An Iterative Double Auction for Mobile Data Offloading

George Iosifidis, Lin Gao, Jianwei Huang, and Leandros Tassiulas

***Abstract*—Mobile data offloading through complementary network technologies such as WiFi and femtocell can significantly alleviate network congestion and enhance user QoS. In this paper we consider a market where mobile network operators (MNOs) lease third-party deployed WiFi or femtocell access points (APs) in order to offload the traffic of their mobile users in a dynamic fashion. We assume that each MNO can employ multiple APs and each AP can concurrently serve traffic from different operators. We design a mechanism that ensures the efficient operation of the market by allowing MNOs to maximize their offloading benefits and APs to minimize their offloading costs. We take into account the special characteristics of the wireless network such as the coupling of each MNO's offloading decisions, and the capacity constraint of each AP. The proposed market scheme does not require prior information about the MNOs and APs, incurs minimum communication overhead, and creates non-negative revenue for the market broker.**

I. Introduction

Today we are witnessing an unprecedented worldwide growth of mobile data traffic that is expected to reach 10.8 exabytes/month in 2016, an 18-fold increase compared to 2011 [1]. These developments pose new challenges to mobile network operators (MNO) who have to significantly enhance their infrastructure accordingly. However, traditional network expansion methods such as acquiring new spectrum licences and upgrading technology (e.g., from WCDMA to LTE/LTE-A) are costly [2] and time-consuming, and more importantly are expected to be outpaced in less than 4 years by the continuing traffic increase [1]. Clearly, operators must find novel methods to address this problem, and *mobile data offloading* appears as one of the most attractive solutions.

Mobile data offloading refers to the technique of routing the data traffic of mobile users of a macrocellular network using alternative means such as WiFi or femtocell networks. Nowadays, there is consensus that data offloading is a cost-effective and energy-prudent method that benefits both the operators and the mobile users (MUs). Therefore, it is not surprising that many MNOs have already deployed their own WiFi access points (APs) to complement their traditional macrocellular network (e.g. AT&T [3]), or initiated collaborations with existing WiFi networks (as O2 did with BT [4]). This approach is facilitated by technological advances such as the HotSpot 2.0 protocol which addresses related security issues [5]. Interestingly, recently operators began offering seamless hybrid macrocellular - WiFi connection services [6].

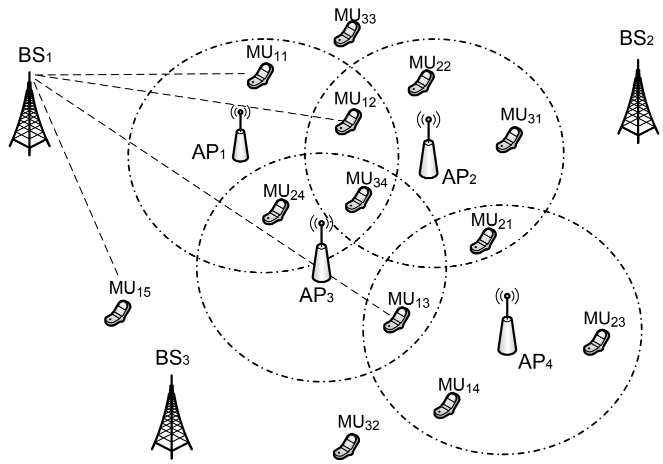

Fig. 1. Each BS is managed by a different MNO and serves a set of MUs. MU_{ij} is the MU j associated with BS i. Each MU is covered by multiple APs and each AP serves more than one MUs.

Nevertheless, in order to fully reap the benefits of offloading, the MNOs need to ensure that their clients will be able to offload data as frequently as possible. However, a ubiquitous access point deployment by the MNOs is very expensive and even impractical in some cases (e.g., due to site acquisition issues). An ideal method to overcome this obstacle and ensure the high availability of APs is the *employment* (leasing) of WiFi and femto APs, which are already installed in homes or other venues (e.g., companies, stores) by third parties. This strategy will allow operators to handle mobile data traffic with reduced capital and operational expenditures (CAPEX and OPEX), and increase their network capacity on-demand by responding to spatiotemporal traffic variations.

However, AP owners are expected to ask for (monetary) compensation, since admitting macrocellular traffic will consume their limited wireless resources and broadband connection capacity for their own (internal) traffic demand. Hence, we need to understand: *how much traffic each AP should admit for each MNO and how much to charge*? or, from the perspective of the MNOs: *how much traffic each MNO should offload to each AP and how much to pay?* In this paper, we design a mechanism that addresses these issues from an economic perspective, by taking into account the particular challenges of the wireless data offloading.

Specifically, we envision an *offloading market* where a set of MNOs (the buyers) compete to lease a set of APs (the sellers)

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for the offloading service. We assume that the marketplace is managed by a centralized *broker* who can be a state-managed clearing house or a private company, similar as those in the secondary spectrum market [7]. WiFi and femtocell AP owners offer their services (i.e., offloading traffic for the MNOs) with certain reimbursements. As shown in Figure 1, we look at the general case where each AP can serve more than one MNOs, and each MNO may lease multiple APs at different locations to offload the traffic of its users (APs are overlapping). The MNOs declare how much they are willing to pay each AP. The broker collects the MNOs' requests and the APs' offers, and determines how much traffic of each operator will be offloaded to each access point and at what price.

The challenge here is to design a market mechanism tailored to the wireless offloading problem which, at the same time, satisfies the desirable economic properties. We consider the realistic scenario of a market with incomplete information, i.e., where the broker is not aware of the actual needs of the MNOs and the APs. Therefore, he must employ an incentive compatible mechanism that induces the buyers (MNOs) and the sellers (APs) to reveal truthfully their needs. With this information, the broker tries to maximize the efficiency of the market by properly matching the buyers and the sellers. At the same time, the broker is not willing to loose money. Hence, the mechanism should be (weakly) budget balanced, i.e., the total payments from the buyers should not exceed the aggregate payments to the sellers.

