Evolutionarily Stable Open Spectrum Access in a Many-Users Regime

Xu Chen* and Jianwei Huang*

***Abstract*—In this paper, we consider the open spectrum access mechanism design with both complete and incomplete network information in a many-users regime. We propose an evolutionary spectrum access mechanism with complete network information, and show that the mechanism achieves an equilibrium that is both evolutionarily stable and globally stable. With incomplete network information, we propose a distributed learning mechanism, where each user utilizes local observations to estimate the channel quality and learns to adjust its spectrum access strategy adaptively over time. Numerical results show that the proposed mechanisms achieve efficient spectrum sharing among the users, and are robust to the perturbations of users' channel selections.**

I. INTRODUCTION

Open spectrum sharing is a promising paradigm for improving spectrum utilization [1], as demonstrated by the successful deployments of Wi-Fi and bluetooth devices over the unlicensed ISM bands. Since the spectrum access is open, the population of wireless users is expected to be large, which would lead to "the tragedy of commons" without proper regulation [1]. Moreover, the diverse spectrum access behaviors in large user population may lead to an instable spectrum access. These motivate us to design efficient and stable spectrum sharing mechanisms in a many-users regime.

The user competition for common spectrum resources has been studied using *non-cooperative game theory*. Liu and Wu in [2] modeled the interactions among spatially separated users as congestion games with resource reuse. Nie and Comniciu in [3] designed a self-enforcing distributed spectrum access mechanism based on potential games. Southwell and Huang in [4] studied the channel selection dynamics of the spectrum access game. Law *et al.* in [5] studied the price of anarchy of spectrum access game, and showed that users' selfish choices may significantly degrade the system performance. A common assumption of all the above work is that users are *fully rational* and thus can select channels by computing the best responses. To have the full rationality, a user needs to have a high computational power to collect and analyze the network information in order to predict other users' behaviors. This is often not feasible in today's network, due to the computation limitation and energy constraint of wireless devices.

Without knowing the channel information, users need to learn the environment and adapt the channel selection decisions accordingly. Law *et al.* in [6] and Maskery *et al.* in [7] used no-regret learning to solve this problem, assuming that the users' channel selections are common information. The learning process converges to a correlated equilibrium, wherein the common observed history serves as a signal to coordinate all users' channel selections. When users' channel selections are not observable, Li in [8] applied reinforcement learning to analyze an Aloha-type spectrum access.

*The authors are with the Department of Information Engineering, the Chinese University of Hong Kong, Hong Kong; Email:{cx008,jwhuang}@ie.cuhk.edu.hk. This work is supported by the General Research Funds (Project Number 412509 and 412511) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

In this paper, we consider the open spectrum access mechanism design with and without complete network information (i.e., channel statistics and user selections). The common assumption in both scenarios is *bounded rationality*, where users choose channels only based on *better responses* instead of *best responses*. This requires much less computation power than the full rationality case, and thus matches the reality of wireless communications better.

We first propose an evolutionary game approach for open spectrum access with the complete network information, where each user takes a better response to evolve its spectrum access decision over time. We analyze the stability of the mechanism based on the concept of evolutionarily stable strategy (ESS). We then propose a distributed learning mechanism for open spectrum access with incomplete information, which does not require any prior knowledge of channel statistics or information exchange among users. In this case, each user estimates the channel quality based on local observations, and *learn* to adjust its channel selection strategy adaptively. The main results and contributions of this paper are as follows:

- *Evolutionary spectrum access mechanism*: we formulate the open spectrum access over multiple heterogeneous time-varying channels as an evolutionary spectrum access game, and study the evolutionary dynamics of spectrum access.
- *Evolutionary dynamics and stability*: we show that the evolutionary spectrum access mechanism converges exponentially to the evolutionary equilibrium, and prove that it is an ESS and is globally asymptotically stable.
- *Learning mechanism with incomplete information*: we further propose a leaning mechanism without the knowledge of channel statistics and user information exchange. We show that the learning mechanism converges to the same evolutionary equilibrium on the time average.

The rest of the paper is organized as follows. We introduce the system model in Section II. After briefly reviewing the evolutionary game theory in Section III, we present the evolutionary spectrum access mechanism with complete information in Section IV. Then we introduce the learning mechanism in Section V. We illustrate the performance of the proposed mechanisms through numerical results in Section VI and finally conclude in Section VII. Due to space limitations, the proofs details are given in our online technical report [9].

