Energy Conservation and Interference Mitigation: From Decoupling Property to Win-Win Strategy

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***Abstract*— This paper studies the problem of conserving the energy of mobile terminals in a multi-cell TDMA networks supporting bursty real-time sessions. The associated optimization problem involves joint scheduling, rate control, and power control. To tackle the complexity, we propose a decomposition method that decouples the overall problem into two sub-problems: intra-cell energy optimization and inter-cell interference control. This decomposition results in a "win-win" situation: it reduces the energy consumptions and inter-cell interference at the same time. Simulations show that our decomposition method can achieve an energy reduction of more than 70% compared to the simplistic maximum transmit power policy. It can achieve an energy reduction of more than 50% compared to the case where only intra-cell energy optimal transmission is performed. We also derive an interesting decoupling property assuming that the interference power stays constant over a TDMA frame: if the idle power consumption of mobile terminals is no less than their circuit power consumption during transmission, or when both are negligible, then the energy-optimal transmission rates of the mobile terminals are independent of the inter-cell interference power level.**

***Index Terms*— Energy-efficiency, power control, rate control, scheduling, multi-cell wireless system.**

I. Introduction

Green wireless is an important area of focus to reduce the carbon footprint and energy consumption of information technology (IT) industry. There are more than 4 billion mobile phones in the world [1] and wireless devices and equipment consume 9% of the total energy of IT: as much as 6.1 TWh/year [2]. Future wireless systems such as 3GPP-LTE or WiMAX2 are evolving to support mobile broadband services that demand higher capacity. In most cases, this is achieved at the expense of higher energy consumption. Besides the environmental concern, there is also the need to lengthen the battery lifetime of these mobile terminals.

Since RF transmission consumes a significant amount of energy of mobile terminals, a focus will be on reducing their transmit energy consumption. This paper is an attempt toward that direction. The problem we focus on is as follows:

Consider a time-division-multiple-access (TDMA) cellular network. In each cell, a base station serves a number of mobile terminals. The transmissions of these terminals do not overlap in time. However, the transmissions of terminals in different cells may overlap and interfere with one another. Each terminal has a certain traffic requirement, specified in terms of number of bits per TDMA frame. How do we schedule the transmissions so as to minimize the total energy consumption while satisfying the traffic requirements of all users?

The gist of the problem is as follows. In the absence of interference, for a transmission, Shannon's capacity formula states that $x = w \log\left(1 + \frac{p \cdot G}{\sigma^2}\right)$, where x is the data rate, w is the bandwidth, p is the transmit power, G is the channel gain, and σ^2 is the noise power. Suppose that the transmission is turned on for T seconds within a frame. Then, the number of bits delivered per frame is $b = xT = wT \log\left(1 + \frac{E \cdot G}{T\sigma^2}\right)$, where E is the energy consumption per frame. From this expression, we immediately see a tradeoff between T and the transmit energy E. Specifically, if we allow more time to deliver b bits, we can reduce the transmit energy. The interplay between different transmissions enter the picture in two ways:

1) Within a cell, each frame has a finite amount of airtime. If one transmission uses more airtime to reduce its transmit energy, there is less airtime left for other transmissions. Thus, their transmit energies trade off against each other.
2) Across cells, the interference to a transmission depends on simultaneous transmissions in other cells. Scheduling the simultaneous transmissions in a judicious manner can reduce their mutual interference. This in turn will have the effect of reducing the energy requirements. This can be intuitively seen from $b = wT \log\left(1 + \frac{E \cdot G}{T(\sigma^2 + q)}\right)$, where q is the interference; that is, all things being equal, there is a tradeoff between E and q.

Thus, to minimize the total energy consumption, we not only have to consider the airtime devoted to each transmission, but also the scheduling of the transmissions. In practice, besides the transmit energy E, wireless devices also consume circuit energy when they transmit, and "idle" energy when they do not. The relative magnitudes of these energies have a subtle but important effect on the solution to our problem.

As will be elaborated in Section II, finding the optimal solution to the above problem is non-trivial. In this paper, we propose a decomposition method that decouples the overall

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problem into two sub-problems along the line of 1 and 2 above. That is, we first consider the sub-problem of intra-cell airtime allocation, assuming interference is constant throughout a frame. After the airtimes (and target SINRs) for the transmissions in different cells are fixed, we then consider the scheduling of the transmissions of the overall network, as well as setting the transmit powers of the terminals to fulfill the target SINRs. Based on the solution to the second sub-problem, we then adjust the inter-cell interferences to the different cells and solve the first sub-problem again. The process is iterated by alternating between these two modules.

