Game Theoretical Analysis of Cognitive Radio Networks: An NCEL Perspective

(Invited Review Paper)

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***Abstract*—We provide an overview of the research activities related to game theoretical analysis of cognitive radio networks in Network Communications and Economics Lab (NCEL) at the Chinese University of Hong Kong. Our focus is to study how distributed and strategic users and networks interact in various new networking and business scenarios enabled by the cognitive radio technology. We will summarize the key recent research results related to hierarchical-access, dynamic exclusive use, and hybrid models. We will also highlight the research challenges for future research.**

I. Introduction

Wireless spectrum has been a tightly controlled resource worldwide since the early part of the 20th century. The traditional way of regulating the spectrum is to assign each wireless application its own slice of spectrum at a particular location. This means that every new commercial service, from satellite broadcasting to wireless local-area network, has to compete for licenses with numerous existing sources, creating a state of "spectrum drought" [1]. However, recent technology advances of smart technologies in software-defined, frequency-agile, or cognitive radios [2], [3], together with reforms of the government regulation policies, start to enable more flexible and efficient spectrum sharing.

In cognitive radio networks, wireless devices and networks can sense, adapt, and efficiently utilize the spectrum resource to achieve the communication targets. When end-users and network operators have individual selfish objectives, it is natural to analyze their interactions using game theory. Even when users want to cooperate, game theory still provides a powerful mathematical framework for designing distributed algorithms with fast convergence, robust performance, and limited information exchange requirements (e.g., [4]).

Various new networking and business models have been proposed to allow the coexistence of primary (licensed) and secondary (unlicensed) users in the same spectrum. The proposed mechanisms can be loosely characterized in three categories: open-sharing, hierarchical-access, and dynamic exclusive use ([5], [8], [13]–[17]). Open-sharing supports all users to share the spectrum resource with equal rights (i.e., without differentiating primary and secondary users). Hierarchical-access encourages the secondary users to access the spectrum without affecting the performance of the primary users. Dynamic exclusive use allows a primary user to dynamically transfer and trade the usage right of its licensed spectrum to secondary operators or users. Depending on the technology and policy considerations of a specific network scenario, one mechanism might be more suitable than the others.

In this paper, we provide an overview of various research activities related to the game theoretical analysis of cognitive radio networks in Network Communications and Economics Lab (NCEL) at the Chinese University of Hong Kong. Most of the research activities have been jointly pursued with distinguished collaborators from City University of Hong Kong, Ecole Polytechnique Fdrale de Lausanne, Northwestern University, Tsinghua University, University of Michigan, and The University of New South Wales. We will focus our discussions on the hierarchical-access models, dynamic exclusive models, and a hybrid of the above two models. We will both summarize the recent progresses and discuss the open issues for future research. Table I in the next page summarizes various results that have been studied by NCEL members.

In each of the following sections, we will try to answer the following two questions:

- What are the new networking and business models enabled by the cognitive radio technology?
- What are the impacts of the game theoretical interactions of users and networks on the network performance?

II. Hierarchical-Access

In the hierarchical-access models, the secondary users (SUs) are allowed to use the spectrum as long as their operations do not negatively impact the performance of the primary users (PUs). There are two basic forms of hierarchical-access: spectrum underlay and spectrum overlay. In spectrum underlay, the SUs transmit simultaneously with the PUs such that the total interference to the PUs is below a threshold. Our previous work in [8], [9] proposed several auction mechanisms to allocate allowable interference among a group of SUs. In

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TABLE I
SUMMARY OF COGNITIVE RADIO GAME MODELS STUDIED BY NCEL MEMBERS

Network Models		Open Access	Hierarchical Access		Dynamic Exclusive Use		Hybrid
			Overlay	Underlay	Leasing	Negotiation	
Game	Dynamics	Static			Dynamic		
	Models	Supermodular	Congestion	Auction	Multi-leader	Bargaining	Stackelberg
References		[5]	[6], [7]	[8], [9]	[10]	[11]	[12]
Sections in This Paper		–	II	–	III	–	IV

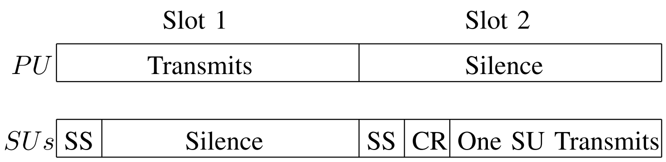

Fig. 1. Activities of PU and SUs of a channel in two time slots.

spectrum overlay, the SUs will sense, locate, and transmit in the "spectrum holes", i.e., the frequency-space-time blocks that are temporarily unused by the PUs.

