Revenue Management for Cognitive Spectrum Underlay Networks: An Interference Elasticity Perspective

Jianwei Huang[1] and Xiwei Huang[2]
[1]Information Engineering Department, Chinese University of Hong Kong, China, jwhuang@ie.cuhk.edu.hk
[2]Linkage Technology Co. Ltd., Nanjing, China, huangxw@liangchuang.com

***Abstract*—We study the problem of revenue management in a cognitive spectrum underlay network, where the primary user chooses the bandwidth and tolerable interference levels for the secondary users. We show that the interference received by each secondary user is the same in a large system regime, and a simple pricing scheme achieves the efficient resource allocation at the market clearing equilibrium. We further show that the key for the primary user to maximize revenue is to consider the secondary users' *power-interference elasticity*, which is a new concept proposed and is different from all elasticity concepts used in the previous networking literature. When this elasticity is negative (positive, respectively), the primary user should increase (decrease, respectively) the bandwidth and tolerable interference level to increase revenue.**

I. Introduction

Recent advances in cognitive radio technology have enabled wireless devices and networks to locate and exploit under-utilized spectrum. Most existing studies in this field have focused on how to improve the network performance by allowing primary (licensed) and secondary (unlicensed) users to efficiently and flexibly share the spectrum. However, in practice a primary user may not have enough incentives to share the spectrum with the secondary users in a fear of potential degradation of its Quality of Services[1].

In this paper, we consider a scenario where a primary user can collect revenue from the secondary users. This provides the necessary incentive for the primary user to share the spectrum with the secondary users. We consider a spectrum underlay network where the secondary users can transmit simultaneously with the primary user, subject to total bandwidth and tolerable interference constraints. The primary user collects the revenue through charging the secondary users proportional to their generated interference. The primary user can maximize its revenue by adjust the bandwidth and tolerable interference constraints as long as certain physical constraints are not violated.

The key contribution of this paper is to propose the concept of *power-interference elasticity*, which is the key for the primary user's revenue maximization decision. In the existing literature, Shenker [2] was the first to introduce the concept of elasticity into networking, where he refereed *elasticity* as the applications' ability to adapt their sending rates according to the available resource. Yuksel and Kalyanaraman [3] further developed this idea and defined a *utility-bandwidth elasticity*, which is always nonnegative. The authors also calculated the corresponding optimal pricing to maximize total utility. Marbach and Berry [4] proposed an optimal pricing scheme by price discriminating users with respect to the *power-price elasticity,* but the market equilibrium is not necessarily reached. By contrast, the *power-interference elasticity* defined in this paper is about the relationship between the demand (power) and the environment (interference), instead of the utility and the resource (bandwidth).

[1]It is widely known that the broadcasting industry in USA is strongly opposed to the FCC's intention of allowing public access of the white space in the TV spectrum [1].

Our paper is related to the large body of work on uplink power control with pricing for CDMA networks (e.g., [5]–[10] and a recent survey in [11]). The key difference here is that we focus on the spectrum underlay model, which has a total received interference power constraint at the primary user's receiver. The transmission power constraints of the secondary users are not the performance bottleneck of the system and thus are not considered here.

Another closely related literature is [12], where the authors considered the problem of social optimal resource allocation through pricing. The focus of this paper, however, is revenue optimization of the primary user by adjusting the total bandwidth and interference power constraint. By comparison, the bandwidth and interference constraints are fixed in [12]. Moreover, we focus on a large system limit here, where the total power P and bandwidth B increase in fixed proportion to the number of users M. In this case, the interference received by each user equals P/B and thus is user-independent. We study the changes of the total utility of secondary users and the revenue of the primary user at the market equilibrium with respect to both P and B.

This paper is organized as follows. We describe the system model in Section II, and introduce the concepts of power-interference elasticity in Section III. We analyze the impact of bandwidth and tolerable interference constraints on the total network utility and the revenue in Section IV. We present the simulation results are shown in Section V and conclude in Section VI.

II. System Model

We consider a primary licensed user who owns a wireless spectrum of bandwidth $\bar{B}$. The primary user is able to tolerate a maximum interference temperature $\bar{P}/\bar{B}$. The primary user can allow the secondary users to share a total spectrum of bandwidth B with a total generated interference power at the primary user's receiver as P, as long as $B \leq \bar{B}$ and $P/B \leq \bar{P}/\bar{B}$.