A suitable scheme for this setting is a double auction mechanism. Unfortunately, double auctions are notoriously hard to design and implement [8]. They can be inefficient and applicable only to certain simplified settings, e.g., for bidders with single-unit demands (McAfee auction [9]), or they can be budget imbalanced with a high computational complexity (VCG auction [10]). In our case, the double auction design problem is further perplexed due to the following realistic issues that we explicitly take into account:

- ($I1$) *The offloading benefit (utility) for each MNO is AP-specific*. For example, an AP that is located at the boundary of an MNO's cell is the most important one, since it can offload the traffic that otherwise would be very costly for the MNO to serve directly due to wireless fading, low spectral efficiency, etc.
- ($I2$) *The offloading decisions of the MNOs are coupled*. The accrued benefit from offloading a given amount of traffic to a certain AP depends on how loaded the MNO already is, which in turn depends on its decisions of offloading traffic to other APs.
- ($I3$) *The APs are heterogeneous*. That is, different APs may have different costs for serving cellular traffic from the same MNO. Besides, the same AP may also have different costs for serving traffic from different MNOs due to different quality of service (QoS) requirements.
- ($I4$) *The offloading decisions of the APs are coupled*. The AP's cost for offloading certain amount of data for one MNO also depends on the total traffic that the AP has committed to offload for other MNOs.

In order to overcome the difficulties in double auction design without compromising the system modeling, we choose an alternative method based on the framework of *Network Utility Maximization* (NUM) [11]. Our starting point is the work of Kelly *et. al.* [12] which introduced a Walrasian auction for link capacity allocation in networks. In that scheme, multiple buyers (the nodes) bid for bandwidth, and a single seller (the network) determines the unit price for each link so as to balance demand and supply. Here, we use an extended version of this one-side approach (i.e. with one seller, the network) [12] for a more general setting with many sellers (the APs) and many buyers (the MNOs). Moreover, the prices in our scheme not only reflect the APs' capacity constraints but also their offloading costs[1].

Specifically, our proposed mechanism is an iterative algorithm that enables the broker to gradually reach the socially efficient solution, without any prior knowledge for the market. The MNOs and APs submit request and offer bids respectively, in each round, responding to the prices announced by the broker. A basic assumption of [12] is that bidders (i.e., the MNOs and APs in our scheme) are *price-takers*. This means that they do not anticipate (or cannot estimate) the impact of their bids on the prices. Price-taking behavior is often observed in markets with many buyers and sellers [13], or when each bidder is not aware of the decisions of other bidders and/or system parameters [14].

In summary, the main contributions of this paper are as follows:

1) We study a market model where multiple operators (M-NOs) compete to lease multiple (possibly overlapping) access points (APs) for data offloading. Each MNO can concurrently lease several APs and each AP can offload traffic for several MNOs at the same time. Hence, our model is very general.
2) We apply an iterative double auction scheme which is efficient (maximizes the social welfare), weakly budget balanced (the broker does not lose money), individually rational (MNOs and APs are willing to participate), and incentive compatible (MNOs and APs reveal their truthful needs/demands) under the assumption of price-taking behavior.
3) The proposed scheme has low computational complexity, induces small communication overheads and clears the market for general MNO utility (benefit) functions and AP cost functions (only concavity is required). The broker does not need to know these functions in advance (*incomplete market information*).
4) The introduced framework considers important realistic issues of the mobile data offloading problem, ($I1$) – ($I4$), which, as we explain in details in Section II, have been overlooked until now by the related works.

The rest of this paper is organized as follows. In Section

[1]For the distributed version of [12], it can be argued that there are multiple bandwidth sellers, i.e. the links. However, the links only balance the traffic, i.e. they do not have cost functions and do not submit bids.

II, we review the literature and emphasize how it differs to our work. In Section III, we provide the system model and formally introduce the problem. In Section IV, we present the mechanism and prove its properties, and in Section V we provide numerical results. We conclude in Section VI.

II. Related Work

The benefits of macrocellular data offloading to WiFi networks have recently been studied [15]–[17]. Approximately 80% of mobile data traffic is generated and consumed indoors [18], and hence can be offloaded to APs. Because of their small spatial coverage areas, the APs transmit with a much higher efficiency than macrocellular base stations, and therefore decrease the data delivery cost [19]. The offloading benefits depend on the availability of APs that are open and have extra capacity to offload cellular traffic. Interestingly though, the problem of incentivizing WiFi open access has received very little attention until today.

Another option for offloading are femtocell access points (FAPs) [20]. This presumes that FAPs operate in the so-called open access mode and admit traffic from non-registered macrocellular users. However, FAP owners are expected to be reluctant to serve other users without proper compensation [21]. This compensation can be either a price discount [22], or a direct payment from the operator. Very few related works in this area study monopolistic markets (i.e., with one operator) [23], or do not consider the challenges, $(I1)-(I4)$, that arise for the MNOs [24], [25].

Markets with many buyers and sellers under incomplete information are usually cleared through double auctions. One option is to use the VCG mechanism which exhibits a very high computational complexity and yields a budget imbalanced outcome [10]. Another prominent scheme is the McAfee mechanism [9] which has recently been proposed for spectrum allocation in secondary spectrum markets [26] or for traffic relaying [27]. However, this mechanism was originally designed for single-unit demands/offers of homogeneous items, and there are very few extensions for multiple or heterogenous items [26], [27]. In all cases, the outcome is inefficient, which is an inherent characteristic of McAfee auction.

In this paper, we adopt a different approach and use a market mechanism based on the NUM framework [11] and particularly on the scheme in [12] for bandwidth allocation in networks. This is a Walrasian auction which maximizes the market welfare under the assumption that bidders are price-takers. The latter is a valid assumption for large markets where the bid of each player has an infinitesimal impact on the prices, or for the case the bidders have limited knowledge about the market (e.g., number of players, system resources) and hence cannot estimate their impacts [13], [14].

At the expense of this assumption, we apply an iterative double auction which satisfies all the desirable economic properties. Our work substantially departs from the algorithm in [12]. Namely, the resource sellers (the APs) in our work, unlike the respective sellers (the links) in [12], try to maximize their own net benefit instead of simply balancing the traffic. This renders the proposed scheme appropriate for the many-to-many interactions setting considered here, and leads to a different behavior compared with [12]. A similar approach was followed in our previous work [28] for bandwidth allocation in peer-to-peer networks where however the objectives and the solution method were significantly different.

III. System Model and Problem Statement

In this section, we introduce the system model, and formulate the data offloading problem as a market design problem where the objective is to maximize the social welfare.

