Dynamic Channel Selection in Cognitive Radio Network with Channel Heterogeneity

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***Abstract*—We consider the channel selection problem in a cognitive radio network with heterogenous channel availabilities at different nodes. We formulate the maximum channel selection (MCS) problem as a binary integer nonlinear optimization problem, with an objective of maximizing the total channel utilization for all secondary nodes. We first prove that MCS problem is NP-complete. Then we design a centralized greedy channel selection (GCS) algorithm. The GCS algorithm is polynomial in computational complexity, and achieves a close-to-optimal (higher than 95%) numerical performance. We further propose a distributed priority order channel selection algorithm, which has significantly less signaling overhead compared with the GCS algorithm. We study the performance of the distributed algorithm both theoretically and numerically.**

I. Introduction

The wireless spectrum is traditionally allocated through static licensing, which leads to severe spectrum under-utilization. Recent field measurements indicate that the utilization of various assigned spectrum bands varies from 15% to 85% [1]. Cognitive radio technology has been recently proposed as a promising solution to address the conflict between the spectrum under-utilization and the scarcity of the total available spectrum resource.

In a cognitive radio (CR) network, an unlicensed secondary user senses a wide range of frequency band and selects an idle channel to transmit. The channel selection problem is challenging as the availabilities of the channels dynamically change over time and locations [2]. Also, very often multiple secondary users compete to transmit in the same idle channel simultaneously and thus cause collision. The focus of this paper is to design channel selection algorithms that minimize collisions among secondary users and achieve a good channel utilization.

Channel selection and media access control (MAC) protocol design in CR networks have received considerable attention recently. Wang *et al*. in [3] proposed two MAC mechanisms to support voice service in CR networks considering a single channel. Xiao *et al*. in [4] proposed two opportunistic channel selection schemes with a single secondary node. Compared with [3], [4], our paper considers channel selection of multiple secondary source destination pairs over multiple channels. Song *et al*. in [5] proposed a stochastic selection scheme, where each secondary source-destination pair (user) adaptively adjusts the selection probability based on previous number of successfully transmissions. Compared with [5], we will consider a more challenging case where the source and destination of the same secondary user may have different channel availabilities, and thus need to agree on which channel to use for the data transmission.

Heterogenous channel availability among secondary nodes was considered in [6], where cooperative relay was introduced to assist transmissions and meet the heterogeneous traffic demands of different nodes. Huang *et al*. in [7] proposed two spectrum access schemes to achieve a high throughput for the secondary users while considering the channel heterogeneity. An auction based channel allocation was proposed in [8], where secondary users bid channel access opportunity at each time slot based on their channel condition, traffic, and payoff function to maximize throughput. The above papers focused on the communications between the secondary access point and secondary users, instead of the secondary source and destination nodes here.

In our paper, we address the channel selection problem in a multi-channel cognitive radio network. We want to maximize the total channel utilization by considering the heterogeneity of channel availability among secondary nodes (sources and destinations). We formulate this as a maximum channel selection (MCS) problem and propose both centralized and distributed algorithms to solve it. Our main contributions include:

- *NP-complete proof*: We show that the MCS problem is a binary integer nonlinear optimization problem and prove that it is NP-complete.
- *Design of centralized algorithm*: We design a centralized greedy channel selection (GCS) algorithm with polynomial computational complexity. Numerical results show that the GCS algorithm achieves a close-to-optimal (higher than 95%) solution. Such an algorithm serves as a benchmark for distributed algorithms.
- *Design of distributed algorithm*: We propose a distributed low-complexity priority order channel selection algorithm that achieves a high channel utilization.
- *Theoretical analysis and simulation validation*: An analytical model is developed to study the performance of the distributed algorithm. We also validate the theoretical analysis through extensive simulations.

The remainder of the paper is organized as follows. We present network model and problem formulation in Section II. We then propose the centralized and distributed algorithms in

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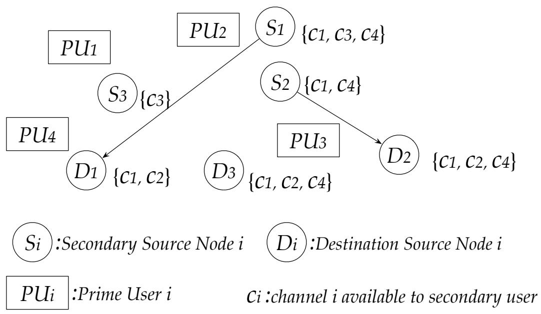

Fig. 1. A cognitive radio network with heterogeneous channel availabilities at different secondary nodes.

