Economic Analysis of 4G Upgrade Timing

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Abstract—As the successor to the 3G standard, the 4G cellular standard provides much higher data rates to address cellular users’ ever-increasing demands for high-speed multimedia communications. This paper analyzes the cellular operators’ timing of network upgrades, by considering user subscription dynamics induced by switching from 3G to 4G technologies. Being the first to upgrade 3G to 4G service, an operator increases its market share but takes more risk or upgrade cost as 4G technology matures over time. This paper first studies a 4G monopoly market with one dominant operator and some small operators, where the monopolist decides its upgrade time by trading off increased market share and upgrade cost. The paper also considers a 4G competitive market and develops a game theoretic model for studying operators’ interactions. The analysis shows that operators select different upgrade times to avoid severe competition. One operator takes the lead to upgrade, using the benefit of a larger market share to compensate for the larger cost of an early upgrade. This result matches well with many industry observations of asymmetric 4G upgrades. The paper further shows that the availability of 4G upgrade may decrease both operators’ profits due to increased competition. Perhaps surprisingly, the profits can increase with the upgrade cost.

Index Terms—networks economics, 4G networks economics, 4G upgrade timing, user dynamics, game theory, Nash equilibrium.

1 INTRODUCTION

The third generation (3G) of cellular wireless networks was launched during the last decade. It has provided users with high-quality voice channels and moderate data rates (up to 2 Mbps). However, 3G service cannot seamlessly integrate the existing wireless technologies (e.g., GSM, wireless LAN, and Bluetooth) [2], and cannot satisfy users’ fast growing needs for high data rates. Thus, most major cellular operators worldwide plan to deploy fourth-generation (4G) networks to provide much higher data rates (up to hundreds of megabits per second) and integrate heterogeneous wireless technologies. 4G technology is expected to support new services such as high-quality video chat and video conferencing [5].

One may expect competitive operators in the same cellular market to upgrade to a 4G service at about the same time. However, many industry examples show that symmetric 4G upgrades do not happen in practice, even when multiple operators have obtained the necessary spectrum and technology patents for upgrade ([5], [6]). In South Korea, for example, Korean Telecom took the lead to deploy the world’s first 4G network using WiMAX technology in 2006, whereas SK Telecom started to upgrade using more mature LTE technology in 2011. In US, Sprint deployed the first 4G WiMAX network in late 2008, Verizon waited until the beginning of 2011 to deploy its 4G LTE network, and AT&T formally deployed its 4G LTE network in 2013 [5]. In China, China Mobile and China Unicom are the two dominant cellular operators, and China Mobile has decided to be the first to deploy 4G LTE network during 2012-2013 [6]. Thus, the key question we want to answer in this paper is the following: How do the cellular operators decide the timing of an upgrade to 4G networks?²

In this paper, we analyze the timing of operators’ 4G upgrades in different settings, including both a 4G monopoly market and a 4G competitive market. Operators need to pay the cost of 4G upgrade, which decreases over time as 4G technology matures. There are two key factors that affect the operators’ upgrade decisions: namely, 4G upgrade cost and user switching cost. An existing 3G user can switch to the 4G service of the same operator or of a different operator, depending on how large the switching cost is. In a monopoly market where only a dominant operator can choose to upgrade to 4G, this operator can use the 4G service to capture a larger market share from small operators. In a competitive market where multiple operators can choose to upgrade, we analyze the operators’ interactions as a non-cooperative game. We study how the users’ inter-network switching cost affects the operators’ upgrade decisions, and our findings are consistent with the asymmetric upgrades observed in the industry.

Our key results and contributions are as follows.

1 We define the upgrade time to be the first time when the 4G service is offered by the operator in the market.
A revenue-sharing model between operators: Most existing works only study a single network’s revenue by exploring the network effect (e.g., [4], [9]), and the results may not apply to a competitive market. In Section 3, we study two interconnected networks, where their operators share the revenue of the inter-network traffic.

Monopolist’s optimal timing of 4G upgrade: By upgrading early, the 4G monopolist in Section 4 obtains a large market share and a large revenue because of 4G’s Quality of Service (QoS) improvement, but it cannot benefit from the cost depreciation over time. When the upgrade cost is relatively low, it upgrades at the earliest available time; otherwise it postpones its upgrade.

Competitive operators’ symmetric upgrades under no inter-network switching: In Section 5, we develop a game theoretic model for studying competitive operators’ interactions. In this situation, the operators upgrade simultaneously, irrespectively of whether the upgrade cost is high or low.

