Regulating Wireless Access Pricing

Kwanfong Leung and Jianwei Huang
Department of Information Engineering, The Chinese University of Hong Kong
Email: {lkf009,jwhuang}@ie.cuhk.edu.hk

***Abstract*—Today's wireless networks are interconnected. A network provider can charge other network providers *access prices* for serving communication sessions originated from other networks. Meanwhile, a network provider also charges its users directly for using the network services. This paper looks at the proper choices of user pricing and access pricing from a regulator's point of view. We first derive the social optimal user prices when the regulator can directly control them. We characterize and explain how the social optimal user prices change with network's market shares and bandwidth costs. When such direct control is not possible or desirable, we further derive the regulator's optimal choice of access prices, which align the profit-maximizing behaviors of the networks providers with the social optimality objective. We further characterize the relationship among the socially optimal access pricing, the market share, and the users' utility elasticities.**

I. Introduction

Today's wireless networks are interconnected. A subscriber of one network can communicate with subscribers in many other networks. Inter-network communications bring benefits to all networks involved, as no single network needs to build an infrastructure covering the entire market. However, serving users from other networks brings negative impacts on a network's own available resource. This can be compensated by the *access pricing* charged between network providers. Meanwhile, a user who initiates a communication session needs to pay the *user pricing* to its own network provider.

This paper focuses on the interaction between user pricing and access pricing in multiple interconnected wireless communication networks. Users communicate with each other through providers' base stations. Users experience different channel conditions to their base stations, and hence require different amount of network resources to achieve the same data rate. The uplink and downlink transmissions of the same communication session may also consume different amount of resources depending on the locations of users and base stations. These unique features make the analysis more challenging compared with the existing access pricing study for wireline networks.

In this paper, we study the user pricing and access pricing issues from a regulator's point of view. The main results and contributions of this work include:

- *Wireless market model:* we study a market model that captures the unique wireless relationship among user locations, channel conditions, network resources, and the communication data rates.
- *Social optimal user pricing:* We compute the optimal user prices that achieve the social optimality, and illustrate how the user prices change with the network costs and market shares.
- *Social optimal access pricing:* We show that a regulator can achieve the same social optimum by setting the access prices and letting providers determine their user prices locally. We also characterize the relationship between the socially optimal access pricing and the market share as well as users' utility elasticities.

This work is supported by the General Research Funds (Project Number 412710) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

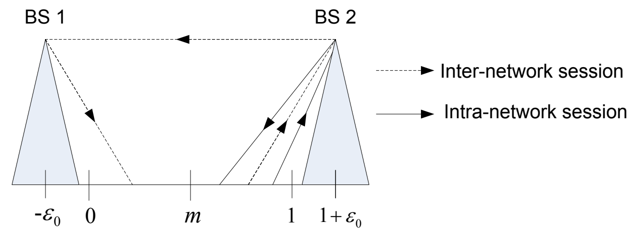

Fig. 1. An illustration of the wireless market.

Our work fills in the gap between the existing studies on wireless network competitions and network access pricing. Existing studies on network access prices considered either a traditional wireline network [1] or a hybrid market of a wireline telephone network and a wireless cellular network [2]. References [3], [4] studied how to determine the access pricing among hierarchical Internet Service Providers with a specific traffic model. None of the above results captured the impacts of channel conditions and transmission power. Existing work on wireless network competition (e.g., [5]–[8]) did not consider access prices between operators. Reference [9] studied the access price in mobile networks without considering users' different channel conditions. Moreover, the access pricing in [9] is on a per call basis and does not consider the impact of flexible data rate.

Next, we will introduce the system model in Section II. Sections III and IV demonstrate how the regulator can compute the social optimal user pricing and access pricing. We finally conclude in Section V.

