Competitive Charging Station Pricing for Plug-in Electric Vehicles

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***Abstract*—This paper considers the problem of charging station pricing and station selection of plug-in electric vehicles (PEVs). Every PEV needs to select a charging station by considering the charging prices, waiting times, and travel distances. Each charging station optimizes its charging price based on the prediction of the PEVs' charging station selection decisions, in an attempt to maximize its profit. To obtain insights of such a highly coupled system, we consider a one-dimensional system with two charging stations and Poisson arriving PEVs. We propose a multi-leader-multi-follower Stackelberg game model, in which the charging stations (leaders) announce their charging prices in Stage I, and the PEVs (followers) make their charging station selections in Stage II. We show that there always exists a unique charging station selection equilibrium in Stage II, and such equilibrium depends on the price difference between the charging stations. We then characterize the sufficient conditions for the existence and uniqueness of the pricing equilibrium in Stage I. Unfortunately, it is hard to compute the pricing equilibrium in closed form. To overcome this challenge, we develop a low-complexity algorithm that efficiently computes the pricing equilibrium and the subgame perfect equilibrium of our Stackelberg game with no information exchange.**

I. Introduction

The growing concerns for climate change and energy security have led to significant industry interests in the development of plug-in electric vehicles (PEVs) [1]-[9]. The battery of a PEV can be conveniently recharged at a charging station, and a charging station has the incentive to optimize the charging price to maximize its profit. With latest mobile applications such as PlugShare [10], a PEV driver can easily identify close-by charging stations in real time. When facing multiple charging station choices, a PEV needs to optimize the selection based on the charging prices, travel distances, and the estimated waiting times at different stations. The collective decisions of PEVs will determine the total demand and the profit of each charging station.

In this work, we aim to answer the following questions: 1) How should a PEV select a charging station based on the charging prices, travel distances, and the estimation of waiting times? 2) How should a charging station optimize its charging price to maximize its profit, based on the competitor's price and the prediction of PEVs' selections? The key challenge for answering these questions is that the station selections of different PEVs are coupled, and such selection often cannot be characterized as a closed form function of the charging prices. To shed some light on this problem, we consider a one-dimensional system with two competing charging stations and Poisson arriving PEVs. In such a system, the charging stations announce their charging prices simultaneously at the beginning of a day, and the PEVs make their selections asynchronously during the day. We formulate the problem as a *multi-leader-multi-follower* Stackelberg game [11], in which the charging stations are the leaders making decisions in the first stage, and the PEVs are the followers making decisions in the second stage. We then analyze this game and characterize the charging stations' pricing and the PEVs' selection behaviors. The main contributions of this paper are summarized as follows.

- *Novel and Practical Model:* To the best of our knowledge, this is the first work that jointly studies competitive charging station pricing and PEV station selection. Our model considers the charging stations with heterogenous service capabilities and asymmetric locations. It also takes into account the waiting time before a PEV receives the charging service.
- *Analysis and Insights:* Our analysis shows that there always exists a charging station selection equilibrium under any fixed charging prices. The equilibrium can be one of five types, depending on the prices. We further characterize the sufficient conditions for the uniqueness of pricing equilibrium.
- *Efficient Algorithm:* We propose an algorithm to compute the equilibrium pricing of the game. Such an algorithm does not require explicit information exchange between the charging stations, is provable convergent, and has a low computational complexity.

The remainder of this paper is organized as follows. Section II discusses the related work. Section III presents the system model. We analyze the system equilibrium and propose an algorithm to achieve the equilibrium in Sections IV and V, respectively. Computer simulation results are provided in Section VI, followed by the concluding remarks in Section VII.

II. Related Work

Prior work on pricing for PEVs can be divided into two categories: single seller pricing (e.g., [3]) and multi-seller pricing (e.g., [4][5]). For example, Tushar *et al.* in [3] considered a smart grid and multiple PEVs groups, and optimized the price using a game theoretical method. Escudero-Garźas and Seco-Granados in [4] studied the competitive pricing of multiple charging stations under the Bertrand's oligopoly model. In their work, the PEVs select their charging stations based on prices and distances. Ban *et al.* in [5] investigated the

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optimal PEV allocation problem and dynamic price control methods for multiple charging stations. However, none of the existing results jointly consider the competition among charging stations and the interaction among PEVs. In our model, a PEV makes a selection according to the charging prices, the travel distances, and the waiting times. Due to the coupling among the decisions of PEVs and charging stations, we propose a multi-leader-multi-follower Stackelberg game in this paper.