Competitive operators’ asymmetric upgrades under practical inter-network switching: In Section 7, users can switch operators. By upgrading early, an operator captures a large market share and the 4G’s QoS improvement which can compensate for the large upgrade cost. The other operator, however, postpones its upgrade to avoid severe competition and benefits from cost reduction. The availability of 4G upgrade may decrease both operators’ profits because of the increased competition, and paradoxically, their profits may increase with the upgrade cost.

2 RELATED WORK
2.1 Network Effect and Network Value
In telecommunications, the network effect is the added value that a user derives from the presence of other users [25]. In a network with \( N \) users, each user perceives a value that increases with \( N \). If each user attaches the same value to the possibility of connecting to any one of the other \( N - 1 \) users, it may be considered that he perceives a network value proportional to \( N - 1 \). Then the total value of the network is proportional to \( N(N - 1) \), or roughly \( N^2 \), which is known as the Metcalfe’s Law [4]. A refined model was suggested by Briscoe et al. [9], where each user perceives a value of order \( \log(N) \). In that model, a user ranks the other users in decreasing order of importance and assigns a value \( 1/k \) to the \( k \)-th user in that order, for a total value \( 1 + 1/2 + \cdots + 1/(N - 1) \approx \log(N) \). The resulting total network value is \( N \log(N) \), which is appropriate for cellular networks shown by quantitative studies [9].

2.2 Network Upgrade
Recently, there has been a growing interest in studying the economics of network upgrades [10]–[12]. Musacchio et al. [10] studied the upgrade timing game between two interconnected Internet Service Providers (ISPs), where one ISP’s architecture upgrade also benefits the other because of the network effect. This free-riding effect may make the second operator postpone its upgrade or even never decide to upgrade. Jiang et al. [11] studied a network security game, where one user’s investment (upgrade) can reduce the propagation of computer viruses to all users. In our problem, however, one operator may benefit from the other’s upgrade only when it also upgrades, letting its 4G users communicate with existing 4G users in other networks. Moreover, our model characterizes the dynamics of users switching between operators and/or services. These dynamics imply that an operator can obtain a larger market share by upgrading earlier, and this weakens the free-riding effect. Sen et al. [12] studied the users’ adoption and diffusion of a new network technology in the presence of an incumbent technology. Our work is different from that study in that we are not focusing on technology competition to attract users, but on the operators’ competition in upgrade timing to obtain greater profits. Moreover, the switching cost is not considered in [12], whereas it is an important parameter of our model.

Our study is related to the literature on the war of attrition [34]–[38], which considered how \( N + K \) firms compete for \( N \) prizes, and analyzed the exit time for low-value firms. A key result is that low-value firms will drop out instantaneously. Our model is different from these models in several aspects. First, our model considers a tighter interplay among operators, where one operator can use 4G service to attract subscribers from another operator. There is no such interactions among firms in the war of attrition games. Second, the user churn in our model makes the revenue rate time-varying for operators, and the maturity of 4G technology over time provides an incentive for a late upgrade. These factors were not considered in [34]–[38]. Because of these differences, our analysis produces some counter-intuitive results, e.g., the availability of the 4G technology may reduce the operators’ profits.

3 SYSTEM MODEL
3.1 Value of Cellular Networks
In this paper, we adopt the \( N \log(N) \) Law [9], where the network value with \( N \) users is proportional to \( N \log(N) \). The operator of a cellular network prefers a large network value; this is because the revenue it obtains by charging users can be proportional to the network value. According to the surveys done by ABIresearch and Validas LLC ([13], [14]), the 4G data service is usually priced significantly higher than the 3G data service, and 4G users usually consume much more data compared with 3G users. The communication between two 4G users is more efficient and more frequent than between two 3G users. Thus the value of a 4G network is larger than a 3G network even when two networks have the
same number of users. Because the average data rate in the 4G service is 5-10 times higher than the 3G (both downlink and uplink), a 4G network can support many new applications. We denote the efficiency ratio between 3G and 4G services as \( \gamma \in (0, 1) \). That is, by serving all its users via QoS-guaranteed 4G rather than 3G services, an operator obtains a larger (normalized) revenue \( N \log(N) \) instead of \( \gamma N \log(N) \).\(^2\) Note that this result holds for a single operator’s network that is not connected to other networks.

Next we discuss the revenues of multiple operators whose networks (e.g., two 3G networks) are interconnected. For the purpose of illustration, we consider two networks that contain \( N_1 \) and \( N_2 \) users, respectively. The whole market covers \( N = N_1 + N_2 \) users. We assume that two operators’ 3G (and later 4G) services are equally good to users, and the efficiency ratio \( \gamma \) is the same for both operators. The traffic between two users can be intra-network (when both users belong to the same operator) or inter-network (when two users belong to different operators), and the revenue calculations in the two cases are different. We assume that the user who originates the communication session (irrespective of whether the same network or to the other network) pays for the communication. This is motivated by the industry observations in EU and many Asian countries.\(^3\) Before analyzing each operator’s revenue, we first introduce two practical concepts in cellular market: “termination rate” and “user ignorance”.