Here we will focus on the discussions of spectrum overlay. Consider a network with a set $\mathcal{M} = \{1, \ldots, M\}$ of channels (owned by M PUs) and a set $\mathcal{K} = \{1, \ldots, K\}$ of SUs. The expected data rate of channel j perceived by SU k is R_{kj}[1]. We assume R_{kj}s for all SUs and channels are common knowledge.

The time is divided into discrete slots. The PU of a particular channel may transmit or remain silent in any given time slot. All SUs can hear each other, and only one SU can transmit during a time slot on one channel (if the corresponding PU is silent). For each channel, a time slot is divided into three phases: Sensing (SS), Contention Resolution (CR), and Transmission, as illustrated in Fig. 1.

- *Sensing (SS)*: At the beginning of a slot, SUs are refrained from transmitting in the channel and can only passively sense the channel for the presence of a PU.
- *Contention resolution (CR)*: If no PU's activity is detected during the sensing phase, a SU who has sensed this channel will start to count down from a random number chosen from a uniform distribution common to all SUs. It will continue to sense the channel for presence of other SUs.
- *Transmission*: A SU will proceed to transmit if its count-down timer expires and no other SUs have started the transmission on the channel. Otherwise, it will remain silent until the end of this current time slot.

A SU's goal is to choose a channel to sense in order to maximize his *expected* data rate. Such optimization not only depends on R_{ij}s but also on the number of users competing for the same channel. Since each idle channel is available to only one SU after the CR phase, each SU sensing channel j has an equal probability $1/n_j$ to succeed in his transmission, where n_j is the number of SUs sensing channel j. This motivates us to model and analyze the cognitive MAC game as a *congestion game* as follows.

Consider the game tuple $(\mathcal{K}, \mathcal{M}, (\Sigma_k)_{k\in\mathcal{K}}, (\pi_{kj})_{k\in\mathcal{K}, j\in\mathcal{M}})$, where $\Sigma_k = \mathcal{M}$ is the set of pure strategies for user k, and $\pi_{ij} = R_{kj}/n_j$ is the payoff (expected data rate) of SU k for sensing channel j. Each SU i wants to choose a channel to maximize his expected data rate, i.e., $\max_{j\in\Sigma_k} \pi_{kj}$. Since primary users do not appear explicitly in the description and analysis of the game, from now on we will simply use "user" to denote "secondary user". Notice that we focus SUs' interactions in a single time slot, and thus the game is a *static game with complete information*.

A pure strategy profile is given by $\boldsymbol{\sigma} = (\sigma_1, ..., \sigma_N)$, where $\sigma_k \in \Sigma_k$ denotes the channel that user i senses. The set of strategy profiles is denoted by $\Pi = \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_K$. The profile $\boldsymbol{\sigma}$ is a *Nash Equilibrium* if and only if no user can improve his payoff by deviating unilaterally, i.e.,

$$\frac{R_{k\sigma_k}}{n_{\sigma_k}} \geq \frac{R_{kl}}{n_l + 1}, \quad \forall k \in \mathcal{K}, \ \forall l \neq \sigma_k.$$

Since our cognitive MAC game belongs to the class of *congestion games* [18], [19], it always has a pure Nash Equilibrium. It is quite common, however, to have multiple Nash Equilibria in a congestion game. Given at least one Nash Equilibrium exists in our game, a natural question to ask is how well this Nash Equilibrium performs. Let us first represent the total expected rate received by all users at a given Nash Equilibrium $\boldsymbol{\sigma}$ as

$$SUM(\boldsymbol{\sigma}) = \sum_{k\in\mathcal{K}} \pi_{k\sigma_k}.$$

The social optimum *opt* of the game is defined to be the maximum total rates received by all N users[2]. Similar as the Nash equilibria, there can be multiple sensing choices that lead to the same social optimum.