The solution found by this decomposition method is guaranteed to be a feasible solution, albeit not necessarily an optimal one. Simulations indicate that this decomposition method can achieve energy reduction of more than 70% compared with the simplistic scheme in which all terminals use a common maximum transmit power. We also derive an interesting decoupling property under the assumption that the inter-cell interference power stays constant over a TDMA frame: if the idle power consumption of mobile terminals is no less than their circuit power consumption, or when both are negligible, then the energy-optimal transmission rates of the mobile terminals are independent of the inter-cell interference power level.

II. System Model

We consider energy efficient uplink communications in wireless cellular networks. Within each cell, the mobile terminals send traffic to the same base station (BS) via Time Division Multiple Access (TDMA). The time is divided into fixed length frames. Within a frame, each mobile terminal is allocated a dedicated time period, during which it is the only uplink transmitter within the cell. There is no interferences among different mobile terminals in the same cell. The concurrent transmissions of different mobile terminals at different cells, however, lead to inter-cell interferences. We would like to choose the proper time allocations and transmission powers for mobile terminals in multiple cells, such that the total energy consumption is minimized while satisfying the Quality of Service (QoS) requirements.

A. Power Consumption Model

We consider a comprehensive mobile terminal power consumption model, which includes the transmit power, the circuit power, and the idling power [3], [4].

The transmit power p depends on the transmission rate x. The relationship can be calculated according to Shannon's capacity formula:

$$x = w\log\left(1+\frac{p\cdot G}{\sigma^2+q}\right) \Leftrightarrow p = \left(\exp\left(\frac{x}{w}\right)-1\right)\frac{\sigma^2+q}{G},$$

where w is the bandwidth, G is the channel gain, σ^2 is the noise power, and q is the inter-cell interference. Given a fixed transmission rate x, a larger inter-cell interference power q leads to a larger transmit power p.

The total power consumption $f(x)$ of a mobile terminal with transmission rate x is given as

$$f(x) = \begin{cases} \left(\exp\left(\frac{x}{w}\right)-1\right)\frac{\sigma^2+q}{\theta G}+\alpha, & \text{if } x>0 \text{ (active)},\\ \beta, & \text{if } x=0 \text{ (idling)}, \end{cases}$$

where θ is the drain efficiency which is defined as the ratio of the output power and the power consumed in the power amplifier, α is the power of the circuit blocks in the transmission chain[1], and β is the power consumed in idle state.

B. Inter-cell Interference

Consider a system with a set of M cells: $\{C(m), 1 \le m \le M\}$. Each cell $C(m)$ contains a set of mobile users (terminals) $\mathcal{A}(m)$. The mobile users within the same cell are allocated different time fractions for uplink transmissions. However, mobile users in different cells may transmit simultaneously and cause interference to each other. Next we calculate the minimal power vector that can support the rate requirements of several simultaneous transmissions.

Let $\mathcal{S}$ denote the set of mobile users that are active simultaneously in the multi-cell network. Based on the assumption of TDMA within each cell, we know that the number of mobile users in set $\mathcal{S}$ is no larger than M, i.e., $|\mathcal{S}| \le M$. Without loss of generality, we only need consider the $|S|$ cells with active mobile users. Let us define an $|\mathcal{S}| \times |\mathcal{S}|$ nonnegative relative-channel-gain matrix $\mathbf{B}_{\mathcal{S}}$ of set $\mathcal{S}$, with entries as follows:

$$b_{mn} = \begin{cases} 0, & \text{if } m=n,\\ \frac{G_{i(n),C(m)}}{G_{i(m),i(m)}}, & \text{if } m\neq n. \end{cases}$$

Here $G_{i(m),i(m)}$ is the channel gain from the mobile user $i(m)$ in cell $C(m)$ to the BS of cell $C(m)$, and $G_{i(n),C(m)}$ is the channel gain from the mobile user $i(n)$ in cell $C(n)$ to the BS of cell $C(m)$. Let $\boldsymbol{\gamma}_{\mathcal{S}} = \left(\gamma_{i(m)} :\ \forall i(m)\ s.t.\ i(m)\in\mathcal{S}\right)$ denote the target SINR vector of the mobile users in set $\mathcal{S}$. Let $\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)$ be the $|S|\times|S|$ diagonal matrix whose diagonal entries are $\boldsymbol{\gamma}_{\mathcal{S}}$. Let $\rho\left(\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)\mathbf{B}_{\mathcal{S}}\right)$ denote the largest real eigenvalue (also called the Perron-Frobenius eigenvalue or the spectral radius) of matrix $\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)\mathbf{B}_{\mathcal{S}}$.