Sections III and IV, respectively. The numerical results are given in Section V. Finally, we conclude in Section VI.

II. Network Model and Problem Formulation

A. Network Model

We consider a cognitive radio network with a set $\mathcal{C} = \{c_1, c_2, \cdots, c_L\}$ of L primary channels. There are $2N$ secondary wireless nodes forming a set $\mathcal{N} = \{1, \cdots, N\}$ of distinct secondary source-destination (S-D) pairs. The sets of source and destination nodes are denoted as $\mathcal{N}_s = \{1, \cdots, N\}$ and $\mathcal{N}_d = \{1, \cdots, N\}$, respectively. An example is shown in Fig. 1, where $L = 4$ and $N = 3$.

Each secondary source (destination) node senses the idle channels around it and decides which channel to transmit (to listen). The availability of the channels depends on the activities of the primary users (over the primary channels) around the secondary node, and thus is location dependent. Let $a^l_{s,i}$ and $a^l_{d,i}$ denote the availability of channel c_l at the source and destination node of pair i, respectively. Then $a^l_{s,i} = 1$ ($a^l_{d,i} = 1$) if channel c_l is available at the source (destination) node of pair i, and zero otherwise. Such heterogenous channel availability makes the channel selection problem challenging.

B. Channel Selection Process

We consider a synchronous time-slotted system. Each time slot consists of three periods: sensing, access, and data transmission. During the sensing period, each secondary node senses the spectrum and determines the list of channels available at its location. The sensing is passive and thus does not generate interference or collision among nodes. Each secondary node will then select one of the available channels as its working channel (i.e., transmission channel for a source or listening channel for a destination). *The focus of this paper is to optimize the channel selection at this stage*. At the beginning of access period, each secondary node tunes its radio to the selected channel. For a destination node, it listens over the selected channel. For a source node, the access period is divided into K mini-slots. At the beginning of access period, it starts to count down according to a randomly and uniformly chosen integer between 1 and K, where K is a large integer known to all nodes. The source node monitors the channel during this count down process. If the channel remains idle till its countdown reaches zero, it will send a Request-To-Send (RTS) message over this channel. If the intended destination happens to listen to the same channel, it will acknowledge the RTS message with a Clear-To-Send (CTS) message over this channel. This S-D pair then claims ownership of the channel and starts data transmission. Other source nodes who monitor this channel will not transmit in the current time slot.

To summarize, an S-D pair needs to satisfy two conditions in order to have a successful data transmission over a channel in a particular time slot: (i) the source has the unique minimum initial counting down value among all source nodes choosing this channel, so that it can send out the RTS without colliding with other nodes; (ii) the destination needs to listen to the same channel during this time slot.

C. Channel Utilization

Now we calculate the channel utilization as a function of the channel selection decisions of all nodes in a particular time slot. Let $\mathcal{M}^l_s \in \mathcal{N}_s$ and $\mathcal{M}^l_d \in \mathcal{N}_d$ be the sets of source and destination nodes selecting channel c_l during the sensing period, respectively. Let $\mathcal{M}^l$ be the set of S-D pairs selecting their common channel c_l, *i.e.*, $\mathcal{M}^l = \mathcal{M}^l_s \bigcap \mathcal{M}^l_d$.

Denote $X_1, X_2, \cdots, X_{|M^l_s|}$ as the initial countdown values of the source nodes. They are independently and uniformly chosen from the set $\{1, \cdots, K\}$. Denote $W = \min\{X_1, X_2, \cdots, X_{|M^l_s|}\}$ as the smallest initial count down value. We define U^l as the utilization of channel c_l, which equals to the probability of having a successful data transmission over channel c_l, *i.e.*,

$$\begin{aligned} U^l &= \sum_{i=1}^{|M^l|} Pr(W = X_i) \sum_{x=1}^{K} \left(Pr(X_i = x) Pr\left(\bigcap_{j=1; j\neq i}^{|M^l_s|} \{X_j > x\} \right) \right) \\ &= \frac{|M^l|}{K|M^l_s|} \sum_{x=1}^{K} \left(\frac{K-x}{K} \right)^{|M^l_s|-1}. \end{aligned} \quad (1)$$

In this paper, we will focus on the asymptotic case where K goes to ∞, such that $\lim_{K\to\infty} \sum_{x=1}^{K} \left(\frac{1}{K} \left(\frac{K-x}{K} \right)^{|M^l_s|-1} \right) = 1$ and $U^l = \frac{|M^l|}{|M^l_s|}$. The total utilization of all channels is

$$U = \sum_{l=1}^{L} U^l = \sum_{l=1}^{L} \frac{|M^l|}{|M^l_s|}, \quad (2)$$

where L is the total number of primary channels. We want to design a channel selection algorithm such that U is maximized.