Our theoretical model is related to the hotelling game [12]-[14], where two geographically separated sellers compete to serve customers at different locations. However, the existing hotelling game models cannot be directly applied to characterize the interplay between charging stations and PEVs. For example, the hotelling games studied in [12]-[14] did not consider the dependence among the customers' decisions. Gallay and Hongler in [15] extended the hotelling game by adding the waiting time cost to the model. However, their model assumed that the sellers choose the same price, and ignored the pricing optimizations of sellers. Different from these hotelling games, our Stackelberg game model characterizes the interdependent decisions of both the sellers (charging stations) and the customers (PEVs).

III. Problem Formulation

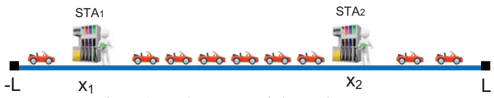


Fig. 1: The considered system.

Consider a one-dimensional system with two competing charging stations, as shown in Fig. 1. The entire system is represented by a line segment, denoted by $[-L, L]$.[1] The charging stations locate at x_1 and x_2 ($-L < x_1 < x_2 < L$), respectively. We allow the two stations to be asymmetrically located on the line, i.e., $x_1 + L \neq L - x_2$. Each charging station serves its customers (i.e., the PEVs waiting in its queue) on a first come first served basis. Assume charging station $i \in \{1, 2\}$ has $k_i \geq 1$ identical charging ports, and the service (charging) time for a single PEV at each port is independent and identically distributed with a mean $\frac{1}{\mu_i}$ ($\mu_i > 0$) and a variance σ_i^2.

The two-stage Stackelberg model works as follows:

• Stage I: The charging stations simultaneously determine their charging prices for the next 24 hours at the beginning of a day, and periodically broadcast the (fixed) prices together with their locations to the potential customers (PEVs) throughout the day (say through the mobile app such as PlugShare).

• Stage II: Given the charging prices and the locations of both charging stations, every PEV independently selects a charging station to recharge its battery.

In this work, we aims to derive the charging stations' pricing equilibrium and the PEVs' charging station selection equilibrium, at which no charging station or PEV has an incentive to change its current choice unilaterally. To this end, we propose a multiple-leader-multiple-follower game model. In Stage I of the game, the charging stations engage in a non-cooperative Charging Station Pricing Game (*CSPG*). In Stage II of the game, the PEVs engage in a non-cooperative Charging Station Selection Game (*CSSG*), under fixed prices from the charging stations.

A. Charging Station Selection Game in Stage II

In *CSSG*, PEVs are the players. A PEV $n \in \mathcal{N}$ decides its station choice $s_n \in \{1, 2\}$, where $\mathcal{N}$ is the set of all PEVs. We denote all PEVs' selections as a strategy profile $\boldsymbol{s} = \{s_n, n \in \mathcal{N}\}$. Since there is no direct communications between PEVs, we assume that a PEV does not know other PEVs' decisions when making its decision. Hence although the PEVs make charging station selection asynchronously during the day, we can model CSSG as a *simultaneous move* (or static) game[2]. Every PEV selects a charging station to maximize its payoff, which equals to the utility minus the cost. Here the utility represents the benefit of getting the vehicle charged, which we assume to be a constant Z that is large enough, such that a PEV is always willing to select one of the two stations. The cost includes three terms: traveling cost, waiting time, and charging cost. Accordingly, the payoff of PEV n is defined as

$$U_n(\boldsymbol{s}) = Z - k_l l_{n,s_n} - k_q q_{s_n}(\boldsymbol{s}) - k_p d p_{s_n} \qquad (1)$$

Here l_{n,s_n} denotes the distance from the PEV's current location to the selected charging station s_n, q_{s_n} represents the estimated waiting time at the selected charging station, d is charging demand[3] and p_{s_n} is the price of the selected charging station. Furthermore, k_l, k_q, and k_p are positive weighting factors. Let x denote the location of PEV n, then $l_{n,s_n} = |x - x_{s_n}|$.

Next we will estimate the waiting time based on queueing theory. The inter-arrival time between two consecutive PEVs arriving at the same charging station depends on two factors: 1) the time interval between the time instances at which these two PEVs decide to seek charging service, and 2) the difference between the travel times to the station. Since a PEV will only consider charging stations close-by, we assume that the difference between the travel times is relatively small[4]. We further assume that the time interval between decision-making of two consecutive PEVs is exponentially distributed [6]. Accordingly, the sequence of PEVs arriving at each charging station can be considered as a Poisson stream. Suppose that the arrival rate of the PEVs coming from a unit line segment is $\lambda > 0$. Let $A_i \subseteq [-L, L]$ be the set of locations of the PEVs who select charging station i. Notice that A_i might include multiple disjoint segments. We use $|A_i|$ to denote the total length of A_i. Then the arrival rate of the PEVs selecting

[1] In fact, the line segment model is also applicable to the scenario where both stations are along a zigzag road.