The following well-known proposition gives the necessary and sufficient condition of checking the feasibility of a target SINR vector $\boldsymbol{\gamma}_{\mathcal{S}}$ and computing the minimum transmit power solutions that achieves this target SINR vector $\boldsymbol{\gamma}_{\mathcal{S}}$.

Proposition 1 ([5]–[8]): A target SINR vector $\boldsymbol{\gamma}_{\mathcal{S}}$ is feasible if and only if $\rho\left(\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)\mathbf{B}_{\mathcal{S}}\right) < 1$. If $\boldsymbol{\gamma}_{\mathcal{S}}$ is feasible, the componentwise minimum transmit power which achieves the target SINR vector $\boldsymbol{\gamma}_{\mathcal{S}}$ is

$$\mathbf{p}_{\mathcal{S}}(\boldsymbol{\gamma}_{\mathcal{S}}) = \left(\mathbf{I}-\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)\mathbf{B}_{\mathcal{S}}\right)^{-1}\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)\mathbf{v}_{\mathcal{S}}, \qquad (1)$$

where $\mathbf{I}$ is an $|\mathcal{S}|\times|\mathcal{S}|$ identity matrix, and vector $\mathbf{v}_{\mathcal{S}} = \left(\frac{\sigma^2}{G_{i(m),i(m)}} :\ \forall i(m)\ s.t.\ i(m)\in\mathcal{S}\right)^T$ is the noise power vector normalized by the channel gain. Furthermore, the corresponding interference power vector of set $\mathcal{S}$ is given by

$$\mathbf{q}_{\mathcal{S}} = \left(\mathbf{I}-\mathbf{B}_{\mathcal{S}}\mathbf{D}\left(\boldsymbol{\gamma}_{\mathcal{S}}\right)\right)^{-1}\boldsymbol{\eta}_{\mathcal{S}}, \qquad (2)$$

where $\boldsymbol{\eta}_{\mathcal{S}} = \left(\sigma^2,\sigma^2,\cdots,\sigma^2\right)^T$ is the noise power vector.

C. Dynamic Sessions

We study a dynamic system with real-time application sessions (e.g., video/voice sessions). Our target is to minimize the average energy consumption per session in a

[1]Circuit power is the additional power expended besides the transmit power when a mobile terminal transmit.

stationary system. We assume that the users' arrival follows a Poisson process with rate λ. Let J and P denote the energy consumption per session and the total power consumption in the system. The following proposition from prior work shows the relation between $E[P]$ and $E[J]$ in a stationary dynamic system:

Proposition 2 ([3]): In a stationary system with user arrival rate λ, we have $E[P] = \lambda E[J]$.

According to Proposition 2, minimizing the average energy consumption per session is equivalent to minimizing the average power consumption of all the users in the system. It is straightforward to extend the above result to a multi-cell system.

The holding time of a real-time session is independent of the allocated transmission rate. For example, allocating a higher transmission rate to a voice session cannot make the phone call end earlier, and the stationary distribution of the number of users in the TDMA system is independent of the transmit powers as long as the rate requirements are satisfied [3]. Therefore, minimizing the energy consumption in a *dynamic* system that supports real time sessions is equivalent to minimizing the energy consumption with a *static* number of users in the TDMA system. In the rest of the paper, we will focus on the average power minimization problem in the multi-cell system with a static number of users.