II. SYSTEM MODEL

We consider a time-slotted wireless network with M unlicensed channels and N users. The channels are heterogeneous and time-varying due to environmental effects such as fading. We assume that the data rate $b_m(t)$ of channel m evolves according to an i.i.d random process in time t with a finite mean B_m and a finite variance σ_m^2. Users are homogeneous, i.e., all users achieve the same data rate on the same channel during the same slot. The heterogeneous user case will be considered in a future work. We first assume that there exists a common control channel for exchanging the channel selection information among users. This restriction will be removed when we discuss the learning mechanism in Section V. The system model is described as follows:

- *Channel contention*: at the beginning of each time slot, each user selects one channel to access, based on the channel selection decision made at the end of last time slot. To avoid collision by multiple concurrent transmissions over the same channel, each user n executes the following two steps:
  1) Count down according to a randomly and uniformly chosen integral backoff time λ_n between 1 and $\lambda_{\max}$, where $\lambda_{\max}$ is a large integer known to all users.
  2) Once the timer expires, monitor the channel and transmit data only if the channel is clear (i.e., no ongoing transmission).

  Suppose that k_m users choose channel m to access. Then the probability that user n (out of the k_m users) grabs the channel m is
  $$\begin{aligned} P_n =& Pr\{\min\{\lambda_1, ..., \lambda_{k_m}\} = \lambda_n\} \\ & \cdot \sum_{\lambda=1}^{\lambda_{max}} Pr\{\lambda_n = \lambda\} Pr\{\min_{i \neq n}\{\lambda_i\} > \lambda | \lambda_n = \lambda\} \\ =& \frac{1}{k_m} \sum_{\lambda=1}^{\lambda_{max}} \frac{1}{\lambda_{max}} \left(\frac{\lambda_{max} - \lambda}{\lambda_{max}} \right)^{k_m - 1}. \end{aligned}$$
  In this study, we will focus on the asymptotic case where λ_{max} goes to ∞, such that $\lim_{\lambda_{max} \to \infty} \frac{1}{\lambda_{max}} \sum_{\lambda=1}^{\lambda_{max}} (\frac{\lambda_{max} - \lambda}{\lambda_{max}})^{k_m - 1} = 1$, and the expected throughput is given as
  $$U_n(t) = \frac{b_m(t)}{k_m}. \quad (1)$$
- *Data transmission*: if a user successfully grabs the channel, it will transmit data packets during the remaining of the time slot. Otherwise, it just keeps silent. For simplicity, we assume that the total countdown time is negligible compared with the total length of the time slot.
- *Channel selection*: in the complete information case, users broadcast the chosen channel IDs to other users through the common control channel, and then make the channel selection decisions based on the evolutionary spectrum access mechanisms. With incomplete information, users update the channel estimations based on the current access results, and make the channel selection decisions according to the distributed learning mechanism (described later).

In this study, we focus on the analysis in the many-users regime. Numerical results show that our algorithms also apply when the number of users is small (for details, see [9]).

III. Overview of Evolutionary Game Theory

Evolutionary game theory was first used in biology to study the change of animal populations, and then later applied in economics to model human behaviors. It is most useful to understand how a large population of users converge to Nash equilibria in a dynamic system [10]. A player in an evolutionary game has bounded rationality, i.e., limited computational capability and knowledge, and improves its decisions as it learns about the environment over time [10].

The evolutionarily stable strategy (ESS) is a key concept to describe the evolutionary equilibrium. An ESS ensures the stability such that the population is robust to perturbations by a small fraction of players. Formally, suppose that a small share $\epsilon \in (0, 1)$ of players in the population deviate to choose a mutant strategy j, while all other players stick to the incumbent strategy i.[1] Let $U(a, \boldsymbol{x}_i)$ denote the payoff of choosing a strategy $a \in \{i, j\}$ given that all other players choosing the same strategy i. Similarly, $U(a, \boldsymbol{x}_{\epsilon j + (1-\epsilon) i})$ denotes the payoff of choosing a strategy $a \in \{i, j\}$ if ϵ fraction of players playing the mutant strategy j while the others playing strategy i.

