Cognitive Mobile Virtual Network Operator: Investment and Pricing with Supply Uncertainty

Lingjie Duan*, Jianwei Huang*, Biying Shou†
*Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong
†Department of Management Sciences, City University of Hong Kong, Hong Kong
email:{dlj008,jwhuang}@ie.cuhk.edu.hk,biying.shou@cityu.edu.hk

***Abstract*—This paper presents the first analytical study of optimal investment and pricing decisions of a cognitive mobile virtual network operator (C-MVNO) under *spectrum supply uncertainty*. Compared with a traditional MVNO who only obtains spectrum by long-term leasing contracts, a C-MVNO can acquire short-term spectrum by both sensing the empty "spectrum holes" of licensed bands and dynamically leasing from the spectrum owner. As a result, a C-MVNO can make flexible investment and pricing decisions to match the current demands of the secondary unlicensed users. Spectrum sensing is typically cheaper than dynamic spectrum leasing, but the obtained useful spectrum amount is random due to primary licensed users' stochastic traffic. The C-MVNO needs to determine the optimal amounts of sensing and leasing spectrum, considering the trade-offs between cost and uncertainty. The C-MVNO also needs to determine the optimal retail price to sell the spectrum to the secondary unlicensed users, taking into account wireless heterogeneity of users such as different maximum transmission power levels and channel gains. We model and analyze these decisions and the interactions between the C-MVNO and secondary users as a multi-stage Stackelberg game. We show several interesting properties of the network equilibrium, such as threshold structures of the optimal investment and pricing decisions, independence between the optimal price and users' wireless characteristics, and fair and predictable spectrum allocations to the users. Compared with the traditional MVNO, spectrum sensing can significantly improve the C-MVNO's expected profit and users' payoffs.**

I. INTRODUCTION

Wireless spectrum is typically considered as a scarce resource, and thus is often allocated through static licensing. Field measurements show that, however, most spectrum bands are often under-utilized even in densely populated urban areas [1]. To resolve this problem, people have proposed using the cognitive radio technology [2], which enables a radio device to flexibly and intelligently change its operational parameters to match the radio environment. The device can then efficiently *sense* and utilize the otherwise wasted "spectrum holes" in time and space without violating the rights of the licensed users [3].

Another approach to achieve more efficient spectrum utilization is dynamic spectrum *leasing*, where a spectrum owner allows secondary network operators or secondary users to use its temporarily unused part of spectrum in exchange of monetary return ([4]–[6]). The dynamic spectrum leasing decisions can be made at a short time scale (comparable to the time scale for sensing) or even real-time ([7], [8]).

In this paper, we study the operation of a cognitive radio network that consists a cognitive mobile virtual network operator (C-MVNO) and a group of secondary unlicensed users. The C-MVNO serves as the interface between the traditional spectrum owner and the secondary users. The spectrum owner has the license of certain spectrum resource, part of which is used to serve the primary users. The C-MVNO can both dynamically lease the *temporarily unused spectrum* from the spectrum owner, and sense the *active bands* used by the primary users for spectrum holes. It then resells the spectrum (i.e., bandwidth) to the secondary users to maximize its profit.

Compared with a traditional MVNO who often obtain spectrum through long-term fixed contracts[1], the key feature of a C-MVNO is that it is able to acquire spectrum in the short-term through both sensing and dynamic leasing. Compared with leasing, the main advantage of spectrum sensing is its low cost. Since sensing is transparent to the spectrum owner, the main sensing cost for the C-MVNO is the sensing time and energy. This sensing cost is typically much lower than the leasing cost; the latter depends on the negotiation between the C-MVNO and the spectrum owner. On the other hand, the sensing result is often uncertain since the C-MVNO has to avoid conflict with the primary users' stochastic traffic [10]. It is thus critical for the C-MVNO to find the right balance between cost and uncertainty.

In this paper, we show that both the C-MVNO and the secondary users will significantly benefit from the availability of sensing. Our key results and contributions are summarized as follows (we will refer to C-MVNO simply as "operator" and secondary users as "users"):

- *A new dynamic decision model*: We model and analyze the interactions between the operator and the users as a four-stage Stackelberg game model. The operator moves first and makes the sensing, leasing, and pricing decisions sequentially. Users share the spectrum using frequency division multiplexing (FDM) or orthogonal frequency division multiplexing (OFDM). They then purchase bandwidth from the operator to maximize their payoffs. The payoff function incorporates the heterogeneity of the users in terms of transmission power levels and channel gains. Despite of the complexity of the model, we are able to fully characterize the unique equilibrium behav-

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[1]There are over 400 mobile virtual network operators owned by over 360 companies worldwide as of February 2009 [9].

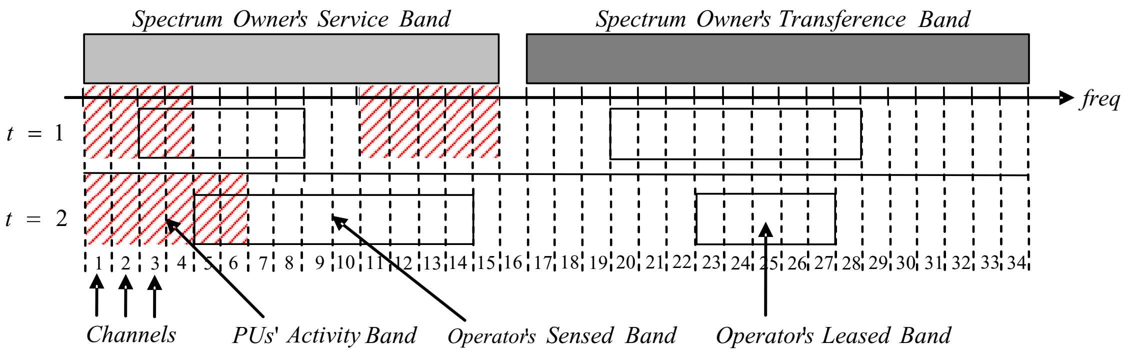

Fig. 1. Operator's Investment in Spectrum Sensing and Leasing

iors of the profit-maximizing operator and the payoff-maximizing users.