III. Problem Formulation and Decoupling

A. Power Minimization in Multi-Cell Networks

Since different users are active at different times in different cells, we will have different concurrent transmission sets in the multi-cell network. Suppose there are a total K concurrent transmission sets, denoted by $\{\mathcal{S}_k, 1 \le k \le K\}$. Each set $\mathcal{S}_k$ is active for a time fraction of t_k within a frame. Let $\mathbf{x}_{\mathcal{S}_k} = \left(x_{i(m)}(k) : \forall i(m) \; s.t. \; i(m) \in \mathcal{S}_k\right)$ denote the instantaneous transmission rate vector of set $\mathcal{S}_k$. The relation between the instantaneous transmission rate vector $\mathbf{x}_{\mathcal{S}_k}$ and the corresponding SINR vector $\boldsymbol{\gamma}_{\mathcal{S}_k}$ is

$$\boldsymbol{\gamma}_{\mathcal{S}_k} = \exp\left(\frac{\mathbf{x}_{\mathcal{S}_k}}{w}\right) - 1. \tag{3}$$

Substituting (3) into (1), then the minimmal power vector $\mathbf{p}_{\mathcal{S}_k}$ that supports the rate vector $\mathbf{x}_{\mathcal{S}_k}$ is

$$\mathbf{p}_{\mathcal{S}_k}(\mathbf{x}_{\mathcal{S}_k}) = \left(\mathbf{I} - \mathbf{D}\left(\exp\left(\frac{\mathbf{x}_{\mathcal{S}_k}}{w}\right) - 1\right)\mathbf{B}_{\mathcal{S}}\right)^{-1} \cdot \mathbf{D}\left(\exp\left(\frac{\mathbf{x}_{\mathcal{S}_k}}{w}\right) - 1\right)\mathbf{v}_{\mathcal{S}}. \tag{4}$$

Recall that $\mathcal{A}(m)$ is the set of mobile users in cell $C(m)$. Let $r_{i(m)}$ be the session rate requirement of user $i(m) \in \mathcal{A}(m)$. The problem of energy savings in multi-cell systems supporting real-time sessions is equivalent to minimizing the average power consumptions of all the mobile users in all the cells. To represent this problem mathematically, we define the following binary coefficients for each user $i(m) \in \mathcal{A}(m)$, $1 \le m \le M$, and $1 \le k \le K$,

$$z_{i(m)}(k) = \begin{cases} 1, & \text{if } i(m) \in \mathcal{S}_k, \\ 0, & \text{if } i(m) \notin \mathcal{S}_k. \end{cases} \tag{5}$$

Problem: Power Minimization in a Multi-cell Network

$$\begin{aligned}
\text{minimize} \;& \sum_{k=1}^{K} t_k \left(\sum_{m=1}^{M} \left(\sum_{i(m)\in\mathcal{A}(m)} \Big((1 - z_{i(m)}(k))\beta_{i(m)} + z_{i(m)}(k)\left(\alpha_{i(m)} + \frac{p_{i(m)}(k)}{\theta}\right)\Big)\right)\right) \\
\text{subject to} \;& \sum_{k=1}^{K} t_k = 1, \\
& \sum_{k=1}^{K} z_{i(m)}(k) x_{i(m)}(k) t_k = r_{i(m)}, \forall i(m), \forall m, \\
\text{variables} \;& x_{i(m)}(k) \ge 0, \;\; \forall k, \forall i(m), \forall m, \\
& t_k \ge 0, \;\; \forall k.
\end{aligned} \tag{6}$$

The objective function in (6) is the average power consumption of all the mobile users in the system and consists of two parts. The first part is the power consumption when the users are idle. The second part is the power consumption when the users are involved in transmissions, where $p_{i(m)}(k)$ is computed according to (4) as a function of $\mathbf{x}_{\mathcal{S}_k}$. The first constraint in (6) states that the total fraction of time that all users are active equals 1. Here, we regard the case where no user is active in any cell as a special concurrent transmission set of $\mathcal{S}_k = \emptyset$. The second constraint in (6) states that the session rate requirement of each user $i(m)$ is satisfied. The variables in (6) are the time fraction variables t_k and the instantaneous rate variables $x_{i(m)}(k)$.

It is challenging to solve Problem (6) directly and optimally. First, if we consider all possible combinations of simultaneous active users, then the number of the total concurrent transmission sets K can be as large as $\prod_{m=1}^{M} (|\mathcal{A}(m)| + 1)$. For example, in a multi-cell network with 19 cells and each cell has 9 mobile users, we have $K \le 10^{19}$. Second, the transmit power $p_{i(m)}(k)$ in the objective function of (6) is a complicated function of the instantaneous rate variables $x_{i(m)}(k)$'s. Such function is different for each user $i(m)$ and each different concurrent transmission set $\mathcal{S}_k$. This adds another level of complication to the problem formulation.

