4]. It is a potential huge market awaiting for the network content providers to explore. However, although providers want to reach the potential large audience, indoor users typically have difficulty in accessing the mobile TV programs with a high quality due to poor cellular signal receptions. Owners of large office buildings and shopping malls may help to "amplify" the signals through special equipments, and receive payments from the mobile TV providers for providing the extra coverage. The two providers need to bargain and reach a proper agreement. So both them can benefit from the development of new technology for distributing contents.

In this study, we are motivated by the above three examples and want to study the interactions among multiple network content providers over content, coverage, and the possible strategies of peering. The network architecture is illustrated in Fig 1.1.

Figure 1.1: Network architecture among advertisers, network content providers, and end users

The network content providers work as intermediaries to offer a platform for advertisers to deliver their advertisement, as well as provide the end users with contents and advertisement. In this study, content quality is seen as a synonym for content popularity. We will assume that network content providers obtain revenue through two approaches: advertisement income based on the agreement with advertisers and the market coverage, and subscription income based on the content quality and the subscription fees. However, the network content providers often can typically only cover a portion of the total market share. Thus, other than increasing the content quality and decreasing the subscription fee, peering with other providers will be a viable option for the providers to increase coverage and revenues. However, for this avenue to be fully exploited, peering agreements must be perceived as fair and beneficial by all providers to the agreement.

Therefore, the present study aims to investigate the peering strategy between the network content providers. The focus will be on the interactions of two network content providers in three cases. In the first baseline "static" case, both providers have fixed quality contents and subscription fees over time and they do not peer with each other. We will examine the users' subscription choices and the corresponding market share. Then we study how providers' peering can increase their revenues with static contents. In the second "dynamic" case, providers can change their contents quality by introducing special content with high quality. Two cases of the special content budget sources will be considered: additional investment and finite budget. We will discuss how the budget sources, the introduction of a special content and switching cost will impact the users' subscription choices and the providers' revenues. It is further assumed that the switching cost will only be incurred if the users switch to other provider from the original subscribing provider and the contract is terminated early [13]. In the third case, both providers consider whether or not to peer in delivering the special content. Two peering agreements for additional investment case are consider: peering over T time slots and peering over one time slot. We first show how the providers' peering agreements in the "dynamic" case and the advertisement revenue functions affect the peering incentive, and compare their strategies with the peering strategy in static case. Then, we discuss the peering incentive for finite budget case.

Our main results and contributions of this study are as follows:

- *General Network Model:* We present a model that captures the interactions among advertisers, network content providers, and users, and explain how users choices influence the network content providers' content strategies, revenues and peering incentive in both static and dynamic content model.

- *Win-win Peering Agreement:* We propose a Nash bargaining based peering framework between providers in both content models, by considering the changes of content, advertisement, and coverage within the peering. We characterize the necessary condition for cooperation to happen, and show that a provider's bargaining power depends on its capability of generating advertisement revenue.
- *Impact of Dynamic Content:* One-time special content induces users to switch providers. We show how the budget source, the switching cost, content evaluation, and time discount factor together determine a user's subscription decision and the providers' revenues. We further show how the introduction of special content changes the peering strategy between the providers, and compare it with the optimal peering strategy in static content model.

The rest of the study is organized as follows. Section 1.2 discusses the related work in the area of content distribution market. Chapter 2 presents the static baseline model and shows the peering strategy between providers through bargaining. The impact of dynamic content without peering between providers is given in Chapter 3 and the peering incentive in dynamic content model is presented in Chapter 4. We conclude in Chapter 5 and summarize the future work directions.

1.2 Literature Review

The research in the field of the interaction among the network providers has been experienced a marked development. Studies on different aspects of the phenomenon, such as the pricing policies of the contents and cooperating or multi-homing with other network content providers have dominated the research effort. Three types of stakeholders are often found during the interaction procedures among providers-Content Providers (CP) (sometimes referred to as Advertisers, as is the case in the present study), Network Providers (NP) and Users. Although the NP offers a platform for the interaction between users and CPs, they can still have diverse interests. From CPs' perspective, the goal is to produce the contents and allow them to reach as many users as possible, yielding higher revenues. In contrast, NPs provide the contents with the access channels to user and desire to acquire the contents exclusively. So they can obtain a competitive advantage in attracting users compared with other providers. Finally, the users are concerned primarily with being able to access the popular contents whilst paying a low subscription fee. These different concerns of the three kinds of stakeholders lead to different research directions, which, in turn, yield various results and implications.

One research direction in this area is that the content distribution market is regarded and analyzed as a two-sided market. Reference [34] and [36] define a two-sided market as an environment whereby any change of the charge price by the NP will directly influence the other stakeholders (CP and Users) and the transactions between them. There are many examples of this kind of market, *i.e.*, the credit card market, the newspaper, the television industry, search engines and internet service industry. References ([6], [33], [35],[10] and [32]) have conducted an framework analysis for the pricing structure of NP. Their findings indicate that the intermediary is often inclined to charge less or subsidize one side of the stakholder (*i.e.*, users) to attract users to subscribe to the service. he view is that the cost of the subsidy will be covered by the increase in the charges for the other stakeholders. This pricing structure stems from the indirect network effect between the two sides. It is based on the assumption that if more users join in the platform NP, the CP will enjoy the increasing potential to sell their products, which will lead to CP paying a high price to subscribe to the platform service. Hence, the platforms are willing to offer some benefits to attract users to join their services. In addition to the pricing structure, reference [17] studied the business model of CP, focusing on the optimal way for the CP to interact with the NP and users. The authors examined whether selling the products to the NP and charging a transaction fee (platform mode) or selling the products directly to users and paying a channel usage fee to the NP (merchant mode) would be preferred option. The findings suggest that the merchant mode is superior to platform mode if the platform aims solely to maximize profits. Moreover, under this scenario, the NP is not responsible for the content quality, making the strategy less risky. Our study can be viewed as one research work on this two-sided market area. But we will focus on the high-level problem (peering) with other network content providers) rather than investigating the optimal price structure of providers or figuring out the providers' business mode.

Another research line is to investigate the content distribution strategy as an exclusivity problem. At present, content Providers can choose to multi-home (non-exclusive contracts) or single-home (exclusive contracts) with the network providers. In a traditional view, the network provider may attain a competitive advantage by entering into an exclusive arrangement with the chosen Content Provider [21], as it can guarantee the return of the cost of acquiring the popular contents and increase the market entry level. Some works ([7] and [20]) analyze the problem based on Hotelling model ([24]), assuming asymmetries in contents quality and costs. Within their model, they analyze the effect of popular content and implicate that if the cost of the popular content is lump-sum based, exclusive contracts will be preferred. Otherwise, if the cost is based on pay-per-view, content providers prefer non-exclusive contracts. This reasoning can be explained from the perspective of CP, whereby, if it cannot estimate the actual number of users accessing the content, the lump-sum payment can decrease the risk of uncertainty and the exclusive contract is optimal.

Reference [39] concludes that exclusivity will induce providers to focus on price and content quality strategies. In return, this interaction between providers will ensure that all users can enjoy the increasing quality contents and lower subscription fees from this interaction between providers. However, as [7] and [28] point out, the bidding wars for the popular content are pretty fierce. The cost is increasing exponentially. This is not beneficial from the view of NP. The development of new technology has greatly affected the media ecosystems [14]. The transaction cost has been significantly reduced. Network providers tend to focus on service innovation rather than vertical integration with other providers. Moreover, reference [12] and [30] point out that exclusivity does not always prompt the network providers' business, rather it hinders the competition and innovation in both contents and associated technology. Reference [8] and [18] argue that as more users will pay and access the popular contents, non-exclusivity can be profitable for popular contents providers.

Although the exclusive contracts can increase the market entry levels, and cosequently hinders the competition between the providers, they may lead to a trend whereby there is no incentive for providers to focus on platform innovation, which will yield restrictions in users' choice of the network providers. On one side, the exclusivity can achieve industry profits but at the expense of social welfare. On the other side, non-exclusivity can reach social welfare optimum whereby all users can access the popular contents, albeit leading to decreased industry profits. Thus, when designing policies, regulators should take these two sides into consideration and aim for optimal balance between social benefits and profits. Reference [22] suggests that if content is high popularity, and network providers' market share difference is large, and the cost for delivering the content in two systems is low, the content producers can sign an exclusive contract. Otherwise, non-exclusive contract is more preferred. However, it is likely that the debate regarding the exclusivity issues will continue in the foreseeable future. The present study gives one direction for solving the exclusivity problem, offering potential peering strategies to providers. The work presented in this thesis includes not only the transfer payment, but also considers the advertisement arrangement between the providers, which has not been investigated in the above studies.

Furthermore, a number of extant studies discuss the interaction between the network providers, identifying two types of relationship: competitive and peering relationship. The former arises naturally when the providers require significant efforts to draw users to subscribe to their network. As their user coverage determines the subscription and advertisement revenue, they are motivated to become more competitive. In contrast, in the cooperative relationship, the providers can increase their coverage with the help of other providers.

Since there are multiple Network Providers that can deliver the CP's contents to users, the market power of NP may be reduced. [19] shows that the social welfare and the content providers' benefit increase in competitive market compared to the welfare in monopoly market. This view implies that the policy should encourage the competition in the market from a global view. However, from the perspective of NP, they need to consider their strategy that would allow them to compete in such a market structure. Some studies ([16], [15] and [5]) investigate how to choose contents and determine advertisement lengths to attract users. These works focus on the endogenous solutions to identify the optimal strategy for providers. They show that NPs will choose the optimal content types due to different settings (*i.e.*, advertising rate, revenue distribution between advertiser and platform). On the other hand, other authors investigate the business model of the network providers. Reference [31] and [26] investigate how to increase revenue through either an advertisement-sponsored only approach or a subscription-and-advertisement-sponsored approach. Finally, the dynamics of competition has also been studied in the past. The findings indicate that, after the ex-post competition for attracting users, when providers change their content quality or prices, some users will switch the providers, but incurring the switching costs. Reference [13] has provided a comprehensively survey of the effect of switching cost for the providers' competition. The results indicate that there is a lock-in effect for the switching cost. Since users need to pay after the first adoption, the early users' preference counts more than later adoptions. Hence, the competition for the future is less fierce and the entry of the market is not easy. However, none of the prior results have considered the peering among providers, which is the reason why the present study focused on this issue.

Rather than competing with other providers, reference [38] studies the possibility of open access to other providers (*i.e.*, internet service). The network providers can also collaborate with each other to deliver the contents. For example, reference [29, 11] examined the incentive for ISPs to interconnect and develop Shapley value based revenue distribution mechanisms. Similarly, reference [23] studies the optimal pricing for CDN service and [19] studies the price and rate allocation of ISPs with content providers' participation. Reference [25] and [27] studied the revenue sharing mechanism between the CP and NP. These works consider the pricing strategies from the engineering perspective. However, most of them assume that each user has a fixed subscription to one provider and can not switch to different providers.

In another study, reference [37] discusses the cooperation between Google and Yahoo, whereby the superior Google shares its search technique with its rival. If they choose to cooperate, the inferior search engine attains search technique and provides a higher quality service. Hence, it can provide better result accuracy with the queries and consequently attract more users and increase the advertisement amount. In other words, the inferior side in the collaborative relationship can benefit from the peering. However, although the superior engine also benefits from increasing the price and the amount of advertisement, some users may switch to the inferior provider that will now offer competitive service, whereby the superior engine will lose some of its former market share. Whether the superior enters the cooperation agreement depends on their assessment of the two effects. If the former effect outweighs the latter, it will cooperate. Otherwise, it will not share the technology. However, as there is no transfer payment between the two engines, the superior engine cannot receive any benefit from the cooperator other than through increase in its own revenues. Reference [40] discusses about the cooperation between two Pay TV providers. The content holder will charge the cooperator a per-subscriber fee and a lump-sum payment. This makes sure that the content holders are always willing to cooperate with the rival. But this transfer is not fair for the cooperators. As the author proved, the per-subscriber fee can raise as high as possible. Hence, there is a possibility that the content holder extracts all the benefit from the cooperator and the cooperator get less or nothing. This is obviously not fair for the cooperator. The cooperation strategy should be a win-in strategy so that both the participants have incentive to stick to the peering. Also, there is no advertisement transfer as our work. In contrast, we consider the interactions of advertisers, network providers, and users. We design a fair revenue distribution mechanism for the providers, where users may switch between providers depending on the contents and subscription fees.

Chapter 2

A Static Baseline Model

We consider a duopoly market of two network providers: A and B. Each provider has a dedicated advertiser, who pays the provider advertising fees based on the provider's coverage. A provider's coverage depends on the number of users subscribing to its service. A provider can attract subscribers by attractive contents or a low monthly subscription fee.