There exists a set $\mathcal{M} = \{1, \ldots, M\}$ of secondary users, who transmit to the same secondary base station. We focus on the case where the secondary base station is co-located with the primary user's receiver. In this case, the total power received from the secondary users at the base station is the same as the total tolerable interference at the primary user's receiver.

A user $i's$ valuation of the resource is characterized by a utility function $U_i(\theta_i, \gamma_i(\boldsymbol{p}))$, which is increasing, strictly concave, and twice continuously differentiable of its signal-to-interference plus noise ratio (SINR) at the base station

$$\gamma_i(\boldsymbol{p}) = \frac{p_i}{n_0 + p_{-i}/B}, \tag{1}$$

where n_0 is the background noise power density, p_i is the power received from user i at the secondary base station, $p_{-i} = \sum_{j \neq i, j \in \mathcal{M}} p_j$, $\boldsymbol{p} = (p_i, p_{-i})$, and θ_i is a user-dependent parameter. We assume that the users choose spread spectrum transmission over the entire allowed bandwidth B, and thus the interference of a user i is the total received power from other secondary users scaled by the bandwidth. To simplify the notations, we denote $U_i'(\theta, \gamma) = \partial U_i(\theta, \gamma)/\partial \gamma$ and $U_i''(\theta, \gamma) = \partial^2 U_i(\theta, \gamma)/\partial \gamma^2$.

The key constraint is that the total power allocation satisfies

$$\sum_{i \in \mathcal{M}} p_i = p_i + p_{-i} \leq P, \tag{2}$$

where P is the total tolerable interference at the primary user's receiver.

The power allocation is performed in a distributed fashion via pricing. The primary user announces a positive unit price π. Each secondary user i submits the demand $p_i^*(p_{-i}, \pi)$ to maximize his surplus

$$\begin{aligned} p_i^*(\pi, p_{-i}, \theta_i) &= \arg\max_{\hat{p}_i \geq 0} S_i(\pi, \hat{p}_i, p_{-i}, \theta_i) \\ &= \arg\max_{\hat{p}_i \geq 0} U_i(\theta_i, \gamma_i(\hat{p}_i, p_{-i})) - \pi \hat{p}_i. \end{aligned} \tag{3}$$

It is clear that secondary users play a noncooperative game here, since a user decision p_i^* depends on the other users' choice p_{-i}. Details of such game theoretical analysis for a finite system can be found in [12].

Here, we focus on a *large system limit* where there are many secondary users sharing a large bandwidth. Mathematically, we focus on the asymptotic behavior as P, B, M go to infinity, while keeping P/M and P/B fixed. We can show that if the utilities are asymptotically sublinear with respect to γ_i (i.e., $\lim_{\gamma_i \to \infty} \frac{1}{\gamma_i} U_i(\theta_i, \gamma_i) = 0$ for all i) and θ_i is continuously distributed in a nonnegative interval $[\underline{\theta}, \bar{\theta}]$, then we can always find a price *market clearing price* π^* such that $\sum_{i=1}^{M} p_i^*(p_{-i}^*, \pi^*) = P$. More importantly, a user i's SINR at the market equilibrium is

$$\gamma_i(p_i) = \frac{p_i}{n_0 + P/B}, \tag{4}$$

i.e., the interference experienced by any secondary user i is a user-independent constant P/B.

The sublinear requirement can be satisfied by many common utility functions, e.g., $\theta \ln(\gamma)$, $\theta \ln(1+\gamma)$, $\theta \gamma^\alpha$ ($\alpha \in (0,1)$), and any upper-bounded utility such as $1 - e^{-\theta\gamma}$. The user-independent property of the interference makes the large system limit analytically more attractable than the finite system. In [12] we showed that this large system limit can be reached with moderate number of users (less than 20) in practice.

In the rest of the paper, we will restrict our study to revenue management at the market clearing price π^*. The results can be easily generalized to the case where the primary user can choose a price that does not clear the market, in which case the primary user may increase the revenue by further price discriminating among users [13].