- *Threshold structures of the optimal investment and pricing decisions*: At the equilibrium, the operator will sense the spectrum if the sensing cost is cheaper than a threshold. It will lease additional spectrum only if the resource obtained through sensing is below a threshold. Finally, the operator will charge a constant price to the users as long as the total bandwidth obtained through sensing and leasing does not exceed a threshold.
- *Fair and predictable resource allocation*: The operator's optimal pricing decision is independent of the users' wireless characteristics. Each user receives a bandwidth allocation that is proportional to its channel gain and transmission power, which leads to the same signal-to-noise (SNR) for all users. As a result, a user can easily predict its Quality of Service without knowing any information of other users.
- *Impact of spectrum sensing*: We compare our results with the operation of a traditional MVNO who only leases spectrum. We show that the availability of sensing always increases the operator's profit in the *expected* sense, despite that the realized profit at a particular time will have some variations depending on the sensing result. Moreover, users can only get better payoffs when sensing is performed by the operator.

The rest of the paper is organized as follows. We introduce the network model and problem formulation in Section II. In Section III, we analyze the four-stage decision model through backward induction. We discuss various insights obtained from the equilibrium analysis together with various numerical examples in Section IV. In Section V, we show the impact of spectrum sensing on both the operator and the users. We conclude with some future research directions in Section VI.

A. Related Work

Some recent work that studied the interactions between cognitive network operators and the secondary users can be found in [8], [11]–[15]. [11] studied the competition among two or more cognitive service providers without considering the users' wireless modeling details. [14] derived users' demand functions based on the acceptance probability model. [15] explored demand functions in both quality-sensitive and price-sensitive buyer population models. Both [14], [15] obtained various interesting results mainly through extensive simulations. Some other work focused on the operators' investment in spectrum resource, and various auction mechanisms for dynamic spectrum allocation are proposed in [12], [13]. Only few papers (e.g., [8]) considered the operator's joint investment and pricing decision problem as we do in this paper. The key difference between our paper and the previous work is that we present the first analytical study that characterizes the optimal investment and pricing decisions under *unreliable* spectrum supply due to sensing.

Our unreliable sensing model is related to the random-yield model in supply chain management (e.g., [16]–[19]), where a supplier provides a random output dependent on the retailer's order size. The unique wireless aspects of the users' payoff functions and demand functions in our problem lead to a completely new solution structure.

II. Network Model

A. Background on Spectrum Leasing and Sensing

We assume that the spectrum owner divides its owned spectrum (bandwidth) into two portions:

- *Service Band*: This band is reserved for serving the spectrum owner's own primary users (PUs). Since the PUs' traffic is stochastic, there will be some unused spectrum which changes dynamically. The operator can sense and utilize the unused portions. There are no explicit communications between the spectrum owner and the operator for this band.
- *Transference Band*: The spectrum owner temporally does not need to use this band. The operator can lease the bandwidth through explicit negotiations with the spectrum owner. No sensing is allowed in this band.

We assume that the operator makes the sensing and leasing decisions in each short time slot. The length of a time slot is much larger than the coherence time of users' channel conditions, such that it is enough to consider the mean channel condition of each user.

The operator's spectrum leasing and sensing operations are illustrated in Fig. 1, where we give a concrete example to illustrate the dynamic opportunities for spectrum sensing, the

TABLE I
KEY NOTATIONS

Symbol	Physical Meaning
B_l	Leasing bandwidth
B_s	Sensing bandwidth
$\alpha \in [0,1]$	Sensing realization factor
C_l	(Unit) leasing cost
C_s	(Unit) sensing cost
$\mathcal{I} = \{1, \cdots, I\}$	Set of secondary users in the cognitive network
π	(Unit) price announced by operator
w_i	User i's bandwidth allocation
r_i	User i's data rate
$P_i^{\max}$	User i's maximum transmission power
h_i	User i's channel gain
n_0	Noise power per unit bandwidth
g_i	User i's wireless characteristic
$\mathtt{SNR}_i = g_i/w_i$	User i's SNR
$G = \sum_{i\in\mathcal{I}} g_i$	Users' aggregate wireless characteristics
R	Operator's profit

uncertainty of sensing outcome, and the impact of sensing on leasing decisions. We assume the bands are divided into small channels labeled as $1-34$. Channel 16 is the guard band between the service and transference bands.

- Time slot $t=1$: PUs' activities occupy channels $1-4$ and $11-15$. The operator does not know this information and senses channels $3-8$. As a result, it only obtains 4 useful channels ($5-8$). It leases additional 9 channels ($20-28$) from the transference band.
- Time slot $t=2$: PUs' activities occupy channels $1-6$. The operator senses channels $5-14$ and obtains 8 useful channels ($7-14$). It leases additional 5 channels ($23-27$) from the transference band.

In this paper we will study the operator's optimal decisions within a particular short time slot. The interaction across time slots is a subject for future research. In the rest of the paper, we will use three terms "spectrum", "bandwidth", and "resource" interchangeably.

B. Notations and Assumptions

We consider a cognitive network with one operator and a set $\mathcal{I} = \{1,\ldots,I\}$ of users. The key notations of this paper are listed in Table I. Some are explained as follows.

- *Investment decisions B_s and B_l*: operator's sensing and leasing bandwidths, respectively.
- *Sensing realization factor α*: it depends on PUs' stochastic traffic. The useful bandwidth obtained through sensing is $B_s\alpha$.
- *Cost parameters C_l and C_s*: operator's leasing and sensing costs per unit bandwidth, respectively. These are fixed system parameters. C_l is determined through the negotiation between the operator and the spectrum owner. C_s depends on the operator's own sensing technologies. When the operator senses spectrum, it needs to spend energy in channel sampling and signal processing ([20]). Sensing over differnt channels needs to be done sequentially due to the potential large number of channels open to sensing and the limited power and hardware capacity of cognitive radios ([21]). The larger sensing bandwidth, the longer time it takes which leads to higher energy cost ([22]). For simplicity, we assume that total sensing cost is linear in bandwidth B_s.
- *Price decision π*: operator's selling price per unit bandwidth to the users.