2.1 Content Qualities and Subscribing Fees

We consider a period of T time slots, where each time slot has a unit length (*e.g.*, representing one month of time). We characterize the contents' popularity in time slot $t \in \{1, \ldots, T\}$ as content quality q_i, where $i \in \{A, B\}$. Higher value of q_i represents the contents are more attractive and popular to users. In this chapter's analysis, we assume that both providers have *static* contents, *i.e.*, $q_{it} = q_{it'} = q_i$ for any $t, t' \in \{1, \ldots, T\}$ and both $i = A$ and $i = B$. Figure 2.1 illustrates one example of the static content quality for T time slots of the two providers.

Without loss of generality, we assume that provider A has a more popular content, *i.e.*, $q_A \geq q_B$. This may reflect the fact that provider A has a larger budget and can purchase/produce higher quality contents than provider B. We will come back to the budget issue in Chapter 3.

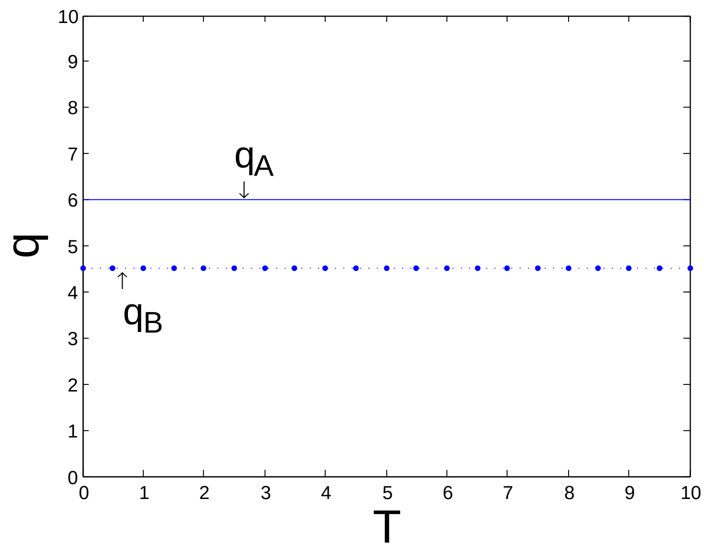

Figure 2.1: Static content quality

During each time slot, provider $i \in \{A, B\}$ charges each of its subscriber p_i. As provider A has better contents for all T time slots, it can charge a higher subscribing fee, *i.e.*, $p_A > p_B$. [1] *We further assume that both p_A and p_B are fixed throughout this study.* This allows us to focus our work on the impact of content choices and providers' peering incentive.

2.2 Users' Utilities

Users may achieve different satisfaction levels by consuming the same contents. We characterize a user with two parameters: θ representing the user's valuation of the content quality, and δ representing the user's time discount factor over future contents. A user's total utility of subscribing and consuming contents

[1] Assume this is not true and $p_A \leq p_B$. Then all users will choose provider A, who offers a better content with a lower fee. Provider B will have no subscribers and will be out of the market. This is apparently not an interesting case and will not be further discussed in this paper.

from provider $i \in \{A, B\}$ over T time slot is

$$U(\theta, \delta) = \theta \sum_{t=1}^{T} \delta^{t-1} q_{it} - p_i T. \tag{2.1}$$

Once the users choose their service providers, they need to sign an T time contract with their providers. For a user who is indifferent of choosing either provider, we have the following relationship between θ and δ:

$$\theta \sum_{t=1}^{T} \delta^{t-1} q_{At} - p_A T = \theta \sum_{t=1}^{T} \delta^{t-1} q_{Bt} - p_B T. \tag{2.2}$$

Based on (2.2), we can compute the boundary evaluation $\theta^*(\delta)$ as a function of δ, which is

$$\theta^*(\delta) = \frac{(p_A - p_B)T}{\sum_{t=1}^{T} \delta^{t-1} q_{At} - \sum_{t=1}^{T} \delta^{t-1} q_{Bt}}. \tag{2.3}$$

Fig 2.2 illustrates one example of the boundary line of $\theta^*(\delta)$. Users with parameters (θ, δ) below the boundary will choose to subscribe to provider B, while the users above the boundary will subscribe to provider A. We can see that $\theta^*(\delta)$ is a decreasing function in terms of δ. There are more users with small value of δ choosing provider B, and more users with large value of δ choosing provider A instead of B. This is because for users with small value δ, they consider current content quality more than future one. Thus, they make their providers' choice mainly based on current content. While for users with large δ, they value the future content quality as current one. Thus, they make their decisions based on the total contract utility. These different preferences on future contents induce the different choice of providers among users.

2.3 Providers' Coverages and Revenues

For the rest of the analysis, we assume that both θ and δ are uniformly distributed in $[0, 1]$. This assumption may not be always true for the real world. However, we argue that the different distribution of users only makes users'

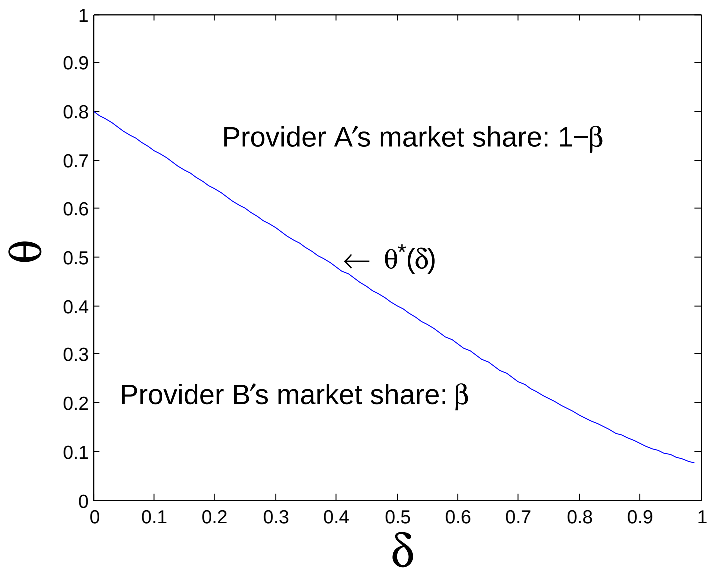

Figure 2.2: Two providers' market shares

choice of providers different. It will not change the analysis method and the result intuition. Without loss of generality, we normalize the total users population to be 1. Then the area under the boundary represents the market share of provider B (denoted as β), and the provider A has a market share of $(1-\beta)$. Since $\theta^*(\delta)$ has been denoted as a function of δ as in Equation (2.3), the area below the boundary line is an integration over δ. Thus, the two providers' market shares are

$$A\text{'s coverage} \quad : \quad 1-\beta = 1 - \int_0^1 \frac{(p_A - p_B)T}{\sum_{t=1}^T \delta^{t-1} q_{At} - \sum_{t=1}^T \delta^{t-1} q_{Bt}} d\delta, \quad (2.4)$$

$$B\text{'s coverage} \quad : \quad \beta = \int_0^1 \frac{(p_A - p_B)T}{\sum_{t=1}^T \delta^{t-1} q_{At} - \sum_{t=1}^T \delta^{t-1} q_{Bt}} d\delta. \quad (2.5)$$

We assume that provider $i \in \{A, B\}$ has a advertisement revenue function $f_i(\cdot)$ per time slot. Here $f_i(\cdot)$ is an increasing function of its market share. If there are no users accessing the contents, no advertisers would like to pay for the advertisement. Thus $f_i(0) = 0$. The two providers' revenues over T time slots are

$$\pi_A = (f_A(1-\beta) + p_A \cdot (1-\beta)) \cdot T, \quad (2.6)$$

$$\pi_B = (f_B(\beta) + p_B \cdot \beta) \cdot T. \quad (2.7)$$

2.4 Content Procurement Strategies

Each provider may change its revenue through contents procurement[2] . We consider two possibilities in the next discussions: peering between providers to share contents and increase coverage (Section 2.5 and Chapter 4), and introducing special content to attract users to switch providers (Chapter 3).

2.5 The Peering and Bargaining of Providers

2.5.1 Peering Agreement

When two providers peer with each other in this static content quality situation, we assume that one provider will purchase the *whole* content from the other provider. Since provider A has a better content (*i.e.*, $q_A \geq q_B$ as assumed in Section 2.1), provider A will be the seller and provider B will be the buyer.

However, these two providers have different concerns when peering. From A's point of view, it wishes to deliver both the content and its advertisement in B's network, so as to increase the advertisement revenue from its dedicated advertiser. Provider A also wishes B to pay for the usage of the content. From B's point of view, it wishes to carry its own advertisement in order to get payment from its own dedicated advertiser. Provider B also wishes A to pay for the additional coverage after peering with B. They need to bargain over and over and reach a win-win peering strategy.

[2] Recall that we have assumed the subscribing fees p_A and p_B are fixed in this work.

Next we describe a general peering agreement. When peering, provider A and B will deliver the same content (*i.e.*, the original content of provider A). As for advertisement, provider A will deliver its own advertisement. Provider B delivers α portion of A's advertisement and (1-α) portion of its own advertisement. Finally, B pays provider A a one-time payment c for peering over T time slots, where c can be either positive or negative. Figures 2.3 and 2.4 illustrate the two providers' contents and advertisements with and without peering.

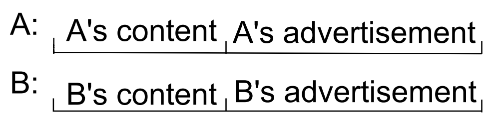

Figure 2.3: Two providers' contents and advertisements without peering

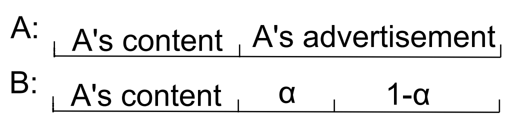

Figure 2.4: Two providers' contents and advertisements with peering

The bargaining variables are the advertisement ratio α and payment c. Figure 2.5 illustrates this bargaining process and the exchange of content and advertisement.

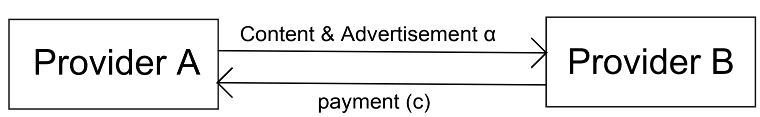

Figure 2.5: Bargaining model

2.5.2 Change of Coverage

Now let us consider how users change their subscriptions when providers peer. Here we assume that all users are freely to choose. This may not be the case where users already sign contracts with their providers. The additional switching cost because of this will be further discussed in Chapter 3. Since now, both providers have the same contents and provider B charges a lower price $p_B < p_A$, then all users will choose to subscribe to provider B. Provider A will get zero subscriber. However, notice that since α part of provider A's advertisement is delivered through B, then provider B's coverage also contributes to the advertisement revenue of A.

2.5.3 Providers' Revenues

Now let us compute the providers' revenues with peering. For provider A, its advertisement can reach all users with α fraction of the time. But it will receive no subscription revenue due to the loss of all subscribers. Hence, provider A's revenue with peering is

$$\pi_A^c(\alpha, c) = \alpha f_A(1)T + c. \tag{2.8}$$

For provider B, it receives all users subscription fees and delivers its own advertisement to all users with $(1 - \alpha)$ fraction of the time. Thus, its revenue with peering is

$$\pi_B^c(\alpha, c) = (1 - \alpha) f_B(1)T + p_B T - c. \tag{2.9}$$

2.5.4 Nash Bargaining Problem

Next we model the bargaining problem based on the Nash Bargaining Solution [9], which abstracts away the procedures of bargaining between the two providers and considers only the set of outcomes.

Definition 1. *A peering strategy (α^*, c^*) is a Nash bargaining solution if it solves the following problem:*

$$maximize_{\alpha \in [0,1], c} \quad F = (\pi_A^c(\alpha, c) - \pi_A) \cdot (\pi_B^c(\alpha, c) - \pi_B), \tag{2.10}$$

where π_A and π_B are the revenue obtained without peering as in (2.6) and (2.7).

The Nash Bargaining Solution is a unique solution that satisfies the axioms of *Pareto efficiency*, *symmetry*, *invariance*, and *independence of irrelevant alternatives* in a peering game. The meaning of these properties is explained as follows.