II. System Model

A. Wireless Market Model

Following the tradition of network access pricing literature (e.g., [1]), we adopt a one-dimensional market model as in Fig. 1. There are two networks, each represented by a base station at one end of the market. Users are uniformly located along the segment [0,1]. The average channel gain between a user and a base station follows the large-scale distance based attenuation. To avoid having an infinitely large channel gain, the base-stations are ϵ_0 away from their closest users. In the rest of the paper, we use the terms "network", "provider", and "base station" interchangeably.

We assume that the user-network associations are fixed, such that users in $[0, m]$ subscribe to network 1 and users in $[m, 1]$ subscribe to network 2. This means that network 1 has a market share of m, and network 2 has a market share of $1-m$. Reference [11] illustrates one such example, where some part of California is covered by AT&T but not Verizon (and vice versa). The analysis based on this restrictive assumption of fixed market share will help us understand the more general scenario of flexible market share.

B. User's Utility, Payment, Payoff, and Demand

A one-way data communication session involves a source user and destination user. The session is *intra-network* when both users belong to the same network, or *inter-network* when the two users belong to different networks. We adopt the α-fair utility function [10] to represent the quality of service (QoS) of a session in terms of its data rate y,

$$u(y) = \frac{y^{1-\alpha}}{1-\alpha},$$

where the utility parameter $\alpha \in (0, 1)$ [1].

If a source user i belonging to network j starts a session with the data rate y_{ij}, then it pays network j the *user pricing* $\pi_j y_{ij}$.[1] The source user i's payoff (which is also the payoff of the communication session) is

$$r_i(y_{ij}, \pi_j) = \frac{y_{ij}^{1-\alpha}}{1-\alpha} - \pi_j y_{ij}.$$

The optimal *demand* of data rate that maximizes utility is

$$y_{ij}^*(\pi_j) = \pi_j^{-1/\alpha},$$

which has a constant elasticity of $1/\alpha$. A small α denotes an elastic application and a large α denotes an inelastic application. Source user i's optimal payoff is

$$r_i(y_{ij}^*(\pi_j), \pi_j) = \frac{\pi_j^{1-1/\alpha}}{1-\alpha} - \pi_j^{1-\frac{1}{\alpha}},$$

which only depends on the network price π_j and is independent of the user's location. This is desirable in practice, as the user only needs to keep track of the total data usage instead of where he conducts the communications.

[1]We assume that the destination node does not need to pay the user pricing. This is similar as the pricing of multimedia messaging services in may countries today.

C. Network Costs

A communication session involves both a uplink transmission (from the source user to its network's base station) and a downlink transmission (from the destination user's base station to the destination user). Each part of the transmissions involves a cost proportional to the bandwidth consumed. For a source user i belonging to network j, the relationship between the transmission rate y_{ij} and the consumed bandwidth B_{ij} depends on the distance between the user and the base station d_{ij}, the uplink transmission power per unit bandwidth P^u, and the background noise density n_0. We assume an Orthogonal Frequency Division Multiple Access scheme with equal power allocation, and no two users (either in a same or different networks) interfere with each other. Thus

$$y_{ij} = B_{ij} \log\left(1 + \frac{P^u h_{ij}(d_{ij})}{n_0}\right),$$

where $h_{ij}(d_{ij})$ is the channel gain depending on the distance d_{ij}. One possible choice is $h_{ij}(d_{ij}) = \frac{1}{d_{ij}{}^{\beta}}$, where β is the channel attenuation factor (usually between 2 to 4). The analysis in this paper can be generalized to other distance based channel models as we keep the function $h_{ij}(d_{ij})$ abstract most of the time. The total bandwidth cost for supporting this uplink transmission is $c_j B_{ij}$. The cost for the downlink transmission can be computed similarly, except that P^u will be replaced by P^d and c_j will be replaced by the cost of the corresponding network. For notation simplicities, we denote

$$g^d(d_{ij}) \equiv \log\left(1 + \frac{P^d h_{ij}(d_{ij})}{n_0}\right),$$
$$g^u(d_{ij}) \equiv \log\left(1 + \frac{P^u h_{ij}(d_{ij})}{n_0}\right).$$