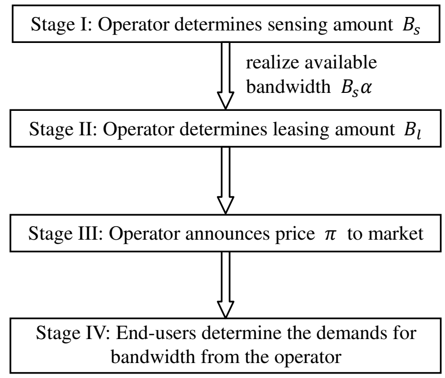

Fig. 2. Four-Stage Stackelberg Game

The following assumptions will be used in the paper:

Assumption 1: α follows a uniform distribution in $[0,1]$.

Uniform distribution allows us to obtain closed-form solutions of the optimal decisions. We can show that the main engineering insights still hold with arbitrary distributions of α, with details in our online technical report [23].

Assumption 2: The sensing cost is non-negligible and is lower bounded by $C_s \geq (1-e^{-2C_l})/4$.

Assumption 2 is used to avoid the trivial case where sensing is so cheap that it is optimal to sense a huge amount of bandwidth. The lower bound is an increasing function of leasing cost C_l. It approaches $C_l/2$ when C_l is small and approaches $1/4$ when C_l is large..

C. A Four-Stage Stackelberg Game

We consider a four-stage Stackelberg game between the operator and the users as shown in Fig. 2. As the Stackelberg leader, the operator first decides the sensing amount B_s in Stage I, and then decides the leasing amount B_l in Stage II based on the sensing result $B_s\alpha$. After that, it announces the price π to the users in Stage III. Finally, the users choose to purchase bandwidths to maximize their individual payoffs in Stage IV.

We note that "sensing followed by leasing" is optimal for the operator's profit maximization. Since sensing is cheaper than leasing, it is always beneficial for the operator to sense first, and only lease additional bandwidth if sensing result is not sufficient. It is straightforward to verify that other possibilities (e.g., leasing before sensing, or simultaneous leasing and sensing) can not increase the operator's profit.

Next we analyze the four-stage Stackelberg game by exploiting the Subgame Perfect Equilibrium (SPE, or simply as *equilibrium* in the rest of the paper). A general technique for determining the SPE is the backward induction ([24]). We start with Stage IV, then proceed to Stage III, and finally to Stages II and I. The backward induction captures the sequential

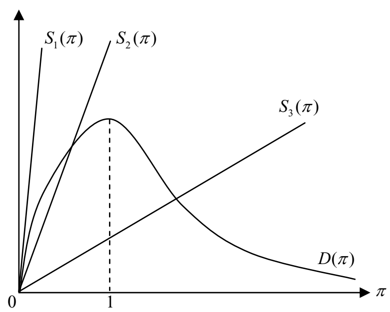

Fig. 3. Different Intersections of $D(\pi)$ and $S(\pi)$

dependence of decisions in four stages. *Most of the proofs are quite lengthy and are given in our online technical report [23].*

III. Backward Induction of the Four-Stage Game

A. Spectrum Allocation in Stage IV

In Stage IV, users determine their bandwidth purchase given the unit price π announced by the operator in Stage III.

We assume that end-users access the spectrum provided by the operator through FDM or OFDM to avoid interferences. User i's achievable data rate (in nats) is ([25]):

$$r_i(w_i) = w_i \ln\left(1 + \frac{P_i^{\max} h_i}{n_0 w_i}\right), \tag{1}$$

where w_i is the allocated bandwidth, $P_i^{\max}$ is user i's maximum transmission power, n_0 is the noise power per unit bandwidth, h_i is the channel gain between user i's own transmitter and receiver in a secondary ad hoc network (assuming frequency-flat fading). Let us define user i's wireless characteristic as $g_i := P_i^{\max} h_i / n_0$ and users' aggregate wireless characteristics as $G = \sum_{i\in\mathcal{I}} g_i$. Thus g_i/w_i is the user i's achievable signal-to-noise ratio (SNR) after he spreads the power evenly across the allocated bandwidth.

In practice, users often have limited choices of modulation and coding schemes. In this paper, we focus on the case where a user's receiver can only correctly decode the message if the SNR is sufficiently high. In this case, a user can only get a limited bandwidth w_i and thus the SNR (g_i/w_i) is naturally high. Under this high SNR assumption, we can approximate the rate in eq. (1) as

$$r_i(w_i) = w_i \ln\left(\frac{g_i}{w_i}\right).$$

The high SNR approximation enables us to obtain closed-form solutions. We will show later in Section IV that the users will indeed operate at the high SNR regime at the equilibrium.

A user i's payoff function is

$$u_i(\pi, w_i) = w_i \ln\left(\frac{g_i}{w_i}\right) - \pi w_i, \tag{2}$$

i.e., the difference between the data rate and the linear payment of purchasing bandwidth from the operator. Payoff $u_i(\pi, w_i)$ is concave in w_i, and the unique bandwidth demand that maximizes the payoff is

$$w_i^*(\pi) = \arg\max_{w_i \geq 0} u_i(\pi, w_i) = g_i e^{-(1+\pi)}, \tag{3}$$

which is positive, linear in g_i, and decreasing in price π. Recall that g_i is linear in channel gain h_i and transmission power $P_i^{\max}$. This means that a user with a better channel condition or a larger transmission power has a larger demand.