[2] Even though the decisions may be made at different points in time, the game is simultaneous because each player has no information about the decisions made before or after his.

[3] We assume that a PEV will consider charging (hence makes station selection decision) when its battery State of Charge (SoC) is below a threshold, and then charges the battery to a target SoC at the charging station. The threshold and target SOCs are assumed to be the same for all PEVs, e.g., 20% and 90%, respectively.

[4] Take charging station 1 as an example. The difference between the travel times of two PEVs is no more than $\frac{L-x_1}{v}$, where v is the speed of PEVs. If v equals to 60 kph and $|L - x_1|$ is 5 km, it will be no more than 5 mins. On the other hand, if a PEV is charged on a standard 120-volt outlet, it usually needs 8 hours to be fully charged. If the PEV uses a dedicated 240 circuit, it may need 3 hours. If the PEV uses a 480V circuit, it needs 20 to 30 mins [16].

charging station i is $|A_i|\lambda$ [5]. To avoid queue explosion at charging station i, we need to have $\frac{|A_i|\lambda}{k_i\mu_i} < 1$. As our focus of this paper is to study the pricing competition between the charging stations, we will assume that each station can serve all the PEVs in the system alone[6]. This leads to the following assumption.

***Assumption** 1:* For any $i \in \{1, 2\}$, we have $\frac{2L\lambda}{k_i\mu_i} < 1$.

By using the theory of $M/G/k$ queue [17], the mean waiting time of a PEV at charging station i is

$$q_i \approx \frac{|A_i|\lambda(\sigma_i^2 + \frac{1}{\mu_i^2})\rho_i^{k_i-1}}{2(k_i-1)!(k_i-\rho_i)^2[\sum_{m=0}^{k_i-1}\frac{\rho_i^m}{m!} + \frac{\rho_i^{k_i}}{(k_i-1)!(k_i-\rho_i)}]} \quad (2)$$

where $\rho_i = \frac{|A_i|\lambda}{\mu_i}$. Clearly, q_i monotonously increases with $|A_i|$.

B. Charging Station Pricing Game in Stage I

In *CSPG*, the charging stations are the players, and they simultaneously determine their charging prices p_1 and p_2. We assume that $p_i \in [p_{\min}, p_{\max}]$, and the unit electricity cost paid by a charging station to the utility company is $c_i \le p_{\min}$. The profit of a charging station includes two parts: 1) the revenue of providing charging service to the PEVs, and 2) the fixed and operational costs. The profit can be written as

$$Q_i(p_i, p_j) = (p_i - c_i)D_i(p_i, p_j) - \check{c}_i. \quad (3)$$

Here $D_i(p_i, p_j) = |A_i|d$ is the total demand of the PEVs selecting charging station i, which depends on the prices of both stations. Constant $\check{c}_i$ is the fixed cost for providing the charging service (e.g., labour cost), which is independent of the number of PEVs requesting the service.

IV. Equilibrium Analysis

Next we will derive the subgame perfect equilibrium (SPE) of the Stackelberg game, which represents a Nash Equilibrium (NE) of every subgame of the game. When the SPE is achieved, neither PEVs nor charging stations have incentives to change their own strategies. Hence SPE corresponds to the stable station selection and pricing outcome. To derive the SPE, we start with Stage II (*CSSG*) and analyze the PEVs' selection given p_1 and p_2. Then, we look at Stage I (*CSPG*) and analyze how charging stations make the pricing decisions, taking the PEVs' responses in Stage II into consideration.

A. Charging Station Selection Game

Under a given price pair (p_1, p_2), every PEV selects a charging station to maximize its payoff in (1). We assume that the number of PEVs is large, hence a single PEV's selection will not affect the sets A_1 and A_2 and the corresponding waiting times at two stations [18]. Let us use $U_n(s_n; A1, A2)$ to denote PEV n's utility, when other PEVs' station choices can be represented by sets A_1 and A_2. Then the NE of *CSSG* can be defined as follows.

[5]If no PEVs select charging station i, $|A_i|$ is zero. In addition, the number of all the PEVs in the system is not fixed since the PEVs are considered to be a Poisson stream.

[6]We will briefly discuss the case where no charging station can serve all the PEVs alone in Section VII.