When two users of the same operator 1 communicate with each other, the calling user only pays operator 1. But when an operator 1’s user calls an operator 2’s user, operator 2 charges a termination rate for the incoming call.\(^4\) We denote the two operators’ revenue-sharing portion per inter-network call as \( \eta \), where the value of \( \eta \in (0, 1) \) depends on the agreement between the two operators or on governments’ regulation on termination rate.

User ignorance is a unique property in the wireless cellular network, where users are often not able to identify which specific network they are calling [27]. Mobile number portability further exacerbates this feature [26]. As the data communication becomes dominant in wireless networks, many recent wireless plans (e.g., provided by AT&T and Verizon) no longer charge intra- and inter-network traffics differently, and this further strengthens user ignorance. Thus a typical user’s evaluation of two interconnected 3G networks does not depend on which network he belongs to, and equals \( \gamma \log(N) \) where \( N = N_1 + N_2 \). We assume a call from any user terminates at a user in network \( i \in \{1, 2\} \) with a probability of \( N_i / N \) as in [26]. The operators’ revenues when they are both providing 3G services are given in Lemma 1.

**Lemma 1:** When operators 1 and 2 provide 3G services, their revenues are \( \gamma N_1 \log(N) \) and \( \gamma N_2 \log(N) \), respectively.

**Proof:** For a user in operator 1’s network, his communication brings \( \frac{N_1}{N} \gamma \log(N) + \frac{N_2}{N} \gamma \log(N) \) amount of revenue to operator 1 and \( (1 - \gamma) \frac{N_1}{N} \gamma \log(N) \) amount of revenue to operator 2. Similarly, for a user in operator 2’s network, his communication brings \( (1 - \gamma) \frac{N_2}{N} \gamma \log(N) + \frac{N_1}{N} \gamma \log(N) \) amount of revenue to operator 1 and \( \frac{N_2}{N} \gamma \log(N) \) amount of revenue to operator 2. Thus operator 1’s total revenue is \( N_1(\gamma \log(N) + \frac{N_2}{N} \gamma \log(N)) + N_2((1 - \gamma) \frac{N_1}{N} \gamma \log(N)) = \gamma N_1 \log(N) \), and operator 2’s total revenue is \( \gamma N_2 \log(N) \) due to symmetry. \( \square \)

Both operators’ revenues are linear in their numbers of users (or market share), and are independent of the sharing portion \( \eta \) of the inter-network revenue. Intuitively, the inter-network traffic between two networks is bidirectional: when a user originates a call from network 1 to another user in network 2, its inter-network traffic generates a fraction \( \eta \) of corresponding revenue to operator 1; when the other user calls back from network 2 to network 1, he generates a fraction \( 1 - \eta \) of the same amount of revenue to operator 1. Thus an operator’s total revenue is independent of \( \eta \). Later, in Section 4, we show that such independence on \( \eta \) also applies when the two operators both provide 4G services or provide mixed 3G and 4G services.

### 3.2 User Churn during Upgrade from 3G to 4G Services

When 4G service becomes available in the market (offered by one or both networks), the existing 3G users have an incentive to switch to the new service to experience a better QoS. Such user churn does not happen simultaneously for all users, as different users have different sensitivities to quality improvements and switching costs. We use two parameters \( \lambda \) and \( \alpha \) to model the user churn within and between operators:

- **Intra-network user churn:** If an operator provides 4G in addition to its existing 3G service, its 3G users need to buy new mobile phones to use the 4G service. The users also spend time to learn how to use the 4G service on their new phones. We use \( \lambda \) to denote the users’ switching rate to the 4G service within the same network as in [20].

- **Inter-network user churn:** If a 3G user wants to switch to another network’s 4G service, he either waits till his current 3G contract expires, or pays for the penalty of immediate contract termination.

\( \text{2. We assume that an operator’s operational cost (proportional to network value) has been deducted already, and thus the revenue in this paper represents a normalized one.} \)

\( \text{3. Our model can also be extended to the case where both involved users in a communication session pay for their communication. This is what happening in US cellular market.} \)

\( \text{4. In the US, termination rate follows “Bill and Keep” and is low. Then operator 1 can keep most of the calling user’s payment. In EU, however, termination rate follows “Calling Party Pays” and is much higher. Then most of the calling user’s payment to operator 1 is used to compensate for the termination rate charged by operator 2 [22]. Though these models have different revenue sharing portions } \eta \text{, our following key results are independent of } \eta \text{ and thus apply to both models.} \)
service. Furthermore, Verizon’s 4G service attracted its own 3G users and other operators’ 3G users at different rates.