III. Elasticities in a Large System

To simplify the notation, let us write $I = P/B$. A user i's surplus is a function of the price π, interference I, and power allocation p:

$$S(\pi, p, I, \theta) = U\left(\theta, \frac{p}{n_0 + I}\right) - \pi p. \tag{5}$$

Here we consider a generic user and omit the user index i.

The *power demand function* $p^o(\pi, I, \theta)$ (i.e., optimal choice of power for a user to maximize its surplus) is

$$\begin{aligned} p^o(\pi, I, \theta) &= \arg\max_{\hat{p} \geq 0} S(\pi, \hat{p}, I, \theta) \\ &= \begin{cases} (n_0 + I)\, g_\theta^{-1}(\pi(n_0 + I)), & U'(\theta, 0) > \pi(n_0 + I) \\ 0, & U'(\theta, 0) \leq \pi(n_0 + I) \end{cases}, \end{aligned} \tag{6}$$

where $g_\theta(\gamma) = U'(\theta, \gamma)$ and the superscript -1 denotes the inverse function. The corresponding *SINR demand function* is $\gamma^o(\pi, I, \theta) = p^o(\pi, I, \theta)/(n_0 + I)$.

Proposition 1: In a large system,

$$\frac{\partial \gamma^o(\pi, I, \theta)}{\partial I} = \frac{\pi}{U''(\theta, \gamma^o(\pi, I, \theta))} < 0 \tag{7}$$

for $\pi < U'(\theta, 0)/(n_0 + I)$.

Proposition 1 shows that a user will always choose a smaller SINR when the interference increases. This is, however, not the case for power demand $p^o(\pi, I, \theta)$.

To facilitate further discussion, we first introduce the *power-price elasticity* used in economics:

Definition 1: Power-price elasticity in a large system is

$$e_\pi(p^o(\pi, I, \theta)) = \frac{\partial p^o(\pi, I, \theta)/p^o(\pi, I, \theta)}{\partial \pi/\pi}. \tag{8}$$

Since the utility function is concave in γ, thus the power demand curve has a negative slope, and the power-price

TABLE I
ELASTICITY OF POWER DEMAND IN A LARGE SYSTEM

$U(\theta,\gamma)$	$\pi < \frac{U'(\theta,0)}{n_0+I}$	$e_\pi(p^o(\pi,I,\theta))$ (power-price elasticity)		$e_I(p^o(\pi,I,\theta))$ (power-interference elasticity)	
$\theta\ln(\gamma)$	$\pi<\infty$	-1	unitary elastic	0	zero
$\theta\ln(\gamma+1)$	$\pi<\frac{\theta}{n_0+I}$	$-\frac{\theta}{\theta-\pi(n_0+I)}<-1$	elastic	$-\frac{\pi(n_0+I)}{\theta-\pi(n_0+I)}<0$	negtive
$\theta\gamma^\alpha$ $(\alpha\in(0,1))$	$\pi<\infty$	$-\frac{1}{1-\alpha}<-1$	elastic	$-\frac{\alpha}{1-\alpha}<0$	negtive
$1-e^{-\theta\gamma}$	$\pi<\frac{\theta}{n_0+I}$	$-\frac{1}{\ln\left(\frac{\theta}{\pi(n_0+I)}\right)}<0$	depends	$-\frac{1-\ln\left(\frac{\theta}{\pi(n_0+I)}\right)}{\ln\left(\frac{\theta}{\pi(n_0+I)}\right)}<1$	depends

elasticity is always negative [14]. The elasticity characteristic $L_{e_\pi(p^o(\pi,I,\theta))}$ is defined as [13]:

$$L_{e_\pi(p^o(\pi,I,\theta))} = \begin{cases} \text{elastic}, & e_\pi \in (-\infty,-1) \\ \text{unitary elastic}, & e_\pi = -1 \\ \text{inelastic}, & e_\pi \in (-1,0) \end{cases} \quad (9)$$

Next we define the *power-interference elasticity* which is new in this paper:

Definition 2: Power-interference elasticity in a large system is

$$e_I\left(p^o(\pi,I,\theta)\right) = \frac{\partial p^o(\pi,I,\theta)/p^o(\pi,I,\theta)}{\partial I/(n_0+I)}. \quad (10)$$

The power-interference elasticity shows how the power demand changes with respect to the change of interference. Although it is possible to give a similar definition of the elasticity characteristic of $e_I(p^o(\pi,I,\theta))$ as in (9), we are more interested in the sign of (10) since it is not necessarily negative as shown below.