D. Problem Formulation

Let $x^l_{s,i}$ and $x^l_{d,i}$ be the binary decision variables for the sources' and destinations' channel selection, *i.e.*,

$$x^l_{s,i} = \begin{cases} 1 & \text{if the source } i \text{ selects channel } c_l; \\ 0 & \text{otherwise.} \end{cases}$$

$$x^l_{d,i} = \begin{cases} 1 & \text{if the destination } i \text{ selects channel } c_l; \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the channel utilization of channel c_l is

$$U^l = \frac{|\mathcal{M}^l|}{|\mathcal{M}^l_s|} = \left(\sum_{i=1}^{N} x^l_{s,i} x^l_{d,i} \right) \Big/ \left(\sum_{i=1}^{N} x^l_{s,i} \right) \quad (3)$$

where $|\mathcal{M}_s^l|$ and $|\mathcal{M}^l|$ are the total number of sources and S-D pairs selecting channel c_l, respectively.

With (3), we can formulate the following maximum channel selection (MCS) problem:

$$\begin{array}{lll}
\max & \sum_{l=1}^{L} \frac{\sum_{i=1}^{N} x_{s,i}^l x_{d,i}^l}{\sum_{i=1}^{N} x_{s,i}^l} & \\
\text{s.t.} & x_{s,i}^l \in \{0,1\}, & \forall\, i \in \mathcal{N}_s, \forall\, l \in \mathcal{L}, \\
& x_{d,i}^l \in \{0,1\}, & \forall\, i \in \mathcal{N}_d, \forall\, l \in \mathcal{L}, \\
& \sum_{l=1}^{L} x_{s,i}^l = 1, & \forall\, i \in \mathcal{A}_s, \\
& \sum_{l=1}^{L} x_{d,i}^l = 1, & \forall\, i \in \mathcal{A}_d, \\
& x_{s,i}^l = 0, & i \in \mathcal{N}_s \setminus \mathcal{A}_s, \forall\, l \in \mathcal{L}, \\
& x_{d,i}^l = 0, & i \in \mathcal{N}_d \setminus \mathcal{A}_d, \forall\, l \in \mathcal{L}, \\
\text{variables} & \boldsymbol{x_s} = \{x_{s,i}^l\}, & \boldsymbol{x_d} = \{x_{d,i}^l\},
\end{array} \tag{4}$$

where $\mathcal{L} = \{1, \cdots, L\}$ is the index set of primary channels, $\mathcal{A}_s = \{i : \exists l \in \mathcal{L}, a_{s,i}^l = 1\}$ and $\mathcal{A}_d = \{i : \exists l \in \mathcal{L}, a_{d,i}^l = 1\}$ are the sets of source and destination nodes at which at least one channel is available, respectively.

The MCS problem is an integer nonlinear 0-1 programming problem and is challenging to solve optimally.

E. Complexity Analysis of the MCS Problem

The classification of the computational difficulty (*e.g.*, NP, NP-hard, NP-complete, etc.) applies not only to optimization problems, but to decision problems where the answer is either Yes or No. There is a convenient conversion between optimization problems and decision problems [10], and the proof of NP-completeness for an optimization problem and its related decision problem are equivalent. We define the decision problem corresponding to the MCS problem as follows.

Definition 1: MCS Decision Problem: Given a set of S-D pairs, the available channels of source and destination nodes, and a number k, does there exist vectors $\boldsymbol{x}_s$ and $\boldsymbol{x}_d$ satisfying all constraints of the MCS problem and the total channel utilization equals k?

Lemma 1: The MCS decision problem is in NP.

Proof: To prove the MCS decision problem is in the complexity class of NP, we show that a solution to an instance of the MCS decision problem can be verified in polynomial time. Suppose we are given two vectors $\boldsymbol{x}_s$ and $\boldsymbol{x}_d$. We can easily verify if they are Yes-instance of the MCS decision problem with a number k by checking (i) whether $\sum_{l=1}^{L} \left(\sum_{i=1}^{N} x_{s,i}^l x_{d,i}^l / \sum_{i=1}^{N} x_{s,i}^l \right) = k$ is satisfied, (ii) whether each node only selects one available channel, and (iii) for all nodes, whether the decision variables corresponding to those unavailable channels are zero. Verifying (i) takes a running time of $O(NL)$, where N and L are the number of S-D pairs and the number of primary channels, respectively. Verifying (ii) and (iii) takes running times of $O(NL)$. Thus, a solution to an instance of the MCS decision problem can be verified in polynomial time. ■

Lemma 2: The MCS decision problem is NP-hard.