Figure 1 illustrates the number of users churning from Verizon’s 3G services to 4G services in each quarter. We can accurately match the intra-network churning data using an exponential function with a rate of \( \lambda = 0.045 \) million per quarter. Similarly, Figure 2 shows how the Verizon’s total subscriber number increases due to user churn from other operators. Note that Verizon’s increase of total user number in Fig. 2 is mainly due to the introduction of 4G LTE service, as its 3G market has already saturated ([21], [39]). Fig. 3 shows that between 2005 and 2010, Verizon’s 3G retail user number increases more and more slowly over time, and such a change becomes negligible before its 4G upgrade (i.e., less than 0.6% increase from 4Q’09 to 1Q’10). After its 4G upgrade, Table 3.2 shows the change (in percentage) of Verizon’s 3G user number. We can see that the 3G user number decreases faster over time, as more and more Verizon’s 3G users churn to its 4G service (which is consistent with Fig. 1). Finally, as Fig. 2 approximates well the number of subscribers switching to Verizon’s 4G service from other operators, we can estimate the parameter of the exponential growth to be \( \alpha \lambda = 0.011 \) million per quarter for inter-network churning. Thus, these industry observations validate our model of exponential user intra- and inter-network churns. The \( \alpha \) value (equal to 0.24 in this example) reflects the transaction cost of switching operators (e.g., breaking existing contract).

Table 1

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Change (in percentage) of Verizon’s 3G user number after the 4G upgrade [21]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q2’11</td>
<td>-0.35%</td>
</tr>
<tr>
<td>Q3’11</td>
<td>-1.72%</td>
</tr>
<tr>
<td>Q4’11</td>
<td>-1.84%</td>
</tr>
<tr>
<td>Q1’12</td>
<td>-2.05%</td>
</tr>
<tr>
<td>Q2’12</td>
<td>-3.88%</td>
</tr>
<tr>
<td>Q3’12</td>
<td>-5.70%</td>
</tr>
</tbody>
</table>

This means that inter-network user churn incurs an additional cost on top of the mobile device update, and thus the switching rate will be smaller than the intra-network user churn. We use \( \alpha \lambda \) to denote the users’ inter-network switching rate to 4G service, where \( \alpha \in (0, 1) \) reflects the transaction cost of switching operators.

We illustrate the process of user churn through a continuous time model. The starting time \( t = 0 \) denotes the time when the spectrum resource and 4G technology are available for at least one operator (see Section 4 for monopoly market and Sections 6 and 7 for competitive market). Similar to [15], [16], we also assume that the portion of users switching to the 4G service follows the exponential distribution (at a rate \( \lambda \) for intra-network churn and a rate of \( \alpha \lambda \) for inter-network churn). The exponential switching model has been widely used in marketing research [15]–[17], and can help us derive closed-form solutions and engineering insights. We assume the user churn model is exogenous given. We do not consider dynamic pricing between strategic operators, and our focus is the upgrade timing in long run. Such a model can reasonably match the existing industry data with properly chosen system parameters. Let us take Verizon’s market data from [21] as an example, which captures the changes of Verizon’s subscribers since its 4G launch in early 2011 (as the second 4G operator in the US). We analyze this data according to the methodology in [19]. After Verizon launched its 4G service, its 3G users no longer churned to Sprint’s 4G service. Furthermore, Verizon’s 4G service attracted its own 3G users and other operators’ 3G users at different rates.

Let us consider an example to illustrate how users switch between two operators and two services. We assume that operator 1 introduces a 4G service at time \( t = T_1 \) while operator 2 decides not to upgrade. The numbers of operator 1’s 4G users and 3G users at any time \( t \geq 0 \) are \( N_1^{4G}(t) \) and \( N_1^{3G}(t) \), respectively. The number of operator 2’s 3G users at time \( t \geq 0 \) is \( N_2^{3G}(t) \). As time \( t \) increases (from \( T_1 \)), 3G users in both networks start to switch to 4G service, and \( \forall t \geq 0 \)

\[
N_1^{3G}(t) = N_1 e^{-\lambda \max(t-T_1,0)}, N_2^{3G}(t) = N_2 e^{-\alpha \lambda \max(t-T_1,0)},
\]