Definition 1 (Efficiency Ratio): The efficiency ratio of a Nash Equilibrium $\boldsymbol{\sigma}$ is the ratio between the total expected rate received at that equilibrium and the social optimum,

$$ER(\boldsymbol{\sigma}) = SUM(\boldsymbol{\sigma})/opt.$$

[1]Different SUs may achieve different data rates on the same channel due to different transmission technologies and channel conditions.

[2]Such social optimum could be achieved, for example, through a coordinated random access protocols (i.e., a centralized scheduler tells each user which channel to sense). We notice that each user still chooses one channel to sense, thus contention can not be completely avoided even at a socially optimal solution.

TABLE II
PRICE OF ANARCHY FOR VARIOUS COGNITIVE MAC GAMES [6]

Games	Symmetric		Asymmetric
	Identical	General	General
PoA	1	$\frac{K}{K+\min(K,M)-1}$	$\frac{1}{K}$

Definition 2 (PoA): The price of anarchy of a family of games is the worst-case efficiency ratio among all pure strategy Nash Equilibria,

$$PoA = \min_{\boldsymbol{\sigma}\in\Pi} SUM(\boldsymbol{\sigma})/opt = \min_{\boldsymbol{\sigma}\in\Pi} ER(\boldsymbol{\sigma}).$$

To facilitate the study of PoA, we classify the cognitive MAC game into several classes of games depending on the heterogeneity of users and channels.

- *Symmetric game*: all users have the same expected rate for any given channel, i.e., $R_{kj} = R_j$ for all user $k \in \mathcal{K}$ and all channel $j \in \mathcal{M}$.
- *Identical game*: a special case of symmetric game where all channels are the same, i.e., $R_{kj} = r$ for all user $k \in \mathcal{K}$ and all channel $j \in \mathcal{M}$.
- *Asymmetric game*: the most general case where R_{kj} can be different for each k and j.

Theorem 1 (Price of Anarchy [6]): The PoA for different cognitive MAC games are summarized in Table II.

Several remarks are in order.

Remark 1: (Symmetric Game) For general symmetric game case, the PoA can be close to one if the number of users K is much larger than the number of channels M. In this case, all channels are sensed by some users. The proof in [6] also suggests that the worst case efficiency loss happens when all users choose to sense one single "big" channel at the Nash Equilibrium. This motivates us to engineer the system (if possible) such that channels are of similar rates in a cognitive network.

Remark 2: (Asymmetric Game) The PoA in the asymmetric game is independent of the number of channels M. To improve the PoA, we may want to limit the number of users (K) competing in a network. In the scenario where we have a large population of SUs competing for the access of channels, we may want to segment them into several different systems (i.e., each user is only allowed to choose from a subset of channels instead of all channels). Moreover, channels of similar rates should be put in the same system in order to avoid the case where all users sense the same big channel.

One key challenge to generalize the above results is to consider more complicated contention/interference relationship among users. In particular, the value of wireless network resources is often location dependent. Users' locality affects the individually perceived interference; far-way users may share the same spectrum without any loss in performance due to spatial reuse. Furthermore, the achievable data rate is a function of the received SINR, which in turn depends on the aggregated interferences from other users/transmitters, not just the number of users as in the standard congestion game. Finally, users can use different random access mechanisms (e.g., slotted aloha) and thus achieve different rates than the ones specified here. Some of the results along these lines have been reported in [7], [20].