A. System Model

We consider a system with a set $\mathcal{M} \triangleq \{1,2,\ldots,M\}$ of base stations (BS) owned by M different MNOs[2], and a set $\mathcal{I} \triangleq \{1,2,\ldots,I\}$ of APs. Each AP can be a WiFi or a FAP operating in a separate carrier, and hence does not interfere with the macrocellular network. Each BS serves a group of mobile users (MUs) which have heavy traffic to send (or receive) and are randomly distributed within its coverage area. Each MU is also covered by one or more APs. We assume that time is slotted and we study the market for one time period. The MUs' location and traffic types may change over consecutive time periods but are considered fixed within each period (slot).

Consider the case where each BS $m \in \mathcal{M}$ would like to offload $x_{mi} \geq 0$ bytes of data through AP $i \in \mathcal{I}$. We define BS m's *offload request* vector to all I APs as $\boldsymbol{x}_m \triangleq (x_{mi})_{i\in\mathcal{I}}$, and total offloaded data of BS m is $X_m = \sum_{i=1}^{I} x_{mi}$. A BS's request depends on the locations and traffics of its MUs. We use $J_m(\boldsymbol{x}_m)$ to denote BS m's utility when offloading traffic $\boldsymbol{x}_m$ to the APs, which equals to BS's cost reduction comparing with the case that it serves $\boldsymbol{x}_m$ directly[3]. We assume that $J_m(\cdot)$ is a positive, increasing, and jointly strictly concave function of vector $\boldsymbol{x}_m$, satisfying the principle of diminishing marginal returns [29].

Our model captures the following important aspects of the offloading problem. First, the BS's offloading benefit is in general AP-specific. It not only depends on the total offloading traffic (X_m), but also depends on which AP offload how much (the vector $\boldsymbol{x}_m$). For example, an AP located at the boundary of the BS's cell is more important since it can offload traffic from MUs that are costly for the BS to serve directly (e.g., due to poor channel condition between the MU and the BS).

Second, for each BS m, the offloading decisions to the different APs are coupled. Clearly, the benefit from offloading traffic to an AP depends on the load of the BS, which in turn depends on its offloading decisions to other APs. In other words, even if two different strategies, $\boldsymbol{x}_m$ and $\hat{\boldsymbol{x}}_m$, suggest equal amount of offloaded data to a certain AP i, $x_{mi} = \hat{x}_{mi}$, the respective utility improvements may differ:

$$J_m(x_{mi}, \boldsymbol{x}_m^{-i}) - J_m(0, \boldsymbol{x}_m^{-i}) \neq J_m(\hat{x}_{mi}, \hat{\boldsymbol{x}}_m^{-i}) - J_m(0, \hat{\boldsymbol{x}}_m^{-i}),$$

[2]For the rest of the paper we will use "BS" and "MNO" interchangeably.

[3]If an MNO manages more than one BSs, then the respective utility function represents the joint offloading benefit.

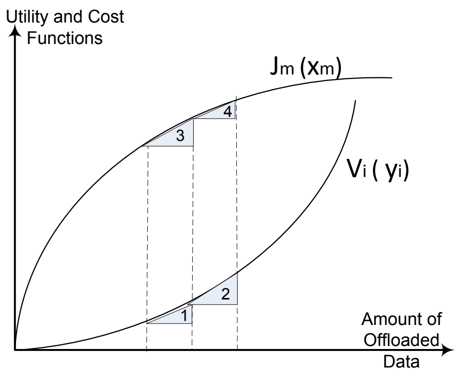

Fig. 2. Concave BS utility function, $J_m(\cdot)$, and convex AP function, $V_i(\cdot)$. The numbered triangles have the same length - corresponding to equal offloaded data increase - but different height (h): $h_2 > h_1$ and $h_4 < h_3$.

where $\boldsymbol{x}_m^{-i} \triangleq (x_{mj})_{j\in\mathcal{I}\setminus\{i\}}$, and $\hat{\boldsymbol{x}}_m^{-i} \triangleq (\hat{x}_{mj})_{j\in\mathcal{I}\setminus\{i\}}$.

Each AP $i \in \mathcal{I}$ responds to offloading requests and admits y_{im} bytes from each BS m in one time period. We define the *admitted traffic* vector $\boldsymbol{y}_i \triangleq (y_{im})_{m\in\mathcal{M}}$. Clearly, a viable market solution exists only if the BSs and APs finally agree on the market outcome, i.e. if $x_{mi} = y_{im}$, $\forall m \in \mathcal{M}, \forall i \in \mathcal{I}$. We use $V_i(\boldsymbol{y}_i)$ to denote the cost incurred by AP i for serving the BSs, which is a positive, increasing and jointly strictly convex function in vector $\boldsymbol{y}_i$. This property captures the fact that as the admitted traffic by each AP increases, its operation cost for admitting one more unit of traffic increases due to the congestion effect and because less of its resources are available for serving its own traffic [29], Figure 2.

The AP's offloading cost depends on its own traffic demand as well as the MUs' traffic characteristics (e.g., average distance from the AP, requested QoS). Similarly, AP's incurred cost for admitting traffic for a certain BS, depends on how loaded the AP already is, i.e. how much traffic offloaded for other BSs. Finally, AP $i \in \mathcal{I}$ has a capacity of C_i bps and hence the maximum amount of data that can be admitted within a certain time period is $C_i \cdot T$ bytes:

$$\sum_{m=1}^{M} y_{im} \leq C_i \cdot T. \tag{1}$$

Without loss of generality, we normalize the slot duration to be $T = 1$. A key difference of our model from previous works is that we use general utility and cost functions that allow us to capture a wide range of wireless offloading systems. For example, if the MUs of a BS are not covered by any AP, then the respective offloading utility component is zero.

B. Problem Statement

Clearly, the objectives of the BSs and APs are conflicting with each other. If they decide independently how much data to offload or to admit, it is very difficult to reach an agreement (i.e., $x_{im} = y_{mi}$, $\forall m \in \mathcal{M}, \forall i \in \mathcal{I}$). Therefore, there is a need for a market controller, a broker, with the task to find the offload request matrix $\boldsymbol{x} \triangleq (\boldsymbol{x}_m)_{m\in\mathcal{M}} = (x_{mi})_{m\in\mathcal{M},i\in\mathcal{I}}$ and the admitted traffic matrix $\boldsymbol{y} \triangleq (\boldsymbol{y}_i)_{i\in\mathcal{I}} = (y_{mi})_{m\in\mathcal{M},i\in\mathcal{I}}$ that ensure the efficient operation of the market. This is achieved when the total benefit for the MNOs is maximized and the aggregate cost for the APs is minimized.