1. An operator can predict such a model based on historical market data and new market survey before committing to the technology upgrade.
2. In the future work, we plan to study the case where an operator can use a low introductory price to promote the 4G service, in order to attract users from the other operators. It can also reduce its 3G service price, to reduce the user churn of its 3G users to other operators’ 4G services. In this case, we can use a utility-based modeling to understand how users churn under different prices, similar as [12].
and operator 1’s 4G service gains an increasing market share,

\[ N_1^{4G}(t) = N - N_1 e^{-\lambda \max(t-T_1,0)} - N_2 e^{-\alpha \lambda \max(t-T_1,0)}. \]  

(2)

We illustrate (1) and (2) in Fig. 4. We can see that operator 1’s early upgrade attracts users from its competitor and increases its market share. Notice that (2) increases with \( \alpha \), thus operator 1 captures a large market share when \( \alpha \) is large (i.e., the switching cost is low).

### 3.3 Operators’ Revenues and Upgrade Costs

Owing to depreciation over time, an operator values the current revenue more than the same amount of revenue in the future. We denote the discount rate over time as \( S \), and the discount factor is thus \( e^{-St} \) at time \( t \) according to [29].

We approximate one operator’s 4G upgrade cost as a one-time investment. This is a practical approximation, as an operator’s initial investment of wireless spectrum and infrastructure can be much higher than the maintenance costs in the future. For example, spectrum is a very scarce resource that is allocated (auctioned) infrequently by government agencies. It is time-consuming and not easy for an operator to obtain sufficient spectrum from another operator for its 4G upgrade.\(^8\) To ensure a good initial 4G coverage, an operator also needs to update many base stations to cover at least a whole city all at once. Otherwise, 4G users would be unhappy with the service, and this would damage the operator’s reputation. That is why Sprint and Verizon covered many markets in their initial launch of their 4G services [31].

More specifically, we denote the 4G upgrade cost at \( t = 0 \) as \( K \), which reduces over time at a rate \( U \). Thus if an operator upgrades at time \( t \), it needs to pay an upgrade cost \( Ke^{-Ut} \), according to [29]. We should point out that the upgrade cost decreases faster than the normal discount rate (i.e., \( U > S \)).\(^9\) This happens because the upgrade cost decreases because of both technology improvement and depreciation. Very often the advance of technology is the dominant factor in determining \( U \), and this is discussed further in Section 7.

Based on these discussions on revenue and upgrade cost, we define an operator’s profit as the difference between its revenue in the long run and the one-time upgrade cost. Without loss of generality, we will normalize an operator’s revenue rate (at any time \( t \)), total revenue, and upgrade cost by \( N \log(N) \), where \( N \) is the total number of users in the market.\(^10\) Because of this normalization, our main results derived in the following sections are robust to the choice of 4G network valuation function. For example, if the majority of 4G traffic is related to mobile Internet instead of direct communications among users, the network effect will be weaker and network value function \( f(N) \) satisfies \( N < f(N) < N \log(N) \).\(^11\) In this case, by normalizing the operator’s revenue rate, total revenue, and upgrade cost by \( f(N) \), we can still derive the same optimal upgrade timing for the monopoly case and prove asymmetric timing equilibrium for the duopoly case by following the same analysis as follows.

### 4 4G MONOPOLY MARKET

We first look at the case where only operator 1 can choose to upgrade from 3G to 4G, while the other operators (one or more) always offer 3G service because of the lack of financial resources or the necessary technology. This can be a reasonable model, for example, for countries such as Mexico and some other Latin American ones, where America Movil is the dominant cellular operator in the 3G market. As the world’s fourth-largest cellular network operator, America Movil has the advantage over other small local operators in winning additional spectrum via auctions and obtaining LTE patents, and it is expected to be the 4G monopolist in that area [28].

The key question in this section is how operator 1 should choose its upgrade time \( T_1 \) from the 3G service to the 4G service. \( T_1 = 0 \) means that operator 1 upgrades at the earliest time that the spectrum and technology are available, and \( T_1 > 0 \) means that operator 1 chooses to upgrade later to take advantage of the reduction in the upgrade cost. Because of user churn from the 3G to the 4G service, the operators’ market shares and revenue rates change after time \( T_1 \). For that reason, we analyze periods \( t < T_1 \) and \( t > T_1 \) separately.