Let us compute the total cost of serving one communication session. If source user i at location d_i is with network 1, then network 1's cost of the uplink transmission is:

$$c_1 B_{i1} = c_1 \frac{y_{i1}}{\log\left(1 + \frac{P^u h_{ij}(d_i)}{n_0}\right)} = c_1 \frac{y_{i1}}{g^u(d_i)}.$$

Serving a user close-by (with a small value of d_i and thus a larger $g^u(d_i)$) costs less compared with serving a user far away. For the downlink traffic, since the destination may be with network 1 or network 2, we need to compute the *expected* downlink cost. In our model, users are uniformly distributed along the segment [0,1], and each of them will have equal probability of receiving data. The expected cost is $c_1 \int_0^m \frac{y_{i1}}{g^d(r)} dr$ to network 1 and $c_2 \int_0^{1-m} \frac{y_{i1}}{g^d(r)} dr$ to network 2. Hence, the total expected cost to support a session initiated by a user i located at d_i in network 1 with data rate y_{i1} is:

$$c_1 \frac{y_{i1}}{g^u(d_i)} + c_1 \int_0^m \frac{y_{i1}}{g^d(r)} dr + c_2 \int_0^{1-m} \frac{y_{i1}}{g^d(r)} dr.$$

The first two terms are related to network 1 and the third term is related to network 2.

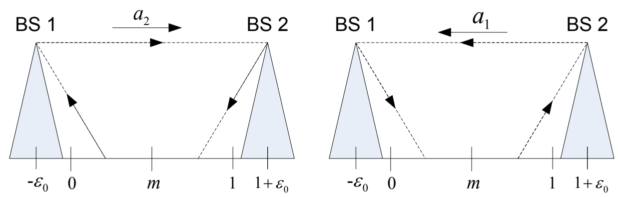

Fig. 2. Illustration of the access pricing. Left: To complete an inter-network traffic initiated from network 1, network 2 charges network 1 a_2 per unit of bandwidth. Right: the access price a_1 is defined similarly.

In the rest of the paper, we denote

$$B^u(m) \equiv \int_0^m \frac{1}{g^u(r)}dr \text{ and } B^d(m) \equiv \int_0^m \frac{1}{g^d(r)}dr,$$

which represent the average bandwidth needed to serve one unit of uplink and a downlink transmission of a network with a market share m, respectively.

D. Access Price

For an inter-network session, the network with the destination user cannot charge the source user, but bears the cost of the downlink transmission. To compensate this additional cost, the destination network charges the source network an *access pricing*. As illustrated in Fig. 2, to complete an inter-network session initiated from network 1, network 1 pays network 2 a_2 per unit of bandwidth consumed in network 2. The access price a_1 is defined similarly.

III. Social Optimal User Pricing

A regulator cares about the social welfare, which is the total payoffs of all entities in the market, i.e., the total user utility minus the total network cost. The payments (either from users to networks or between networks) are internal transfers and do not affect the social welfare. However, a regulator typically cannot directly control how much resources that users consume. In this section, we will look at the case where the regulator maximizes the social welfare by controlling the user pricing π_1 and π_2.[2]

The social welfare $SW(\pi_1, \pi_2, m, c_1, c_2)$ is:

$$SW(\pi_1, \pi_2, m, c_1, c_2) = m\frac{{\pi_1}^{\alpha-\frac{1}{\alpha}}}{1-\alpha} + (1-m)\frac{{\pi_2}^{\alpha-\frac{1}{\alpha}}}{1-\alpha} - \pi_1^{-\frac{1}{\alpha}} f(m, c_1, c_2) - \pi_2^{-\frac{1}{\alpha}} f(1-m, c_2, c_1), \quad (1)$$

where $f(m, c_j, c_{-j})$ represents the total cost in serving sessions originated from network j of a market share m,

$$f(m, c_j, c_{-j}) = c_j B^u(m) + mc_j B^d(m) + mc_{-j} B^d(1-m).$$

Here c_{-j} denotes the per unit bandwidth cost of the network other than network j. For example, if $j = 1$, then $c_{-j} = c_2$.