Proof: We use the way of restriction to prove that the MCS problem is NP-hard. Let $\mathcal{C}_{s,i}$ and $\mathcal{C}_{d,i}$ be the sets of available channels at the source and destination node of pair i. We will restrict the MCS decision problem to an instance of $L = 2, N = 2, \mathcal{C}_{s,1} = \mathcal{C}_{s,2} = \{c_1, c_2\}$, and $\mathcal{C}_{d,1} = \mathcal{C}_{d,2} = \{c_1, c_2\}$. Then we prove that this restricted MCS problem can be transformed to a circuit satisfiability (SAT) problem [11] in polynomial time. In other words, we show that there exists a feasible solution to the restricted MCS decision problem if and only if the SAT problem has a Yes answer. The detailed proof can be found in our online technical report [12]. ■

Lemmas 1 and 2 together lead to the following main result.

Theorem *1:* MCS Problem is NP-complete.

III. A Centralized Greedy Algorithm

Since the MCS problem is NP-complete, it seems impossible to solve it in polynomial time. In this section, we design a centralized greedy channel selection (GCS) algorithm to achieve a close-to-optimal solution. This algorithm serves as a performance benchmark for the distributed algorithm in the next section. In GCS algorithm, we first divide S-D pairs into two sets $\mathcal{I}_{wc}$ and $\mathcal{I}_{oc}$. Any S-D pair in set $\mathcal{I}_{wc}$ has at least one common channel between its source and destination nodes. Any S-D pair in set $\mathcal{I}_{oc}$ does not have common channel between its source and destination nodes. Let $\mathcal{C}_{sd,i}$ be the set of common channels for S-D pair i, and $\mathcal{C}_{sd} = \bigcup_{i \in \mathcal{N}} \mathcal{C}_{sd,i}$. We can then construct a bipartite graph G based on sets $\mathcal{I}_{wc}$ and $\mathcal{C}_{sd}$, and determine the channel selection for S-D pairs in the set $\mathcal{I}_{wc}$ based on a maximum matching. For S-D pairs set $\mathcal{I}_{oc}$, we determine the channel selection to minimize the negative impact on the total channel utilization. The proposed algorithm is shown in Algorithm 1, which consists of three main phases as follows.

A. Initialization Phase (lines 1 to 5)

We initialize the set of common channels $\mathcal{C}_{sd,i}$ for each S-D pair i, the set of all common channels $\mathcal{C}_{sd}$, and the sets of S-D pairs with and without common channels (*i.e.*, $\mathcal{I}_{wc}$ and $\mathcal{I}_{oc}$).

B. Construct a Bipartite Graph (lines 6 to 7)

In the graph theory, the maximum matching is a set of independent edges with the largest possible cardinality. In the proposed algorithm, we use it to match S-D pairs with common channels such that the maximum number of S-D pairs can select different common channels to achieve a high channel utilization. We use the sets $\mathcal{I}_{wc}$ and $\mathcal{C}_{sd}$ to construct a bipartite graph $G = (V(\mathcal{I}_{wc}, \mathcal{C}_{sd}), E)$, such that $E = \{e_{ij} : i \in \mathcal{I}_{wc}, c_j \in \mathcal{C}_{sd,i}\}$. In the bipartite graph, an edge connects a vertex i in $\mathcal{I}_{wc}$ to vertex c_j in $\mathcal{C}_{sd}$ if the channel c_j is a common channel for the S-D pair i. For instance, Fig. 2 shows the bipartite graph corresponding to Fig. 1, where the set of common channels for S-D pair 1 and 2 are $\mathcal{C}_{sd,1} = \{c_1\}$ and $\mathcal{C}_{sd,2} = \{c_1, c_4\}$, respectively. The set of S-D pairs with common channels is $\mathcal{I}_{wc} = \{1, 2\}$, and the set of all common channels is $\mathcal{C}_{sd} = \{c_1, c_4\}$. Given a constructed bipartite graph