Definition 1 ([10])**.** *A strategy i is an **evolutionarily stable strategy** if for every strategy $j \neq i$, there exists an $\bar{\epsilon} \in (0, 1)$ such that $U(i, \boldsymbol{x}_{\epsilon j + (1-\epsilon) i}) > U(j, \boldsymbol{x}_{\epsilon j + (1-\epsilon) i})$ for any $j \neq i$ and $\epsilon \in (0, \bar{\epsilon})$.*

Definition 1 means that the mutant strategy j cannot invade the population when the perturbation is small enough, if the incumbent strategy i is an ESS. It is shown in [10] that any strict Nash equilibrium (i.e., $U(i, \boldsymbol{x}_i) > U(j, \boldsymbol{x}_i), \forall j \neq i$) is also an ESS and hence locally evolutionarily stable.

Several recent results applied the evolutionary game theory to study various networking problems. Niyato and Hossain in [11] investigated the evolutionary dynamics of heterogeneous network selections. Zhang *et al.* in [12] designed incentive schemes for resource-sharing in P2P networks based on the evolutionary game theory. Wang *et al.* in [13] proposed the evolutionary game approach for collaborative spectrum sensing mechanism design in cognitive radio networks. Here we apply the evolutionary game theory to design spectrum access mechanism, which can achieve globally evolutionary stability.

IV. Evolutionary Spectrum Access

We now apply the evolutionary game theory to design an efficient and stable spectrum access mechanism with complete network information. We will show that the spectrum access equilibrium is an ESS, which guarantees that the spectrum access mechanism is robust to random perturbations of users' channel selections.

A. Evolutionary Game Formulation

The evolutionary spectrum access game is formulated as follows:

- Players: the set of users $\mathcal{N} = \{1, 2, ..., N\}$.
- Strategies: each user can access one of the set of channels $\mathcal{M} = \{1, 2, ..., M\}$.

[1]Following convention in the definition of ESS, we consider a symmetric game where all users adopt the same strategy i at the ESS. The definition can be (and will be) easily extended to the case of asymmetric game, where we view the population's collective behavior as a mixed strategy i at the ESS.

- Population state: the user distribution over M channels at time t, $\boldsymbol{x}(t) = (x_m(t), \forall m \in \mathcal{M})$, where $x_m(t)$ is proportion of users selecting channel m at time t. We have $\sum_{m \in \mathcal{M}} x_m(t) = 1$ for all t.
- Payoff: a user's expected throughput $U_n(a_n, \boldsymbol{x}(t))$ when choosing channel $a_n \in \mathcal{M}$, given that the population state is $\boldsymbol{x}(t)$. Since each user has the information of channel statistics, from (1), we have $U_n(a_n, \boldsymbol{x}(t)) = \frac{E[b_{a_n}(t)]}{N x_{a_n}(t)} = \frac{B_{a_n}}{N x_{a_n}(t)}$.

B. Evolutionary Dynamics

Based on the evolutionary game formulation above, we propose an evolutionary spectrum access mechanism in Algorithm 1. The idea is to let those users who have payoffs lower than the targeted average population payoff $\frac{\sum_{i=1}^M B_i}{N}$ to select a better channel, with a probability proportional to the (normalized) channel's "net fitness" $\frac{B_m}{N x_m(t)} - \frac{\sum_{i=1}^M B_i}{N}$. We show that the dynamics of channel selections in the mechanism can be described with the evolutionary dynamics in (2). The proof is given in [9].

Theorem 1. *For the evolutionary spectrum access mechanism, the evolutionary dynamics are given as*

$$\dot{x}_m(t) = \beta \left(\frac{B_m}{N} - \frac{\sum_{i=1}^M B_i}{N} x_m(t) \right), \forall m \in \mathcal{M}, \quad (2)$$

where the rate of strategy adaptation $\beta = \frac{\alpha N}{\sum_{i=1}^M B_i}$ and the derivative is with respect to time.