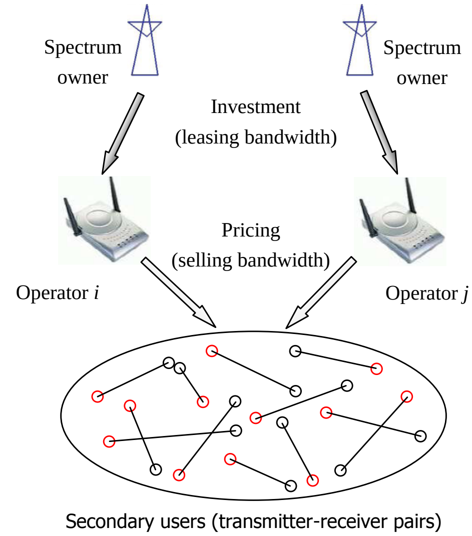

Fig. 2. Network model for the cognitive network operators.

III. DYNAMIC EXCLUSIVE USE

With dynamic exclusive use, the PUs can communicate with the SUs and allow the SUs to share the spectrum resource in exchange of momentary returns. We will focus our study on the approach of "dynamic spectrum leasing" [16], [17], [21]. We will study two secondary cognitive radio network operators, who lease the temporarily unused spectrum resource from the spectrum owners and then compete to serve a group of SUs. These operators serve as the interface between the spectrum owners and the SUs and are called as Cognitive Mobile Virtual Network Operators (C-MVNOs). Compared with a MVNO, a C-MVNO here can dynamically adjust its decisions based on the short-term change of users' demands. A traditional MVNO, however, is often stuck in a long-term contract and thus much less flexible.

We consider two secondary network operators ($i, j \in \{1,2\}$ and $i \neq j$) and a set $\mathcal{K} = \{1,\ldots,K\}$ of users, as shown in Fig. 2. The operators obtain wireless spectrum from different spectrum owners with different unit costs C_i and C_j, respectively, and compete to serve the same set $\mathcal{K}$ of SUs (i.e., duopoly competition). Each user k represents a dedicated transmitter-receiver pair with the channel gain h_k.

The interactions between the two operators and users can be modeled as a *three-stage multi-leader dynamic game*. The operators are the leaders, who simultaneously determine their leasing bandwidths B_i and B_j in Stage I. They then simultaneously announce the prices π_i and π_j to the users in Stage II. Finally, each user k chooses to purchase bandwidth

TABLE III

DUOPOLY'S AND USERS' BEHAVIORS AT THE SUBGAME PERFECT EQUILIBRIUMS (SPES) (ASSUMING $C_i \leq C_j$) [12]

Costs regimes	Low aggregate costs: $C_i + C_j \leq 1$	High and comparable costs: $C_i + C_j > 1$ and $C_j - C_i \leq 1$	Incomparable costs: $C_j > 1 + C_i$
Number of SPEs	Infinite	Unique	Unique
Leasing equilibria (B_i^*, B_j^*)	$B_i^* \geq C_j G e^{-2}$, $B_j^* \geq C_i G e^{-2}$, and $B_i^* + B_j^* = Ge^{-2}$	$\left(\frac{(1+C_j-C_i)G}{2e^{\frac{C_i+C_j+3}{2}}}, \frac{(1+C_i-C_j)G}{2e^{\frac{C_i+C_j+3}{2}}} \right)$	$(Ge^{-(2+C_i)}, 0)$
Pricing equilibrium (π_i^*, π_j^*)	$(1,1)$	$\left(\frac{C_i+C_j+1}{2}, \frac{C_i+C_j+1}{2} \right)$	$(1+C_i, N/A)$
Profits (R_i, R_j)	$R_i = B_i^*(1-C_i)$, $R_j = B_j^*(1-C_j)$	$R_i = \left(\frac{1+C_j-C_i}{2}\right)^2 Ge^{-\left(\frac{C_i+C_j+3}{2}\right)}$, $R_j = \left(\frac{1+C_i-C_j}{2}\right)^2 Ge^{-\left(\frac{C_i+C_j+3}{2}\right)}$	$\left(Ge^{-(2+C_i)}, 0\right)$
User k's SNR	e^2	$e^{\frac{C_i+C_j+3}{2}}$	e^{2+C_i}
User k's payoff	$g_k e^{-2}$	$g_k e^{-\left(\frac{C_i+C_j+3}{2}\right)}$	$g_k e^{-(2+C_i)}$

either w_{ki} or w_{kj} from *only one operator* to maximize its payoff in Stage III.