Specifically, the broker can find the optimal $\boldsymbol{x}$ and $\boldsymbol{y}$ by solving the social welfare maximization (SWM) problem:

$$\text{SWM}: \quad \max_{\boldsymbol{x},\boldsymbol{y}} \sum_{m=1}^{M} J_m(\boldsymbol{x}_m) - \sum_{i=1}^{I} V_i(\boldsymbol{y}_i), \tag{2}$$

s.t.

$$\sum_{m=1}^{M} y_{im} \leq C_i, \; i \in \mathcal{I}, \tag{3}$$

$$x_{mi} = y_{im}, \, m \in \mathcal{M}, \; i \in \mathcal{I}, \tag{4}$$

$$x_{mi} \geq 0, \, y_{im} \geq 0, \; m \in \mathcal{M}, \, i \in \mathcal{I}. \tag{5}$$

Notice that, technically, we could remove from SWM the variables $\boldsymbol{x}$ by using (4). However, we keep this formulation in order to facilitate the analysis and make clear the relation of the SWM problem with the respective problems that each BS and AP solves.[4] The objective function of SWM is strictly concave and the constraint set is compact and convex. Hence, SWM admits a unique optimal solution that can be described using the Karush-Kuhn-Tucker (KKT) conditions [31].

Specifically, we relax the constraints and define the Lagrangian:

$$L(\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{x}, \boldsymbol{y}) = \sum_{m=1}^{M} J_m(\boldsymbol{x}_m) - \sum_{i=1}^{I} V_i(\boldsymbol{y}_i) - \sum_{i=1}^{I} \lambda_i \cdot \Big(\sum_{m=1}^{M} y_{im} - C_i \Big) + \sum_{m=1}^{M} \sum_{i=1}^{I} \mu_{mi} \cdot (y_{im} - x_{mi}),$$

where $\boldsymbol{\lambda} \triangleq (\lambda_i \geq 0)_{i\in\mathcal{I}}$ is the vector of Lagrange multipliers corresponding to constraints (3), and $\boldsymbol{\mu} \triangleq (\mu_{mi} \in \mathcal{R})_{m\in\mathcal{M},i\in\mathcal{I}}$ is the matrix of Lagrange multipliers for constraints (4). The KKT conditions that yield the optimal solution, $\boldsymbol{\lambda}^\circ$, $\boldsymbol{\mu}^\circ$, $\boldsymbol{x}^\circ$, $\boldsymbol{y}^\circ$, for the SWM problem are given by the following set of equations: $\forall\, m \in \mathcal{M}$, $i \in \mathcal{I}$,

$$(A1): \quad \frac{\partial J_m(\boldsymbol{x}_m^\circ)}{\partial x_{mi}} = \mu_{mi}^\circ, \; (A2): \frac{\partial V_i(\boldsymbol{y}_i^\circ)}{\partial y_{im}} = \mu_{mi}^\circ - \lambda_i^\circ,$$

$$(A3): \quad \lambda_i^\circ \cdot \Big(\sum_{m=1}^{M} y_{im}^\circ - C_i \Big) = 0, \; (A4): x_{mi}^\circ = y_{im}^\circ,$$

$$(A5): \quad \mu_{mi}^\circ \cdot (y_{im}^\circ - x_{mi}^\circ) = 0, \, (A6): x_{mi}^\circ, y_{im}^\circ, \lambda_i^\circ \geq 0.$$

However, the direct SWM solution from the broker is not possible due to the limited information the broker has for the market. Namely, we assume that the broker is unaware of the utility and cost functions $J_m(\cdot)$, $m \in \mathcal{M}$, and $V_i(\cdot)$, $i \in \mathcal{I}$. Therefore, he has to organize an auction, and specifically a double auction, in order to elicit this hidden information. Such a scheme should ideally be (i) *efficient*: maximize the social welfare, (ii) *individually rational*: bidders do not get worse by participating, (iii) *incentive compatible*: bidders truthfully reveal (directly or indirectly) their private information, (iv) *(weakly) budget balanced*: total payments to the broker are

[4] This formulation is similar to the *consistency pricing* in [30] where different nodes must agree on a common - system wide - solution

nonnegative. Nevertheless, a fundamental result in mechanism design is that there does not exist a double auction that possesses all theses properties [8].

Here, we take a different approach and assume that bidders are *price takers*. This assumption, which corresponds to a perfect competition market, is reasonable for bidders with bounded computational capabilities and/or limited information, or for large markets with many participants where each one has infinitesimal impact on the market prices.

IV. The IDA Mechanism

In this section, we present an Iterative Double Auction (IDA) mechanism for price taking bidders. We prove that it solves the mobile data offloading problem, taking into account $(I1)-(I4)$, and satisfies the desirable economic properties.

A. IDA Resource Allocation and Pricing Rules

The basic idea of this mechanism is that the broker solves a different optimization problem to determine $\boldsymbol{x}$ and $\boldsymbol{y}$ which, if it is combined with proper pricing (for the BSs) and reimbursement (for the APs) rules, ensures the optimal solution of SWP. This scheme corresponds to a double auction where many buyers (BSs) and many sellers (APs) interact in an iterative fashion until the market reaches an efficient, i.e. market clearing, point.