Algorithm 1 Evolutionary Spectrum Access Mechanism

initialization:
  set the global strategy adaptation factor $0 < \alpha \leq 1$.
  select a random channel for each user.
end initialization

loop for each time slot t and each user $n \in \mathcal{N}$ in parallel:
  compete for the chosen channel and transmit data packets if successfully grabbing the channel.
  broadcast the chosen channel ID to other users through the common control channel.
  receive the information of other users' channel selection and calculate the population state $\boldsymbol{x}(t)$.
  compute the expected payoff $U_n(a_n, \boldsymbol{x}(t)) = \frac{B_{a_n}}{N x_{a_n}(t)}$.
  if $U_n(a_n, \boldsymbol{x}(t)) < \frac{\sum_{i=1}^M B_i}{N}$ **then**
    generate a random value δ according to a uniform distribution on $(0, 1)$.
    if $\delta < \alpha \left(1 - \frac{U_n(a_n, \boldsymbol{x}(t))}{\frac{\sum_{i=1}^M B_i}{N}} \right)$ **then**
      select a new channel m with probability

$$p_m = \frac{\max\left\{ \frac{B_m}{\sum_{i=1}^M B_i} - x_m(t), 0 \right\}}{\sum_{m'=1}^M \max\left\{ \frac{B_{m'}}{\sum_{i=1}^M B_i} - x_{m'}(t), 0 \right\}}.$$

    else select the original channel.
    end if
  end if
end loop

C. Evolutionary Equilibrium

We next investigate the equilibrium of the evolutionary spectrum access mechanism. Note that evolutionary dynamics in (2) are a set of first-order linear differential equations, which can be solved as

$$x_m(t) = \left[x_m(0) - \frac{B_m}{\sum_{i=1}^M B_i} \right] e^{-\beta t (\sum_{i=1}^M B_i)} + \frac{B_m}{\sum_{i=1}^M B_i}. \quad (3)$$

From (3), we have

Theorem 2. *The evolutionary spectrum access mechanism converges exponentially to the evolutionary equilibrium $\boldsymbol{x}^* = \left(x_m^* = \frac{B_m}{\sum_{i=1}^M B_i}, \forall m \in \mathcal{M} \right)$.*

This implies the fast convergent property of the evolutionary spectrum access mechanism. Next we study evolutionary stability of the equilibrium.

In general, the equilibrium of the evolutionary dynamics may not be an ESS [10]. For our model, we can prove the following.

Theorem 3. *For the evolutionary spectrum access mechanism, the evolutionary equilibrium $\boldsymbol{x}^*$ is an ESS.*

The proof is given in [9]. Actually we can obtain a stronger result than Theorem 3. Typically, an ESS is only locally asymptotically stable (i.e., stable within a limited region around the ESS) [10]. For our case, we show that the evolutionary equilibrium $\boldsymbol{x}^*$ is globally asymptotically stable (i.e., stable in the entire feasible region of a population state $\boldsymbol{x}$, $\{\boldsymbol{x} = (x_m, m \in \mathcal{M}) | \sum_{m=1}^M x_m = 1$ and $x_m \geq 0, \forall m \in \mathcal{M}\}$).

Theorem 4. *For the evolutionary spectrum access mechanism, the evolutionary equilibrium $\boldsymbol{x}^*$ is globally asymptotically stable.*

This is proved by showing that $V(\boldsymbol{x}(t)) = \sum_{m=1}^M (x_m(t) - x^*)^2$ is a strict Lyapunov function. Since the ESS is globally asymptotically stable, the evolutionary spectrum access mechanism is robust to any degree of (not necessarily small) random perturbations of channel selections. Since the users are homogeneous, it is easy to verify that the ESS $\boldsymbol{x}^*$ is also max-min fair and all the users achieve the same throughput.

V. Learning Mechanism For Open spectrum access

For the evolutionary spectrum access mechanism in Section IV, we assume that each user has the perfect knowledge of channel statistics and the population state by information exchange on a common control channel. Such mechanism leads to significant communication overhead and energy consumption, and may even be impossible in some systems. We thus propose a learning mechanism for open spectrum access with incomplete information. The challenge is how to achieve an evoltuionarily stable state based on user's local observations only.

A. Leaning Mechanism For Open Spectrum Access

The proposed learning process has two sequential stages: *initial channel estimation* and *access strategy learning* (see Algorithm 2). Each stage is defined over a sequence of decision periods $T = 1, 2, ...$, where each decision period consists of K time slots.