Proposition 2: In the large system limit,

$$e_I\left(p^o(\pi,I,\theta)\right) = e_\pi\left(p^o(\pi,I,\theta)\right)+1 \quad (11)$$

for $\pi < U'(\theta,0)/(n_0+I)$.

Proposition 2 shows the simple relationship between the power-interference and the power-price elasticities. Moreover, the sign of $e_I(p^o(\pi,I,\theta))$ depends simply on the elasticity characteristic of $e_\pi(p^o(\pi,I,\theta))$.

Table I shows both elasticities for some common utility functions.

Similarly, we can define the aggregated power-price elasticity (aggregate power-interference elasticity, respectively) in a large system as $e_{A\pi}\left(\sum_{i=1}^M p_i^o(\pi,I,\theta_i)\right)$ ($e_{AI}\left(\sum_{i=1}^M p_i^o(\pi,I,\theta_i)\right)$, respectively) by substituting $p^o(\pi,I,\theta)$ in Definition 1 (Definition 2, respectively) with $\sum_{i=1}^M p_i^o(\pi,I,,\theta_i)$. It is easy to show that

$$e_{AI}\left(\sum_{i=1}^M p_i^o(\pi,I,\theta_i)\right) = e_{A\pi}\left(\sum_{i=1}^M p_i^o(\pi,I,\theta_i)\right)+1. \quad (12)$$

IV. THE IMPACT OF BANDWIDTH B AND INTERFERENCE P ON THE TOTAL UTILITY AND REVENUE

First consider the impact of B and P on secondary users' total utility $\sum_{i\in\mathcal{M}} U_i(\theta_i,\gamma_i)$.

Theorem 1: In the large system limit, the secondary users' total utility is maximized at the market clearing price. Moreover, the total utility and the active users' SINRs are increasing in P and B.

This show that allowing more bandwidth or more tolerable interference to the secondary users will increase the secondary users' QoS. This is intuitive, as more bandwidth means less interference and more tolerable interference (from the primary user's point of view) means higher transmission power (for the secondary users), both will lead to high SINRs of the secondary users.

The impact of B and P on the revenue of the primary user, however, is more complicated.

Theorem 2: In the large system limit,

$$\partial R/\partial B \begin{cases} >0, & e_{AI}\left(\sum_{i=1}^M p_i^o(\pi^*,I,\theta_i)\right)<0 \\ =0, & e_{AI}\left(\sum_{i=1}^M p_i^o(\pi^*,I,\theta_i)\right)=0 \\ <0, & e_{AI}\left(\sum_{i=1}^M p_i^o(\pi^*,I,\theta_i)\right)>0 \end{cases}, \quad (13)$$

where π^* is the market clearing price.

Theorem 3: In the large system limit,

$$\partial R/\partial P \begin{cases} >0, & e_{AI}\left(\sum_{i=1}^M p_i^o(\pi^*,I,\theta_i)\right)<0 \\ =0, & e_{AI}\left(\sum_{i=1}^M p_i^o(\pi^*,I,\theta_i)\right)=0 \\ <0, & e_{AI}\left(\sum_{i=1}^M p_i^o(\pi^*,I,\theta_i)\right)>0 \end{cases},$$

where π^* is the market clearing price. If n_0 is negligible compared with interference for all i, then revenue R does not change with P, i.e., $\partial R/\partial P = 0$.

Theorems 2 and 3 show that the aggregated power-interference elasticity is important for the primary user's revenue maximization decision. First, if it is negative, the primary user should increase P and B until it becomes zero, or the resource is exhausted, or the interference temperature is reached. Second, if it is positive, the manger should decrease P and B until it becomes zero, or the last user is indifferent in joining or quiting the system (but is still active), or the interference temperature is reached. Finally, if it is zero, nothing needs to be done since the revenue is already maximized.