The regulator's objective is to choose π_1 and π_2 to maximize the social welfare. From (1), it is clear that the social welfare is decoupled across π_1 and π_2. We can also show that the social welfare is quasi-concave in both π_1 and π_2, and thus the optimal user prices can be obtained through solving the first order conditions.

[2]The access prices a_1 and a_2 cancel out and do not affect social welfare.

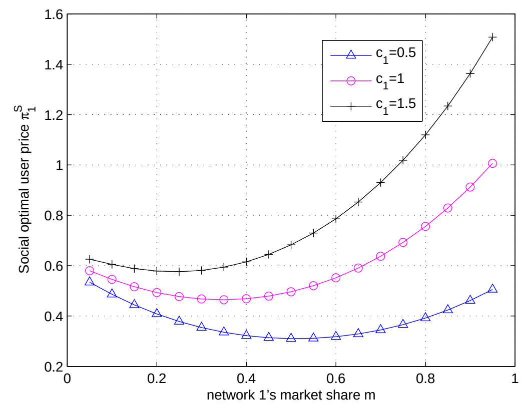

Fig. 3. Social optimal user price π_1^S of network 1 with different market share m and cost c_1. Here $h_{ij}(d_{ij}) = d_{ij}^{-3}$, $P^d = P^u = 1$, and $c_2 = 1$.

Proposition 1: Socially optimal user prices are $\pi_1^S = f(m, c_1, c_2)/m$ and $\pi_2^S = f(1-m, c_2, c_1)/(1-m)$.

Observation 1: The social optimal user prices do not depend on the utility parameter α.

Proposition 1 works for any channel function. Next, we study the impacts of market share and bandwidth costs with a particular choice of channel function of $h_{ij}(d_{ij}) = d_{ij}^{-3}$. We have two observations from the results in Fig. 3.

Observation 2: A network's social optimal price first decreases and then increases with its market share.

Figure 3 illustrates this observation for network 1, which has a market share m. When m is small, only users close to BS1 are associated with network 1 (see Fig. 1). Since the average channel condition of these users to BS1 is high, the average cost of serving one unit of data rate is low. When the market share m increases, it is beneficial for the regulator to decrease the price π_1^S to induce more demands of the users, which increases users' utility with a relatively small increase in the total cost.

When the market share m is large, many users associated with network 1 are located far away from BS1, and thus the average cost of serving users is high. Any additional user joining the network due to the increase of market share requires the highest cost to serve. As a result, it is beneficial for the regulator to increase the price so as to decrease the operational cost, with a small penalty of users' utility decrease.

Comparing three curves in Figure 3, we have the following.

Observation 3: A network's social optimal price increases in its bandwidth cost.

IV. Social Optimal Access Pricing

Very often the regulator cannot even control the user pricing in practice. This may be due to the complexity of the regulation, or due to the fact that the government just does not want to micro-manage the telecommunication industry. However, the regulator can still achieve the social optimality by setting the access pricing.

Mathematically, we can model the system as a two-stage decision process. In stage one, the regulator determines the access pricing a_1 and a_2 between two network operators. In stage two, each network j chooses the user pricing π_j to maximize its profit, given a_1 and a_2. We will look at the stage 2 problem first.