We assume that users share the spectrum using FDM (frequency division multiplexing) to avoid mutual interferences. If a user k obtains bandwidth w_{ki} from operator i, then it achieves a data rate (in nats) of [22]

$$r_k(w_{ki}) = w_{ki} \ln \left(1 + \frac{P_k^{\max} h_k}{n_0 w_{ki}} \right), \tag{1}$$

where $P_k^{\max}$ is user k's maximum transmission power, n_0 is the noise power per unit bandwidth, h_k is the channel gain between user k's transmitter and receiver (assuming frequency-flat fading). Here we assume that user k spreads its power $P_k^{\max}$ across the entire allocated bandwidth w_{ki}. To simplify later discussions, we let

$$g_k = P_k^{\max} h_k / n_0,$$

thus g_k/w_{ki} is the user k's signal-to-noise ratio (SNR). The rate in (1) is calculated based on the Shannon capacity. We further define $G = \sum_{k \in \mathcal{K}} g_k = \sum_{k \in \mathcal{K}} P_k^{max} h_k / n_0$ as the *aggregate wireless characteristics* of the user population.

In practice, users often have limited choices of modulation and coding schemes. We will focus on the case where a user's receiver can only correctly decode the message if the SNR is sufficiently high (i.e., $\texttt{SNR} \gg 1$). Under this high SNR assumption, we can approximate the rate in eq. (1) as

$$r_k(w_{ki}) = w_{ki} \ln \left(\frac{g_k}{w_{ki}} \right).$$

A user k receives the following payoff by purchasing resource w_{ki} from operator i,

$$u_k(\pi_i, w_{ki}) = w_{ki} \ln \left(\frac{g_k}{w_{ki}} \right) - \pi_i w_{ki}. \tag{2}$$

The user will choose to purchase from one of the operators who leads to a larger payoff, depending on prices π_i and π_j.

For an operator i, it wants to choose bandwidth B_i and price π_i to maximize its profit,

$$\max_{B_i} \left[\max_{\pi_i} \ \pi_i \min \left(Q_i \left(\pi_i, \pi_j \right), B_i \right) \right] - B_i C_i,$$

where $Q_i(\pi_i, \pi_j)$ is the total demand that users request from operator i and is determined jointly by the prices of both operators.

Our solution concept for the dynamic game is the subgame perfect equilibrium (SPE), which constitutes a Nash equilibrium for every subgame[3]. All SPEs of the game are summarized as follows.

Theorem 2 (Equilibrium Behavior [12]): The two operators' leasing decisions, pricing decisions, and profits as well as the users' SNRs and payoffs at all SPEs are summarized in Table III.

Several interesting remarks are in order.

Remark 3: Operators' investment decisions B_i^* and B_j^* are linear in G, and thus increasing in the user population, users' channel gains, and transmission powers.

Remark 4: The equilibrium prices $\left(\pi_i^*, \pi_j^*\right)$ are always symmetric (when both operators announce positive prices) and independent of G. This shows that the "average" resource allocation per user is "constant" at the equilibrium and does not change with the user population.

Remark 5: Each user $k \in \mathcal{K}$ achieves the same SNR independent of g_k, and achieves a payoff linear in g_k. In fact, a user doesn't need to know anything about the total number and payoffs of other users in the network. It can simply predict its QoS if it knows C_i and C_j.

We are also interested in understanding the impact of competition on the operators' profits and users' payoffs. As a benchmark, we will consider the *coordinated* case where there is a single entity controlling the investment and pricing decisions of both operators to maximize the total profit. We can define Price of Anarchy similar as in Section II by

[3]Our game has three subgames: Stage III alone, Stages II and III together, and the whole game including all three stages.