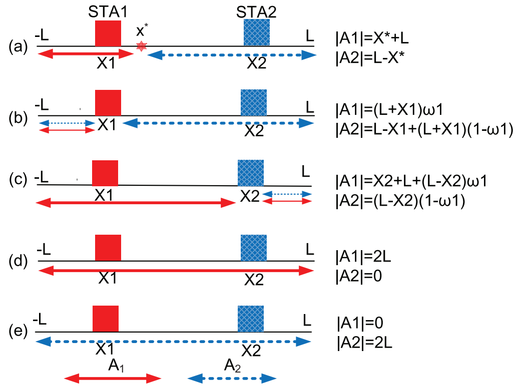


Fig. 2: A_1 and A_2 under different values of $p_1 - p_2$.

***Definition** 1:* A strategy profile $\boldsymbol{s}^* = \{s_n^*, \forall n \in \mathcal{N}\}$ is an NE of the *CSSG* if $U_n(s_n^*; A_1, A_2) \ge U_n(s_n'; A_1, A_2)$ for every $n \in \mathcal{N}$, where s_n' is the different charging station selection than s_n^*.

If $U_n(i; A_1, A_2) \ge U_n(j; A_1, A_2)$, PEV n prefers charging station i over charging station j. When a PEV at a certain location is indifferent of selecting between the two charging stations, i.e., $U_n(1; A_1, A_2) = U_n(2; A_1, A_2)$, we term the corresponding location the *indifference point*.

***Lemma** 1:* Let x^* denote one indifference point in our system. If $x_1 < x^* < x_2$, then we have $A_1 = [-L, x^*]$ and $A_2 = (x^*, L]$ [7], where x^* corresponds to the unique root of the following equation in the range of $[x_1, x_2]$,

$$k_p d(p_1 - p_2) + k_l(2x - x_1 - x_2) + k_q(q_1(x + L) - q_2(L - x)) = 0. \quad (4)$$

The proof of Lemma 1 is omitted due to space limit.[8]

With Lemma 1, we can characterize the NE of *CSSG* in Theorem 1. First we define two thresholds as

$$\theta_1^L = -\frac{k_q(q_1(L + x_2) - q_2(L - x_2)) + k_l(x_2 - x_1)}{k_p \cdot d},$$

and

$$\theta_1^R = \frac{k_q(q_2(L - x_1) - q_1(x_1 + L)) + k_l(x_2 - x_1)}{k_p \cdot d}.$$

***Theorem** 1:* If $p_1 - p_2 \in (\theta_1^L, \theta_1^R)$, then the indifference point is *unique* and the NE strategy of a PEV at location x is 1 if $x \le x^*$ and 2 otherwise, where x^* is the unique root of (4) in the range of (x_1, x_2).

Theorem 1 shows that if $p_1 - p_2 \in (\theta_1^L, \theta_1^R)$, all the PEVs left to the unique indifference point are willing to select charging station 1, and the rest PEVs are willing to select charging station 2, as shown in Fig. 2 (a). The location of the indifference point depends on the price difference $p_1 - p_2$, the service rates, and the charging stations' locations. As $p_1 - p_2$ increases, x^* will move closer to x_1 and more customers are

[7]x^* can belong to either A_1 or A_2, and the result does not change.

[8]All the proofs in this paper can be found in http://itec.hust.edu.cn/∼yuanwei/charging-technical-report.pdf.

attracted by charging station 2. Once $p_1 - p_2$ reaches θ_1^R, the indifference point will be location x_1.

When $p_1 - p_2$ further increases beyond θ_1^R, a new type of equilibrium will emerge, where some PEVs in $[-L, x_1]$ select charging station 1 while other PEVs in $[-L, x_1]$ select charging station 2. For these PEVs, whether selecting station 1 or station 2 no longer depends on its location; it only depends on the value of $|A_1|$ and $|A_2|$. This is illustrated in Fig. 2 (b), and the fraction of PEVs in $[-L, x_1]$ select charging station 1 (denoted by the red arrow) needs to be properly chosen, such that $U_n(1; A_1, A_2) = U_n(2; A_1, A_2)$ for all PEVs in this range, which corresponds to a mixed strategy equilibrium. Similarly, when $p_1 - p_2$ decreases below θ_1^L, a similar equilibrium will emerge, as shown in Fig. 2 (c).