- *Pareto efficiency* : There is no other outcome $F(\alpha, c)$ such that $F(\alpha, c) \geq F(\alpha^*, c^*)$.
- *Symmetry* : If the revenues without peering satisfies $\pi_A = \pi_B$, we have $F'(\alpha, c) = Max((\pi_A^c(\alpha, c) - \pi_B)(\pi_B^c(\alpha, c) - \pi_A)) = F(\alpha^*, c^*)$. The property assures that if the two providers are indistinguishable, the outcome will not discriminate between them.
- *Invariance* : A linear transformation (Θ) of the two providers' revenue function with peering ($\pi_A^c(\alpha, c)$ and $\pi_B^c(\alpha, c)$) will not alter the outcome of the bargaining process. We have $F(\Theta(\alpha^*), \Theta(c^*)) = \Theta(F(\alpha^*, c^*))$.
- *Independence of irrelevant alternatives* : The solution only depends on the two determined variables (α and c).

It is clear that both providers should achieve revenues no worse than their non-peering revenues (*i.e.*, π_A and π_B) at the Nash bargaining solution. Otherwise, at least one of the two providers does not have the incentive to bargain. This means that a peering agreement can be achieved if and only if the following condition holds:

$$\alpha f_A(1) + (1 - \alpha) f_B(1) > f_A(1 - \beta) + f_B(\beta) + (p_A - p_B)(1 - \beta). \qquad (2.11)$$

With a proper choice of c, condition (2.11) can ensure that both providers get better payoffs through peering.

The optimal solution of (2.10) depends on the revenue functions $f_A(\cdot)$ and $f_B(\cdot)$. As an illustrative example, we consider linear advertisement revenue functions and assume $f_A(x) = k_A \cdot x$ and $f_B(x) = k_B \cdot x$. Higher values of

k_A and k_B lead to higher values of advertisement revenue with the same user coverage. Next we summarize the optimal solution of (2.10) depending on three possible relationships between k_A and k_B, with detailed proofs given in Appendix A.1.

Scenario 1. $k_A = k_B$.

In this case, both providers have the same advertisement revenue function. The advertisements from both advertisers are equally important. If we plug $k_A = k_B$ into condition (2.11), then the left hand side (LHS) equals k_A and the right hand side (RHS) equals $k_A + (p_A - p_B)(1 - \beta)$. Since $p_A > p_B$, we know that the LHS actually is less than RHS, and thus condition (2.11) does not hold. This means that providers will not choose to peer in this case. This is because the peering can not generate more advertisement revenue. Instead, due to all of provider A's users switching to the weaker provider B, the total subscription fees received from the two providers are less compared with the fees without peering. Hence, the total revenues will be less after peering. In return, the two providers have no incentive to peer.

Scenario 2. $k_A > k_B$.

In this case, provider A has a stronger ability in generating advertisement revenue than B. We can show that the optimal advertising strategy is $\alpha^* = 1$, in which both providers deliver the same advertisement originally belonging to provider A. With $\alpha^* = 1$, condition (2.11) becomes

$$k_A > k_A(1 - \beta) + k_B\beta + (p_A - p_B)(1 - \beta),$$

which means that the subscription fees p_A and p_B need to satisfy

$$p_A - p_B < \frac{(k_A - k_B)\beta}{1 - \beta}. \tag{2.12}$$

so that the providers want to peer. When the providers want to peer, we can further show that the optimal payment strategy c^* from provider B to provider

A is

$$c^* = \frac{1}{2}\big((p_A + p_B)(1-\beta) - (k_A + k_B)\beta\big) \cdot T, \tag{2.13}$$

which can be either positive or negative. For example, if the revenue income is much larger than the user subscription fee, *i.e.*, $(k_A+k_B)\beta > (p_A+p_B)(1-\beta)$, then $c^* < 0$. The negative value of c^* means that provider A should compensate B to stick to the peering. This is because provider A's increase in advertisement income with peering is much larger than provider B's revenue increase by getting more subscribers. Then provider A should share the additional income with B. Otherwise, provider B has no incentive to purchase A's content.

Scenario 3. $k_A < k_B$.

In this case, provider A has a weaker capability in generating advertisement revenue than B. We can show that the optimal advertising strategy is $\alpha^* = 0$, in which the two providers deliver their own advertisements. With $\alpha^* = 0$, condition (2.11) becomes

$$k_B > k_A(1-\beta) + k_B\beta + (p_A - p_B)(1-\beta),$$

which is equivalent to

$$p_A - p_B < k_B - k_A, \tag{2.14}$$

which can be satisfied under proper values of p_A and p_B. When the providers want to peer, we can further show that the optimal payment strategy c^* from provider B to provider A is

$$c^* = \frac{1}{2} \cdot \big(k_A + k_B + p_1 + p_2\big)(1-\beta) \cdot T > 0. \tag{2.15}$$

In this case, provider A loses all the subscribers and can not get any advertisement and users' subscription revenues. As a result, the payment c^* should be positive and provider B should compensate A's loss. Otherwise, provider A has no incentive to peer.

Scenario 2 and 3 show that if the providers have different advertisement revenue functions, the peering will be desirable. This is because that the two providers can peer through proper advertisement arrangement, so that they can generate more advertisement revenue. When this increase of advertisement revenues is larger than the loss of the total subscription fees, the two providers will peer. Otherwise, there will be no peering no matter who deliver whose advertisement. As for the advertisement strategy during peering, we have proved that who has stronger ability in generating advertisement revenue will contributes more to the total advertisement revenues, and in return, the stronger provider has the power to determine the optimal advertisement strategy. Thus, the stronger provider can receive more revenues during peering and need to compensate the cooperator and share the additional income.

The analysis of other advertisement revenue functions (*i.e.*, convex or concave functions) is similar as the methodology used in linear function situation. We would like to further discuss them in Section 4.1.

Chapter 3

Impact of Dynamic Content

In this chapter, we consider how a provider can change its coverage (and thus the revenue) by introducing some special content (*i.e.*, content with a very high quality) over a short time period. Without loss of generality, we assume provider A introduces the special content with a high quality $q_s > q_A$ in the first time slot.

However, introducing (purchasing) a special content in one time slot needs additional budget from the provider. We consider two ways for introducing the special content. The first one is that provider A has some additional fund and wants to make use of it to increase its revenue. It invests this additional fund for certain special content without decreasing other contents quality of the remaining $(T-1)$ time slots. The other case is provider A has a finite budget. Introducing the special content will decrease the content quality of the remaining $(T-1)$ time slots. The first case considers the effect of incremental budget change. The second case is a special case of a generalization of our problem where providers can use content choice as a strategy.

This dynamic change in content quality will alter the content payoff for users. Thus, it will induce some users to switch the service providers. However, when users subscribe the service, they need to sign a contract of duration T with the providers and commit to pay for the whole contract. Therefore, if users want to switch to other providers, they will not enjoy the remaining contents and incur a switching cost if the original contract is terminated early [13]. We characterize users with different remaining contract time z, which is uniformly distributed between [0,T] by assumption. This assumption comes from the fact that users may subscribe to providers at different time and thus have various remaining contract time. Then, the switching cost will be the subscription fee times the remaining contract time, $p_i \cdot z$ for both $i = A$ and $i = B$.

Next, we will discuss the impact of dynamic content and switching cost based on the two budget cases, and show their influences on providers' coverages and revenues.

3.1 Additional Investment for Dynamic Content

3.1.1 Content Change

In this case, provider A spends an additional fund on purchasing the special content. This spending does not come from the original budget for regular contents. Thus, the introduction of special content will not affect the qualities of other contents. Figure 3.1 illustrates one example of this change in content quality, where q_A and q_B are the regular content quality of provider A and B.

3.1.2 Change of Coverage

This variation in content over time causes users to switch. Users will make their decisions based on the content quality of the next T time slots. Once they switch the provider, they can achieve more payoff from the new provider in the next T time slots and thus have on incentive to switch again during the

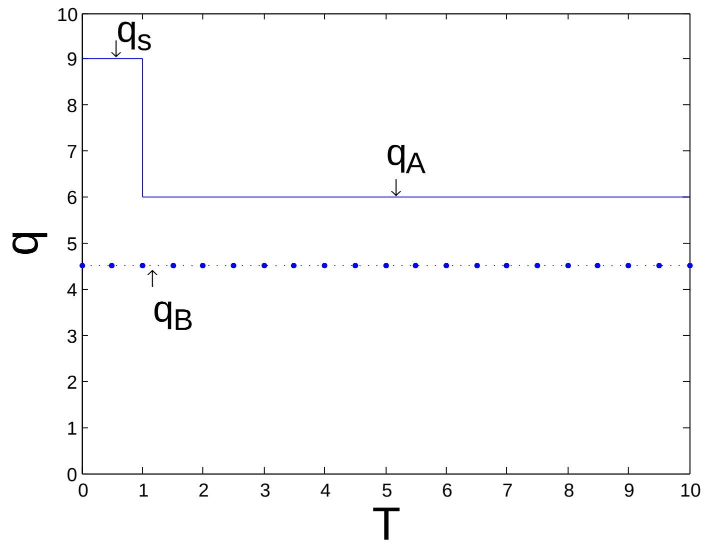

Figure 3.1: Example of dynamic content in Additional Investment Case

T time slots[1] .

Apparently, the number of switching users depends on the magnitude of extra content benefit and users' switching cost. We will discuss how these two parameters affect users' choice of providers in the following.

Switching users of Provider A

Now, let us consider if any user wants to switch from provider A to provider B. Recall that the utility of the next T time slots of a user subscribing to provider A is

$$U_{A,static} = \theta \sum_{t=1}^{T} \delta^{t-1} q_A - p_A T$$

before introducing the special content and

$$U_{A,dynamic} = \theta q_s + \theta \sum_{t=2}^{T} \delta^{t-1} q_A - p_A T$$

[1]Actually, users can switch multiple times during the T time slots. Their switching decisions are based on the utility they achieve for next T time slots. Thus, the number of switching users at time t, $t \in (1, \cdots, T)$ depends on the providers' content utility in [t, t+T], which is related to the future content choice of providers. The peering problem of multiple switching times is more complicated. The present study will give a preliminary analysis to investigate users' switching decisions and their influence on providers.

after introducing the special content. The utility change is

$$U_{A,dynamic} - U_{A,static} = \theta(q_s - q_A) > 0$$

This means that provider A offers higher content utility and user will get more benefit for the next T time slots if sticking to A. Thus, no users will switch from provider A to B during the T time slots.

Switching users of Provider B

On the other hand, some original subscribers of provider B might want to switch to provider A due to the extra value of the special content.

As users have different switching cost and different preferences on contents quality and their availability, they are distributed in these three dimensions (z, θ and δ). Therefore, we can first figure out the size of switching users S_{A2B} in terms of θ and δ for certain switching cost. Then, we will find out the boundary cost $p_B \cdot z_{A2B}$ under which users will pay to switch. Lastly, by integrating S_{A2B} over z with $z \leq z_{A2B}$, the total number of switching users for next T time slots is found out.

Following the above procedures, we will first find out the users with what value of θ and δ will pay the cost to switch. A user will only switch from provider B to provider A if his utility of next T time slots improves after the switching, *i.e.*,

$$\theta(q_s + \sum_{t=2}^{T} \delta^{t-1} q_A) - p_A \cdot T - p_B \cdot z \geq \theta \sum_{t=1}^{T} \delta^{t-1} q_B - p_B \cdot T. \tag{3.1}$$

Then, users who are indifferent in terms of switching to provider A or staying with provider B have a parameter $\theta_{B2A}(\delta, z, q_s)$ that satisfies, *i.e.*,

$$\theta_{B2A}(\delta, z, q_s) = \frac{(p_A - p_B)T + p_B \cdot z}{(q_s + \sum_{t=2}^{T} \delta^{t-1} q_A) - \sum_{t=1}^{T} \delta^{t-1} q_B}, \tag{3.2}$$

which is shown by two dimensions of θ and δ in Fig 3.2 (with $z = 0.5$). Here we need to have $\theta \geq \theta_{B2A}(\delta, z, q_s)$, otherwise no user with this switching cost will switch from provider B to A.