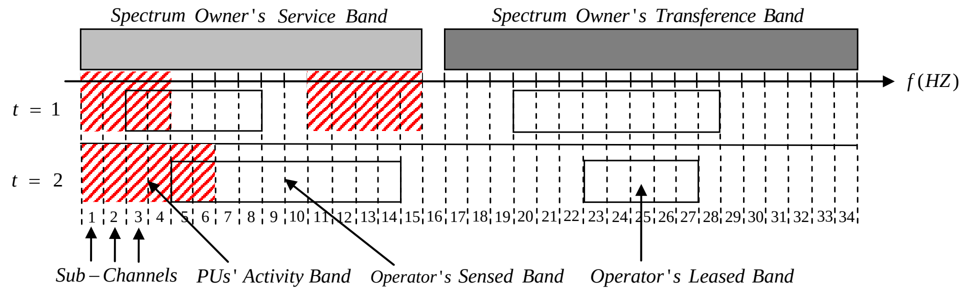

Fig. 3. Operator's Investment in Spectrum Sensing and Leasing

comparing the total profits achieved in the duopoly case and the coordinated case.

Theorem 3 (Price of Anarchy [12]): The PoA of the duopoly competition is 0.75 and is achieved in the low aggregate cost regime ($0 < C_i + C_j \leq 1$).

Theorem 3 suggests that the most intensive competition happens when the costs for obtaining bandwidth are low for both operators. In that case, both operators compete fiercely to attract the users and thus lead to the most efficiency loss.

The competition, on the other hand, always leads to the same or higher payoffs for all users compared with the coordinated case.

In the above analysis, we assume that users' channel conditions are fixed. This is obviously a simplified assumption, since the channel conditions in general depend on the transmission frequency band. With the heterogeneity of the channel conditions, the pricing equilibrium will mostly likely not be symmetric anymore. We can also consider the case where the operators can obtain resource through both spectrum leasing (dynamic exclusive use model) and spectrum sensing (spectrum overlay model). Next section is along the second direction focusing on a single operator.

IV. Hybrid Access

Here we will consider a C-MVNO who obtains spectrum resource through both sensing and leasing from the spectrum owner, and then sell the resource to the SUs. When the operator senses spectrum, it needs to spend time and energy on channel sampling and signal processing. Sensing over different channels often needs to be done sequentially due to the potentially large number of channels open to opportunistic spectrum access and the limited power/hardware capacity of cognitive radios. The larger sensing bandwidth and the more channels, the longer time and higher energy it requires ([23]). For simplicity, we assume that sensing cost is linear in the sensing bandwidth B_s with unit cost C_s. Leasing cost C_l is determined through the negotiation between the operator and the spectrum owner and is assumed to be larger than C_s.

As an example, we consider a spectrum owner who divides its licensed spectrum into two types as in Fig. 3:

- *Service Band*: This band is reserved for serving the spectrum owner's PUs. Since the PUs' traffic is stochastic, there will be some unused spectrum dynamically changing over time. The operator can sense and utilize the unused portions. There are no explicit communications between the spectrum owner and the operator.
- *Transference Band*: The spectrum owner temporarily does not use this band. The operator can lease the bandwidth through explicit communications with the spectrum owner. No sensing is allowed in this band.

Due to the short-term property of both sensing and leasing, the operator needs to make both the sensing and leasing decisions in each time slot. In Fig. 3, the spectrum owner's entire band is divided into small 34 channels[4]. In time slot 1, PUs use channels $1-4$ and $11-15$. The operator is unaware of this and senses channels $3-8$. As a result, it obtains 4 unused channels $(5-8)$. It leases additional 9 channels $(20-28)$ from the transference band. In time slot 2, PUs change their behavior and use channels $1-6$. The operator senses channels $5-14$ and obtains 8 unused channels $(7-14)$. It leases additional 5 channels $(23-27)$ from the transference band. Our analytical study will focus on the operator's optimal decisions in a single time slot.