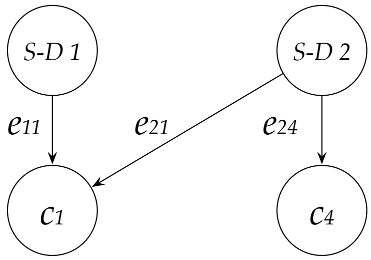

Fig. 2. Illustration of a bipartite graph

Algorithm 1: The Greedy Channel Selection Algorithm

1 **for** $i := 1$ **to** N **do**
2 $\quad \mathcal{C}_{sd,i} = \{c_l : a^l_{s,i} a^l_{d,i} = 1, l \in \mathcal{L}\}$;
3 $\mathcal{C}_{sd} = \bigcup_{i \in \mathcal{N}} \mathcal{C}_{sd,i}$;
4 $\mathcal{I}_{wc} = \{i : |\mathcal{C}_{sd,i}| > 0, i \in \mathcal{N}\}$;
5 $\mathcal{I}_{oc} = \{i : |\mathcal{C}_{sd,i}| = 0, i \in \mathcal{N}\}$;
6 Construct a bipartite graph $G = (V(\mathcal{I}_{wc}, \mathcal{C}_{sd}), E)$, where $E = \{e_{ij} : i \in \mathcal{I}_{wc}, c_j \in \mathcal{C}_{sd,i}\}$;
7 $H \leftarrow$ maximum bipartite matching of graph G ;
8 $\mathcal{I}^m_{wc} = \{i : \ \exists j, \ \ e_{ij} \in H\}$; $\mathcal{I}^u_{wc} = \{i : \ \forall j, \ e_{ij} \notin H \}$;
9 **for** $i := 1$ **to** N **do**
10 $\quad$ **if** $i \in \mathcal{I}^m_{wc}$ **then**
11 $\quad\quad$ **for** $j := 1$ **to** L **do**
12 $\quad\quad\quad$ **if** $e_{ij} \in H$ **then**
13 $\quad\quad\quad\quad x^j_{s,i} = 1; x^j_{d,i} = 1;$
14 $\quad\quad\quad\quad x^l_{s,i} = 0; x^l_{d,i} = 0; \forall l \in \mathcal{L}, l \neq j;$
15 $\quad$ **else**
16 $\quad\quad$ **if** $i \in \mathcal{I}^u_{wc}$ **then**
17 $\quad\quad\quad$ Randomly select a channel $j' \in \mathcal{C}_{sd,i}$;
18 $\quad\quad\quad x^{j'}_{s,i} = 1; x^{j'}_{d,i} = 1$;
19 $\quad\quad\quad x^l_{s,i} = 0; x^l_{d,i} = 0; \forall \ l \in \mathcal{L}, l \neq j'$;
20 **while** $\mathcal{I}_{oc} \neq \emptyset$ **do**
21 $\quad$ Randomly select $i \in \mathcal{I}_{oc}$;
22 $\quad j^* = \arg\max_{j \in \mathcal{C}_{s,i}} \left(\sum_{i' \in \mathcal{N}_s, i' \neq i} x^j_{s,i} \right); \ x^{j^*}_{s,i} = 1;$
23 $\quad x^l_{s,i} = 0; \forall \ l \in \mathcal{L}, l \neq j^*$;
24 $\quad$ Randomly select $j \in \mathcal{C}_{d,i}$;
25 $\quad x^j_{d,i} = 1; x^l_{d,i} = 0; \forall \ l \in \mathcal{L}, l \neq j;$
26 $\quad \mathcal{I}_{oc} \leftarrow \mathcal{I}_{oc} \setminus \{i\}$;

G, we can compute the maximum bipartite matching in G [9] and assign to H (line 7). If the maximum bipartite matching is not unique, we randomly assign one of them to H, which does not impact the maximum number of utilized channels.

C. Channel Selection for S-D Pairs with Common Channels (lines 8 to 19)

We select the channels for S-D pairs in $\mathcal{I}_{wc}$ based on the maximum bipartite matching H. First, we divide the S-D pairs in $\mathcal{I}_{wc}$ into two sets $\mathcal{I}^m_{wc}$ and $\mathcal{I}^u_{wc}$, which consist of matched and unmatched S-D pairs, respectively. Take Fig. 2 as an example, where we have the maximum bipartite matching $H = \{e_{11}, e_{24}\}, \mathcal{I}_{wc} = \{1, 2\}, \mathcal{I}^m_{wc} = \{1, 2\}$, and $\mathcal{I}^u_{wc} = \emptyset$.