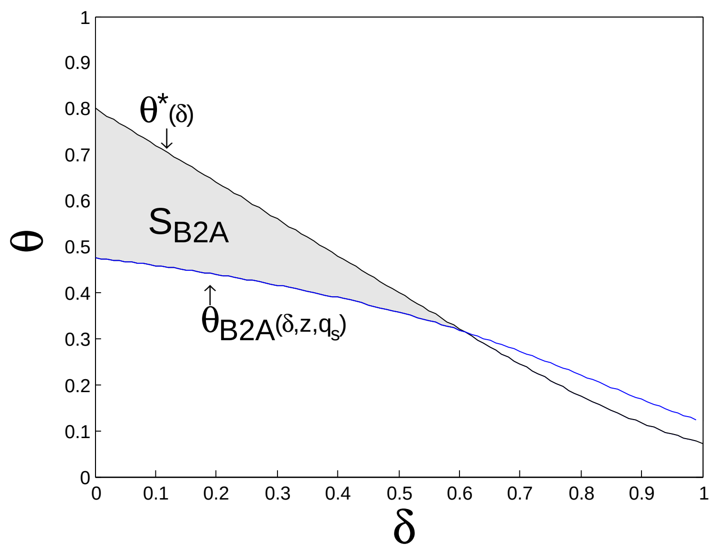

Figure 3.2: Switching users without peering in Additional Investment Case

We observe that users with parameter (θ, δ) above the boundary $\theta_{B2A}(\delta, z, q_s)$ will choose provider A. In fact, all users who are above the curve $\theta^*(\delta)$ choose provider A even without the special content. Only users who are below the curve $\theta^*(\delta)$ and above the curve $\theta_{B2A}(\delta, z, q_s)$ are the switching users. The gray area S_{B2A} denotes the size of switching users. We further observe that switching users have a large value of θ and a small value of δ. This is because these users value the content utility and its availability more. So they can achieve more utility from switching to provider A. Furthermore, users below the boundary $\theta_{B2A}(\delta, z, q_s)$ will stick to the original providers (either A or B).

Besides, we notice that $\theta_{B2A}(\delta, z, q_s)$ is a decreasing function of δ. As provider A increases the total contents' utility, more users with large δ who value future content as current content will chooses provider A. In additional, function $\theta_{B2A}(\delta, z, q_s)$ decreases in the special content quality q_s. It means that the line $\theta_{B2A}(\delta, z, q_s)$ will move downwards if q_s increases. Thus, more users will switch and the gray area will expand. The higher the quality of special content is, the higher utility the users can obtain. Then, more users will switch. So from provider A's view, it would like to spend its additional

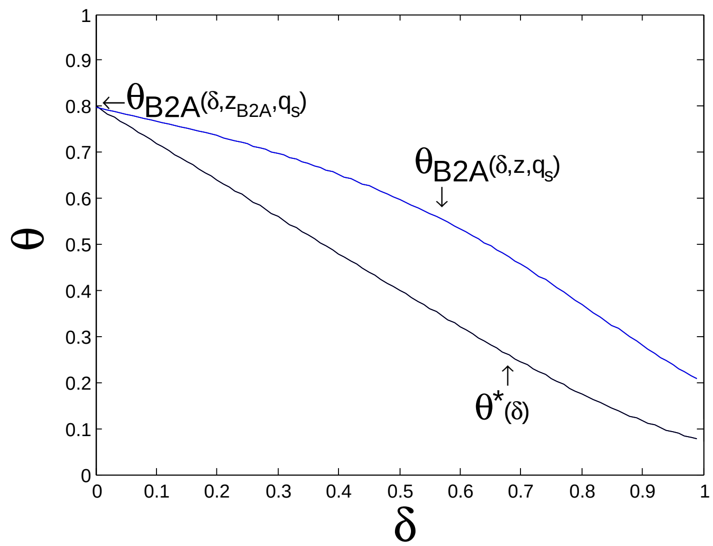

Figure 3.3: Relationship between $\theta(\delta, z, q_s)$ and z

fund to enhance the content quality and increase its coverage.

However, as the switching cost increases, fewer users will pay such a high cost to switch. Next, we will find out the threshold value of z_{A2B}, with which the switching users are indifferent from staying with B or switching to A.

We observe that function $\theta_{B2A}(\delta, z, q_s)$ increases in z. Fewer users with large z will switch and the gray area will reduce. This is because they only need to pay a small cost and they can receive more content payoff from switching. When z increases to z_{B2A}, there is no switching users and the gray area reduces to one point, which is represented by the point of $\theta(\delta, z_{B2A}, q_s)$.

Thus, Let $\theta^*(\delta)$ equal to $\theta_{B2A}(\delta, z, q_s)$, which is

$$\frac{(p_A - p_B)T}{\sum_{t=1}^{T} \delta^{t-1} q_{At} - \sum_{t=1}^{T} \delta^{t-1} q_{Bt}} = \frac{(p_A - p_B)T + p_B \cdot z}{(q_s + \sum_{t=2}^{T} \delta^{t-1} q_A) - \sum_{t=1}^{T} \delta^{t-1} q_B}. \quad (3.3)$$

Substituting $\delta = 0$ into (3.3), we have

$$z_{B2A} = (\frac{p_A}{p_B} - 1) \cdot (\frac{q_s - q_B}{q_A - q_B} - 1) \cdot T. \quad (3.4)$$

For users with $z > z_{B2A}$, their benefit from switching is less than the large switching cost they pay even after A introduced the special content. Hence, they will not switch. For users with $z \leq z_{B2A}$, users with large θ and small δ illustrated in Fig 3.2 who can afford the cost will switch.

Therefore, the total number of switching users from B to A is the integration of S_{B2A} in terms of z with $z \leq z_{B2A}$, which is

$$S_n = \int_0^{z_{B2A}} \frac{S_{B2A}}{T} \quad dz. \tag{3.5}$$

These switching users will receive higher content utility for the next T time slots. Therefore, they have no incentive to reconsider their subscription choice during the T time slots.

Let γ denote the coverage of provider B after A introducing special content. Then, the coverages for the two providers without peering are

$$\begin{aligned} A\text{'s coverage} &: \quad 1-\gamma = 1-\beta+S_n, \\ B\text{'s coverage} &: \quad \gamma = \beta - S_n. \end{aligned}$$

Timing of Special Content

Now, we will discuss the effect of placement of special content during the contract time. As the regular content quality will not decrease, users of provider A will still achieve higher content utility even the special content is placed at the last time slot T. So A's original subscribers will not switch. In contrast, for provider B's users, the benefit is not so much as the case when special content is placed at the first time slot. So fewer users will switch. But this does not change the insight of users' switching decisions. Next we will focus on the case that the special content is placed at the first time slot. The analysis for other cases is similar and will not be covered.

3.1.3 Providers' Revenue

The two providers' revenues after provider A introducing the special content are as follows:

$$\pi_{As} = (f_A(1-\gamma) + p_A \cdot (1-\gamma)) \cdot T, \tag{3.6}$$

$$\pi_{Bs} = (f_B(\gamma) + p_B \cdot \gamma) \cdot T. \tag{3.7}$$

3.2 Finite Budget for Dynamic Content

3.2.1 Content Change

In this case, provider A has finite budget and no additional fund to increase content quality. But it can also introduce the special content with decreasing other content quality to q'_A for the remaining $(T-1)$ time slots.

To simplify the analysis, we assume that the budget and the content quality has a linear relationship. Then, the finite budget constraint means that

$$q_s + q'_A(T-1) = q_A T. \tag{3.8}$$

Figure 3.4 illustrates one example of the change of content after provider A introducing the special content by finite budget. Where q_A is the static content quality of provider A without special content.

3.2.2 Change of Coverage

The dynamic contents of provider A will also change users' content utility and induce some to pay the switching cost to switch. Users will make their choice of subscription according to the change of contents and their switching cost when the special content comes.

Next, we will discuss how the finite budget of provider, the special content and the switching cost affect users' choice of subscription in details.

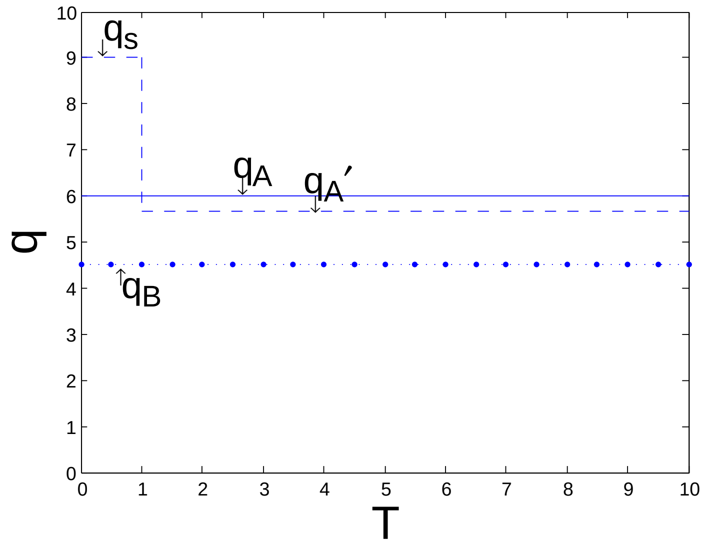

Figure 3.4: Example of dynamic content in Finite Budget Case

Switching Users of Provider A

Recall that the utility of a user subscribing to provider A is

$$U_{A,static} = \theta \sum_{t=1}^{T} \delta^{t-1} q_A - p_A T$$

before introducing the special content and

$$U'_{A,dynamic} = \theta q_s + \theta \sum_{t=2}^{T} \delta^{t-1} q'_A - p_A T$$

after introducing the special content in finite budget case. The utility change is

$$\begin{aligned} & U'_{A,dynamic} - U_{A,static} \\ = \; & \theta q_s + \theta \sum_{t=2}^{T} \delta^{t-1} q'_A - \theta \sum_{t=1}^{T} \delta^{t-1} q_A \\ = \; & \theta \left(q_s - q_A + \sum_{t=2}^{T} \delta^{t-1} \left(q'_A - q_A \right) \right) \\ \geq \; & \theta \left(q_s - q_A + (T-1) \left(q'_A - q_A \right) \right) \\ = \; & 0. \end{aligned}$$

The inequality follows from $\delta < 1$ and $q'_A < q_A$, and the last equality follows from (3.8). This means that provider A provides higher content utility and user will get more benefit if sticking to A. Thus, no users will switch from provider A to B in this case, which is the same as in additional investment case.

Switching Users of Provider B

On the other hand, some original subscribers of provider B might want to switch to provider A due to the special content. The analysis of figuring out provider B's switching users is similar with the method used in Section 3.1.2. Thus, we will skip the details of proof and give the results directly.

Then, users who are indifferent in terms of switching to provider A or staying with provider B in this finite budget case have a parameter θ equals to

$$\theta'_{B2A}(\delta, z, q_s) = \frac{(p_A - p_B)T + p_B \cdot z}{(q_s + \sum_{t=2}^{T} \delta^{t-1} q'_A) - \sum_{t=1}^{T} \delta^{t-1} q_B}, \tag{3.9}$$

which is shown by two dimensions of θ and δ in Fig 3.5 (with $z = 0.5$). Here we need to have $\theta \geq \theta'_{B2A}(\delta, q_s, z)$, otherwise no user will switch from B to A. The gray area S'_{B2A} denotes this portion of switching users in finite budget case.

We also notice that users who incur a small switching cost and value current content and its availability more will switch, which is similar with the switching users in additional investment case.

However, comparing equation (3.2) with (3.9), as provider A's regular content quality decreases to q'_A, the value of $\theta_{B2A}(\delta, z, q_s)$ in additional investment case is less than the value of $\theta'_{B2A}(\delta, z, q_s)$ in finite budget case under the same special content quality. It means that more users will switch in the former case than in the latter case. The reason is that provider A spends more budget other than the original budget in the additional investment case to increase

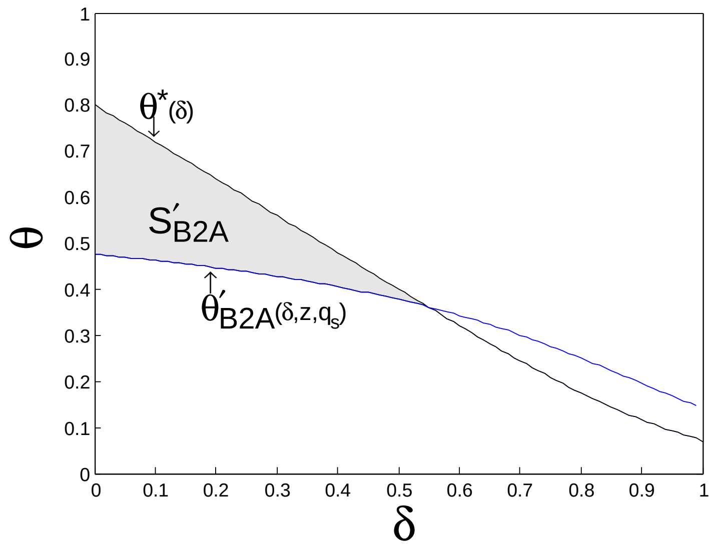

Figure 3.5: Switching users without peering in Finite Case

content quality. Moreover, the higher the special content quality is, the lower the regular contents quality is in finite case. Hence, the difference between the two values of θ_{B2A} and θ'_{B2A} is larger, which leads to larger difference of the number of switching users.