We consider a cognitive network with one operator and a set $\mathcal{K} = \{1, \ldots, K\}$ of SUs. We model the interactions as a *four-stage dynamic Stackelberg game*. The operator (leader) determines the amount of bandwidth to be sensed, B_s, during the Stage I. Due to the stochastic nature of the spectrum owner's traffic, the actually useable bandwidth is $B_s\alpha$, where α is a random variable in $[0, 1]$. The operator then determines the leasing bandwidth amount, B_l, in Stage II. During Stage III, the operator determines the linear unit price π to sell the bandwidth $B_s\alpha + B_l$. Finally in Stage IV, each user (follower) k determines its bandwidth demand w_k to maximize its surplus $w_k \ln\left(1 + P_k^{\max} h_k / \left(n_0 w_k\right)\right) - \pi w_k$. Again we define $g_k = P_k^{\max} h_k / n_0$ and $G = \sum_{k \in \mathcal{K} g_k}$. Denote the total demand from the users as $Q(\pi)$. The operator will choose the sensing, leasing, and pricing decisions to maximize its profit,

$$\max_{B_s} E_\alpha \left[\max_{B_l} \left[\max_{\pi} \ \pi \min\left(Q(\pi), B_s\alpha + B_l\right) \right] - C_s B_s - C_l B_l \right].$$

The solution concept is again SPE. We can show that there exists a unique SPE for the Stackelberg game, with structures

[4] Channel 16 is the guard band between the service and transference bands.

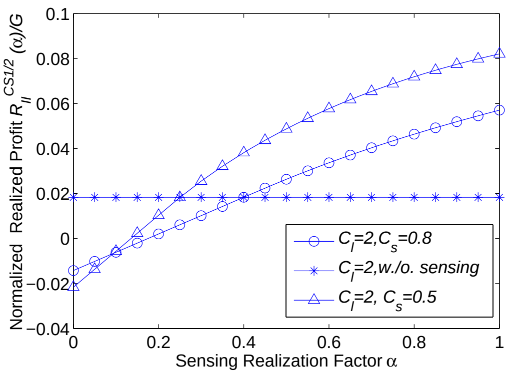

Fig. 4. Operator's normalized optimal realized profit as a function of the sensing realization factor α. The baseline is the case without sensing.

similar as the ones for the duopoly competition case.

Theorem 4 (Equilibrium Behavior [10]): At the unique SEP, the sensing and leasing decisions B_s^* and B_l^* are both linear in G, the price π^* is independent of G and is non-increasing in the realization of α, and each user i receives the same SNR.

Next we will compare with the SPE with the baseline approach where sensing is not allowed.

Theorem 5 (Sensing Realization Factor [10]): The operator's realized profit (i.e., the profit for a given α) is a non-decreasing function in α. Furthermore, there exists a threshold $\alpha_{th} \in (0,1)$ such that the operator's realized profit is larger than the baseline approach if $\alpha > \alpha_{th}$.

Figure 4 shows the realized profit as a function of α for different costs together with the benchmark approach (w./o. sensing). The "crossing" feature of the two increasing curves is due to the fact that the optimal sensing amount B_s^* is larger under a cheaper sensing cost ($C_s = 0.5$), which leads to larger realized profit loss (gain, respectively) when $\alpha \rightarrow 0$ ($\alpha \rightarrow 1$, respectively). This shows the tradeoff between improvement of expected profit and the large variability of the realized profit.

For the users, we can show that their profits are the same or higher compared with the benchmark approach where no sensing is performed.

One way to generalize the above model is to consider the case where the spectrum owner also wants to maximize its revenue by optimizing the leasing cost C_j. Then the game becomes a five-stage dynamic game.

V. Conclusion

In this paper, we provide an overview of various research activities related to the game theoretical analysis of cognitive radio networks in Network Communications and Economics Lab (NCEL) at the Chinese University of Hong Kong. We have discussed three models in details: spectrum overlay in the hierarchical-access model, spectrum leasing in the dynamic exclusive use model, and a hybrid model with both spectrum sensing and leasing. In all models, we have described the new networking and business models induced by the cognitive radio technology, as well as the impacts of the game theoretical interactions of the users and networks on the network performance. We have also outline various directions to generalize the existing studies. For more information about the related research, please also check http://ncel.ie.cuhk.edu.hk.

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