The mechanism consists of two stages during each iteration. In the first stage, each BS $m \in \mathcal{M}$ submits a bid $p_{mi} \geq 0$, $i \in \mathcal{I}$, for each AP, and each AP $i \in \mathcal{I}$ submits a bid $\alpha_{im} \geq 0$ for every BS $m \in \mathcal{M}$. These bids signal the offloading needs and serving costs for the BSs and the APs respectively, and are used as input in the allocation rule. Later on we will explain the precise relationship between these bids and the actual BSs' payments and APs' reimbursements. In the second stage, the broker determines the allocation (how much traffic each AP will admit from each BS) based on the bids from two sides by solving the broker allocation problem (BAP):

$$\text{BAP}: \quad \max_{\boldsymbol{x},\boldsymbol{y}} \sum_{m=1}^{M}\sum_{i=1}^{I}\Big(p_{mi}\log x_{mi} - \frac{\alpha_{im}}{2}y_{im}^2\Big), \tag{6}$$

s.t.

$$\sum_{m=1}^{M} y_{im} \leq C_i,\ i \in \mathcal{I}, \tag{7}$$

$$x_{mi} = y_{im},\, m \in \mathcal{M},\ i \in \mathcal{I}, \tag{8}$$

$$x_{mi} \geq 0,\, y_{im} \geq 0,\ m \in \mathcal{M},\, i \in \mathcal{I}. \tag{9}$$

Notice that the objective function is motivated by the allocation rule of [12], with the additional convex component capturing the (convex) increasing cost functions of the APs. We also define the bid vectors $\boldsymbol{p}_m \triangleq (p_{mi})_{i\in\mathcal{I}}$ and $\boldsymbol{\alpha}_i \triangleq (\alpha_{im})_{m\in\mathcal{M}}$ for every BS $m \in \mathcal{M}$ and AP $i \in \mathcal{I}$ respectively.

The BAP problem has the same constraint set as the SWM problem, and a different yet strictly concave objective function. Hence, it admits a unique optimal solution. We define the corresponding Lagrange function as

$$L(\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{x},\boldsymbol{y}) = \sum_{m=1}^{M}\sum_{i=1}^{I}\Big(p_{mi}\log x_{mi} - \frac{\alpha_{im}}{2}y_{im}^2\Big) - \sum_{i=1}^{I}\lambda_i\cdot\Big(\sum_{m=1}^{M}y_{im} - C_i\Big) + \sum_{m=1}^{M}\sum_{i=1}^{I}\mu_{mi}\cdot(y_{im}-x_{mi}),$$

and denote the optimal solution of the BAP problem as $\boldsymbol{x}^*$, $\boldsymbol{y}^*$, $\boldsymbol{\lambda}^*$, and $\boldsymbol{\mu}^*$. The corresponding KKT conditions yield a set of equations $(B1)-(B6)$, where $(B3)-(B6)$ are identical to $(A3)-(A6)$ of SWM but the first two sets differ:

$$(B1): x_{mi}^* = \frac{p_{mi}}{\mu_{mi}^*},\quad (B2): y_{im}^* = \frac{\mu_{mi}^* - \lambda_i^*}{\alpha_{im}}, \tag{10}$$

for every $m \in \mathcal{M}$, $i \in \mathcal{I}$. It is important to note that (10) defines the **allocation rule** of our proposed mechanism.

Comparing equations $(A1)-(A6)$ and $(B1)-(B6)$, we observe that if the BSs and the APs submit the following bids:

$$p_{mi} = x_{mi}^*\cdot\frac{\partial J_m(\boldsymbol{x}_m^*)}{\partial x_{mi}},\quad \alpha_{im} = \frac{1}{y_{im}^*}\cdot\frac{\partial V_i(\boldsymbol{y}_i^*)}{\partial y_{im}}, \tag{11}$$

then the previously described two-stage scheme yields a solution identical to the one of problem SWM in a single iteration, i.e., $\boldsymbol{x}^\circ \triangleq \boldsymbol{x}^*$ and $\boldsymbol{y}^\circ \triangleq \boldsymbol{y}^*$. Clearly, the task of the market designer here is to *derive the proper payment and reimbursement rules that will induce the players, i.e., the BSs and APs, to bid according to (11).*

We now look at the bidders' behavior in the first stage. Let $h_m(\boldsymbol{x}_m)$ denote the BS m's payment to the broker for the service it receives from the APs (vector $\boldsymbol{x}_m$). Similarly, let $l_i(\boldsymbol{y}_i)$ denote the AP i's reimbursement from the broker for the data it offloads (vector $\boldsymbol{y}_i$). Clearly, the payments and reimbursements depend on the respective bids through the auction allocation rule. The bidders are rational, price-taking entities and optimize their bids by maximizing their utility and cost functions.

Specifically, given the allocation rule defined in $(B1)$ and $(B2)$, BSs and APs solve their own optimization problems in order to find their optimal bids. Each BS $m \in \mathcal{M}$ finds the optimal bid vector $\boldsymbol{p}_m^*$ by solving the following problem:[5]

$$\text{BSP}: \quad \max_{\boldsymbol{p}_m\succeq\boldsymbol{0}} \big(J_m(\boldsymbol{x}_m) - h_m(\boldsymbol{x}_m)\big), \tag{12}$$

which yields the following optimality conditions:

$$\frac{\partial J_m(\boldsymbol{x}_m)}{\partial x_{mi}} = \mu_{mi}\frac{\partial h_m(\boldsymbol{x}_m)}{\partial p_{mi}},\quad \forall i \in \mathcal{I}. \tag{13}$$

Similarly, each AP $i \in \mathcal{I}$ finds the optimal bid vector $\boldsymbol{\alpha}_i^*$ by solving the following problem:

$$\text{APP}: \quad \max_{\boldsymbol{\alpha}_i\succeq\boldsymbol{0}} \big(-V_i(\boldsymbol{y}_i) + l_i(\boldsymbol{y}_i)\big), \tag{14}$$

which yields the following optimality conditions:

$$\frac{\partial V_i(\boldsymbol{y}_i)}{\partial y_{im}} = \frac{\alpha_{im}^2}{\lambda_i - \mu_{mi}}\frac{\partial l_i(\boldsymbol{y}_i)}{\partial \alpha_{im}},\quad \forall m \in \mathcal{M}. \tag{15}$$

Comparing the best responses of BSs, (13), and APs, (15), with those required by the socially optimal solution, i.e. (11),

[5]Here $\boldsymbol{p}_m \succeq \boldsymbol{0}$ means $\boldsymbol{p}_m$ is a non-negative vector, i.e., $p_{mi} \geq 0, \forall i \in \mathcal{I}$.

we can immediately obtain the **pricing rules** that will lead to the solution of SWM (i.e., the socially optimal solution):

$$h_m(\boldsymbol{p}_m) = \sum_{i=1}^{I} p_{mi}, \quad l_i(\boldsymbol{\alpha}_i) = \sum_{m=1}^{M} \frac{(\lambda_i - \mu_{mi})^2}{a_{im}}, \qquad (16)$$

where we have written the payment $h_m(\boldsymbol{p}_m)$ by each BS m and the compensation $l_i(\boldsymbol{\alpha}_i)$ to each AP i, as functions of the respective bids. These rules are intuitive: *each BS pays exactly its bid*, i.e., the amount it declared that is willing to pay. On the other hand, the reimbursement can be written as $l_i(\boldsymbol{\alpha}_i) = \sum_{m=1}^{M} y_{im}(\lambda_i - \mu_{mi})$: *each AP is reimbursed proportionally to the amount of data that it offloaded* weighted by a factor indicating the congestion on its link (i.e., λ_i) and the difference between the requested and admitted traffic (i.e., μ_{mi}, which, by $(B1)$ and $(B2)$, is equal to $\sqrt{p_{mi} \cdot \alpha_{im}}$ whenever there is no link congestion).