Since the edges in the bipartite graph G connect S-D pairs to channels, the maximum matching represents a set of channel selection that can achieve the maximum number of utilized channels. For an S-D pair in the matched set $\mathcal{I}^m_{wc}$, we allocate the channel c_j such that $e_{ij} \in H$ (lines 10 to 14). In Fig. 2, we select channel c_1 for S-D pair 1 and channel c_4 for S-D pair 2. For an S-D pair in the unmatched set $\mathcal{I}^u_{wc}$, we randomly select one of its common channels for it (lines 16 to 19). This random allocation does not impact the maximum number of utilized channels.

D. Channel Selection for S-D Pairs without Common Channels (lines 20 to 26)

For the S-D pairs without common channels, it is impossible for them to have successful data transmissions. On the other hand, each source of such a pair still will select a channel to access by the description of the protocol. The objective of this part of the algorithm is then to minimize the negative impact of these pairs on the system total channel utilization. According to the objective function of the MCS problem in (4), we know that adding one source to the channel with the largest number of senders introduces the least degradation of the objective function value. Motivated by this, a source node of S-D pair $i \in \mathcal{I}_{oc}$ will choose a channel j^* within its available channels such that $j^* = \arg\max_{j \in \mathcal{C}_{s,i}} (\sum_{i' \in \mathcal{N}_s, i' \neq i} x^j_{s,i'})$ (lines 21 to 23).

For an S-D pair in set $\mathcal{I}_{oc}$, its destination node's channel choice does not affect the objective function of MCS problem and thus can be arbitrary (lines 24 to 25).

E. Computational Complexity

We analyze the runtime complexity of Algorithm 1 in terms of the number of S-D pairs N and primary channels L. In the initialization phase, it takes a running time of $O(NL)$ to initialize the sets of $\mathcal{C}_{sd,i}$, $\mathcal{C}_{sd}$, $\mathcal{I}_{wc}$, and $\mathcal{I}_{oc}$. Maximum bipartite matching problem can be solved with a running time of $O(|E| \cdot \sqrt{|V|})$ in [9], where $|E|$ is the number of edges in the graph and $|V|$ is the number of vertices of the graph. Given N and L, the complexity of solving the maximum matching H in the case of dense bipartite graph is bounded by $O(N^2\sqrt{N + L})$. With the maximum matching H, it takes a running time of $O(N)$ to select the channel for S-D pairs in $\mathcal{I}_{wc}$. For the S-D pairs without common channels in $\mathcal{I}_{oc}$, it takes a running time of $O(L)$ to find the channel j^* using the linear searching, and takes a running time of $O(1)$ to assign the channels to these S-D pairs. Typically, we have $L < N^2$. Therefore, the computational complexity of the greedy algorithm mainly depends on the complexity of solving the maximum matching problem, and the overall complexity is bounded by $O(N^2\sqrt{N + L})$.

IV. A Distributed Algorithm

A centralized algorithm can provide a high channel utilization and system performance at the expense of large information exchange overhead. In this Section, we propose a distributed algorithm that is more suitable for practical implementation.

TABLE I
EXAMPLE OF CHANNEL PRIORITY ORDER

h	Time slot t	Channel priority order
h=1	$t=0$	$c_1 > c_2 > \cdots > c_L$
	$t=1$	$c_2 > c_3 > \cdots > c_L > c_1$
	$t=2$	$c_3 > c_4 > \cdots > c_L > c_1 > c_2$

A. Priority Order Channel Selection Algorithm

In the distributed algorithm, the channel selection is based on a predetermined channel priority order. At each given time slot, each secondary node orders all channels based on the same priority order, then chooses the channel that has the highest priority among all its available channels. The priority order changes over time as follows:

The rule of channel priority order : without loss of generality, assume that channel h has the highest priority in time slot $t = 0$. The priority order of all the channels changes according to a simple circular left shift rule between two adjacent time slots as follows:

$$c_{(h+t-1 \bmod L)+1} > c_{(h+t \bmod L)+1} > \cdots > c_{(h+t+L-2 \bmod L)+1} \quad (5)$$

where $(x \bmod y)$ represents modulo operation, which returns the remainder when the first argument is divided by the second argument. A numerical example of $h = 1$ is given in Table I.

Since the channel availabilities of secondary nodes are time varying, dynamically changing the channel priority order according to (5) can help to achieve a good fairness. For a node just joining the network, the only information it needs to know is the current highest priority channel. It can broadcast a request message when joining the network, and any existing node in the network can provide such priority information. There is no need for further information exchange during later time slots. Therefore, this algorithm can be easily implemented in a distributed fashion.