Equation (3) shows that every user achieves the same SNR:

$$\texttt{SNR}_i = \frac{g_i}{w_i^*(\pi)} = e^{(1+\pi)}.$$

User i's optimal payoff is

$$u_i(\pi, w_i^*(\pi)) = g_i e^{-(1+\pi)},$$

which is also linear in g_i. Finally, the total users' demand is

$$\sum_{i\in\mathcal{I}} w_i^*(\pi) = G e^{-(1+\pi)}. \tag{4}$$

B. Optimal Pricing Strategy in Stage III

In Stage III, the operator determines the optimal pricing strategy considering users' total demand (4), given the total bandwidth $B_s\alpha + B_l$ obtained in Stages I and II. The operator's profit is

$$R(B_s, \alpha, B_l, \pi) = \min\left(\pi \sum_{i\in\mathcal{I}} w_i^*(\pi), \pi\left(B_l + B_s\alpha\right)\right) - B_s C_s - B_l C_l, \tag{5}$$

which is the difference between the revenue and total cost. The $\min$ operation is due to the fact that the total demand can not be larger than the total bandwidth supply. The values of B_s, α, and B_l are already given in this stage. The objective of Stage III is to find the optimal price $\pi^*\left(B_s, \alpha, B_l\right)$ that maximizes profit, that is,

$$R_{III}(B_s, \alpha, B_l) = \max_{\pi \geq 0} R(B_s, \alpha, B_l, \pi). \tag{6}$$

The subscript "III" denotes the best profit in Stage III.

Since the only optimization variable in this Stage is price π, to solve (6) we only need to consider

$$\max_{\pi \geq 0} \min\left(\pi \sum_{i\in\mathcal{I}} w_i^*(\pi), \pi\left(B_l + B_s\alpha\right)\right). \tag{7}$$

The solution of (7) depends on the bandwidth investment in Stages I and II. Let us define $D(\pi) = \pi \sum_{i\in\mathcal{I}} w_i^*(\pi)$ and $S(\pi) = \pi(B_l + B_s\alpha)$. Figure 3 shows the three possible relationships between these two terms. Here $S_j(\pi)$ (for $j = 1, 2, 3$) represents three possible choices[2] of $S(\pi)$ depending on the bandwidth $B_l + B_s\alpha$:

- $S_1(\pi)$: No intersection with $D(\pi)$;
- $S_2(\pi)$: intersect once with $D(\pi)$ where $D(\pi)$ has a non-negative slope;
- $S_3(\pi)$: intersect once with $D(\pi)$ where $D(\pi)$ has a negative slope.

The first two cases correspond to the "Excessive Supply" regime, where $\max_{\pi\geq 0} \min\left(S(\pi), D(\pi)\right) = \max_{\pi \geq 0} D(\pi)$, i.e., the max-min solution occurs at the maximum point of $D(\pi)$ (in which case $\pi^* = 1$). The third case corresponds

[2]We ignore the common interactions of all curves at the origin.

TABLE II
OPTIMAL PRICING DECISION AND PROFIT IN STAGE III

Given Total Bandwidth After Stage II	Optimal Price $\pi^*(B_s, \alpha, B_l)$	Optimal Profit $R_{III}(B_s, \alpha, B_l)$
Excessive Supply Regime: $B_l + B_s\alpha \geq Ge^{-2}$	$\pi^{ES} = 1$	$R_{III}^{ES}(B_s, \alpha, B_l) = Ge^{-2} - B_sC_s - B_lC_l$
Conservative Supply Regime: $B_l + B_s\alpha < Ge^{-2}$	$\pi^{CS} = \ln\left(\frac{G}{B_l + B_s\alpha}\right) - 1$	$R_{III}^{CS}(B_s, \alpha, B_l) = (B_l + B_s\alpha)\ln\left(\frac{G}{B_l + B_s\alpha}\right) - B_s(\alpha + C_s) - B_l(1 + C_l)$

TABLE III
OPTIMAL LEASING DECISION AND PROFIT IN STAGE II

Given Sensing Result $B_s\alpha$ After Stage I	Optimal Leasing Amount B_l^*	Optimal Profit $R_{II}(B_s, \alpha)$
(CS1) $B_s\alpha \leq Ge^{-(2+C_l)}$	$B_l^{CS1} = Ge^{-(2+C_l)} - B_s\alpha$	$R_{II}^{CS1}(B_s, \alpha) = Ge^{-(2+C_l)} + B_s(\alpha C_l - C_s)$
(CS2) $B_s\alpha \in \left(Ge^{-(2+C_l)}, Ge^{-2}\right]$	$B_l^{CS2} = 0$	$R_{II}^{CS2}(B_s, \alpha) = B_s\alpha \ln\left(\frac{G}{B_s\alpha}\right) - B_s(\alpha + C_s)$
(ES3) $B_s\alpha > Ge^{-2}$	$B_l^{ES3} = 0$	$R_{II}^{ES3}(B_s, \alpha) = Ge^{-2} - B_sC_s$

to the "Conservative Supply" regime, where the max-min solution occurs at the unique intersection point of $D(\pi)$ and $S(\pi)$.

Based on this observation, *we can summarize the optimal pricing decision and the corresponding optimal profit of Stage III in Table II.*

Note that some bandwidth is left unsold at the optimal price in the Excessive Supply regime (i.e., $S(\pi^*) > D(\pi^*)$). This is because the acquired bandwidth is too large, and selling everything will lead to a very low price which hurts the total profit. Apparently the profit can be improved if the operator acquires less bandwidth in earlier investment stages. It is thus intuitive to believe that the equilibrium of the four-stage game must lie in the Conservative Supply regime if we have non-neglegible sensing cost as in Assumption 2. Later analysis in Stages II and I will confirm this intuition.