To formally characterize the mixed strategy equilibrium, we allows a player to randomly select between two strategies. Let ω_1 and $1-\omega_1$ denote the probability of selecting charging stations 1 and 2, respectively. Then we use $(\omega_1, 1-\omega_1)$ to represent the mixed strategy of a PEV of choosing station 1 with probability ω_1 and choosing station 2 with probability $1-\omega_1$, and define two more thresholds as

$$\theta_2^L = -\frac{k_q q_1(2L) + k_l(x_2 - x_1)}{k_p \cdot d},$$

and

$$\theta_2^R = \frac{k_q q_2(2L) + k_l(x_2 - x_1)}{k_p \cdot d}.$$

***Theorem** 2:* If $p_1 - p_2 \in [\theta_1^R, \theta_2^R)$, then the NE strategy of the PEV at location x is 2 if $x \in [x_1, L]$, and $(\omega_1, 1-\omega_1)$ if $x \in [-L, x_1)$, where ω_1 is the unique root of the following equation in the range of $[0, 1]$,

$$k_q(q_1((x_1+L)\omega_1) - q_2(L - x_1 + (x_1+L)(1-\omega_1))) + k_p d(p_1 - p_2) + k_l(x_1 - x_2) = 0. \quad (5)$$

If $p_1 - p_2 \in (\theta_2^L, \theta_1^L]$, then the NE strategy of the PEV at location x is 1 if $x \in [-L, x_2]$, and $(\omega_1, 1-\omega_1)$ if $x \in (x_2, L]$, where ω_1 is the unique root of the following equation in the range of $[0, 1]$,

$$k_q(q_2((L-x_2)(1-\omega_1)) - q_1(x_2 + L + (L-x_2)\omega_1)) + k_p d(p_2 - p_1) + k_l(x_1 - x_2) = 0. \quad (6)$$

We can show that when $p_1 - p_2$ decreases, the probability ω_1 will increase accordingly. If $p_1 - p_2$ reaches θ_2^L, then ω_1 increases to 1. At this moment, all the PEVs will select charging station 1. If $p_1 - p_2$ further decreases, the PEVs will not change their selections. In this case, there is no indifference point in the system, and all PEVs will select charging station 1, as shown in Fig. 2 (d). Similarly, if $p_1 - p_2 \geq \theta_2^R$, all PEVs select charging station 2, as shown in Fig. 2 (e).

***Theorem** 3:* If $p_1 - p_2 \in [\theta_2^R, p_{\max} - p_{\min}]$, then the NE strategy for any PEV is 2. If $p_1 - p_2 \in [p_{\min} - p_{\max}, \theta_2^L]$, then the NE strategy of any PEV is 1.

According to Theorems 1 to 3, we can conclude that *CSSG* always has a unique NE.

B. Charging Station Pricing Game

Now we analyze the SPE of the Stackelberg game, by investigating the NE of *CSPG* given the NE of *CSSG*. To this end, we first derive the total demand for each charging station. Take charging station 1 as an example. Let $\Delta p = p_1 - p_2$. According to Theorems 1 to 3, the total demand of charging station 1 is

$$D_1 = \begin{cases} 2L\lambda d, & \text{if } \Delta p \in [p_{\min} - p_{\max}, \theta_2^L], \\ (\omega_1(L - x_2) + L + x_2)\lambda d, & \text{if } \Delta p \in (\theta_2^L, \theta_1^L], \\ (L + x^*)\lambda d, & \text{if } \Delta p \in (\theta_1^L, \theta_1^R), \\ (x_1 + L)\omega_1\lambda d, & \text{if } \Delta p \in [\theta_1^R, \theta_2^R), \\ 0, & \text{if } \Delta p \in [\theta_2^R, p_{\max} - p_{\min}]. \end{cases} \quad (7)$$

where x^* is the indifference point. Let $\mathcal{B}_i(p_j)$ (with $i \neq j$) be the *best response* of charging station i to price p_j, i.e.,

$$\mathcal{B}_i(p_j) \in \arg \max_{p_i \in [p_{\min}, p_{\max}]} Q_i(p_i, p_j) \quad (8)$$

Let (p_1^*, p_2^*) be the NE of *CSPG*, where the prices are mutual best responses. This means for any $i \in \{1, 2\}$, we have

$$\mathcal{B}_i(\mathcal{B}_j(p_i^*)) = p_i^*. \quad (9)$$

***Theorem** 4:* There exists a *unique* pure NE in *CSPG*, if $\mathcal{B}_i(p_j)$ monotonically increases in p_j, and $\mathcal{B}_i(p_j) - p_j$ strictly monotonically decreases in p_j for $i, j \in \{1, 2\}$ and $i \neq j$.[9]

We can use Theorem 12.5 of [19] to prove the existence of NE, and then prove the uniqueness by contradiction. In Theorem 4, $\mathcal{B}_i(p_j) - p_j$ is the optimal price offset of charging station i, when responding to its opponent's price p_j. The condition in Theorem 4 implies that when the opponent's price increases, a charging station's optimal price offset will decrease. Both conditions in Theorem 4 are verified via our simulations. In the remainder of this paper, we only consider the case where both conditions in Theorem 4 are satisfied.