B. Performance Analysis

To analyze the performance of the distributed algorithm, we consider a simple two-state Markov model for the primary channels and independent channel availabilities among secondary nodes. Our future work will consider performance analysis with dependent channel availabilities.

Let $\alpha^l_{s,i}, \beta^l_{s,i}, \alpha^l_{d,i}, \beta^l_{d,i}$ $(i \in \mathcal{N}, l \in \mathcal{L})$ be the parameters of the channel availability model for the source and destination nodes of S-D pair i, respectively. Figure 3 shows the availability model of channel c_l for the source node of pair i. When a channel c_l is in the "Off" state (i.e., no active primary user on the channel), it is available for the source node i during this time slot. Otherwise, it is not available.

Thus, the expected total channel utilization is

$$E[U] = E\left[\sum_{l=1}^{L}\left(\left(\sum_{i=1}^{N} x^l_{s,i}x^l_{d,i}\right) \Big/ \left(\sum_{i=1}^{N} x^l_{s,i}\right)\right)\right]$$

$$= \sum_{l=1}^{L}\sum_{i=1}^{N} Pr(x^l_{s,i}=1)Pr(x^l_{d,i}=1)E\left[\frac{1}{\sum\limits_{k=1,k\neq i}^{N} x^l_{s,k}+1}\right], \quad (6)$$

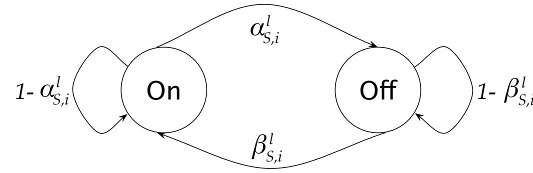

Fig. 3. Channel availability model

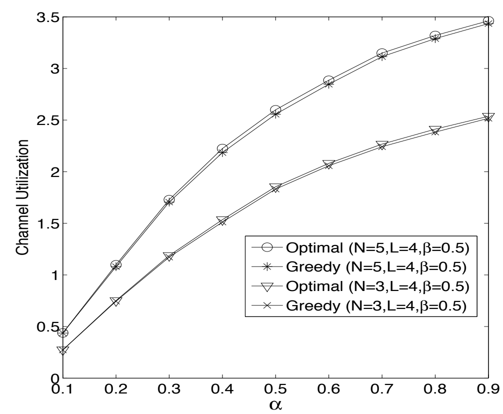

Fig. 4. The comparison of optimal and greedy solutions

where the binary random variables $x^l_{s,i}$ and $x^l_{d,i}$ indicate whether the channel c_l is selected by the source and destination i, respectively.

To compute (6), next we calculate the probability of selecting channel c_l at the source and destination nodes of pair i (*i.e.*, $Pr(x^l_{s,i} = 1)$ and $Pr(x^l_{d,i} = 1)$). In an arbitrary time slot, the channel c_l is selected at the source (or destination) node of pair i only when this channel is available to this node and all higher priority channels are not available to this node. Let $\mathcal{C}_{l,H}$ be the set of indices of channels with higher priority than c_l at this time slot. Thus, we have

$$Pr(x^l_{s,i}=1) = \frac{\alpha^l_{s,i}}{\alpha^l_{s,i}+\beta^l_{s,i}} \prod_{l'\in\mathcal{C}_{l,H}} \frac{\beta^{l'}_{s,i}}{\alpha^{l'}_{s,i}+\beta^{l'}_{s,i}}, \quad (7)$$

$$Pr(x^l_{d,i}=1) = \frac{\alpha^l_{d,i}}{\alpha^l_{d,i}+\beta^l_{d,i}} \prod_{l'\in\mathcal{C}_{l,H}} \frac{\beta^{l'}_{d,i}}{\alpha^{l'}_{d,i}+\beta^{l'}_{d,i}}. \quad (8)$$

The last term in (6) can calculated based on the probability mass of the binary random variable $x^l_{s,k}$ as in (9) and (10):

$$Pr(x^l_{s,k}=1) = \frac{\alpha^l_{s,k}}{\alpha^l_{s,k}+\beta^l_{s,k}} \prod_{l'\in\mathcal{C}_{l,H}} \frac{\beta^{l'}_{s,k}}{\alpha^{l'}_{s,k}+\beta^{l'}_{s,k}}, \quad (9)$$

$$Pr(x^l_{s,k}=0) = 1 - Pr(x^l_{s,k}=1). \quad (10)$$

V. NUMERICAL RESULTS

We have conducted extensive simulations to evaluate the performance of the proposed centralized and distributed algorithms. We set $\alpha^l_{s,i} = \alpha^l_{d,i} = \alpha$, and $\beta^l_{s,i} = \beta^l_{d,i} = \beta$ for $i \in \mathcal{N}, l \in \mathcal{L}$. We repeat each experiment for 20 runs with different random seeds and calculate the average value.