If we assume that the primary user is the only seller in the spectrum market of a certain time period at certain geographic area, and the secondary users can not transfer the usage rights among themselves, then the primary user can further improve the revenue if he can separate the users into groups by the individual power-interference elasticities. The improvement can be achieved by a third-degree price discrimination, i.e., different groups are charged different unit prices [13]. One thing to notice is that the secondary users' behavior in our problem depends heavily on the interference level, thus the primary user should be careful in assigning resources for different groups, i.e., the values of P, B, and the ratio P/B. This is rather unique for our problem.

Theorem 3 also shows that when n_0 is very small compared with interference, the primary user could decrease P (and thus decrease interference temperature) while keeping the revenue unchanged. However this will eventually breaks down when P/B is close to n_0.

V. Numerical Study

Here we present some numerical results illustrating how the primary user's revenue changes with respect to P and B for various utility functions with different elasticities.

We consider 10 symmetric secondary users with $\theta_i = 10$ for each user. Figure 1 shows the rate utility $\theta_i \ln(1+\gamma)$, which always has a negative power-interference elasticity. As a result, the revenue increases in both B and P. Figure 2 shows the exponential utility $1 - \exp(-\theta\gamma)$, which has an positive power-interference elasticity under the system parameters here. The revenue decreases in both B and P. These results are consistent with Theorems 2 and 3.

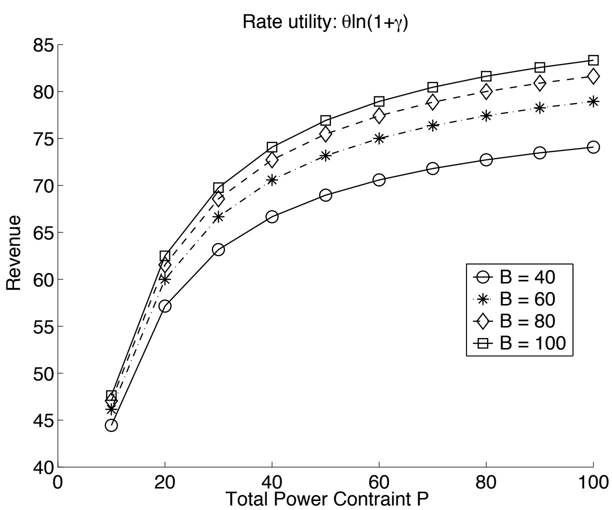

Fig. 1. Revenue with the elastic rate utility function

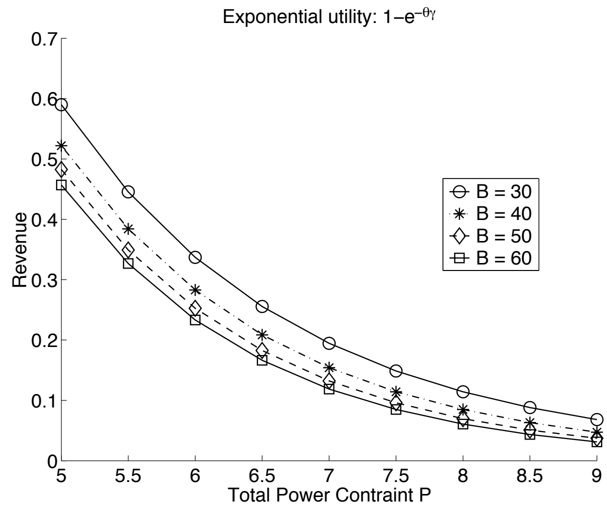

Fig. 2. Revenue with the inelastic exponential utility function

VI. Summary

In this paper, we consider the problem of revenue management in a cognitive spectrum underlay network. The primary user maximizes its revenue by adjusting the bandwidth and interference constraints to the secondary users. To solve this problem, we propose the new concept of *power-interference elasticity*, which shows the simple connection with the traditional power-price elasticity in microeconomics. We show that in order to maximize the revenue, the primary user should allocate a higher (lower, respectively) bandwidth and interference power constraint whenever the aggregate power-interference elasticity of the secondary users is negative (positive, respectively).

Future research directions include detailed analysis of the revenue management in a finite system where the interference is not user-independent.

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