A. Networks' Profit-Maximizing User Pricing Given Fixed Access Prices: $\pi_1^*(a_1, a_2)$ and $\pi_2^*(a_1, a_2)$

We begin by deriving the network profits. Consider a session originated from a user i located at d_i in network 1. Network 1's profit from this session equals the payment received from user i minus the total expected cost for either an intra-network session and an inter-network session, i.e.,

$$\pi_1^{1-\frac{1}{\alpha}} - c_1 \frac{\pi_1^{-\frac{1}{\alpha}}}{g^u(d_i)} - c_1 \int_0^m \frac{\pi_1^{-\frac{1}{\alpha}}}{g^d(r)} dr - a_2 \int_0^{1-m} \frac{\pi_1^{-\frac{1}{\alpha}}}{g^d(r)} dr.$$

For an inter-network session originating from a user i located at d_i in network 2, network 1's profit equals the access price payment received from network 2 minus the cost in supporting the downlink communication, i.e.,

$$(a_1 - c_1) \int_0^m \frac{\pi_2^{\frac{-1}{\alpha}}}{g^d(r)} dr.$$

Combining the above analysis, the profits of network 1 (R_1) and of network 2 (R_2) are

$$\begin{aligned} R_1(\pi_1, \pi_2) &= m\pi_1^{1-\frac{1}{\alpha}} - c_1 \pi_1^{-\frac{1}{\alpha}} B^u(m) - m c_1 \pi_1^{-\frac{1}{\alpha}} B^d(m) \\ &- m a_2 \pi_1^{-\frac{1}{\alpha}} B^d(1-m) + (1-m)(a_1 - c_1) \pi_2^{-\frac{1}{\alpha}} B^d(m) \end{aligned}$$

and

$$\begin{aligned} R_2(\pi_1, \pi_2) &= (1-m)\pi_2^{1-\frac{1}{\alpha}} - c_2 \pi_2^{-\frac{1}{\alpha}} B^u(1-m) \\ &- (1-m) c_2 \pi_2^{-\frac{1}{\alpha}} B^d(1-m) - (1-m) a_1 \pi_2^{-\frac{1}{\alpha}} B^d(m) \\ &+ m(a_2 - c_2) \pi_1^{-\frac{1}{\alpha}} B^d(1-m). \end{aligned}$$

By optimizing R_1 over π_1 and optimizing R_2 over π_2, we have the following results.

Proposition 2: For any given access prices a_1 and a_2 set by the regulator, networks 1 and 2 set the following user prices to maximize their individual profits,

$$\pi_1^*(a_2) = \frac{c_1 B^u(m) + m c_1 B^d(m) + a_2 B^d(1-m)}{m(1-\alpha)}, \qquad (2)$$

and

$$\pi_2^*(a_1) = \frac{c_2 B^u(1-m) + (1-m) c_2 B^d(1-m) + a_1 B^d(m)}{(1-m)(1-\alpha)}. \qquad (3)$$

Observation 4: A network j's profit-maximizing user pricing π_j^* depends on the rival network's access pricing a_{-j} and is independent of its own access pricing a_j.

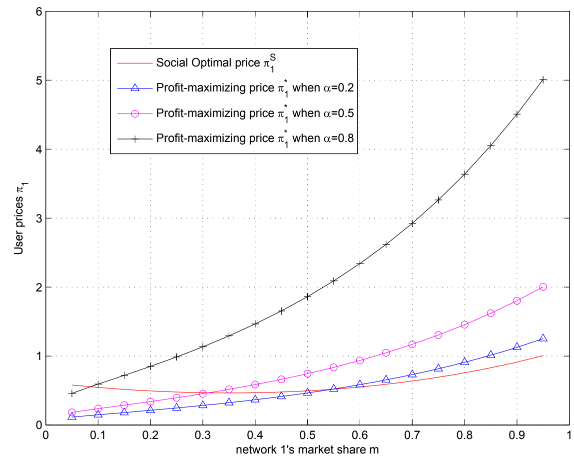

Fig. 4. Social optimal user price π_1^S and profit-maximizing user price π_1^* with different values of market share m and utility parameter α. Here $h_{ij}(d_{ij}) = d_{ij}^{-3}$, $P^d = P^u = 1$, and $c_1 = c_2 = 1$.

Before studying the regulator's optimal choice of access pricing, let us consider the case where no access pricing is set, i.e., $a_1 = a_2 = 0$. We can compare networks' profit maximizing user pricing (π_1^* and π_2^*) with the social optimal user pricing (π_1^S and π_2^S) computed in Section III.