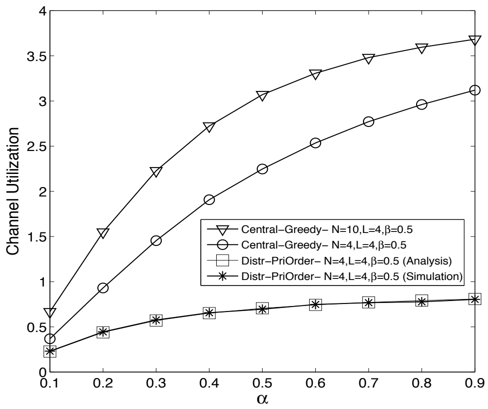

Fig. 5. The comparison of centralized and distributed algorithms

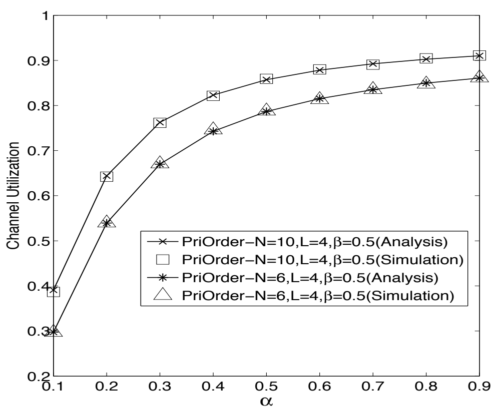

Fig. 6. Channel utilization with different channel availability

Figure 4 compares the proposed centralized greedy algorithm and the optimal solution of the MCS problem (obtained using exhaustive search). We observe that the greedy algorithm achieves a performance very close to the optimal one with a maximum performance loss of 5%.

Figure 5 compares between the centralized greedy algorithm and the distributed priority order channel selection algorithm. The centralized greedy algorithm is better than the distributed algorithm at the expense of more message exchanging.

Figure 6 demonstrates the impact of parameter α on the channel utilization. A larger α leads to a higher channel availability, which increases the probability that an S-D pair has common channels and leads to a higher channel utilization.

Figure 7 shows the impacts of parameters N and L. With a larger number of users N, the probability that a channel becomes a common channel of at least one S-D pair increases, which leads to a higher channel utilization. In addition, a larger number of channels L leads to a performance increase.

VI. Conclusions

In this paper, we consider the channel selection problem in a multi-channel cognitive radio network with heterogeneous channel availabilities. This problem is challenging to tackle and yet very critical in achieving good performance in a cognitive network. We formulate the maximum channel selection (MCS) problem as a binary integer nonlinear optimization problem, with an objective of maximizing the total channel utilization for all secondary nodes. After showing that the problem is NP-complete, we propose a centralized greedy algorithm that achieves close-to-optimal performance with a polynomial complexity.

We further propose a distributed priority based algorithm that significantly decreases the signaling overhead compared with the centralized algorithm. Simulation results of the distributed algorithm match the theoretical analysis very well. We also characterize the impact of various system parameters on the performance of the distribution algorithm. Examples of other distributed algorithms with detailed performance analysis can be found in our technical report [12].

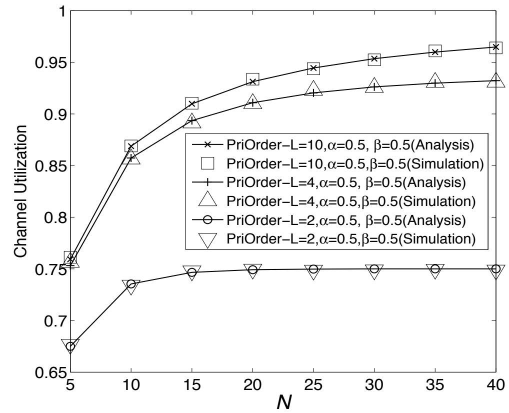

Fig. 7. Channel utilization with different number of S-D pairs

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