- **Before 4G upgrade (\( t < T_1 \)):** Operator 1’s and other operators’ market shares do not change over time. Operator 1’s revenue rate at time \( t \) is

  \[ \pi_1^{3G-3G}(t) = \frac{N_1}{N}, \]

- **After 4G upgrade (\( t > T_1 \)):** Operator 1 upgrades at time \( T_1 \), and the operators’ market shares change over time. We denote the new market share of operator 1 at time \( t > T_1 \) as \( N_1(t) \).
which is independent of time $t$. Its revenue during this time period is
\[
\pi_{1, t < T_1}^{3G-3G} = \int_0^{T_1} \pi_1^{3G-3G}(t)e^{-St}dt = \frac{\gamma N_1}{SN}(1 - e^{-ST_1}).
\]  
(3)

- **After 4G upgrade ($t > T_1$):** Operator 1’s market share increases overall time, and the other operators’ total market share (denoted by $N_{3G}^3(t)/N$) decreases over time. We denote operator 1’s numbers of 3G users and 4G users as $N_1^{3G}(t)$ and $N_1^{4G}(t)$, respectively, and have $N_1^{3G}(t) + N_1^{4G}(t) = N$. This implies that $N_1^{4G}(t) = N - N_1e^{-\alpha(t-T_1)}$, and
\[
N_1^{3G}(t) = N_1e^{-\alpha(t-T_1)} - (N - N_1)e^{-\alpha(t-T_1)}.
\]
Note that a 3G user’s communication with a 3G or a 4G user is still based on the 3G standard, and only the communication between two 4G users can achieve a high 4G standard QoS. Operator 1’s revenue rate is
\[
\pi_1^{4G-3G}(t) = \frac{\gamma N_1^{3G}(t)}{N} + \frac{N_1^{4G}(t)}{N}
\cdot \left(\frac{N_1^{4G}(t) + \gamma N_1^{3G}(t)}{N} + \frac{\gamma N_1^{3G}(t)}{N}\right),
\]
which is independent of the revenue sharing ratio $\eta$ between the calling party and receiving party. Operator 1’s revenue during this time period is then
\[
\pi_{1, t > T_1}^{4G-3G} = \int_{T_1}^{\infty} \pi_1^{4G-3G}(t)e^{-St}dt,
\]  
(4)
where $t \to \infty$ is an approximation of the long-term 4G service provision (e.g., one decade) before the emergence of the next generation standard. This approximation is reasonable since the revenue in the distant future becomes less important because of discounting.

Figure 4 illustrates how the numbers of users of operators’ different services change over time. Before operator 1’s upgrade (e.g., $t < T_1$ in Fig. 4), the number of total users in each network does not change;\(^{12}\) after operator 1’s upgrade, operator 1’s and the other operators’ 3G users switch to the new 4G service at rates $\lambda$ and $\alpha\lambda$, respectively.

By considering (3), (4), and the decreasing cost $Ke^{-UT_1}$, operator 1’s long-term profit when choosing an upgrade time $T_1$ is
\[
\pi_1(T_1) = \pi_{1, t < T_1}^{3G-3G} + \pi_{1, t > T_1}^{4G-3G} - Ke^{-UT_1},
\]
\[
e^{-ST_1}\left(\frac{1}{S} + (1 - \gamma)\frac{N_1}{2\lambda + S} + (1 - \gamma)\frac{N_2}{2\alpha\lambda + S}\right) - e^{-ST_1}\left(2(1 - \gamma)\frac{N_1}{\lambda + S} + (2 - \gamma)\frac{N_2}{\alpha\lambda + S}\right) - Ke^{-UT_1} + 2e^{-ST_1}(1 - \gamma)\frac{N_1(N - N_1)}{(1 + \alpha)\lambda + S} + N_1\gamma\frac{N}{NS}(1 - e^{-ST_1}).
\]  
(5)

We can show that $\pi_1(T_1)$ in (5) is strictly concave in $T_1$, thus we can compute the optimal upgrade time $T_1^*$ by solving the first-order condition. The optimal upgrade time depends on the following upgrade cost threshold in the monopoly 4G market,
\[
K_{th}^{mono} = (1 - \gamma)\frac{N_1^2}{2\alpha\lambda + S} + \frac{\gamma N_2}{3\lambda + S} + \frac{2N_1(N - N_1)}{(1 + \alpha)\lambda + S}
\]
\[
\frac{U}{S} + \frac{1 - \gamma N_1}{N} - (2 - \gamma)\frac{N - N_1}{\alpha\lambda + S}.
\]  
(6)

**Theorem 1:** Operator 1’s optimal upgrade time in a 4G monopoly market is:

- **Low cost regime** (upgrade cost $K \leq K_{th}^{mono}$): operator 1 upgrades at $T_1 = 0$.
- **High cost regime** ($K > K_{th}^{mono}$): operator 1 upgrades at
\[
T_1^* = \frac{1}{U - S}\log\left(\frac{K}{K_{th}^{mono}}\right) > 0.
\]  
(7)

Intuitively, an early upgrade gives operator 1 a larger market share and enables him to get a higher revenue via the more efficient 4G service. Such advantage is especially obvious in the low cost regime where the upgrade cost $K$ is small.