C. Optimal Leasing Strategy in Stage II

In Stage II, the operator decides the leasing amount B_l given the sensing result $B_s\alpha$. The objective is to solve

$$R_{II}(B_s, \alpha) = \max_{B_l \geq 0} R_{III}(B_s, \alpha, B_l).$$

We can decompose this problem into two subproblems based on the two supply regimes in Table II,

1) Choose B_l such that the total bandwidth falls into the Excessive Supply regime in Stage III:

$$R_{II}^{ES}(B_s, \alpha) = \max_{B_l \geq \max\{Ge^{-2} - B_s\alpha, 0\}} R_{III}^{ES}(B_s, \alpha, B_l). \tag{8}$$

2) Choose B_l such that the total bandwidth falls into the Conservative Supply regime in Stage III:

$$R_{II}^{CS}(B_s, \alpha) = \max_{0 \leq B_l \leq Ge^{-2} - B_s\alpha} R_{III}^{CS}(B_s, \alpha, B_l). \tag{9}$$

We can first solve subproblem (8), and prove that its optimal solution always lies at the lower boundary of the feasible set (i.e., $B_l^* = \max\left\{Ge^{-2} - B_s\alpha, 0\right\}$). In the case where $B_s\alpha \leq Ge^{-2}$, we can show that the optimal objective value of (8) is always no larger than the optimal objective value of (9), and thus it is enough to consider the Conservative Supply regime only. In the case where $B_s\alpha > Ge^{-2}$, the total bandwidth is already in the Excessive Supply regime as defined in Table II, and it is optimal to not to lease in Stage II.

Based on this observation, *the optimal leasing decision and the corresponding optimal profit in Stage II can be summarized in Table III.*

Table III contains three cases based on the value of $B_s\alpha$: (CS1), (CS2), and (ES3). The first two cases involve solving the subproblem (9) in the Conservative Supply regime, and the last one corresponds to the Excessive Supply regime. Although the decisions in cases (CS2) and (ES3) are the same (i.e., zero leasing), we still treat them separately since the profit expressions are different.

It is clear that we have an optimal *threshold* leasing policy here: the operator wants to achieve a total bandwidth equal to $Ge^{-(2+C_l)}$ whenever possible. When the bandwidth obtained through sensing is not enough, the operator will lease additional bandwidth to reach the threshold; otherwise the operator will not lease.

D. Optimal Sensing Strategy in Stage I

In Stage I, the operator needs to decide the optimal sensing amount to maximize its expected profit by taking the uncertainty of the sensing realization factor α into account. The operator needs to solve the following problem

$$R_I = \max_{B_{s \geq 0}} R_{II}(B_s),$$

where $R_{II}(B_s)$ is obtained by taking the expectation of α over the profit functions in Stage II (i.e., $R_{II}^{CS1}(B_s, \alpha)$, $R_{II}^{CS2}(B_s, \alpha)$, and $R_{II}^{ES3}(B_s, \alpha)$ in Table III).

To derive function $R_{II}(B_s)$, we will consider the following three intervals:

1) Case I: $B_s \leq Ge^{-(2+C_l)}$. In this case, we always have $B_s\alpha \leq Ge^{-(2+C_l)}$ for any value $\alpha \in [0, 1]$, which corresponds to case (CS1) in Table III. The expected profit is

$$\begin{aligned} R_{II}^1(B_s) &= E_{\alpha \in [0,1]}\left[R_{II}^{CS1}(B_s, \alpha)\right] \\ &= Ge^{-(2+C_l)} + B_s\left(\frac{C_l}{2} - C_s\right), \end{aligned}$$

which is a linear function of B_s.

TABLE IV
CHOICE OF OPTIMAL SENSING AMOUNT IN STAGE I

	Optimal Sensing Decision B_s^*	Expected Profit R_I
High Sensing Cost Regime: $C_s \geq C_l/2$	$B_s^* = 0$	$R_I^H = Ge^{-(2+C_l)}$
Low Sensing Cost Regime: $C_s \in \left[(1-e^{-2C_l})/4, C_l/2\right]$	$B_s^* = B_s^{L*}$, solution to eq. (11)	R_I^L in eq. (12)

2) Case II: $B_s \in \left(Ge^{-(2+C_l)}, Ge^{-2}\right]$. Depending on the value of α, $B_s\alpha$ can be in either case (CS1) or case (CS2) in Table III. The expected profit is

$$\begin{aligned} R_{II}^2(B_s) =& E_{\alpha\in\left[0,\frac{Ge^{-(2+C_l)}}{B_s}\right]}\left[R_{II}^{CS1}(B_s,\alpha)\right] \\ &+ E_{\alpha\in\left[\frac{Ge^{-(2+C_l)}}{B_s},1\right]}\left[R_{II}^{CS2}(B_s,\alpha)\right] \\ =& \frac{B_s}{2}\ln\left(\frac{G}{B_s}\right) - \frac{B_s}{4} \\ &+ \frac{B_s}{4}\left(\frac{Ge^{-(2+C_l)}}{B_s}\right)^2 - B_sC_s, \end{aligned}$$

which is a concave function of B_s.

3) Case III: $B_s \geq Ge^{-2}$. Depending on the value of α, $B_s\alpha$ can be any of the three cases in Table III. The expected profit is

$$\begin{aligned} R_{II}^3(B_s) =& E_{\alpha\in\left[0,\frac{Ge^{-(2+C_l)}}{B_s}\right]}\left[R_{II}^{CS1}(B_s,\alpha)\right] \\ &+ E_{\alpha\in\left[\frac{Ge^{-(2+C_l)}}{B_s},\frac{Ge^{-2}}{B_s}\right]}\left[R_{II}^{CS2}(B_s,\alpha)\right] \\ &+ E_{\alpha\in\left[\frac{Ge^{-2}}{B_s},1\right]}\left[R_{II}^{ES3}(B_s,\alpha)\right] \\ =& \left(\frac{G}{e^2}\right)^2\frac{e^{-2C_l}-1}{4B_s} - B_sC_s + \frac{G}{e^2}, \end{aligned}$$

which achieves it maximum at $B_s = Ge^{-2}$.

We can verify that Case II always achieves higher optimal profit than Case III, and thus there is no need to consider Case III with $B_s \geq Ge^{-2}$. This means that the optimal sensing will only lead to either case (CS1) or case (CS2) in Stage II, which corresponds to the Conservative Supply regime in Stage III. This confirms our previous intuition that equilibrium is always in the Conservative Supply regime since some resource is wasted in the Excessive Supply regime (see discussions in Section III-B).