B. IDA Algorithm

With the proper allocation rule (10) and payment rule (16), the BSs and APs can compute the optimal bids in one round and achieve an efficient market equilibrium, if they know the complete network information (mainly including the BSs' utility functions and APs' cost functions). However, as the BSs and APs do not have this information, there is a need for an iterative algorithm that will gradually adjust the market operation point to reach the desirable one.

The proposed scheme consists of the following consecutive steps in each iteration. The broker solves the BAP problem to determine the prices, and the BSs and APs solve their own problems to determine their bids. Notice that the BAP problem is parameterized by the bids of the buyers and sellers. Solving BAP yields the matrices $\boldsymbol{x}$ and $\boldsymbol{y}$ and the Lagrange multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, which further determine the prices $h_m(\cdot)$ and $l_i(\cdot)$. Similarly, APP and BSP problems are parameterized by the Lagrange multipliers of the BAP problem. We exploit the decomposable structure of BAP, and solve it by employing a primal-dual Lagrange decomposition method [31]. This enables its parallelization and hence its faster execution.

The scheme is described in details in Algorithm 1. First, the broker initializes the primal variables (i.e., $\boldsymbol{x}$ and $\boldsymbol{y}$) and dual variables (i.e., $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$) so as to satisfy the complementary slackness constraints ($A3$ and $A5$), and announces the latter to the BSs and APs (lines $1-3$). Accordingly, the BSs and APs calculate their optimal bids by solving their respective optimization problems (lines $7-8$). The broker collects the new bids and checks the termination condition (line 9). Termination happens when the updated bids are equal to the bids that were submitted in the previous round[6]. Accordingly, the broker finds the new values for $\boldsymbol{x}$ and $\boldsymbol{y}$ using the allocation rule of problem BAP (10) (line 10), and calculates the payments. Next, the new values of the primal variables are used for the update, through a gradient descent method, of the dual variables $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, (line 12). The new dual variables are announced to the bidders which will use them to update their bids in the next step, in case the algorithm has not converged.

[6]In order to facilitate the numerical analysis, instead of strict equality we check whether their difference is smaller than $\epsilon > 0$ which is a very small number.

Algorithm 1: Iterative Double Auction (IDA)

output: $\boldsymbol{x}^*$, $\boldsymbol{y}^*$, $\boldsymbol{\lambda}^*$, $\boldsymbol{\mu}^*$

1 $t \leftarrow 0$;
2 Initialize $x_{mi}^{(0)}$, $y_{im}^{(0)}$, $\mu_{mi}^{(0)}$, $\lambda_i^{(0)}$, $\forall m \in \mathcal{M}, i \in \mathcal{I}$;
3 Announce $\mu_{mi}^{(0)}$ and $\lambda_i^{(0)}$, $\forall m \in \mathcal{M}, i \in \mathcal{I}$;
4 conv_flag $\leftarrow 0$;
5 **while** conv_flag $= 0$ **do**
6 $t \leftarrow t + 1$;
7 Each BS m computes the optimal bids $\boldsymbol{p}_m^{(t)}$ by (12);
8 Each AP i computes the optimal bids $\boldsymbol{\alpha}_i^{(t)}$ by (14);
9 The broker collects all bids and checks termination:
10 **if** $|p_{mi}^{(t)} - p_{mi}^{(t-1)}| < \epsilon$ and $|\alpha_{im}^{(t)} - \alpha_{im}^{(t-1)}| < \epsilon$, $\forall m \in \mathcal{M}, i \in \mathcal{I}$ **then**
conv_flag $\leftarrow 1$;
end
11 The broker computes the new $\boldsymbol{x}^{(t)}$, $\boldsymbol{y}^{(t)}$ by (10);
12 The broker computes $h_m(\boldsymbol{x}_m^{(t)})$ and $l_i(\boldsymbol{y}_i^{(t)})$, $\forall m, i$;
13 The broker uses gradient update for dual vars:
$$\lambda_i^{(t+1)} = \big(\lambda_i^{(t)} + s_t \cdot (\textstyle\sum_{m=1}^{M} y_{im}^{(t)} - C_i)\big)^+, \forall i \in \mathcal{I},$$
$$\mu_{mi}^{(t+1)} = \mu_{mi}^{(t)} - s_t \cdot (y_{im}^{(t)} - x_{mi}^{(t)}), \forall m \in \mathcal{M}, i \in \mathcal{I},$$
where s_t is a properly selected step size [31];
14 The broker announces $\boldsymbol{\mu}^{(t+1)}$ and $\boldsymbol{\lambda}^{(t+1)}$;
end

C. Convergence Analysis of IDA Mechanism

Algorithm 1 converges to the optimal solution of problem SWM under some mild conditions. We assume that the time slot of the update is very small (or equivalently, the step size is very small), hence we approximate the algorithm with its continues-time counter-part. Specifically, we consider that the Lagrange multipliers are updated according to the following differential equations (based on the gradient updates):

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}t} = \Big(\sum_{m=1}^{M} y_{im} - C_i\Big)^+, \quad \frac{\mathrm{d}\mu_{mi}}{\mathrm{d}t} = x_{mi} - y_{im},$$

where $(\cdot)^+$ is the projection onto the nonnegative orthant.