In this paper, we focus on designing a heuristic algorithm to solve Problem (6) based on one assumption. For each cell $C(m)$, we assume the interference experienced by the BS, $q(m)$, remains constant within a time frame. With this assumption, the users' transmission schedule in one cell does not affect the transmissions in other cells. Without loss of generality, we will simply assume that the transmission schedule of the mobiles in each cell is fixed based on the arrival order of the corresponding sessions. Furthermore, we will tackle Problem (6) by solving intra-cell average power minimization and inter-cell power control separately.

B. Intra-Cell Average Power Minimization

For a given cell, the intra-cell average power minimization problem (assuming constant interference from other cells) turns out to be a convex optimization problem. Let us consider cell $C(m)$. The session rate requirement of mobile user $i(m) \in \mathcal{A}_m$ is $r_{i(m)}$. If the instantaneous transmission rate of $i(m)$ is $x_{i(m)}$, then the time fraction that user $i(m)$ need to support the session rate requirement is is $r_{i(m)}/x_{i(m)}$.

Problem: Intra-Cell Average Power Minimization:

$$\begin{aligned}
\text{minimize} \quad & \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right)-1}{\theta G_{i(m)i(m)}} \left(\sigma^2+q(m)\right) \right. \\
& \left. +\alpha_{i(m)} + \sum_{j(m)\in\mathcal{A}(m)\setminus\{i(m)\}} \beta_{j(m)} \right) \\
& + \left(1-\sum_{i\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}}\right) \sum_{i\in\mathcal{A}(m)} \beta_{i(m)} \\
\text{subject to} \quad & \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \leq 1 \\
\text{variables} \quad & x_{i(m)} \geq 0, \forall i(m) \in \mathcal{A}(m).
\end{aligned} \tag{7}$$

Since we consider the uplink transmissions, the base station is the common receiver for all mobile users in $\mathcal{A}(m)$. Thus the inter-cell interference power $q(m)$ is the same for every user.

Problem (7) can be shown to be equivalent to,

$$\begin{aligned}
\text{minimize} \quad & \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right)-1}{\theta G_{i(m)i(m)}} \left(\sigma^2+q(m)\right) \right. \\
& \left. +\alpha_{i(m)} - \beta_{i(m)} \right) \\
\text{subject to} \quad & \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \leq 1 \\
\text{variables} \quad & x_{i(m)} \geq 0, \forall i(m) \in \mathcal{A}(m).
\end{aligned} \tag{8}$$

Problem (8) is a convex optimization problem. The optimal solution of Problem (8) in general depends on the inter-cell interference power $q(m)$. To simplify notation, let $\delta_{i(m)} = \alpha_{i(m)} - \beta_{i(m)}$.

Next we show that the optimal transmission rate solution to (8) and the inter-cell interference power can be de-coupled if $\delta_{i(m)} \leq 0$.

C. Decoupling Property When $\delta_{i(m)} \leq 0$

If $\delta_{i(m)} \leq 0$, then the idling power $\beta_{i(m)}$ is no smaller than the circuit power $\alpha_{i(m)}$. It can also approximate the case where both the circuit power and the idling power are negligible. In this case, we have the following theorem:

Theorem 1: If $\delta_{i(m)} \leq 0$, the optimal instantaneous transmission rate solutions and the optimal target SINRs of the intra-cell power minimization problem (8), (i.e., $x^*_{i(m)}(\forall i(m) \in \mathcal{A}(m))$ and $\gamma^*_{i(m)}(\forall i(m) \in \mathcal{A}(m))$), are independent of the inter-cell interference power level, the circuit power, and the idling power.

Proof: When $\delta_{i(m)} \leq 0$, the optimal solution to Problem (8) is achieved when the inequality constraint is tight, i.e., $\sum_{i\in\mathcal{A}} \frac{r_{i(m)}}{x_{i(m)}} = 1$. In this case, minimizing

$$\sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right)-1}{\theta G_{i(m)i(m)}} \left(\sigma^2+q(m)\right) + \delta_{i(m)} \right)$$

is equivalent to minimizing

$$\sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right)-1}{\theta G_{i(m)i(m)}} \left(\sigma^2+q(m)\right) \right).$$

Furthermore, $\sigma^2+q(m)$ becomes a common scaling factor in the objective function and thus can be removed. Therefore, Problem (8) can be simplified as

$$\begin{aligned}
\text{minimize} \quad & \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right)-1}{G_{i(m)i(m)}} \right) \\
\text{subject to} \quad & \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} = 1 \\
\text{variables} \quad & x_{i(m)} \geq 0.
\end{aligned} \tag{9}$$

This completes the proof. ■

Theorem 1 will be referred to the "decoupling property" for $\delta_{i(m)} \leq 0$, which enables us to decouple the intra-cell average power optimization from the inter-cell power control.