There can be three kinds of pricing NEs in *CSPG*: 1) $(p_{\max}, p_{\max})$ as in Fig. 3, 2) $(p_{\min}, p_{\min})$ as in Fig. 4, and 3) an *inner* NE as in Fig. 5. In these figures, we draw the best response functions of both charging stations, and the intersection of the curves correspond to the NE. In Fig. 3, $p_{\max}$ is quite small. When p_1 and p_2 reach $p_{\max}$, each charging station still has the incentive to further increase its price, as its opponent's price is low and its optimal price offset is still positive. Since the price is upper bounded by $p_{\max}$, the unique pricing equilibrium is $(p_{\max}, p_{\max})$. As shown in Fig. 4, if $p_{\min}$ is quite large, each charging station still has the incentive to further decrease its price when $p_{\min}$ is reached. Hence $(p_{\min}, p_{\min})$ is the unique NE of *CSPG*. Fig. 5 shows that under proper choices of $p_{\min}$ and $p_{\max}$, we will have a unique inner NE.

V. Computing the Equilibrium

To obtain the SPE of our game, we need to identify the equilibrium of *CSPG* in Stage I, given the equilibrium of *CSSG* in Stage II. The key challenge is that we cannot express the equilibrium of *CSSG* in closed form. In Section IV, we have characterized the SPE by finding the intersection of the two best response functions. Such an approach requires us to fully characterize the best response functions numerically, and hence has a high computational complexity. In this section, we propose a low complexity algorithm to compute the SPE.

[9] $\mathcal{B}_i(p_j)$ does not need to be a strictly monotonic function, as shown in Fig. 3 and Fig. 4.

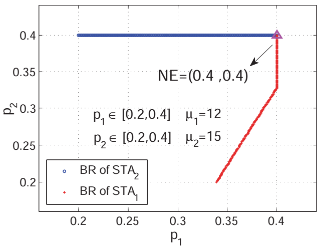


Fig. 3: A maximum boundary pricing equilibrium.

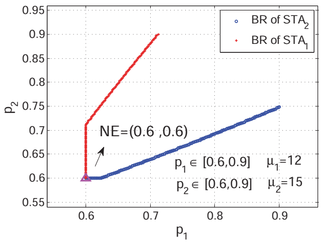


Fig. 4: A minimum boundary pricing equilibrium.

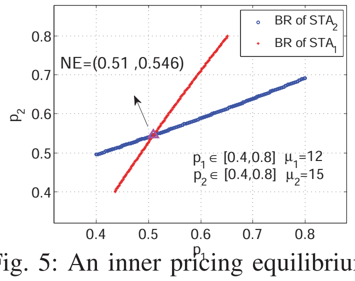


Fig. 5: An inner pricing equilibrium.

Assume that the Stackelberg game has a unique SPE with pricing decisions (p_1^*, p_2^*) (i.e., the sufficient conditions in Theorem 4 are satisfied). Define $\Theta_i(p_i) = \mathcal{B}_i(\mathcal{B}_j(p_i)) - p_i$. We have

***Proposition** 1:* $\Theta_i(p_i) < 0$ if $p_i > p_i^*$, and $\Theta_i(p_i) > 0$ if $p_i < p_i^*$.

Proposition 1 enables a station to know whether its current price is larger or smaller than the equilibrium price, by evaluating the function of Θ_i. This motivates us to propose Algorithm 1, where station 1 adjusts its price to approach p_1^*. If $\Theta_1 < 0$, station 1 will reduce p_1; if $\Theta_1 > 0$, station 1 will increase p_1. By iteratively adjusting p_1, our Directional SPE Search Algorithm (DSSA) algorithm will converge to the equilibrium price p_1^*.

More specifically, in round t of DSSA (Algorithm 1), $p_1(t)$ is updated by a step size $\delta(t)$ according to the sign of $\Theta_1(p_1(t))$. If $p_1(t)$ leaps over p_1^*, then DSSA changes the search direction and reduces its step size. Mathematically, we have

$$p_1(t+1) = [p_1(t) + d(t)\delta(t)]_{p_{\min}}^{p_{\max}}\ {}^{10}, \tag{10}$$

where

$$\delta(t) = \begin{cases} \delta(t-1), & \text{if } \Theta_1(p_1(t)) \cdot \Theta_1(p_1(t-1)) > 0, \\ \alpha\delta(t-1), & \text{if } \Theta_1(p_1(t)) \cdot \Theta_1(p_1(t-1)) < 0, \end{cases} \tag{11}$$

and

$$d(t) = \begin{cases} 1, & \text{if } \Theta(p_1(t)) > 0, \\ -1, & \text{if } \Theta(p_1(t)) < 0. \end{cases} \tag{12}$$

In (11), α is a constant in $(0,1)$, and $\Theta_1(p_1(t)) \cdot \Theta_1(p_1(t-1)) < 0$ indicates that DSSA has leaped over p_1^* and the search direction should be changed. Once the algorithm converges to the equilibrium price p_1^*, station 2's equilibrium price is $p_2^* = \mathcal{B}_2(p_1^*)$.[11]

[10] Here $[x]_b^a = \min(\max(x,b),a)$.