The method of calculation of the size of switching users is also similar with the methodology used in Section 3.1.2. Thus, we would like to skip the details also and present the results straightly.

As the most likely switching users ($\delta = 0$) do not care the future contents quality, the threshold value of z depends only on the special content quality, which means that the value of z_{B2A} is the the same in both budget cases. Therefore, the total number of switching users from B to A in finite case is

$$S'_n = \int_0^{z'_{B2A}} \frac{S'_{B2A}}{T} \quad dz. \tag{3.10}$$

Let γ' denote the coverage of provider B after A introducing special content in finite budget case. Then, the coverages for the two providers without peering are

$$
\begin{aligned}
A\text{'s coverage} &: \ 1-\gamma' = 1-\beta+S_n', \\
B\text{'s coverage} &: \ \gamma' = \beta - S_n'.
\end{aligned}
$$

Timing of Special Content

Now, let us discuss the influence of placement of special content for this budget case. The choice of provider B's users is similar as the one in additional investment case, where users make their decision based on the valuation of special content quality and the switching cost they incur. However, some original subscribers of provider A will switch to B as the regular content quality has decreased. The placement of special content will affect A's users switching decisions, which is different from the case in additional investment case.

Taking the extreme case for example that the special content is placed at the last time slot T, the benefit of special content is not obtained immediately. So users who incur a small switching cost and value current content quality more will switch to provider B. Thus, from the provider's perspective, it should try to avoid decreasing current content quality. Therefore, the provider will use the future budget to introduce the special content and thus will not affect the current users' subscription. That means providers introduce the special content at the first time slot and decrease future content quality in finite budget case, whereby the special content holder will not lose users, and can attract as many users as possible instead. The following analysis focuses on the case that the special content will be introduced at the first time slot.

3.2.3 Providers' Revenue

The two providers' revenues after A introducing special content in finite budget case are as follows

$$\pi'_{As} = (f_A(1-\gamma') + p_A \cdot (1-\gamma') \cdot T, \tag{3.11}$$

$$\pi'_{Bs} = (f_B(\gamma') + p_B \cdot \gamma') \cdot T. \tag{3.12}$$

Summary

Obviously, the special content can give an advantage to the content right holder (provider A) in both budget cases. It helps the provider to attract users who value current content quality more and only need to pay a small switching cost. The higher quality the special content is, the larger the number of switching users will be. The switching users have increased provider A's coverage and thus contributed to A's increasing revenues. Besides, there are more users switching the provider in additional budget case than in finite budget case. The reason is that provider A spends more budget other than the original budget in the former case. In contrast, in the latter case, provider A aims to increasing its coverage by choosing contents and using the current resources.

As for the switching cost, it plays a lock-in effect and helps providers to keep their subscribers. Users with long unfinished contract time will incur a large switching cost. This kind of users is not easy to change their provider. Thus, the providers would like to sign a long time contract with users. So it can decrease users' incentive to switch even other providers introduce some high quality contents. On the other hand, users with short unfinished contract time only need to pay a small switching cost. They are easier to switch the providers. Thus, they would like to sign a short time period of contract with the providers. Then, they can be more flexile to choose the providers. So they can increase their payoff by switching multiple times.

Chapter 4

Peeing in Dynamic Model

In this section, we will study how the introduction of dynamic content changes the providers' incentive to peer.

As discussed in Chapter 3, there are two ways for introducing the special content: additional investment and finite budget. In both cases, the two providers can peer over either T time slots or just over the special content time. Combing these two aspects leads to four scenarios in Table 4.1.

	Additional Investment Case	Finite Budget Case
Peering over T time slots	I	II
Peering over one time slot	III	IV

Table 4.1: Peering scenarios in Dynamic Content Model

For scenarios (III) and (IV), the two providers only peer over one time slot during which the special content is introduced at Provider A. For scenarios (I) and (II), the format of peering agreement is similar as the static content case, *i.e.*, over the advertisement ratio α and payment c.

As the analysis methodology of peering incentive in finite budget case is similar with the analysis used in additional investment case, we will first investigate scenario (I) and (III) in details for additional investment case, and then discuss the peering incentive for scenario (II) and (IV).

4.1 Peering over T Time Slots

4.1.1 Content Change, Advertisement Sharing, and Payment

As provider A has better contents, both providers deliver A's contents as illustrated in Fig 4.1.

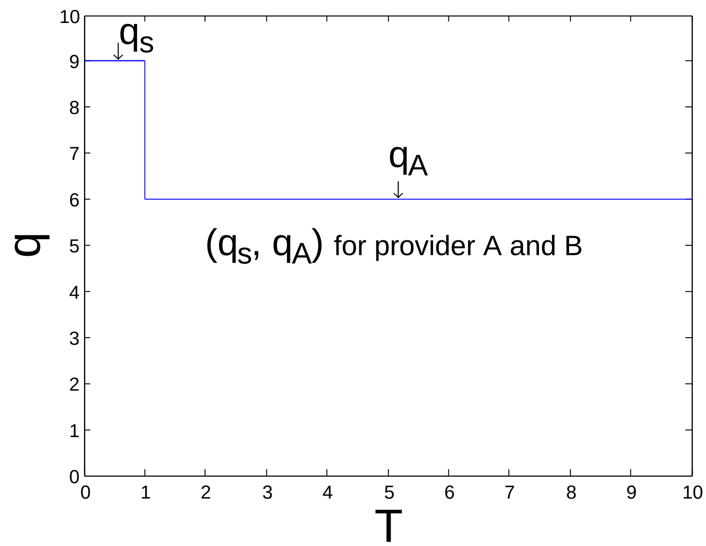

Figure 4.1: The common contents of both providers when peering over T time slots

As for the advertisement arrangement, provider A will deliver its own advertisement. Provider B delivers α portion of A's advertisement and $(1-\alpha)$ portion of its own advertisement. Meanwhile, B pays provider A a one-time payment c for the one time slot peering. Two providers need to bargain over the advertisement splitting ratio α and the payment c similarly as described in Figures 2.3, 2.4, and 2.5.

4.1.2 Change of Coverage

As both providers have the same contents, no users will switch from provider B to A as they can access the same high quality contents with provider B while paying a low monthly subscription fee. On the other hand, some existing subscribers of provider A will switch to B if the benefit in terms of reduced subscription fee is larger than the switching cost, *i.e.*, ,

$$(p_A - p_B)T > p_A z$$

which is equivalent to

$$z \leq (1 - \frac{p_B}{p_A}) \cdot T \tag{4.1}$$

Let us define $z_{A2B} \equiv (1 - \frac{p_B}{p_A}) \cdot T$. Then, users with $z > z_{A2B}$ will incur a large switching cost and thus will not switch and stay with provider A for the next T time slots. On the other hand, users with $z \leq z_{A2B}$ only need to pay a small cost and thus they will switch to provider B.

Let γ_c^T denote the new coverage of provider B after providers peering over T time slots in dynamic model. Then, the coverages for the two providers with peering are

$$A\text{'s coverage} \quad : \quad 1 - \gamma_c^T = (1 - \beta) \cdot (1 - \frac{z_{A2B}}{T}), \tag{4.2}$$

$$B\text{'s coverage} \quad : \quad \gamma_c^T = \beta + (1 - \beta) \cdot \frac{z_{A2B}}{T}. \tag{4.3}$$

4.1.3 Providers' Revenue

The two providers' revenues after peering are as follows:

$$\pi_{As}^T(\alpha, c) = \Big(\alpha f_A(1) + (1-\alpha) f_A(1 - \gamma_c^T) + p_A(1 - \gamma_c^T)\Big) \cdot T + c,$$

$$\pi_{Bs}^T(\alpha, c) = \Big((1-\alpha) f_B(\gamma_c^T) + p_B \cdot \gamma_c^T\Big) \cdot T - c.$$

4.1.4 Nash Bargaining Problem

Now, let us solve the bargaining problem in this dynamic content model using the Nash Bargaining solution, which satisfies several appealing properties illustrated in Section 2.5.

Definition 2. *A peering strategy* (α^*, c^*) *is a Nash bargaining solution if it solves the following problem:*

$$maximize_{\alpha \in [0,1], c} \quad \left(\pi^T_{As}(\alpha, c) - \pi_{As}\right) \cdot \left(\pi^T_{Bs}(\alpha, c) - \pi_{Bs}\right), \tag{4.4}$$

where π_{As} *and* π_{Bs} *are the revenues obtained without peering as in (3.6) and (3.7).*

Each providers should achieve a revenue no worse than its non-peering revenue, otherwise it does not have incentive to bargain. This means that peering will take place if and only if

$$\pi^T_{As}(\alpha, c) + \pi^T_{Bs}(\alpha, c) > \pi_{As} + \pi_{Bs}, \tag{4.5}$$

under which it is always possible to choose a proper value of payment c to increase the revenues of both providers.

We notice that both (4.4) and (2.10) are functions of α and c. Therefore, the optimal solution of (4.4) can be solved using a similar methodology as in Section 2.5.

Not surprisingly, the solution depends on the advertisement functions $f_A(\cdot)$ and $f_B(\cdot)$. Next we also use linear advertisement revenue functions as an illustrated example, *i.e.*, $f_A(x) = k_A x$ and $f_B(x) = k_B x$, which are further used for the discussion in Section 4.2.4. Higher values of the linear coefficients k_A and k_B lead to higher values of advertisement revenue with the same user coverage. We have three cases depending on the relationship between k_A and k_B. The detailed proofs can be found to in Appendix A.2

Scenario 1. $k_A = k_B$.

In this case, two providers have the equal capability of generating the advertisement revenue. Plugging $k_A = k_B$ into condition (4.5) and we get

$$(p_A - p_B) \cdot (\gamma - \gamma^T_c) > 0. \tag{4.6}$$

Since subscription fess $p_A > p_B$ and coverages $\gamma < \gamma_c^T$, condition (4.6)cannot be satisfied. Thus, providers will not peer in this case.

Scenario 2. $k_A > k_B$.

In this case, provider A has a higher capability of generating the advertisement revenue. We can show that the optimal advertising strategy is $\alpha^* = 1$, under which condition (4.5) becomes

$$p_A - p_B < (k_A - k_B) \cdot \frac{\gamma}{\gamma_c^T - \gamma}, \tag{4.7}$$

which can be satisfied under proper subscription fees.

Comparing (4.7) with the corresponding condition (2.12) in the static case, both peering conditions depend on satisfying proper relationship between $p_A - p_B$ and $k_A - k_B$. In both models, the overall subscription revenues of both providers will decrease with peering as some users will switch to provider B with the lower subscription fee. To counter balance this, the two providers need to generate more total advertisement revenues so that both providers can be better off (with a proper payment transfer c).

Furthermore, we notice that the peering incentive depends on the provider B's coverage before and after peering, *i.e.*, γ and γ_c^T. In the case where the special content is introduced at Provider A and there is no peering (see Section 3), the number of users switching from B to A increases with the special content quality q_s, which means that provider A's coverage $(1 - \gamma)$ increases. Then, the right hand side of (4.5) will increases. On the other hand, from (4.1), we observe the number of switching users will not change with q_s during peering as both providers deliver the same content. It means that provider A's coverage $(1 - \gamma_c^T)$ with peering does not change. Therefore, the left hand side of (4.5) will not change. Therefore, (4.5) is harder satisfied, whereby providers' peering incentive is lower when the special content quality increases.

When (4.7) is satisfied and the providers choose to peer, we can show that the optimal payment strategy c^* from provider B to provider A is

$$c^* = \frac{1}{2}\Big((p_A + p_B)(\gamma_c^T - \gamma) - (k_A + k_B)\gamma\Big) \cdot T, \tag{4.8}$$

which can be either positive or negative. The insight of the value of c^* can be referred the discussion in Section 2.5.

Scenario 3. $k_A < k_B$.

In this case, provider B has a higher capability of generating advertisement revenue. We can show the optimal advertising strategy is $\alpha^* = 0$, under which condition (4.5) becomes

$$p_A - p_B < k_B - k_A, \tag{4.9}$$

which can be satisfied under proper values of p_A and p_B.

We note that the peering condition (4.9) is the same as the corresponding condition (2.14) in the static case. The special content quality q_s does not affect the peering condition in this case.

When providers choose to peer, we can show that the optimal payment c^* from provider B to provider A is

$$c^* = \frac{1}{2} \cdot (k_A + k_B + p_A + p_B)(\gamma^c - \gamma) \cdot T > 0. \tag{4.10}$$

Discussion

Now, let us discuss the incentive of peering over T time slots of scenario (II), whereby provider A introduces the special content by finite budget. The analysis is similar with the methodology used in Section 4.1.