Observation 5: With $a_1 = a_2 = 0$, the profit-maximizing user prices π_1^* and π_2^* always lead to a smaller social welfare comparing with the one achieved under π_1^S and π_2^S, except for one value of the market share.

Figure 4 shows the social optimal user price π_1^S (which is independent of the utility parameter α as shown in Observation 1) and the profit-maximizing user price π_1^* for three different values of α under different market shares. For each choice of α, π_1^* only intersects with π_1^S once, and the prices are different for all other values of m. For example, when $\alpha = 0.2$ and market share $m < 0.55$, we have $\pi_1^* < \pi_1^S$ and users' demand is larger in the profit-maximizing case than in the social optimal case. It is the other way around when $m > 0.55$. Neither case is desirable from the regulator's point of view.

B. Social Optimal Access Pricing: a_1^S and a_2^S

Now consider the stage 1 problem. The regulator can set the proper access pricing a_1^S and a_2^S so that the networks' profit-maximizing behavior is aligned with the social optimality objective, i.e., $\pi_1^*(a_2^S) = \pi_1^S$ and $\pi_2^*(a_1^S) = \pi_2^S$.

By comparing the values of π_1^S and π_2^S in Proposition 1 and the values of π_1^* and π_2^* in (2) and (3), we have the following.

Proposition 3: The social optimal access prices are

$$\begin{aligned} a_1^S &= \frac{(1-m)(1-\alpha)\pi_2^S - c_2 B^d(1-m)}{(1-m) B^d(1-m)} - \frac{c_2 B^d(1-m)}{B^d(m)}, \\ a_2^S &= \frac{m(1-\alpha)\pi_1^S - c_1 B^d(m)}{m B^d(m)} - \frac{c_1 B^d(m)}{B^d(1-m)}, \end{aligned}$$

where π_1^S and π_2^S are defined in Proposition 1.

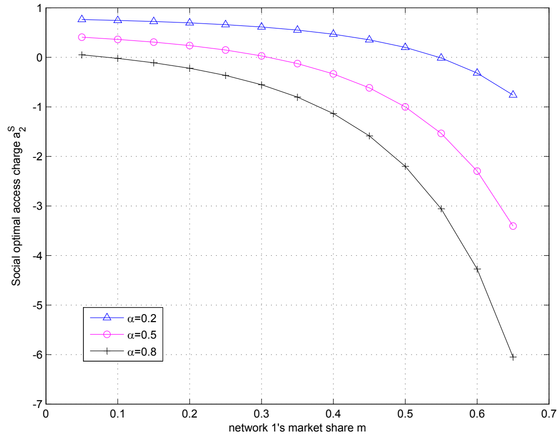

Fig. 5. Social optimal access price a_2^S with different market share m and utility parameter α. Here $h_{ij}(d_{ij}) = d_{ij}^{-3}$, $P^d = P^u = 1$, and $c_1 = c_2 = 1$.

Figure 5 shows the social optimal access price a_2^S with different market share of network 1 m and utility parameter α. We have the following observation.

Observation 6: The social optimal access price of a network decreases in its rival network's market share.

Observation 6 can be explained as follows. When a network's market share (e.g., m of network 1) increases, its average bandwidth cost of serving users also increases. Due to the profit-maximzing nature of the network, it tends to increase the user pricing (π_1^*) significantly to compensate such cost increase (see Fig. 4). Thus the access price charged by the other network (network 2) should decrease to provide incentive for the network to maintain the social optimal user pricing.