[11] We can write another version of the algorithm, where station 2 evaluates the value of $\Theta_2(p_2)$ and adjusts p_2 until convergence.

Algorithm 1: Directional SPE Search Algorithm

Input: $L, x_1, x_2, \mu_1, \mu_2, k_1, k_2, \lambda, p_{\min}, p_{\max}, \alpha, \delta(0), \varepsilon$
Output: p_1^*, p_2^*
if $|\Theta_i(p_{\min})| \le \varepsilon\ (i = 1,2)$ **then**
  $p_1^* = p_2^* = p_{\min}$ and terminate;
if $|\Theta_i(p_{\max})| \le \varepsilon\ (i = 1,2)$ **then**
  $p_1^* = p_2^* = p_{\max}$ and terminate;
Set $\Theta_1(p_1(0)) = 1$, $t = 1$ and randomly choose $p_1(1)$ from $(p_{\min}, p_{\max})$;
while $|\Theta_1(p_1(t))| > \varepsilon$ **do**
  Update $d(t)$ and $\delta(t)$ with (12) and (11), respectively;
  Update $p_1(t+1)$ with (10), and $t = t+1$;
$p_1^* = p_1(t)$ and $p_2^* = \mathcal{B}_2(p_1^*)$;

***Theorem** 5:* The DSSA algorithm converges to the unique SPE of our Stackelberg game under the two sufficient conditions in Theorem 4.

To prove Theorem 5, the key idea is to show that the interation of p_1 in DSSA can be characterized by a pseudocontraction mapping. To implement DSSA in practice, each station needs to evaluate the best response functions of both stations. If the stations' locations, service rates, feasible price range are public information, there is no need for any information exchange between the stations.

VI. Numerical Results

In this section, we use Matlab simulations to verify the previous equilibrium analysis and demonstrate the effectiveness of the proposed algorithm. Some system parameters are chosen as follows: $L = 10$, $x_1 = -8$, $x_2 = 5$, $k_1 = k_2 = 2$, $d = 50$, $k_p = 2$, $k_q = 2.5$, $k_l = 1.5$, $\lambda = 1$, $c_1 = c_2 = 0.1$, $\check{c}_1 = \check{c}_2 = 1$, and $\varepsilon = 0.0001$.

A. Equilibrium of the Charging Station Selection Game

To illustrate the results for the station selection equilibrium in Stage II, Fig. 6 and Fig. 7 show the indifference point location x^* and the probability of selecting charging station 1 ω_1 under various values of $p_1 - p_2$, respectively. In this simulation, $\mu_1 = 12$, $\mu_2 = 15$, and $p_1 - p_2$ changes from -1.0 to 0.84. Accordingly, we can compute the four thresholds in Theorems 1 to 3 as $\theta_2^L = -0.538$, $\theta_1^L = -0.286$, $\theta_1^R = 0.300$, and $\theta_2^R = 0.346$. Fig. 6 shows that x^* decreases from x_2 to x_1 when $p_1 - p_2$ increases from θ_1^L to θ_1^R. Fig. 7 shows the values of ω_1 of the PEVs in $[-L, x_1]$ or $[x_2, L]$. More specifically, when $p_1 - p_2 \in [\theta_2^L, \theta_1^L]$, ω_1 of the PEVs in $[x_2, L]$ ranges from 1 to 0. In other words, the PEVs in $[-L, x_2]$ all select station 1, and the PEVs in $(x_2, L]$ select station 1 with probability ω_1 and station 2 with probability $1 - \omega_1$. When $p_1 - p_2 \in [\theta_1^R, \theta_2^R]$, ω_1 of the PEVs in $[-L, x_1]$ ranges from 1 to 0. For all other price ranges, we will have a pure strategy equilibrium as in Fig. 2.(a), Fig. 2 (d), and Fig. 2 (e).