As discussed in 4.1.4, the number of switching users only depends on the subscription fee and switching cost when providers choose to peer. Thus, the two providers' coverages after peering in finite budget case is the same as the coverages in additional investment case. The only difference is the two providers' coverages before peering, which provider B's coverage is lareger in finite budget case, *i.e.*, $\gamma' > \gamma$.

Then, substituting the value of γ by γ' into the peering conditions discussed in the three scenarios. We notice that the difference is for scenario that the special content holder A has a stronger ability in generating advertisement revenue. The condition in finite budget case is looser than in additional investment case. This is because the right hand side of (4.7) will decrease when γ increases to γ', which means the providers' peering condition is looser in finite budget case. This is because the left hand of (4.5) does not change and the right hand side of (4.5) has decreased in finite budget case, whereby 4.5) is easier satisfied. Then, providers' peering incentive is higher in finite budget case than in additional investment case.

Other advertisement revenue functions

Convex advertisement revenue function

Here we consider convex advertisement revenue functions, and assume $f_A(x) = k_A \cdot x^2$ and $f_B(x) = k_B \cdot x^2$. Next we summarize the optimal solution of (4.4) depending on two possible relationships between k_A and k_B. The detailed proofs can be found in Appendix A.2.

Scenario 1. $k_A \geq k_B$.

In this case, provider A has a no weaker capability of generating the advertisement revenue than provider B. We can show that the optimal advertising strategy is $\alpha^* = 1$, under which condition (4.5) becomes

$$p_A - p_B < \frac{2k_A \cdot \gamma - (k_A + k_B)\gamma^2}{\gamma_c^T - \gamma}, \tag{4.11}$$

which can be satisfied under proper subscription fees.

When (4.11) is satisfied and the providers choose to peer, we can show that the optimal payment strategy c^* from provider B to provider A is

$$c^* = \frac{1}{2}\Big((p_A + p_B)(\gamma_c^T - \gamma) + (k_A - k_B)\gamma^2 - 2k_A\gamma\Big) \cdot T, \qquad (4.12)$$

which can be either positive or negative. The meaning of the value of c^* can be referred the discussion in Section 2.5.

Scenario 2. $k_A < k_B$.

In this case, provider A has a weaker capability of generating advertisement revenue. The choice of users will affect the peering strategy.

- $\gamma_c^T = \frac{k_B - k_A}{k_A + k_B}$: Condition (4.5) can not be satisfied. This is because the two providers can not generate more advertisement revenue during peering. Then providers will not peer.

- $\frac{k_B - k_A}{k_A + k_B} < \gamma_c^T < 1$: It means that if more users switch to provider B, the optimal advertising strategy should be that the two providers deliver the same advertisement ($\alpha^* = 1$). Therefore, the peering condition and payment is the same as in case of $k_A \geq k_B$.

- $0 \leq \gamma_c^T < \frac{k_B - k_A}{k_A + k_B}$: It means that if fewer users switch and more users stay with provider A, the optimal advertising strategy is that the two providers deliver their own advertisement ($\alpha^* = 0$). The peering condition becomes $p_A - p_B < k_B(\gamma_c^T + \gamma) - k_A(2 - \gamma_c^T - \gamma)$, where the payment becomes $c^* = \frac{1}{2}\Big((p_A + p_B)(\gamma_c^T - \gamma) + (\gamma_c^T - \gamma)(k_B(\gamma_c^T + \gamma) + k_A(2 - \gamma_c^T - \gamma))\Big)$.

We find that in the case of convex advertisement revenue function, providers have a higher incentive to deliver the content holder's advertisement. Convex revenue function means that there exists a phenomenon of scale of economies in market share, which providers can benefit much more from lager users coverage. Thus, providers can generate more total revenue through proper advertisement arrangement, whereby providers will peer under proper subscription fees.

Concave advertisement revenue function

Here we consider concave advertisement revenue functions, and assume $f_A = -k_A(x-1)^2 + k_A$ and $f_B = -k_B(x-1)^2 + k_B$. Again, we summarize the optimal solution of (4.4) depending on two possible relationships between k_A and k_B. The detailed proofs can be found in Appendix A.2

Scenario 1. $k_A \leq k_B$.

In this case, provider A has a no stronger capability of generating the advertisement revenue than provider B. We can show that the optimal advertising strategy is $\alpha^* = 0$,, under which condition (4.5) becomes

$$p_A - p_B < k_B(2 - \gamma_c^T - \gamma) - k_A(\gamma_c^T + \gamma), \tag{4.13}$$

which can be satisfied under proper subscription fees.

When (4.13) is satisfied and the providers choose to peer, we can show that the optimal payment strategy c^* from provider B to provider A is

$$c^* = \frac{1}{2}\Big((p_A+p_B)(\gamma_c^T-\gamma)+(\gamma_c^T-\gamma)(k_A(\gamma_c^T+\gamma)+k_B(2-\gamma_c^T-\gamma))\Big)\cdot T, \tag{4.14}$$

which can be either positive. Provider B need to compensate A.

Scenario 2. $k_A > k_B$.

In this case, provider A has a stronger capability of generating advertisement revenue. Then, the choice of users will affect the peering strategy.

- $\gamma_c^T = \frac{k_A - k_B}{k_A + k_B}$: Condition (4.5) can not be satisfied. This is because the two providers can not generate more advertisement revenue during peering. Then providers will not peer.

- $\frac{k_A - k_B}{k_A + k_B} < \gamma_c^T < 1$: It means that if more users switch to provider B during peering, the optimal advertising strategy is that the two providers deliver different advertisement ($\alpha^* = 0$). The peering condition and payment is the same as in case of $k_A \leq k_B$.

- $0 \le \gamma_c^T < \frac{k_A - k_B}{k_A + k_B}$: It means that if fewer users switch to provider B, the optimal advertising strategy is that the two providers deliver the same advertisement ($\alpha^* = 1$). The peering condition becomes $p_A - p_B < \frac{(k_A+k_B)\gamma^2 - 2k_B\gamma}{\gamma_c^T - \gamma}$, where the payment is $c^* = \frac{1}{2}\Big((p_A + p_B)(\gamma_c^T - \gamma) - (k_A - k_B)\gamma^2 + 2k_B\gamma\Big) \cdot T$

Also, we observe that in the case of concave advertisement revenue function, providers have a higher incentive to deliver their own advertisement. Concave revenue function means that providers can generate large advertisement revenue by themselves, which providers would like to deliver their own advertisement. So providers can generate more total revenue and benefit from the peering under proper subscription fees.

4.2 Peering over One Time Slot

4.2.1 Content Change, Advertisement Sharing, and Payment

In this case, both provider will deliver the special content during the first time slot as illustrated in Fig 4.2. After the first time slot, they will no longer peer with each other and thus will deliver their original contents for the rest $T - 1$ time slots.

During the first time slot, provider A will deliver its own advertisement. Provider B delivers α portion of A's advertisement and $(1 - \alpha)$ portion of its own advertisement. Meanwhile, provider B pays provider A a one-time payment c for the one time slot peering. Two providers need to bargain over the advertisement splitting ratio α and the payment c similarly as described in Figures 2.3, 2.4, and 2.5.

In the remaining $T - 1$ time slots, both providers will deliver their own advertisement without any payment transfer.

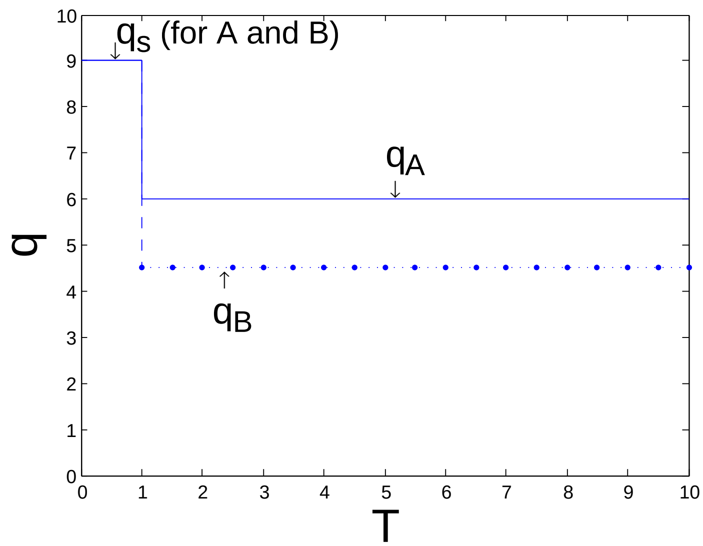

Figure 4.2: Change of Contents with peering over one time slot

4.2.2 Change of Coverage

When two providers share the special content, no users will switch from provider B to A as they can access the special content with provider B while paying a low monthly subscription fee. On the other hand, some existing subscribers of provider A will switch to B if the benefit in terms of reduced subscription fee is larger than the switching cost, *i.e.*,

$$\theta(q_s + \sum_{t=2}^{T} \delta^{t-1} q_B) - p_B \cdot T - p_A \cdot z \geq \theta(q_s + \sum_{t=2}^{T} \delta^{t-1} q_A) - p_A \cdot T,$$

which is equivalent to

$$\theta \leq \frac{(p_A - p_B)T - p_A \cdot z}{\sum_{t=2}^{T} \delta^{t-1} q_A - \sum_{t=2}^{T} \delta^{t-1} q_B}. \tag{4.15}$$

Intuitively, those users who have small θ values will care more about the monthly subscription fee than the content quality, and thus have the incentive to switch from provider A to provider B for the smaller monthly payment p_B. Condition (4.15), of course, also depends on the remaining contract time z of the switching users (*i.e.*, switching penalty).

As we have $z_{A2B} = (1 - \frac{p_B}{p_A})T$, if $z > z_{A2B}$, (4.15) does not hold as the right hand side is negative. This means that users with large switching cost will stay with provider A and not switch. In the case of $z \le z_{A2B}$, the right hand side of (4.15) becomes positive. In this case, users who do not value contents much (*i.e.*, with small θ values) will switch to provider B and enjoy a lower subscription fee. Users who are indifferent between switching to B and staying with A have a parameter θ at satisfies

$$\theta^c_{A2B}(\delta, z) = \frac{(p_A - p_B)T - p_A \cdot z}{\sum_{t=2}^{T} \delta^{t-1} q_A - \sum_{t=2}^{T} \delta^{t-1} q_B},$$

which is shown in Fig 4.3 (with $z = 0.5$).

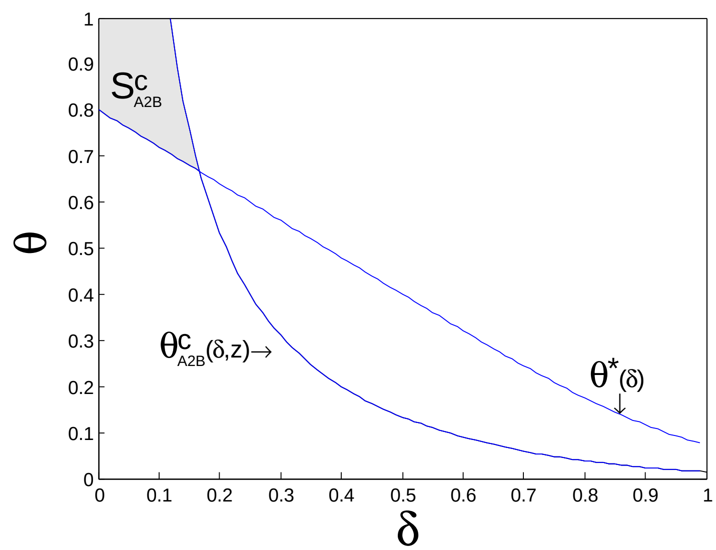

Figure 4.3: Switching users with peering over one time slot

Here we need to have $\theta \le \theta^c_{A2B}(\delta, z)$, otherwise no user will switch from A to B. For these users with $z = 0.5$, users with parameter (θ, δ) on the left hand side of the boundary $\theta^c_{A2B}(\delta, z)$ will choose provider B. In fact, users who are below the curve $\theta^*(\delta)$ choose provider B even without peering. Only users who are above the curve $\theta^*(\delta)$ and on the left side of the curve $\theta^c_{A2B}(\delta, z)$ are the switching users. The gray area S^c_{A2B} denotes the portion of switching users.

We notice that the curve $\theta^c_{A2B}(\delta, z)$ decreases in terms of δ. The reason is that users with smaller δ value current content and subscription fee more. Thus, more users are willing to switch. We further notice that function $\theta^c_{A2B}(\delta, z)$ decreases in z, *i.e.*, the curve moves left with a larger value of z. The switching cost becomes larger and fewer users will switch.