To obtain the optimal profit R_I, we only need to maximize

$$R_{II}(B_s) = \begin{cases} R_{II}^1(B_s), & \text{if } 0 \leq B_s \leq Ge^{-(2+C_l)}; \\ R_{II}^2(B_s), & \text{if } Ge^{-(2+C_l)} < B_s \leq Ge^{-2}. \end{cases} \quad (10)$$

Figure 4 shows two possible shapes for the function $R_{II}(B_s)$. The vertical dashed line represents $B_s = e^{-(2+C_l)}$. For illustration purpose, here we assume $G=1$, $C_l=2$, and $C_s=\{0.8,1.2\}$. When the sensing cost is large (i.e., $C_s = 1.2 > C_l/2$), $R_{II}(B_s)$ achieves its optimum at $B_s = 0$ and thus it is optimal not to sense. When the sensing cost is small (i.e., $C_s = 0.8 < C_l/2$), $R_{II}(B_s)$ achieves its optimum at $B_s > e^{-(2+C_l)}$ and it is optimal to sense.

Based on this observation, *the optimal sensing decision and the corresponding optimal profit in Stage I can be summarized in Table IV.*

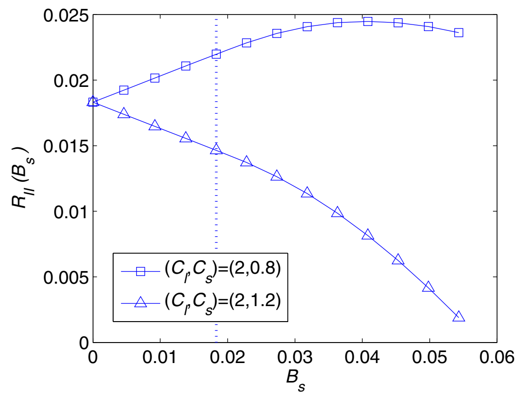

Fig. 4. Expected Profit in Stage II Under Different Sensing and Leasing Costs

Table IV shows that the sensing decision is made in the following two cost regimes:

- *High sensing cost regime* where $C_s > C_l/2$, it is optimal not to sense. Intuitively, the coefficient $1/2$ is due to the uniform distribution assumption of α, i.e., on average obtaining one unit of available bandwidth through sensing costs $2C_s$.
- *Low sensing cost regime* where $C_s \in \left[\frac{1-e^{-2C_l}}{4}, \frac{C_l}{2}\right]$: we prove that the optimal sensing amount B_s^{L*} is the unique solution to the following equation:

$$2\ln\left(\frac{G}{B_s}\right) - 3 - 4C_s - \left(\frac{G}{B_se^{(2+C_l)}}\right)^2 = 0. \quad (11)$$

We can further show that B_s^{L*} lies in the interval of $\left[Ge^{-(2+C_l)}, Ge^{-2}\right]$. Moreover, we can also show that B_s^{L*} is linear in G. Finally, the operator's optimal expected profit is

$$R_I^L = \frac{B_s^{L*}}{2}\ln\left(\frac{G}{B_s^{L*}}\right) - \frac{B_s^{L*}}{4} + \frac{1}{4B_s^{L*}}\left(\frac{G}{e^{2+C_l}}\right)^2 - B_s^{L*}C_s. \quad (12)$$

IV. EQUILIBRIUM SUMMARY AND NUMERICAL RESULTS

Based on the discussions in Section III, we summarize the operator's equilibrium sensing/leasing/pricing decisions and the equilibrium resource allocations to the users in Table V. Several interesting observations are as follows.

Observation 1: Both the optimal sensing amount B_s^* (equal to either 0 or B_s^{L*}) and leasing amount B_l^* are linear in the users' aggregate wireless characteristics $G = \sum_{i\in\mathcal{I}} P_i^{\max}h_i/n_0$.

The linearity enables us to normalize optimal leasing and sensing by users' aggregate wireless characteristics, and study

TABLE V
OPERATOR'S AND USERS' EQUILIBRIUM BEHAVIORS

Sensing Cost Regimes	High Sensing Cost: $C_s \geq \frac{C_l}{2}$	Low Sensing Cost: $\frac{1-e^{-2C_l}}{4} \leq C_s \leq \frac{C_l}{2}$	
Optimal Sensing Amount B_s^*	0	$B_s^{L*} \in \left[Ge^{-(2+C_l)}, Ge^{-2}\right]$, solution to eq. (11)	
Sensing Realization Factor α	$0 \leq \alpha \leq 1$	$0 \leq \alpha \leq Ge^{-(2+C_l)}/B_s^{L*}$	$\alpha > Ge^{-(2+C_l)}/B_s^{L*}$
Optimal Leasing Amount B_l^*	$Ge^{-(2+C_l)}$	$Ge^{-(2+C_l)} - B_s^{L*}\alpha$	0
Optimal Pricing π^*	$1 + C_l$	$1 + C_l$	$\ln\left(\frac{G}{B_s^{L*}\alpha}\right) - 1$
Expected Profit R_I	$R_I^H = Ge^{-(2+C_l)}$	R_I^L in eq. (12)	R_I^L in eq. (12)
User i's SNR	$e^{(2+C_l)}$	$e^{(2+C_l)}$	$\frac{G}{B_s^{L*}\alpha}$
User i's Payoff	$g_i e^{-(2+C_l)}$	$g_i e^{-(2+C_l)}$	$g_i(B_s^{L*}\alpha/G)$

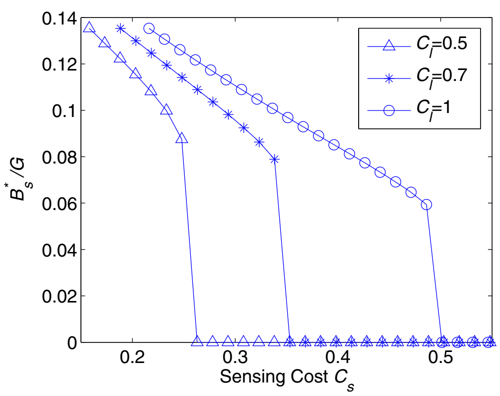

Fig. 5. B_s^* in terms of C_l/C_s.