We prove the convergence of our two-sided algorithm following the rationale of the proof in [32, Ch. 22], which was used to prove the one-sided version of our algorithm in [12]. Specifically, we define the following Lyapunov function:

$$Z(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_{i=1}^{I} \frac{(\lambda_i - \lambda_i^*)^2}{2} + \sum_{m=1}^{M} \sum_{i=1}^{I} \frac{(\mu_{mi} - \mu_{mi}^*)^2}{2}. \quad (17)$$

It suffices to prove the convergence, as we can show

$$\frac{\mathrm{d}Z(\boldsymbol{\lambda}, \boldsymbol{\mu})}{\mathrm{d}t} \leq 0.$$

Specifically, by applying the chain rule, we have:

$$\frac{\mathrm{d}Z(\boldsymbol{\lambda},\boldsymbol{\mu})}{\mathrm{d}t} = \sum_{i=1}^{I}(\lambda_i-\lambda_i^*)\frac{\mathrm{d}\lambda_i}{\mathrm{d}t} + \sum_{m=1}^{M}\sum_{i=1}^{I}(\mu_{mi}-\mu_{mi}^*)\frac{\mathrm{d}\mu_{mi}}{\mathrm{d}t},$$

which can be written as:

$$\frac{\mathrm{d}Z(\boldsymbol{\lambda},\boldsymbol{\mu})}{\mathrm{d}t} = \sum_{i=1}^{I}(\lambda_i-\lambda_i^*)\cdot\Big(\sum_{m=1}^{M} y_{im}-C_i\Big)^{+} + \sum_{m=1}^{M}\sum_{i=1}^{I}(\mu_{mi}-\mu_{mi}^*)\cdot(x_{mi}-y_{im}),$$

or

$$\frac{\mathrm{d}Z(\boldsymbol{\lambda},\boldsymbol{\mu})}{\mathrm{d}t} \leq \sum_{i=1}^{I}(\lambda_i-\lambda_i^*)\cdot\Big(\sum_{m=1}^{M} y_{im}-C_i\Big) + \sum_{m=1}^{M}\sum_{i=1}^{I}(\mu_{mi}-\mu_{mi}^*)\cdot(x_{mi}-y_{im}).$$

After some simple algebraic manipulations, we get:

$$\begin{aligned}\frac{\mathrm{d}Z(\boldsymbol{\lambda},\boldsymbol{\mu})}{\mathrm{d}t} \leq & \sum_{i=1}^{I}(\lambda_i-\lambda_i^*)\Big(\sum_{m=1}^{M} y_{im}-\sum_{m=1}^{M} y_{im}^*\Big) \\ &+ \sum_{m=1}^{M}\sum_{i=1}^{I}(\mu_{mi}-\mu_{mi}^*)(x_{mi}-y_{im}-x_{mi}^*+y_{im}^*) \\ &+ \sum_{i=1}^{I}(\lambda_i-\lambda_i^*)\Big(\sum_{m=1}^{M} y_{im}^*-C_i\Big) \\ &+ \sum_{m=1}^{M}\sum_{i=1}^{I}(\mu_{mi}-\mu_{mi}^*)(x_{mi}^*-y_{im}^*).\end{aligned}$$

Using the complementary slackness and $(A1)$, $(A2)$, we have:

$$\begin{aligned}\frac{\mathrm{d}Z(\boldsymbol{\lambda},\boldsymbol{\mu})}{\mathrm{d}t} \leq \sum_{m=1}^{M}\sum_{i=1}^{I}\Big[&(y_{im}-y_{im}^*)\Big(\frac{\partial V_i(\boldsymbol{y}_i^*)}{\partial y_{im}}-\frac{\partial V_i(\boldsymbol{y}_i)}{\partial y_{im}}\Big) \\ &+(x_{mi}-x_{mi}^*)\Big(\frac{\partial J_m(\boldsymbol{x}_m)}{\partial x_{mi}}-\frac{\partial J_m(\boldsymbol{x}_m^*)}{\partial x_{mi}}\Big)\Big].\end{aligned}$$

The above inequality is satisfied due to the following property that holds for each concave function $f(\cdot)$ [31]:

$$f(y) \leq f(x) + \nabla f(x)^T(y-x). \tag{18}$$

D. Properties of IDA Mechanism

From the analysis above, it is clear that the IDA algorithm induces the BSs and the APs to bid truthfully, (12) and (14), and according to the socially optimal bids, (11). In the sequel, we prove that the algorithm is also weakly budget balanced and individually rational. Specifically, the following lemma holds for the IDA algorithm.

Lemma 1. *The IDA mechanism is weakly budget balanced.*

Proof: The budget balance $\Lambda(\cdot)$ is defined as:

$$\Lambda(\boldsymbol{p},\boldsymbol{\alpha}) = \sum_{m=1}^{M} h_m(\boldsymbol{p}_m) - \sum_{i=1}^{I} l_i(\boldsymbol{\alpha}_i),$$

or, if we use (16), we get

$$\Lambda(\boldsymbol{p},\boldsymbol{\alpha}) = \sum_{m=1}^{M}\sum_{i=1}^{I} p_{mi} - \sum_{m=1}^{M}\sum_{i=1}^{I}\frac{(\lambda_i-\mu_{mi})^2}{a_{im}}.$$

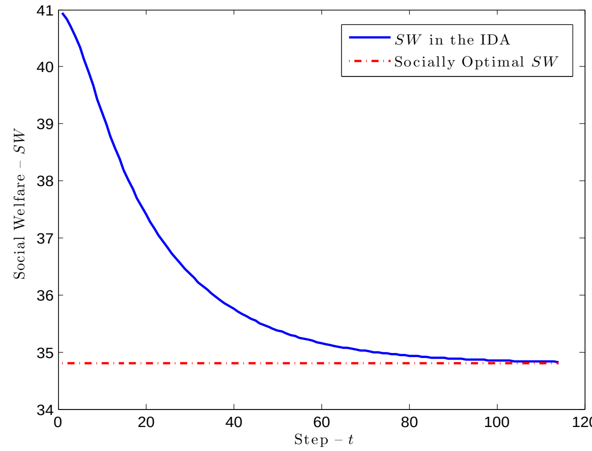

Fig. 3. Evolution of social welfare produced by the IDA.