In the first stage, each user initially estimates the channel quality by accessing all the channels in a randomized round-robin manner. Let $\mathcal{M}_n$ (equals to $\varnothing$ initially) be the set of channels accessed by user n and $\mathcal{M}_n^c = \mathcal{M} \backslash \mathcal{M}_n$. At beginning of each decision period, user n randomly chooses a channel $m \in \mathcal{M}_n^c$ to access. At end of the period, user n can estimate the mean channel data rate by sample averaging as

$$\tilde{B}_{m,n}(0) = \frac{\sum_{t=1}^{K} b_m(t) I_{\{a_n(t,T)=m\}}}{\sum_{t=1}^{K} I_{\{a_n(t,T)=m\}}}, \quad (4)$$

where $I_{\{a_n(t,T)=m\}}$ is an indicator function and equals 1 if the user n successfully grabs the channel m at time slot t. Note that there are K time slots within each decision period, and thus the user will be able to have a fairly good estimation of the channel quality if K is reasonably large. Then user n updates the set of accessed channels as $\mathcal{M}_n = \mathcal{M}_n \cup \{m\}$. When all the channels are accessed, i.e., $\mathcal{M}_n = \mathcal{M}$, the stage of initial channel estimation ends. Thus, the total time slots for the first stage is MK.

In the second stage, at each period $T \geq 1$, each user $n \in \mathcal{N}$ selects a channel m to access according to a mixed strategy $\boldsymbol{f}_n(T) = (f_{1,n}(T), ..., f_{M,n}(T))$, where $f_{m,n}(T)$ is the probability of user n choosing channel m and is computed as

$$f_{m,n}(T) = \frac{(1-\gamma)\sum_{\tau=0}^{T-1} \gamma^{T-\tau-1} \tilde{B}_{m,n}(\tau)}{\sum_{i=1}^{M} (1-\gamma)\sum_{\tau=0}^{T-1} \gamma^{T-\tau-1} \tilde{B}_{i,n}(\tau)}, \forall m \in \mathcal{M}. \quad (5)$$

Here $0 < \gamma < 1$ is the memory weight, $\tilde{B}_{m,n}(\tau)$ is user n's estimation of channel m at period τ (see (6) and (7) later), and $1-\gamma$ is used for normalization only. The update in (5) means that each user adjusts its mixed strategy according to its weighted average estimations of all channels' qualities. When user n chooses channel m to access at period τ, it can obtain the sample averaging estimation of the quality of channel m as in (4), i.e.,

$$\tilde{B}_{m,n}(\tau) = \frac{\sum_{t=1}^{K} b_m(t) I_{\{g_n(t,\tau)=1\}}}{\sum_{t=1}^{K} I_{\{g_n(t,\tau)=1\}}}. \quad (6)$$

For the unchosen channels $m' \neq m$ at this period, user n can empirically estimate the quality of this channel according to its past memories as

$$\tilde{B}_{m',n}(\tau) = (1-\gamma) \sum_{\tau'=0}^{\tau-1} \gamma^{\tau-\tau'-1} \tilde{B}_{m',n}(\tau'). \quad (7)$$

Note that if $\tilde{B}_{m',n}(\tau) = 0$ is adopted for update, it would result in memory decay for the unchosen channels (due to the memory weight γ), and hence lead to a bad performance due to over-exploitation of the accessed channels.

We show that the learning mechanism converges to the ESS on time average (The proof is given in [9]).

Theorem 5. *For the learning mechanism, when the length of each decision period K is large enough, the expectation of the population state $\boldsymbol{x}(T)$ converges to the ESS $\boldsymbol{x}^*$ almost surely as the number of periods T goes to infinity.*

Algorithm 2 Learning Mechanism For Open Spectrum Access

initialization:
    set the global memory weight γ and the set of accessed channels $\mathcal{M}_n = \varnothing$.
end initialization

for each user $n \in \mathcal{N}$ in parallel **do**
    Initial Channel Estimation Stage:
    while $\mathcal{M}_n \neq \mathcal{M}$ **do**
        choose a channel m from the set $\mathcal{M}_n^c$ randomly.
        compete to access the channel m at each time slot of the decision period.
        estimate the mean data rate $\tilde{B}_{m,n}(0)$ by (4).
        set $\mathcal{M}_n = \mathcal{M}_n \cup \{m\}$.
    end while

    Access Strategy Learning Stage:
    loop for each time period T:
        choose a channel m to access according to the mixed strategy $\boldsymbol{f}_n(T)$ in (5).
        compete to access the channel m at each time slot of the decision period.
        estimate data rates of the chosen channel m and the unchosen channels $m' \neq m$ by (6) and (7), respectively.
    end loop
end for

VI. Simulation Results

In this section, we investigate the proposed mechanisms by simulations. The results demonstrate that the proposed mechanisms can achieve efficient and stable spectrum access for open spectrum sharing.