B. Equilibrium of the Charging Station Pricing Game

Now we consider Stage I of our Stackelberg game. We first show that the two conditions in Theorem 4 are usually satisfied in the simulations. For the illustration purpose, we only provide the results in the case of $p_{\min} = 0.4$ and $p_{\max} = 0.8$. Fig. 5 shows the best response curves of charging stations 1 and 2,

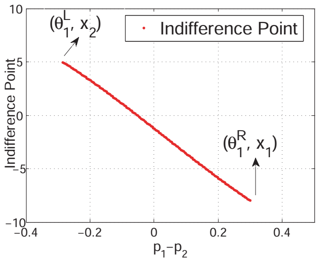


Fig. 6: x^* vs. $p_1 - p_2$.

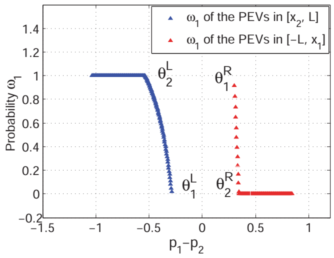


Fig. 7: ω_1 vs. $p_1 - p_2$.

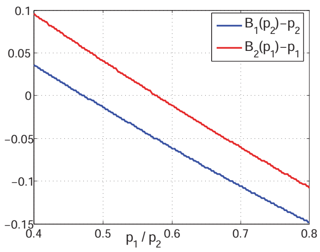


Fig. 8: $\mathcal{B}_j(p_i)) - p_i$.

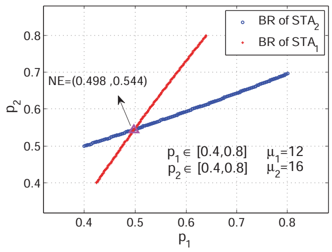


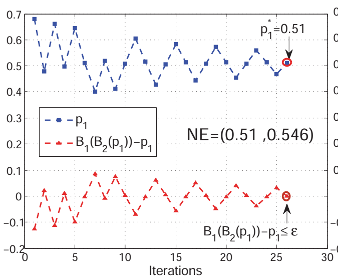
Fig. 9: Another inner pricing equilibrium.

which are monotonically increasing. Fig. 8 shows $\mathcal{B}_1(p_2)) - p_2$ and $\mathcal{B}_2(p_1) - p_1$, which are strictly decreasing.

Next we study the inner pricing equilibrium in more details. Fig. 5 and Fig. 9 illustrate two inner pricing equilibriums under different parameter settings. In Fig. 5, $\mu_2 = 15$ and $x_2 = 5$. In Fig. 9, $\mu_2 = 16$ and $x_2 = 4$. At the NE in both figures, station 2 chooses a higher price than that of station 1. This is because station 2 is closer to the center of the line segment (i.e., $L - x_2 > x_1 + L$) and has a larger service rate (i.e., $\mu_1 < \mu_2$). Hence more PEVs prefer to choose station 2 if the prices are the same from both stations. Station 2 can take advantage of this and announce a higher price to increase its profit at the equilibrium. Furthermore, when μ_2 increases and x_2 decreases, the advantage of station 2 over station 1 is enlarged. Accordingly, the price difference $p_2 - p_1$ increases at the equilibrium in Fig. 9 as compared to that in Fig. 5.

C. The DNSA Algorithm

Finally, we demonstrate the convergence and efficiency of the proposed DSSA. Here we consider the scenarios corresponding to Fig. 5 and Fig. 9. Fig. 10 and Fig. 11 illustrate the convergence of DSSA under two different system parameter settings. We set $\delta(0)$ equal to $0.5(p_{\max} - p_{\min})$ and α equal to 0.9 in both figures. When $p_1 > p_1^*$, $\mathcal{B}_1(\mathcal{B}_2(p_1)) - p_1$ is negative. When $p_1 < p_1^*$, $\mathcal{B}_1(\mathcal{B}_2(p_1)) - p_1$ is positive. In both cases, the system converges to the equilibrium in less than 30 iterations.



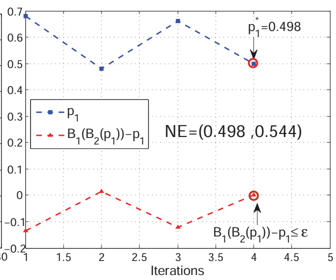


Fig. 10: Iterations of DSSA (1).Fig. 11: Iterations of DSSA (2).

VII. Conclusion and Future Work

This work studies the charging station pricing and PEV station selection through a two-stage Stackelberg game. We prove the uniqueness of the charging station selection equilibrium in Stage II, and propose the sufficient conditions for the uniqueness of the pricing equilibrium in Stage II. We also develop an efficient algorithm to compute the equilibrium of the entire game.

In our future work, we will consider the charging stations with limited service capacity (i.e., $\frac{2L\lambda}{k_i\mu_i} \geq 1$), in which case the waiting time at one charging station may go to infinity if too many users choose this station. This will lead to a new pricing competition game between the stations.

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