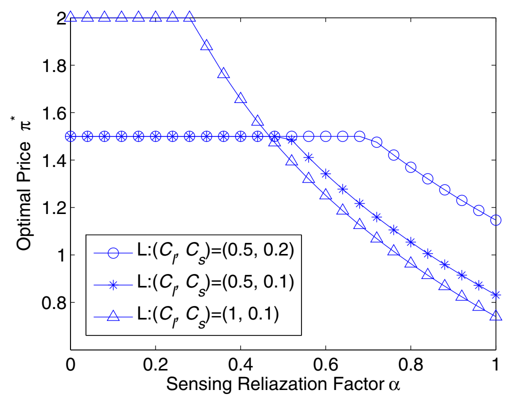

Fig. 7. π^* in terms of $C_l/C_s/\alpha$.

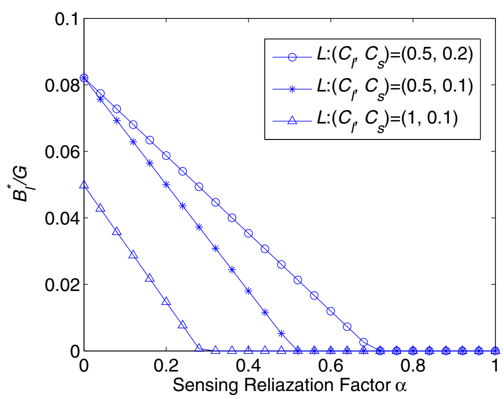

Fig. 6. B_l^* in terms of $C_l/C_s/\alpha$.

the relationships between the normalized optimal decisions and other system parameters as in Figs. 5 and 6.

Figure 5 shows how the normalized optimal sensing decision B_s^*/G changes with the costs. For a given leasing cost C_l, the optimal sensing decision B_s^* decreases as the sensing cost C_s becomes more expensive, and drops to zero when $C_s \geq C_l/2$. For a given sensing cost C_s, the optimal sensing decision B_s^* increases as the leasing cost C_l becomes more expensive, since sensing becomes relatively cheaper and more attractive.

Figure 6 shows how the normalized optimal leasing decision B_l^*/G depends on the costs C_l and C_s as well as the sensing realization factor α in the low sensing cost regime (denoted by "L"). In all cases, a higher value α means more bandwidth is obtained from sensing and there is less need to lease. Figure 6 confirms the threshold structure of the optimal leasing decisions in Section III-C, i.e., no leasing is needed whenever the bandwidth obtained from sensing reaches a threshold. Comparing different curves, we can see that the operator chooses to lease more as leasing becomes cheaper or sensing becomes more expensive. For the high sensing cost regime, the optimal leasing amount is only determined by C_l and independent of C_s and α, and thus is not shown here.

Observation 2: The optimal pricing decision π^* in Stage III is independent of users' aggregate wireless characteristics G.

Observation 2 is closely related to Observation 1. Since the total bandwidth is linearly proportional to the users' aggregate characteristics, the "average" resource allocation per user is "constant" at the equilibrium. This implies that the price must be independent of the user population change, otherwise the resource allocation to each individual user will change with the price accordingly.

Observation 3: The optimal pricing decision π^* in Stage III is non-increasing in α.

First, in the low sensing cost regime where the sensing result is poor (i.e., α is small as the third column in Table V), the operator will lease additional resource such that the total bandwidth reaches the threshold $Ge^{-(2+C_l)}$. In this case, the price is a constant and is independent of the value of α. Second, when the sensing result is good (i.e., α is large as in the last column in Table V), the total bandwidth is large enough. In this case, as α increases, the amount of total bandwidth increases, and the optimal price decreases to maximize the profit.

Figure 7 shows how the the optimal price changes with various costs and α in the low sensing cost regime. It is clear that the price is first a constant and then starts to decrease when α is larger than a threshold. The threshold decreases in the optimal sensing decision of B_s^{L*}: a smaller sensing cost or a higher leasing cost will lead to a higher B_s^{L*} and thus a smaller threshold.

Observation 4: At the equilibrium, the operator will sense the spectrum if the sensing cost is cheaper than a threshold. Furthermore, the operator will lease additional spectrum only if the spectrum obtained through sensing is below a threshold. Finally, the operator will charge a constant price to the users as long as the total bandwidth obtained through sensing and leasing does not exceed a threshold.

Observation 5: Each user i achieves the same SNR independent of g_i and a payoff linear in g_i.

Observation 5 shows that users achieve fair and predictable resource allocation at the equilibrium. In fact, a user does not need to know anything about the total number and payoffs of other users in the system. It can simply predict its QoS based on the cost structure of the network (C_s and C_l). Such property is highly desirable in practice.

Observation 6: Users achieve high SNR at the equilibrium.

At the equilibrium, any user i achieves an SNR equal to either $e^{(2+C_l)}$ or $G/(B_s^{L*}\alpha)$, both of which are larger than e^2. This means that the approximation ratio $\ln(\mathtt{SNR}_i)/\ln(1+\mathtt{SNR}_i)$ is larger than 94%, and can be close to one if the price π is high.

V. The Impact of Unreliable Sensing

The key difference between our model and most previous models (e.g., [8], [11]–[15]) is the possibility of obtaining resource through the cheaper but less reliable approach of spectrum sensing. Here we will elaborate the impact of sensing on the performances of the operator and the users by comparing with the baseline case where sensing is not possible. Note that in the high sensing cost regime it is optimal not to sense, as a result, the performance of the operator and users will be no different from the baseline case. Hence we will focus on the low sensing cost regime. We have the following observations.

Observation 7: The operator's optimal expected profit always benefits from the availability of spectrum sensing in the low sensing cost regime.