IV. The DSP Algorithm

Theorem 1 motivates us to propose an algorithm, called Decoupling Scheduling and Power control (DSP), to achieve energy-efficient transmissions in a multi-cell system. We will define a global metric $\delta = \max_m \max_{i(m)\in\mathcal{A}_m} \delta_{i(m)}$. Different values of δ will lead to different executions in the algorithm.

A. DSP Algorithm When $\delta \leq 0$

Because of the decoupling property when $\delta \leq 0$, we will optimize the average power consumption in two steps:

- Step 1 (intra-cell average power minimization): Each cell $C(m)$ solves Problem (9) to determine the optimal instantaneous rate and the optimal target SINR of each mobile user in $\mathcal{A}(m)$.
- Step 2 (inter-cell power control): Given the optimal target SINRs of the mobile users in each cell, we can get the optimal target SINR vector for the links that are active simultaneously in different cells. The component-wise minimum power solution that satisfies the target SINR vector can be determined.

In Step 1, each cell $C(m)$ solves the convex optimization problem (9) utilizing the Lagrangian method. Let φ denote the Lagrangian multiplier of the constraint in (9). The Lagrangian function is

$$\boldsymbol{L}(\mathbf{x},\varphi) = \sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right)-1}{G_{i(m)i(m)}} \right) + \varphi \left(\sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} - 1 \right).$$

Since Problem (9) is convex, the necessary and sufficient conditions for an optimal solution is the KKT conditions:

$$\nabla_{\mathbf{x}} \boldsymbol{L}(\mathbf{x},\varphi) = 0, \quad \text{and} \quad \left(\sum_{i(m)\in\mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} - 1 \right) = 0.$$

From $\nabla_{\mathbf{x}} \boldsymbol{L}(\mathbf{x}, \varphi) = 0$, we have

$$\varphi^* = \frac{1}{G_{i(m)i(m)}} \left(\exp\left(\frac{x^*_{i(m)}}{w} \right) \left(\frac{x^*_{i(m)}}{w} - 1 \right) + 1 \right), \tag{10}$$

where φ^* is the optimal Lagrange multiplier and $x^*_{i(m)}$ is the optimal instantaneous rate solution to (9). Given the parameters of $r_{i(m)}$, $G_{i(m)i(m)}$, and w, the optimal Lagrange multiplier φ^* can be computed by the Newton's method, which guarantees superlinear convergence (faster than exponential) [9]. After obtaining φ^*, $x^*_{i(m)}$ can be calculated by solving (10). An efficient way to solve (10) is to tabulate the Lambert W function [10], which is defined as

$$W(y) \exp(W(y)) = y.$$

Then $x^*_{i(m)}$ is given by

$$x^*_{i(m)} = \left(W\left(\frac{\varphi^* G_{i(m)i(m)} - 1}{e} \right) + 1 \right) w. \tag{11}$$

The time fraction for user $i(m)$ to be active is $\frac{r_{i(m)}}{x^*_{i(m)}}$. Given the instantaneous rate solution $x^*_{i(m)}$, the target SINR $\gamma^*_{i(m)}$ then can be determined by equation (3).

In Step 2, optimal power control is performed across multiple cells to determine the optimal transmit powers for the mobile users in each cell. We have obtained the instantaneous rate $x^*_{i(m)}$, the target SINR $\gamma^*_{i(m)}$, and the active time fraction $\frac{r_{i(m)}}{x^*_{i(m)}}$ of each user in each cell. Because the scheduling order in each cell is determined by its arrival order, so we can determine all the concurrent transmission sets $\{\mathcal{S}_k, 1 \le k \le K\}$ and their active fractions of time $\{t_k, 1 \le k \le K\}$ in the frame. According to Proposition 1, the componentwise minimum transmit power solutions of each set $\mathcal{S}_k$ which achieve the target SINR vector $\boldsymbol{\gamma}^*_{\mathcal{S}_k}$ is given in (4).