A. Evolutionary Spectrum Access with Complete Network Information

We consider a wireless network consisting $M = 5$ channels. The data rate $b_m(t)$ follows the Rayleigh distribution with the mean data rate B_m equals 10, 40, 50, 20 and 80, Mbps, respectively. We set the strategy adaptation factor $\alpha = 0.5$ in the simulations. We implement the evolutionary spectrum access mechanism with the number of users $N = 100$ and 200, respectively. The results are shown in Figures 1 and 2. From these figures, we see that

- *Fast convergence:* the algorithm takes less than 20 iterations to converge in all cases (see Figure 1).
- *Convergence to ESS*: in all three cases, the algorithm converges to the ESS $\boldsymbol{x}^* = \left(\frac{B_1}{\sum_{i=1}^M B_i}, ..., \frac{B_M}{\sum_{i=1}^M B_i}\right)$ (see Figure 1). At the ESS $\boldsymbol{x}^*$, each user will achieve the same expected payoff $U_n(a_n^*, \boldsymbol{x}^*) = \frac{\sum_{i=1}^M B_i}{N}$ (see Figure 1).
- *Asymptotic stability*: to investigate the stability of the evolutionary spectrum access mechanism, we let a fraction of users play the mutant strategies when the system is at the ESS $\boldsymbol{x}^*$. At time slot $t = 30$, $\epsilon = 0.5$ and 0.9 fraction of users will randomly choose a new channel. The result is shown in Figure 2. We see that the algorithm is capable

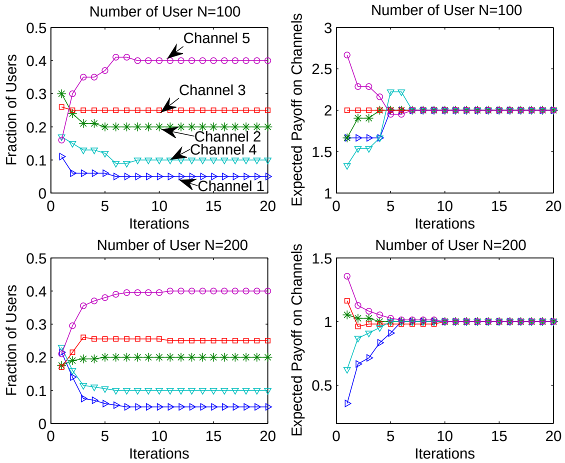

Fig. 1. The fraction of users and the expected payoff of accessing different channels with the number of users $N = 100$ and 200, respectively.

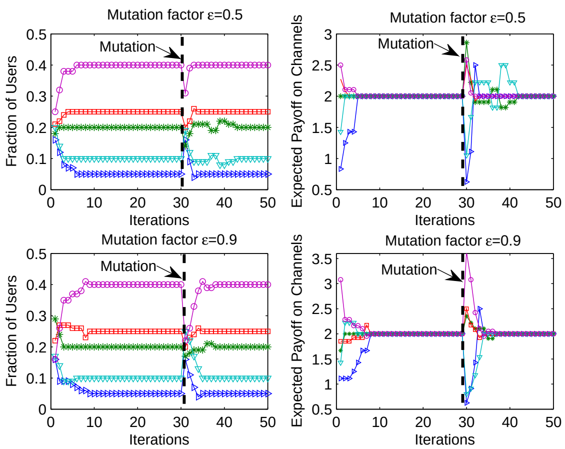

Fig. 2. Stability of the evolutionary spectrum access mechanism. Fraction of users in total $N = 100$ users who choose mutant channels randomly at time slot 30 equal to 0.5 and 0.9, respectively.

to recover the ESS $\boldsymbol{x}^*$ quickly after the mutation occurs. This demonstrates that the evolutionary spectrum access mechanism is robust to the perturbations in the network.