Next we focus on the high cost regime, and explore how the network parameters affect operator 1’s upgrade time.

**Observation 1:** Operator 1’s optimal upgrade time $T_1^*$ increases with the upgrade cost $K$, and decreases with $\alpha$ (i.e., increases with the users’ inter-network switching cost).

**Proof:** Equation (7) shows that $T_1^*$ is increasing in $K$. Next we show the relation between $T_1^*$ and $\alpha$. In order to show that $T_1^*$ is decreasing in $\alpha$, we need to first show that $T_1^*$ is decreasing in $K_{th}^{mono}$, and then show that $K_{th}^{mono}$ is increasing in $\alpha$. Notice that (7) shows that $T_1^*$ is decreasing in $K_{th}^{mono}$. Consequently, we only need to examine the relation between $K_{th}^{mono}$ and $\alpha$. Notice that the partial derivative $\partial K_{th}^{mono}/\partial \alpha$ is linear in $\gamma$, and its values at the two boundaries of $\gamma \in (0, 1)$ are both positive.

\[
\frac{\partial K_{th}^{mono}}{\partial \alpha}\big|_{\gamma=0} = \frac{2\lambda(N - N_1)}{U\gamma N} + \frac{1}{(2\alpha\lambda + S)^2} > 0,
\]
and
\[
\frac{\partial K_{th}^{mono}}{\partial \alpha}\big|_{\gamma=1} = \frac{N - N_1}{U(\alpha\lambda + S)^2} > 0.
\]
Thus $\partial K_{th}^{mono}/\partial \alpha$ is positive for any $\gamma$, and $K_{th}^{mono}$ is increasing in $\alpha$. This completes the proof.

**Observation 2 (Figure 5):** When $K_{th}^{mono} < K < \frac{1}{U}K_{th}^{mono}$, $T_1^*$ first increases and then decreases in $U$. When $K \geq \frac{1}{U}K_{th}^{mono}$, $T_1^*$ monotonically decreases in $U$.

Figure 5 shows $T_1^*$ as a function of $U$ and $K$. When $K$ is large or $U$ is large, operator 1 wants to postpone its upgrade until the upgrade cost decreases significantly. A larger $U$ (thus a faster cost-decreasing rate) strengthens its willingness to postpone. When $K$ is small and $U$ is small, the upgrade cost is small and does not decrease.
fast enough. In this case, operator 1 chooses to upgrade early if the revenue increase can compensate for the small upgrade cost.

By checking the first order derivative of $T^*_1$ in (7) over $\gamma$ under different $N_1/N$ values, we have the following.

**Observation 3 (Figure 6):** When operator 1’s original market share $N_1/N$ is large, $T^*_1$ increases in the efficiency ratio $\gamma$ (between 3G and 4G). When $N_1/N$ is small, $T^*_1$ decreases in $\gamma$.

Figure 6 shows $T^*_1$ as a function of $\gamma$ and $N_1/N$. When $N_1/N$ is large, operator 1 cannot attract many users from other operators and has a smaller incentive to upgrade. As $\gamma$ increases (and thus the QoS gap between 3G and 4G shrinks), it is less interested in 4G service and thus postpones its upgrade. When $N_1/N$ is small, operator 1 has limited market share and can attract many more users by upgrading early. As $\gamma$ increases, operator 1 obtains a higher revenue between its existing 3G users and the increasing number of 4G users. The other small operators without market power, however, do not benefit from this, as they lose their market shares due to operator 1’s 4G service. Thus $T^*_1$ decreases in $\gamma$.

### 5 4G Competitive Market: Duopoly Model and Game Formulation

In this section, we focus on the competition between multiple operators who can choose to upgrade to 4G services. To make the analysis tractable and to derive clear engineering insights, we focus on the case of two operators (duopoly) in this paper. This analysis serves as the first step in understanding the more general oligopoly case. This duopoly model is reasonable in a country like China, where China Mobile and China Unicom are the two dominant cellular operators in the 3G market. A similar situation exists in several other Asian and European countries as well.