Notice that (10) is satisfied at the equilibrium. Hence,

$$\Lambda(\boldsymbol{p},\boldsymbol{\alpha}) = \sum_{m=1}^{M}\sum_{i=1}^{I} x_{mi}^*\cdot\lambda_i^* \geq 0.$$

■

Additionally, the IDA mechanism ensures the voluntarily participation of the bidders since they are guaranteed to have at least zero net utility for all the possible market outcomes.

Lemma 2. *The IDA mechanism is individually rational (IR).*

Proof: For each BS $m \in \mathcal{M}$, the IR condition can be translated to the following constraint:

$$J_m(\boldsymbol{x}_m^*) - \sum_{i=1}^{I} p_{mi} \geq 0, \text{ or } J_m(\boldsymbol{x}_m^*) - \sum_{i=1}^{I} x_{mi}^*\mu_{mi}^* \geq 0,$$

which can be written, using (13), as

$$J_m(\boldsymbol{x}_m^*) \geq \sum_{i=1}^{I} x_{mi}^*\cdot\frac{\partial J_m(\boldsymbol{x}_m^*)}{\partial x_{mi}}. \tag{19}$$

Since $J_m(\cdot)$ is strictly concave and $J_m(\mathbf{0})=0$, the inequality (19) is always satisfied according to the condition (18).

Similarly, for each AP $i \in \mathcal{I}$, the IR condition

$$-V_i(\boldsymbol{y}_i^*) + l_i(\boldsymbol{\alpha}_i) \geq 0 \tag{20}$$

is satisfied according to the property (18). ■

V. Simulations

Now we provide numerical results to validate our theoretical analysis. To facilitate the presentation of the results, we consider a small market with $M=5$ BSs and $I=5$ APs. These results can be directly extended to a larger number of BSs and APs. The BSs' utility functions are defined as

$$J_m(\boldsymbol{x}_m) = 10\cdot\sum_{i=1}^{5}\log(\theta_{mi}x_{mi}),\ m=1,\ldots,5,$$

where $\theta_{mi} \geq 0$ represents the offloading efficiency of AP i for BS m (i.e., the offloading benefit is AP-specific). If BS m has no MUs within the coverage area of AP i, then $\theta_{mi}=0$.

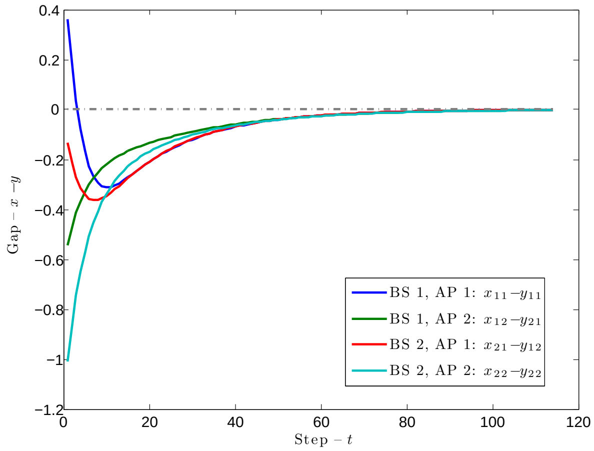

Fig. 4. Evolution of the gap between the requested data traffic by BSs (x) and the admitted by APs (y).

TABLE I
ADMITTED DATA y_{im} AND PARAM. ρ_{im} FOR EACH AP-BS PAIR

(ρ_{im}, y_{im})	AP 1	AP 2	AP 3	AP 4	AP 5
BS 1	.74, 3.3	.61, 3.5	.95, 2.5	.86, 2.8	.99, 2.7
BS 2	.82, 2.8	.56, 3.1	.60, 3.4	.93, 2.6	.97, 2.7
BS 3	.94, 2.7	.79, 2.7	.71, 3.2	.76, 3.1	.94, 2.5
BS 4	.93, 2.4	.61, 3.2	.64, 2.9	.76, 3.0	.80, 3.0
BS 5	.61, 3.5	.92, 2.2	.72, 2.8	.66, 3.3	.67, 3.8

The APs' cost function are defined as

$$V_i(\boldsymbol{y}_i) = 0.1 \cdot \sum_{m=1}^{5} e^{\rho_{im} y_{im}}, \; i = 1, \ldots, 5$$

where $\rho_{im} \geq 0$ captures the fact that each AP may incur different cost by serving a different BS (i.e. APs are heterogeneous). The capacity of each AP is set to $C = 15$ Mbps. The time interval length is normalized to $T = 1$. Parameters ρ_{im} and θ_{mi} are chosen randomly and independently with a uniform probability from interval $[0.5, 1]$.

In Fig. 3, we plot the social welfare achieved by the algorithm in each iteration. We observe that the social welfare gradually converges to the optimal one, which is the solution of SWM (dotted line). In Fig. 4, we present the convergence of $\boldsymbol{x}$ and $\boldsymbol{y}$. Specifically, we plot the gaps between $\boldsymbol{x}$ and $\boldsymbol{y}$ for 4 BS-AP pairs (BS1-AP1, BS1-AP2, BS2-AP1 and BS2-AP2), and observe that the gaps gradually converge to zero. These two figures imply that our IDA algorithm elicits the true hidden information (i.e., the utility and cost functions), and converges to the socially optimal solution. Finally, in Table I, we provide the offloaded data amounts and the respective values of ρ_{im}. We see that the highest the value of ρ_{im} is, the more costly the offloading is for the APs and hence the less traffic it admits.

VI. Discussion and Conclusions

In this paper, we considered a market where MNOs lease third-party owned WiFi or femtocell APs to offload their mobile data traffic. This is a promising solution for increasing the user perceived network capacity in a dynamic and scalable fashion, with low CAPEX and OPEX costs. Today, the technologies to implement such solutions are already in place (e.g., secure offloading methods). Data offloading can alleviate congestion of 2G/3G cellular networks, and also serve as a low-cost auxiliary technology for the emerging 4G networks.

We proposed an iterative double auction mechanism, which satisfies the desirable economic properties, and maximizes the welfare of the market, under the assumption of price-taking bidders. There are very interesting directions for future work. First, one can study what is the impact of strategic, price-anticipating behavior in the market outcome. Similarly, it is challenging to study how colluding behaviors will affect the algorithm. Also, it is important to consider practical implementation issues such as how to hardwire this algorithm to APs and BSs so as to communicate with the broker and execute IDA algorithm in real-time fashion.

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