Assuming S^c_{A2B} denote the size of the gray area. Thus, the total number of switching users from A to B is the integration of S^c_{A2B} over z ($z \in [0, z_{A2B})$, which is

$$S_p = \int_0^{z_{A2B}} \frac{S^c_{A2B}}{T} \quad dz.$$

Apparently, the number of switching users in this one time slot peering is smaller than the switching number in T time slots'peering, which is $S_p < (1-\beta) \cdot \frac{z_{A2B}}{T}$. This is because in the latter peering case, switching users make their decisions only based on the subscription fee and their switching cost. In contrast, in the former case, provider A offers higher content quality for the remaining $(T-1)$ time slots. Users make their decisions based on not only these two factors, but also their valuation for future content quality. Thus, fewer users switch in this one time slot peering.

Let γ^O_c denote the coverage of provider B after the providers sharing the special content. Then, the coverages for the two providers with peering are

$$A\text{'s coverage} \; : \; 1 - \gamma^O_c = (1-\beta) - S_p, \tag{4.16}$$

$$B\text{'s coverage} \; : \; \gamma^O_c = \beta + S_p. \tag{4.17}$$

4.2.3 Providers' Revenue

The two providers' revenues after peering are as follows:

$$\begin{aligned}
\pi^O_{As}(\alpha, c) &= \alpha f_A(1) + (1-\alpha) f_A(1-\gamma^O_c) + c \\
&\quad + f_A(1-\gamma^O_c)(T-1) + p_A \cdot (1-\gamma^O_c) \cdot T, \\
\pi^O_{Bs}(\alpha, c) &= (1-\alpha) f_B(\gamma^O_c) + f_B(\gamma^O_c)(T-1) \\
&\quad + p_B \cdot \gamma^O_c \cdot T - c.
\end{aligned}$$

The revenue of provider A equals to the summation of the peering advertisement revenue for the first time slot, the advertisement revenue for the remaining (T-1) time slots, the subscription revenues from users, and the payment c from provider B due to peering. Provider B's revenue is calculated similarly, except that it pays provider A instead of gets paid.

Apparently, provider A's coverage with peering is smaller than the coverage without peering ($1-\gamma_c^O < 1-\gamma$), while provider B has increased its coverage by peering. Therefore, the two providers need a proper advertisement splitting and transfer payment strategy to balance their interests.

4.2.4 Nash Bargaining Problem

Again, let us solve the bargaining problem in this dynamic content using the Nash Bargaining solution, which satisfies several appealing properties illustrated in Section 2.5.

Definition 3. *A peering strategy (α^*, c^*) is a Nash bargaining solution if it solves the following problem:*

$$maximize_{\alpha\in[0,1],c} \quad \left(\pi_{As}^O(\alpha,c)-\pi_{As}\right)\cdot\left(\pi_{Bs}^c(\alpha,O)-\pi_{Bs}\right), \tag{4.18}$$

where π_{As} and π_{Bs} are the revenues obtained without peering as in (3.6) and (3.7).

Also, each provider should achieve revenue no worse than its non-peering revenue, otherwise it does not have incentives to bargain. This means that peering will take place if and only if

$$\pi_{As}^O(\alpha,c)+\pi_{Bs}^O(\alpha,c) > \pi_{As}+\pi_{Bs}, \tag{4.19}$$

under which it is always possible to choose a proper value of payment c to increase the revenues of both providers.

We notice that problem (4.18) is function of α and c as problem (2.10) and (4.4) . Therefore, the optimal solution of (4.18) can be solved using a similar methodology as in Section 2.5.

By using the illustrated linear advertisement revenue functions in Section 4.1.4, we have three cases depending on the relationship between k_A and k_B. The detailed proofs are given in Appendix A.3. The analysis of other advertisement revenues functions (*i.e.*, convex or concave functions) is similar as the methodology used in linear function, which can be referred to Section 4.1.

Scenario 1. $k_A = k_B$.

In this case, two providers have the equal capability of generating the advertisement revenue. Plugging $k_A = k_B$ into condition (4.19) and we get $(p_A - p_B) \cdot (\gamma - \gamma_c^O) > 0$, which can not be satisfied as $p_A > p_B$ and $\gamma < \gamma_c^O$. Thus, providers will not peer.

Scenario 2. $k_A > k_B$.

In this case, provider A has a higher capability of generating the advertisement revenue. We can show that the optimal advertising strategy is $\alpha^* = 1$, under which condition (4.19) becomes

$$p_A - p_B < (k_A - k_B) \cdot \frac{\Big(\gamma \cdot T - \gamma_c^O (T-1)\Big)}{(\gamma_c^O - \gamma)T} \tag{4.20}$$

which can be satisfied under proper subscription fees.

We observe that the contract time can not be too large, *i.e.*, $T < \frac{\gamma^O}{\gamma^O - \gamma}$. Otherwise, (4.20) can not be satisfied. This is because the two providers just peer over one time slot, provider A has lost coverage and its advertisement can not cover provider B's users over the remaining $(T-1)$ time slots. Then, provider A will lose revenues after sharing the special content. The larger the value of T is, the more revenue provider A will lose in the remaining $(T-1)$ time slots. Hence, provider A need to balance its interest between current and future revenues. This is different from the case of peering over T time slots, where provider A does not need to consider the future revenue effect as providers also peer over the remaining $(T-1)$ time slots. Therefore, the contract time T plays an important role in deciding providers' peering decision. When $T \geq \frac{\gamma^O}{\gamma^O-\gamma}$, both providers have no incentive to peer.

When (4.7) is satisfied and providers choose to peer, we can show that the optimal payment strategy c^* from provider B to provider A is

$$c^* = \frac{1}{2}\Big((k_A+k_B)(\gamma_c^O(T-1)-\gamma T)+(p_A+p_B)(\gamma_c^O-\gamma)T\Big). \tag{4.21}$$

Scenario 3. $k_A < k_B$.

In this case, provider A has a lower capability of generating the advertisement revenue. We can show that the optimal advertising strategy is $\alpha^* = 0$, under which condition (4.19) becomes

$$p_A - p_B < k_B - k_A, \tag{4.22}$$

which can be satisfied under proper values of p_A and p_B.

We notice that condition (4.22) in one time slot peering, condition (4.9) in T time slots' peering, and condition (2.14) in the static case are the same. The explanation why they are the same can be found in Section 4.1.4.

When providers choose to peer, we can show that the optimal payment strategy c^* from provider B to provider A is

$$c^* = \frac{1}{2}\cdot(k_A+k_B+p_A+p_B)(\gamma_c^O-\gamma)\cdot T > 0. \tag{4.23}$$

Discussion

Now, let us discuss the incentive of peering over one time slot of scenario (IV), whereby provider A introduces the special content by finite budget. The analysis is similar with the methodology used in Section **??**. The only difference is the two providers' coverages, which will change the peering condition slightly.

As discussed in 3.2, provider B's coverage is γ' before peering and $\gamma_c'^O$ after peering. And we have $\gamma'^{>\gamma}$ as fewer users will switch to A, and $\gamma_c'^O > \gamma_c^O$ as provider B can attract more users due to the due to the decreasing quality of provider A's regular content in finite case.

Then, substituting these two news values (γ' and $\gamma_c'^O$) into the values of (γ and γ_c^O) discussed in the three peering scenarios. The difference is for the scenario that the special content holder A has a stronger ability in generating advertisement revenue. We find that the peering condition becomes $T < \frac{\gamma'^O}{\gamma'^O - \gamma'}$. Thus, providers need to take the future revenue into when peering over one time slot. When $T \geq \frac{\gamma'^O}{\gamma'^O - \gamma}$, both providers have no incentive to peer.

Chapter 5

Summary and Future Work

In content delivering market, network content providers aim to increase their market shares and revenues. The content popularity and market coverage have significant impacts on a network content provider's revenue. Thus, the goal of the present study is to characterize the popularity as the content quality through the analysis of how provider's peering can affect content quality they offer and change their market coverage. The findings identify the peering conditions and optimal peering strategies for sharing contents.

Firstly, a static baseline model is considered, whereby network content providers have static content and do not cooperate. In this case, we derive the coverage of the providers based on the quality of the contents and user subscription fees. Subsequently, peering and content sharing are considered as means of helping providers to improve their revenues. The key insight is that peering will be desirable when the providers' total revenue is increased and properly shared by inter-provider financial transfers. In the case of linear advertisement functions, peering will take place when providers have different abilities in generating advertisement revenue and set subscription fees properly. Moreover, the provider with stronger ability to generate advertisement revenue will contribute more to the total advertisement revenues. Then, the stronger provider has the power to determine the optimal advertisement strategy.

Secondly, the dynamic content model is considered, whereby a provider can introduce some high quality special content for a short period of time in order to attract users. Two cases of the special content budget sources are discussed: additional investment and finite budget. The findings indicate that the budget sources, the special content timing, the switching cost, the valuation of content, and time discount factor all play important roles in deciding the benefit of special content. The special content can give a competitive advantage to the content right holder (provider A in this study) in both cases. It helps the provider to attract users who incur a small switching cost and value content quality and its availability more. Moreover, there are more users switching the provider in additional budget case than in finite budget case. The reason is that provider A spends more budget other than the original budget in the former case. In contrast, in the latter case, provider A aims to increasing its coverage by choosing contents and using the current resources.

Lastly, we consider the peering incentive in dynamic content model and compare the optimal peering strategy with the the one in static content case. Two peering agreements for additional investment case are first analyzed: Peering over T time slots and Peering over One time slot. We discuss different advertisement revenue functions and present providers' peering incentive. In the case of linear advertisement revenue function, the findings show that the difference between static model and the two peering cases in dynamic model is for the scenario that the special content right holder has a stronger ability in generating advertisement revenue. For peering over T time slots case, the peering condition is stricter if the special content quality increases. Then, we identify that the peering incentive in finite budget case is higher than in additional case. While for peering over one time slot case, we show that when providers consider the future revenues more in both special content budget cases, their peering incentive is lower.

There are several ways to extend this work. One direction is to consider the case where both providers can purchase special contents. Then, it is possible for users to switch the service providers multiple times. These users will engage in a content utility maximization problem according to the special contents quality, the providers' regular contents quality and subscription fees over the T time slots. Similarly, the providers need to reconsider their peering strategy based on users' reaction. They will engage in a game theoretical interaction in terms of the timing, content quality, the amount of special content and users' behavior. The other direction is to consider the strategic interactions between advertisers and content providers. For example, a case where an advertiser has the choice to work with more than one content provider to maximize its revenue should be considered. They will choose a provider based on its coverage and price for advertisement. Advertisers' choice will influence the network content providers' capability in generating advertisement revenues and peering incentive with other providers. Thus, when peering, the providers should not only consider the effect on users, but also the effect on advertisers when peering. Finally, as a part of the present study, the strategy by which the providers can jointly optimize the subscription fees with the contents to become more attractive in the market will be considered. Since a lower subscription fee can attract more users, when one of the network providers introduces special content, the content right holder attains a competitive advantage. The other providers need to find out some way to retain their market share and standing. Thus, in this case, increasing content quality and decreasing subscription fee will be viable options. It will be interesting to investigate this joint optimization problem of the providers.

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Appendix A

Proof of Optimal Peering Strategy

A.1 Proof of Static Optimal Peering Strategy

The maximization problem for (2.10) is not a convex optimization problem. However, we can transform problem (2.10) into the following equivalent minimization problem:

$$\text{minimize}_{\alpha\in[0,1],c} \quad F(c,\alpha) = -\log\Big((\pi_A^c(\alpha,c)-\pi_A)\cdot(\pi_B^c(\alpha,c)-\pi_B)\Big), \quad \text{(A.1)}$$

We first show that the above minimization problem is a convex minimization problem with respect to c and α. Then, we use the KKT necessary and sufficient conditions to find out the optimal solutions for (A.1).