B. DSP Algorithm When $\delta > 0$

When $\delta > 0$, i.e., the circuit power is great than the idling power for at least one mobile user, the intra-cell power minimization problem is given in (8). The optimal instantaneous rate solution to (8) is dependent on the inter-cell interference power $q(m)$. This motivates us to use an iterative method to minimize the energy consumption in the multi-cell network. At the beginning of each iteration, we replace $q(m)$ with the average interference power $\hat{q}(m)$ obtained from the previous iteration for every cell $C(m)$. For the first iteration, the estimated interference power $\hat{q}(m)$ is the averaged interference power of the previous frame.

the DSP algorithm for the case of $\delta > 0$ involves an iteration that alternates between two steps: Step 1 and Step 2. In Step 1, each cell $C(m)$ solves Problem (8) using the Lagrangian method, where $q(m)$ is replaced by $\hat{q}(m)$. The Lagrangian function of (8) is given by

$$\boldsymbol{L}(\mathbf{x}, \varphi) = \sum_{i(m) \in \mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} \left(\frac{\exp\left(\frac{x_{i(m)}}{w}\right) - 1}{\theta G_{i(m)i(m)}} \left(\sigma^2 + \hat{q}(m)\right) + \delta_{i(m)} \right) + \varphi \left(\sum_{i(m) \in \mathcal{A}(m)} \frac{r_{i(m)}}{x_{i(m)}} - 1 \right).$$

Similarly, the KKT conditions are applied to solve formulation (8). Compared to (10) and (11), the optimal Lagrange multiplier φ^* and the instantaneous rate $x^*_{i(m)}$ under the case of $\delta_{i(m)} > 0$ are modified to

$$\varphi^* = \frac{\sigma^2 + \hat{q}(m)}{\theta G_{i(m)i(m)}} \left(\exp\left(\frac{x^*_{i(m)}}{w} \right) \left(\frac{x^*_{i(m)}}{w} - 1 \right) + 1 \right) - \delta_{i(m)},$$

$$x^*_{i(m)} = \left(W\left(\frac{\varphi^* \theta G_{i(m)i(m)} - \left(\sigma^2 + \hat{q}(m)\right)}{e\left(\sigma^2 + \hat{q}(m)\right)} \right) + 1 \right) w.$$

In Step 2, given the instantaneous rate $x^*_{i(m)}$, the target SINR $\gamma^*_{i(m)}$, and the active time fraction $\frac{r_{i(m)}}{x^*_{i(m)}}$ obtained in step 1, the concurrent transmission sets $\{\mathcal{S}_k, 1 \le k \le K\}$ and their active fractions of time $\{t_k, 1 \le k \le K\}$ can be determined. The transmit power vector $\mathbf{p}_{\mathcal{S}_k}$ and the interference power vector $\mathbf{q}_{\mathcal{S}_k}$ for each set $\mathcal{S}_k$ can be determined according to equations (1) and (2), respectively. We use the averaged interference power vector in the current frame to serve as the estimate interference power in the next iteration, which is given by$\hat{\mathbf{q}} = \sum_{k=1}^{K} t_k \mathbf{q}_{\mathcal{S}_k}$. The mth element in vector $\hat{\mathbf{q}}$ is the averaged interference power experienced by the BS in cell $C(m)$, $\hat{q}(m)$. The DSP algorithm will continue until the average power consumptions of all the mobile users in all the cells cannot be further improved.

V. Simulation Results

We carry out extensive simulations to evaluate the performance of the proposed DSP algorithm. We simulate a multi-cell network with a frequency reuse factor of 3. That is one of every 3 cells use the same channel. We consider a 7-cell network operated on the same channel. The radius of each cell is 300 m. Inside each cell, 23 mobile users are uniformly distributed. The session rate requirement of each mobile user is 350 kbps. The bandwidth is 1 MHz. The frame length is normalized to be 1 s. The maximum output power is 27.5 dBm. The drain efficiency is 0.2. The noise power density is -174 dBm/Hz. The idling power and the circuit power are 25 mW and 30 mW, respectively, i.e. the case when $\delta > 0$ and the algorithm in Section IV-B is used. The power related parameters are cited from [3], [11]. We adopt the distance-based path loss model with a path loss exponent of 4.

We compare the performances of the following three transmission policies:

- Maximum power transmission: each mobile user transmits with the maximum transmit power.
- Single-EOT: the Single-cell Energy Optimal Transmission policy proposed in [3] [2].
- DSP: Decoupling Scheduling and Power control.