In fact, in the extreme case where the average cost has increased so much due to a very large market share, the access price from its rival network needs to be negative (i.e., network 2 pays to network 1 for using network 2's resource, as the case of $\alpha = 0.8$ and $m = 0.6$ in Fig. 5) to reach the social optimality. For example, assume that network 1 is a cellular network that has a large coverage area, a large market share, and an average of not very high channel condition to the users. Network 2 is a commercial Wi-Fi service provider that has a small coverage area, a small market share, and an average of excellent channel condition to the users. Then in order to maintain the social optimal user pricing, i.e., keeping the user price of the cellular network π_1^* low enough, the Wi-Fi service provider needs to pay the cellular provider for a file transfer from a cellular user to a Wi-Fi user, as the cellular network is bearing most of the network costs in supporting the communication session. Notice that here we consider the case where the regulator determines the access pricing; it will be a quite different story if the networks themselves optimize the access pricing to maximize their profits.

Observation 7: The social optimal access price increases in the elasticity of the users' utilities.

We also note from Fig. 5 that the utility parameter α has a significant impact on the social optimal access pricing. A smaller α (e.g., a higher elasticity) means a higher optimal access price (under the same market share). Since today's wireless networks are becoming more data-centric, we can expect that the overall users' utility functions (determined by the applications) will become more elastic and thus the optimal access price will become higher.

V. CONCLUSION AND FUTURE WORK

In this paper, we studied the choice of wireless user pricing and access pricing from a regulator's perspective. In the case where the regulator can directly control the user pricing, we show that a network's social optimal user pricing is always increasing in the cost per unit bandwidth. The dependancy on the market share is more complicated and trades off network cost and user utility. When the regulator can only control the access price, we show that it is still possible to achieve the social optimality after taking the networks' profit-maximizing behaviors into consideration. The social optimal access pricing decreases with the rival network's market share and increases in the elasticity of users' utility functions.

Our results can help to understand the case of multiple access networks. For example, in the case of three networks, A, B, and C, where B is located in the middle and the other two are located at the two ends, from the result of this research work, we know that the access price set by B decreases as market shares of A and C increase.

In the future, we will further study a completely deregulated market, where users can freely choose networks, and networks can set both the user pricing and access pricing. We will characterize the loss of social welfare in that case, and find out the scenarios where regulation is most desirable.

REFERENCES

[1] J. Laffont, P. Rey, P. and J. Tirole, *Network competition: I. Overview and nondiscriminatory pricing*, Rand Journal of Economics, vol. 29, no. 1, pp. 1–37, 1998.

[2] J. Wright, *Access Pricing under Competition: an Application to Cellular Networks*, Journal of Industrial Economics, vol. 50, no. 3, pp. 289–315, 2002.

[3] Y. Cheung, D. Chiu, and J. Huang, *Can Bilateral ISP Peering Lead to Network-Wide Cooperative Settlement*, IEEE ICCN, 2008.

[4] R.T.B. Ma, D.M. Chiu, J.C.S. Lui, V. Misra, D. Rubenstein, *Interconnecting Eyeballs to Content: A Shapley Value Perspective on ISP Peering and Settlement*, Network Economics Workshop, 2008.

[5] J. Jia and Q. Zhang, *Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market*, ACM MobiHoc, 2008.

[6] E. Altman, A. Kumar, C. Singh, R. Sundaresan, *Spatial SINR Games Combining Base Station Placement and Mobile Association*, IEEE INFOCOM, April 2009.

[7] G. S. Kasbekar, E. Altman, and S. Sarkar, *A Hierarchical Spatial Game over Licenced Resources*, International Conference on Game Theory for Networks, May 2009.

[8] D. Niyato and E. Hossain, *A game theoretic analysis of service competition and pricing in heterogeneous wireless access networks,* IEEE Transactions on Wireless Communications, vol. 7, no. 12, pp. 5150–5155, December 2008.

[9] M. Armstrong and J. Wright, *Mobile Call Termination*, The Economic Journal, 2008.

[10] J. Mo and J. Walrand, *Fair end-to-end window-based congestion control*, IEEE/ACM Transactions on Networking, vol. 8, no. 5, pp.556–567, 2000.

[11] J. Cohn, *Cell Phone Coverage Maps - AT&T, Verizon, Sprint, T-Mobile* [online], available from World Wide Web:http://www.deadcellzones.com/