It is clear that the constraint set in above minimization problem is convex. To prove function $F(c,\alpha)$ is jointly convex, we just need to show that the Hessian of the function is positive semi-definite, i.e.,

$$\mathbf{H}(F(c,\alpha)) \;=\; \begin{bmatrix} \frac{\partial^2 F}{\partial^2 c} & \frac{\partial^2 F}{\partial c\partial\alpha} \\ \frac{\partial^2 F}{\partial c\partial\alpha} & \frac{\partial^2 F}{\partial^2\alpha} \end{bmatrix} \succeq 0. \quad \text{(A.2)}$$

The Hessian of (A.2) is positive semi-definite if $\frac{\partial^2 F}{\partial^2 c} \geq 0$, $\frac{\partial^2 F}{\partial^2\alpha} \geq 0$ and $\frac{\partial^2 F}{\partial^2 c}\cdot\frac{\partial^2 F}{\partial^2\alpha} - (\frac{\partial^2 F}{\partial\alpha\partial C})^2 \geq 0$. By calculating the derivatives of F function, we have the following results:

$$\begin{aligned}
\frac{\partial^2 F}{\partial^2 c} &= \frac{T^2}{(\pi_A^c - \pi_A)^2} + \frac{T^2}{(\pi_B^c - \pi_B)^2} \geq 0, \\
\frac{\partial^2 F}{\partial^2 \alpha} &= \frac{(f_A(1)T)^2}{(\pi_A^c - \pi_A)^2} + \frac{(f_B(1)T)^2}{(\pi_B^c - \pi_B)^2} \geq 0, \\
\frac{\partial^2 F}{\partial \alpha \partial c} &= \frac{f_A(1) \cdot T}{(\pi_A^c - \pi_A)^2} + \frac{f_B(1) \cdot T}{(\pi_B^c - \pi_B)^2},
\end{aligned}$$

thus:

$$\frac{\partial^2 F}{\partial^2 c} \cdot \frac{\partial^2 F}{\partial^2 \alpha} - (\frac{\partial^2 F}{\partial \alpha \partial C})^2 = \frac{((f_A(1) - f_B(1))T)^2}{(\pi_A^c - \pi_A)^2(\pi_B^c - \pi_B)^2} \geq 0.$$

Hence, the Hessian of $F(c, \alpha)$ function is positive semi-definite and (A.1) is a convex optimization problem. We can use the following KKT necessary and sufficient conditions to figure out the optimal solutions:

$$\begin{aligned}
\frac{\partial F}{\partial c} &= 0, \\
\frac{\partial F}{\partial \alpha} + \mu_1 - \mu_2 &= 0, \\
\mu_1(\alpha - 1) &= 0, \\
\mu_2(-\alpha) &= 0, \\
\mu_1, \mu_2 &\geq 0.
\end{aligned}$$

Where μ_1 and μ_2 are Lagrange multiplier of the constraint of α. Next we discuss the three possible cases of the optimal solutions:

Case I: $f_A(1) = f_B(1)$

Substituting both the revenue functions into the KKT conditions, then they become:

$$\begin{aligned}
\frac{-T}{\pi_A^c(\alpha,c)-\pi_A}+\frac{T}{\pi_B^c(\alpha,c)-\pi_B} &= 0,\\
\frac{-f_A(1)\cdot T}{\pi_A^c(\alpha,c)-\pi_A}+\frac{f_B(1)\cdot T}{\pi_B^c(\alpha,c)-\pi_B} &= \mu_2-\mu_1,\\
\mu_1(\alpha-1) &= 0,\\
\mu_2(-\alpha) &= 0,\\
\mu_1,\mu_2 &\geq 0.
\end{aligned}$$

Rearranging the above equations, we can get the following relationship:

$$\begin{aligned}
\pi_A^c(\alpha,c)-\pi_A &= \pi_B^c(\alpha,c)-\pi_B,\\
\mu_1-\mu_2 &= 0,\\
\mu_1(\alpha-1) &= 0,\\
\mu_2(-\alpha) &= 0,\\
\mu_1,\mu_2 &\geq 0.
\end{aligned}$$

If $\mu_1 = \mu_2 \neq 0$, then $\alpha^* = 0$ and $\alpha^* = 1$, which can not be true. Hence, $\mu_1 = \mu_2 = 0$. The optimal advertising strategy α^* and payment strategy c^* can be figured out by the first equation. The relationship between them is defined by:

$$\begin{aligned}
c^* = \frac{1}{2}\cdot\Big(&(1-\alpha)f_B(1)-f_B(\beta)-\alpha f_A(1)\\
&+f_A(1-\beta)+(p_A+p_B)(1-\beta)\Big)\cdot T.
\end{aligned}$$

That is, provider A can deliver any length of its advertisement to B. It can get the revenue through the a proper payment from provider B.

Case II: $f_A(1) > f_B(1)$

Following *Case I* procedure, we substitute the revenue functions into the KKT conditions and get

$$\begin{aligned} \pi_A^c(\alpha, c) - \pi_A &= \pi_B^c(\alpha, c) - \pi_B, \\ f_A(1) \cdot T - f_B(1) \cdot T &= \mu_1 - \mu_2 > 0, \\ \mu_1(\alpha - 1) &= 0, \\ \mu_2(-\alpha) &= 0, \\ \mu_1, \mu_2 &\geq 0. \end{aligned}$$

From the second equation, we have $\mu_1 > \mu_2$. If $\mu_1 > \mu_2 \neq 0$, $\alpha^* = 0$ and $\alpha^* = 1$, which can not be true. Hence, $\mu_1 > \mu_2 = 0$, thus, $\alpha^* = 1$. The optimal advertising strategy is that the two providers both deliver A's advertisement. The optimal payment strategy is defined by:

$$c^* = \frac{1}{2}(-f_A(1) + f_A(1-\beta) - f_B(\beta) + (p_A + p_B)(1-\beta)) \cdot T.$$

Case III: $f_A(1) < f_B(1)$

Again, following *Case I* procedure, we substitute the revenue function into the KKT conditions and get

$$\begin{aligned} \pi_A^c(\alpha, c) - \pi_A &= \pi_B^c(\alpha, c) - \pi_B, \\ f_A(1) \cdot T - f_B(1) \cdot T &= \mu_1 - \mu_2 < 0, \\ \mu_1(\alpha - 1) &= 0, \\ \mu_2(-\alpha) &= 0, \\ \mu_1, \mu_2 &\geq 0. \end{aligned}$$

From the second equation, we have $\mu_1 < \mu_2$. If $\mu_2 > \mu_1 \neq 0$, $\alpha^* = 0$ and $\alpha^* = 1$, which can not be true. Hence, $\mu_2 > \mu_1 = 0$ and $\alpha^* = 0$. The optimal advertising strategy is that the two providers deliver their own advertisement. The optimal payment strategy is defined by:

$$c^* = \frac{1}{2} \cdot \Big(f_B(1) - f_B(\beta) + f_A(1-\beta) + (p_A + p_B)(1-\beta)\Big) \cdot T.$$

A.2 Proof of Strategy for Peering over T Time Slot

The maximization problem of (4.4) is similar as problem of (2.10) in terms of α and c. It is also not a convex optimization problem. Therefore, problem (4.4) can be solved by a similar methodology used in the evolvement of problem (2.10). Now, let us transform problem (4.4) into the following equivalent minimization problem:

$$\text{minimize}_{\alpha\in[0,1],c} \quad V(c,\alpha) = -\log\Big((\pi^T_{As}(\alpha,c)-\pi_{As})\cdot(\pi^T_{Bs}(\alpha,c)-\pi_{Bs})\Big), \quad \text{(A.3)}$$

Then, we first show that the minimization problem of (A.3) is a convex minimization problem with respect to α and c. Lastly, as in problem of (2.10), we use the KKT necessary and sufficient conditions to find out the optimal solutions for (A.3). As the procedures and analysis for solving (4.4) is similar with the method used in solving (2.10), we would like to skip the details of calculation and show the results straightly.

We find hat the solution depends on the advertisement revenue functions $f_A(\cdot)$ and $f_B(\cdot)$. We have three cases depending on the relationship between between.

Case I: $f_A(1) = f_A(1-\gamma^T_c) + f_B(\gamma^T_c)$.

Provider A's advertisement revenue if reaching all the users equals to the summation of both providers' advertisement revenue if the total users number is spitted when peering. Then, we can show that the optimal advertising strategy α^* and payment strategy c^* can be figured out by the first equation. The relationship between them is defined by:

$$\begin{aligned} c^* \;=\; & \frac{1}{2}\cdot\Big((1-\alpha)f_B(\gamma^T_c) - f_B(\gamma) - \alpha f_A(1) \\ & -(1-\alpha)f_A(1-\gamma^T_c) + f_A(1-\gamma) + (p_A+p_B)(\gamma^T_c-\gamma)\Big)\cdot T. \end{aligned}$$

That is, as in the static case, provider A can deliver any length of its advertisement to B when peering over T time slots. And it can get the revenue through the a proper payment from provider B.

Case II: $f_A(1) > f_A(1-\gamma_c^T) + f_B(\gamma_c^T)$.
Provider A's advertisement revenue if reaching all the users is greater than the summation of both providers' advertisement revenue if the total users number is spitted during peering. Then, we can show that he optimal advertising strategy is that the two providers both deliver A's advertisement for the first time slot, which is $\alpha^* = 1$. The optimal payment strategy is defined by:

$$\begin{aligned} c^* &= \frac{1}{2} \cdot \Big(- f_B(\gamma) - f_A(1) + f_A(1-\gamma) \\ &\quad + (p_A + p_B)(\gamma_c^T - \gamma) \Big) \cdot T. \end{aligned}$$

Case III: $f_A(1) < f_A(1-\gamma_c^T) + f_B(\gamma_c^T)$.
Provider A's advertisement revenue if reaching all the users is less than the summation of both providers' advertisement revenue if the total users number is spitted during peering. Then, the optimal advertising strategy is that the two providers deliver their own advertisement for the first time when peering, which is $\alpha^* = 0$. The optimal payment strategy is defined by:

$$c^* = \frac{1}{2} \cdot \Big(f_B(\gamma_c^T) - f_B(\gamma) - f_A(1-\gamma_c^T) + f_A(1-\gamma) + (p_A + p_B)(\gamma_c^T - \gamma) \Big) \cdot T.$$

A.3 Proof of Strategy for Peering over One Time Slot

The maximization problem of (4.18) is similar as problem of (2.10) and (4.4) in terms of α and c. It is not a convex optimization problem. Thus, problem (4.18) can be solved by a similar methodology used in the evolvement of problem (2.10) and (4.4). Again, we can covert problem (4.18) into the following equivalent minimization problem:

$$\text{minimize}_{\alpha\in[0,1],c} \quad W(c,\alpha) = -\log\Big((\pi^O_{As}(\alpha,c)-\pi_{As})\cdot(\pi^O_{Bs}(\alpha,c)-\pi_{Bs})\Big), \quad (A.4)$$

Then, we first prove that the minimization problem of (A.4) is a convex minimization problem in terms of α and c. Finally, as in problem of (2.10) and (4.4), we can use the KKT necessary and sufficient conditions to find out the optimal solutions for (A.4). As the procedures and analysis for solving (4.18) is similar with the method used in solving (2.10), we would like to skip the details of calculation and present the results directly.

We also find hat the solution depends on the advertisement revenue functions $f_A(\cdot)$ and $f_B(\cdot)$. We have three cases depending on the relationship between between.

Case I: $f_A(1) = f_A(1-\gamma^O_c) + f_B(\gamma^O_c)$.

The meaning of the relation between the two providers' advertisement revenue function referred to the explanation in A.2. Then, we can show that the optimal advertising strategy α^* and payment strategy c^* can be figured out by the first equation. The relationship between them is defined by:

$$\begin{aligned} c^* \;=\; & \frac{1}{2}\cdot\Big((1-\alpha)f_B(\gamma^O_c) - \alpha f_A(1) - (1-\alpha)f_A(1-\gamma^O_c) + (f_B(\gamma^O_c) - f_A(1-\gamma^O_c))(T-1) \\ & -(f_B(\gamma) - f_A(1-\gamma))T + (p_A+p_B)(\gamma^O_c - \gamma)T\Big) \end{aligned}$$

That is, as in the static case and in dynamic case for peering T time slots, provider A can deliver any length of its advertisement to B for the first time slot when peering over one time slot. And it can get the revenue through the a proper payment from provider B.

Case II: $f_A(1) > f_A(1-\gamma^O_c) + f_B(\gamma^O_c)$.

We can show that he optimal advertising strategy is that the two providers both deliver A's advertisement for the first time slot, which is $\alpha^* = 1$. The optimal payment strategy is defined by:

$$\begin{aligned} c^* &= \frac{1}{2} \cdot \Big(- f_A(1) + (f_B(\gamma_c^O) - f_A(1-\gamma_c^O))(T-1) \\ &\quad -(f_B(\gamma) - f_A(1-\gamma))T + (p_A + p_B)(\gamma_c^O - \gamma)T \Big) \end{aligned}$$

Case III: $f_A(1) < f_A(1-\gamma_c^O) + f_B(\gamma_c^O)$.

We can show that the optimal advertising strategy is that the two providers deliver their own advertisement for the first time when peering, which is $\alpha^* = 0$. The optimal payment strategy is defined by:

$$c^* = \frac{1}{2}\Big((f_A(1-\gamma) - f_A(1-\gamma_c^O))T + (f_B(\gamma_c^O) - f_B(\gamma))T + (p_A + p_B)(\gamma_c^O - \gamma)T\Big)$$