[2]Reference [3] considered an isolated single cell network, where the inter-cell interference power is 0. Here we consider multi-cell network extension. In order to make sure the target transmission rate can be achieved under the case that the actual interference power is unknown, we assume the worst case inter-cell interference power. In the worst case, the BS assumes the mobile users in the adjacent cells use maximum transmit power, and the worst case interference distance is twice of the cell radius. So the worst case interference power can be calculated accordingly.

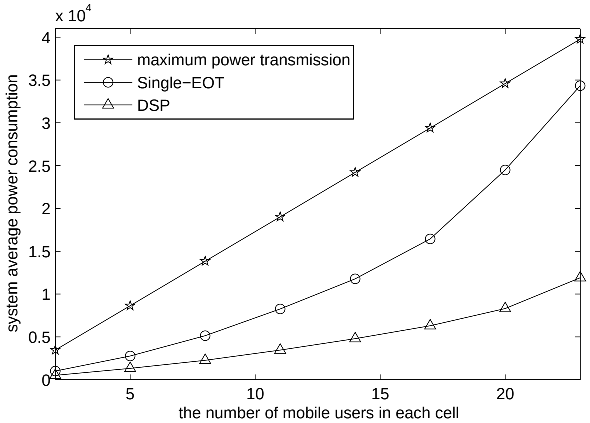

Fig. 1. System power consumptions (The number of users in each cell ranges from 2 to 23)

Figure 1 shows the system power consumptions of the three algorithms as a function of the number of mobile user in each cell. As expected, DSP outperforms single-EOT, which in turn outperforms the maximum transmit power policy. The system power consumptions of the Single-EOT and DSP algorithms increase much slower as the number of mobile users increase. Because the holding time of a real time session is the same among these three algorithms, so the system power reduction ratio is equivalent to the system energy reduction ratio. Compared to the maximum transmit power policy, DSP achieves a power/energy reduction of more than 70% for all the simulated number of mobile users per cell (i.e., ranges from 2 to 23) In single-EOT, the BS makes use of the time resource to adjust the instantaneous transmission rate of each mobile terminal so that the total power/energy consumption can be reduced from a single cell's perspective. However, since BSs of different cells do not cooperate in single-EOT, the power saving is still limited due to conservative estimation of the inter-cell interferences. The DSP algorithm combines the intra-cell average power minimization with inter-cell power control. In Step 1 of the DSP algorithm, after solving the optimization problem in each cell, the instantaneous transmission rate and the target SINR of each mobile can be reduced compared with the maximum power transmission algorithm. In Step 2, after applying joint power control across different cell, the minimum power solutions that satisfy the target SINRs of the concurrent mobile users in all the cells are obtained. DSP leads to a "win-win" situation. It reduces both the transmit power and the inter-cell interference. As a result, the system power/energy consumption reduction ratio can be further improved compared to the Single-EOT algorithm: DSP algorithm achieves a further system power/energy reduction of more than 50% for all the simulated numbers of mobile users per cell.

VI. Conclusion

In this paper, we study the problem of conserving the energy of mobile terminals in multi-cell TDMA networks supporting bursty real-time sessions. The associated optimization problem involves joint scheduling, rate control, and power control.

We propose a decomposition method that decouples the overall problem into two sub-problems: intra-cell energy optimization and inter-cell power control. This decomposition method is guanranteed to find a feasible solution, albeit not an optimal one. The decomposition is motivated and made simple by the following observations:

1) The original optimization problem is too complicated for its directly solution to be implemented today.
2) In cellular networks, the cells that are allocated the same frequency band are usually separate by a distance. Interference is a strong function of distance when the distance is small, but a weak function of distance when the distance is large. If an adjacent cell using the same frequency band is at a large distance, then it does not matter much which of its mobile terminal is transmitting: the interference from it will stay more or less constant throughout a frame. Thus, we could make the approximation that the interference is constant when we make intra-cell airtime allocations to the mobile terminals within a cell.
3) If the idle power is no less than the circuit power, then there is a"decoupling property": the energy-optimal airtime allocations to individual mobile terminals within each cell are independent of the inter-cell interference, assuming the interference stays constant throughout a frame.
4) If the idle power is less than the circuit power, the sub-problems are not decoupled. We can then alternate between the two sub-problems.

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