B. Learning Mechanism For Open Spectrum Access

We evaluate the learning mechanism for open spectrum access in this section. We implement the learning mechanism with the number of users $N = 100$ and $N = 200$, respectively. We set the length of a decision period $K = 50$ time slots, which provides a good estimation of the mean data rate. Since the learning mechanism always converges when $0 < \gamma < 1$ in the simulation, we set the memory factor $\gamma = 0.7$. Figure 3 shows the time average user distribution on the channels converges to the ESS $\boldsymbol{x}^* = \left(\frac{B_1}{\sum_{i=1}^{M} B_i}, ..., \frac{B_M}{\sum_{i=1}^{M} B_i}\right)$, and the time average user's payoff converges the expected payoff $U_n(a_n^*, \boldsymbol{x}^*) = \frac{\sum_{i=1}^{M} B_i}{N}$ at the ESS. Note that users achieve this result without prior knowledge of the statistics of the channels, and the number of users utilizing each channel keeps changing in the learning scheme.

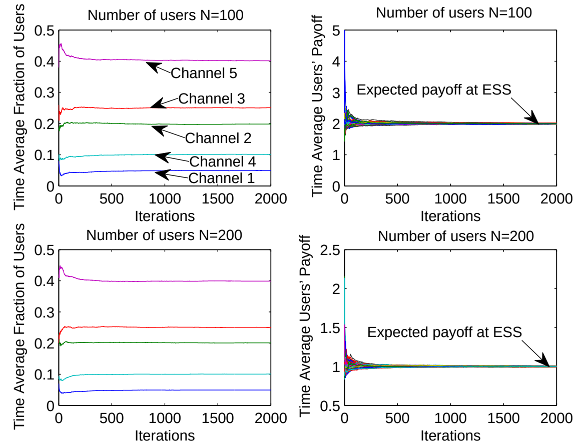

Fig. 3. Learning mechanism for open spectrum access with the number of users $N = 100$ and 200, respectively.

VII. Conclusion

In this paper, we study the problem of open spectrum access of multiple time-varying heterogeneous channels, and propose an evolutionary spectrum access mechanism based on evolutionary game theory. We show that the equilibrium of the mechanism is an evolutionarily stable strategy and is globally stable. We further propose a leaning mechanism, which requires no information exchange among the users. We show that the learning mechanism converges to the evolutionarily stable strategy on the time average. Numerical results show that the proposed mechanisms can achieve efficient and stable spectrum sharing among the users.

References

[1] W. Lehr and J. Crowcroft, "Managing shared access to a spectrum commons," in *IEEE DySPAN*, 2005.

[2] M. Liu and Y. Wu, "Spectrum sharing as congestion games," in *Annual Allerton Conference on Communication, Control, and Computing*, 2008.

[3] N. Nie and Comniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," in *IEEE DySPAN*, 2005.

[4] R. Southwell and J. Huang, "Convergence dynamics of resource-homogeneous congestion games," in *Proceddings of GameNets Conference*, 2011.

[5] L. M. Law, J. Huang, M. Liu, and S. Y. R. Li, "Price of anarchy of cognitive MAC games," in *IEEE GLOBECOM*, 2009.

[6] Z. Han, C. Pandana, and K. J. R. Liu, "Distributive opportunistic spectrum access for cognitive radio using correlated equilibrium and no-regret learning," in *IEEE WCNC*, 2007.

[7] M. Maskery, V. Krishnamurthy, and Q. Zhao, "Decentralized dynamic spectrum access for cognitive radios: Cooperative design of a non-cooperative game," *IEEE Transactions on Communications*, vol. 57, no. 2, pp. 459–469, 2009.

[8] H. Li, "Multi-agent q-learning for aloha-like spectrum access in cognitive radio systems," *IEEE Trans. on Vehicle Technology*, 2009.

[9] X. Chen and J. Huang, "Evolutionarily stable spectrum access in a many-user regime," Tech. Rep. [Online]. Available: http://home.ie.cuhk.edu.hk/~jwhuang/publication/EGTTechReport.pdf

[10] J. M. Smith, *Evolution and the theory of games*. Cambridge University Press, 1982.

[11] D. Niyato and E. Hossain, "Dynamics of network selection in heterogeneous wireless networks: an evolutionary game approach," *IEEE Transactions on Vehicular Technology*, vol. 58, pp. 2008–2017, 2009.

[12] Q. Zhang, H. Xue, and X. Kou, "An evolutionary game model of resources-sharing mechanism in p2p networks," in *IITA Workshop*, 2007.

[13] B. Wang, K. J. R. Liu, and T. C. Clancy, "Evolutionary game framework for behavior dynamics in cooperative spectrum sensing," in *IEEE GLOBECOM*, 2008.