Figure 8 illustrates the normalized optimal expected profit as a function of the sensing cost. We assume leasing cost $C_l = 2$, and thus the low sensing cost regime corresponds to the case where $C_s \in [0.2, 1]$ in the figure. It is clear that sensing achieves a better optimal expected profit in this regime. In fact, sensing leads to 250% increase in profit when $C_s = 0.2$. The benefit decreases as the sensing cost becomes higher. When sensing becomes too costly, the operator will choose not to sense and thus achieve the same profit as in the baseline case.

Observation 8: The operator's realized profit (i.e., the profit for a given α) is a strictly increasing function in α in the low sensing cost regime. Furthermore, there exists a threshold $\alpha_{th} \in (0,1)$ such that the operator's realized profit is larger than the baseline approach if $\alpha > \alpha_{th}$.

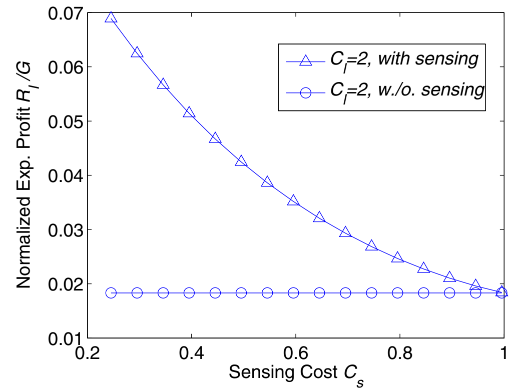

Fig. 8. Operator's Normalized Optimal Expected Profit over Different Sensing/Leasing costs. The baseline is the case without sensing.

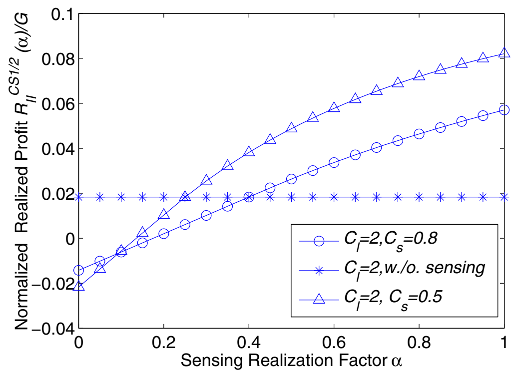

Fig. 9. Operator's Normalized Optimal Realized Profit in α. The baseline is the case without sensing.

The proof sketch of observation 8 is as follows. Based on Table V, we have the following two cases in the low sensing cost regime:

- If $\alpha \leq Ge^{-(2+C_l)}/B_s^{L*}$, then by substituting B_s^{L*} into $R_{II}^{CS1}(B_s,\alpha)$ in Table III, the optimal realized profit is
$$R_{II}^{CS1}(\alpha) = Ge^{-(2+C_l)} - B_s^{L*}C_s + B_s^{L*}\alpha C_l,$$
which is strictly increasing in α.
- If $\alpha \geq Ge^{-(2+C_l)}/B_s^{L*}$, then by substituting B_s^{L*} into $R_{II}^{CS2}(B_s,\alpha)$ in Table III, the optimal realized profit is
$$R_{II}^{CS2}(\alpha) = B_s^{L*}\alpha\left(\ln\left(\frac{G}{B_s^{L*}\alpha}\right) - 1\right) - B_s^{L*}C_s,$$
which can also be shown to be strictly increasing in α.

We can also verify that $R_{II}^{CS1}(\alpha) = R_{II}^{CS2}(\alpha)$ when $\alpha = Ge^{-(2+C_l)}/B_s^{L*}$. Therefore, the realized profit is a continuous and strictly increasing function of α.

Now let us consider the extreme case where $\alpha = 0$. Since the operator obtains no bandwidth through sensing but still incurs some cost, the profit in this case is lower than the baseline case. We also show that the realized profit at $\alpha = 1$ is always larger than the baseline case. Together with the continuity and strictly increasing nature of the realized profit function, we have proven the existence of threshold of α_{th}.

Figure 9 shows the realized profit as a function of α for different costs. The realized profit is increasing in α in both sensing cases. The "crossing" feature of the two increasing curves is because the optimal sensing B_s^* is larger under a cheaper sensing cost ($C_s = 0.5$), which leads to larger realized profit loss (gain, respectively) when $\alpha \to 0$ ($\alpha \to 1$, respectively). This shows the tradeoff between improvement of expected profit and the variability of the realized profit.

Observation 9: Users always benefit from the availability of spectrum sensing in the low sensing cost regime.

In the baseline approach without sensing, the operator always charges the price $1 + C_l$. As shown in Table V, the equilibrium price π^* with sensing is always no larger than $1 + C_l$ for any value of α. Since a user's payoff is strictly decreasing in price, the users always benefit from sensing.

VI. Conclusion and Future Work

The cognitive radio technology enables a mobile virtual network operator to obtain cheaper spectrum through sensing, but also brings uncertainty to the operator's spectrum supply. This paper proposes the first analytical study of the optimal investment (including leasing and sensing) and pricing strategies of a cognitive virtual network operator under spectrum supply uncertainty. We model the interactions between the operator and the users by a four-stage Stackelberg game, which captures the wireless heterogeneity of users due to different maximum transmission power levels and channel gains.

We have discovered several interesting properties of the game equilibrium. For example, the operator's optimal sensing, leasing, and pricing decisions all have nice threshold structures. We also show that the availability of sensing always increases the operator's expected profit, despite that the realized profit in a particular time slot will have some variations depending on the sensing result. Moreover, users always achieve better payoffs when sensing is performed.

In this paper, we have obtained closed-form solutions based on two assumptions: uniform distribution of the sensing realization factor α and high SNR of the users. In our online technical report [23], we have shown that all the major engineering insights (i.e., Observations 1 to 5 in Section IV) remain true when both assumptions are relaxed.

There are several possible ways to extend the results in this paper. For example, when dynamic leasing is done at a relatively large time scale compared with sensing, we need to study a new dynamic decision model with more stages and tight coupling across sequential sensing decisions. It is also interesting to consider the competition between several cognitive virtual network operators. Some preliminary results along the second direction have been obtained in [26].

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