The focus of this and the following section is to understand why in so many existing industry examples (e.g., [5]–[7]) operators choose to upgrade to 4G services at different times even though they have the resources to upgrade simultaneously. In particular, we examine whether such asymmetric upgrades emerge even when the two operators are similar (e.g., having similar market shares before upgrades (e.g., Verizon has 106.3 million users and AT&T has 98.6 million users in the US)). In Section 8, we also examine the case where networks are heterogeneous in nature, and we show that asymmetric operators (e.g., with different market shares) have more incentives to upgrade at different times. Thus in the following analysis we can consider two operators that have the same market shares before the 4G upgrades ($N_1 = N_2$), the same upgrade cost $K$, and the same cost discount rate $U$. We will first derive the operators’ profits under any upgrade decisions, and then analyze the duopoly game where each operator chooses the best upgrade time to maximize its profit.

#### 5.1 Operators’ Long-term Profits

Let us denote two operators’ upgrade times as $T_1$ and $T_2$, respectively. Because the two operators are symmetric, without loss of generality, we assume in the following example (before Lemma 2) that operator 1 upgrades no later than operator 2 (i.e., $T_1 \leq T_2$). To calculate the operators’ profits, we first need to understand how users churn from 3G to 4G services, and how this affects the operators’ revenue rates over time. Figure 7 shows that user churn is different in three phases, depending on how many operators have upgraded.

- **Phase I** ($0 \leq t \leq T_1$): No operator has upgraded and both operators’ market shares do not change. The two operators’ revenue rates are
  \[ \pi_1^{3G-3G}(t) = \pi_2^{3G-3G}(t) = \frac{\gamma}{2} \]

- **Phase II** ($T_1 < t \leq T_2$): Operator 1 has upgraded to 4G service but operator 2 has not. The 3G users of two operators switch to operator 1’s 4G service at different rates. The numbers of users in the operators’ different services are
  \[ N_1^{3G}(t) = \frac{N}{2} e^{-\lambda(t-T_1)}, N_2^{3G}(t) = \frac{N}{2} e^{-\alpha\lambda(t-T_1)}, \]
and
\[ N_{1}^{4G}(t) = N - \frac{N}{2} e^{-\lambda(t-T_1)} - \frac{N}{2} e^{-\alpha\lambda(t-T_1)}. \]
The two operators’ revenue rates are
\[ \pi_{1}^{4G-3G}(t) = \left( 1 - \frac{1}{2} \left( e^{-\lambda(t-T_1)} + e^{-\alpha\lambda(t-T_1)} \right) \right) \left( 1 - e^{-\lambda(t-T_1)} e^{-\alpha\lambda(t-T_1)} \right) + \frac{\gamma}{2} e^{-\lambda(t-T_1)}, \]
and
\[ \pi_{2}^{4G-3G}(t) = \frac{\gamma}{2} e^{-\alpha\lambda(t-T_1)}. \]

- **Phase III** (t > T_2): Both operators have upgraded, and 3G users only switch to the 4G service of their current operator. The numbers of users in operators’ different services are
\[ N_{1}^{3G}(t) = \frac{N}{2} e^{-\lambda(t-T_1)}, \]
\[ N_{1}^{4G}(t) = N - \frac{N}{2} e^{-\lambda(t-T_1)} - \frac{N}{2} e^{-\alpha\lambda(T_2-T_1)}, \]
\[ N_{2}^{3G}(t) = \frac{N}{2} e^{\alpha\lambda(T_2-T_1)-\lambda(t-T_2)}, \]
and
\[ N_{2}^{4G}(t) = \frac{N}{2} e^{\alpha\lambda(T_2-T_1)} \left( 1 - e^{-\lambda(t-T_2)} \right). \]
The two operators’ revenue rates are
\[ \pi_{1}^{4G-4G}(t) = \frac{\gamma}{2} e^{-\lambda(t-T_1)} + \left( 1 - e^{-\lambda(t-T_1)} e^{-\lambda(t-T_2)} \right) \left( 1 - \frac{1}{2} \left( e^{-\lambda(t-T_1)} - \frac{1}{2} \left( e^{-\alpha\lambda(T_2-T_1)-\lambda(t-T_2)} \right) \right), \]
and
\[ \pi_{2}^{4G-4G}(t) = \frac{\gamma e^{-\alpha\lambda(T_2-T_1)-\lambda(t-T_2)}}{2} + \frac{1 - e^{-\lambda(t-T_2)}}{2} \left( 1 - \frac{1}{2} \left( e^{-\lambda(t-T_1)} - \frac{1}{2} \left( e^{-\alpha\lambda(T_2-T_1)-\lambda(t-T_2)} \right) \right), \right. \]
Figure 8 summarizes how users churn in the three phases. Similar to Section 4, we can derive operators’ revenue rates based on users’ churn over time. By integrating each operator’s revenue rate over all three phases, we obtain that operator’s long-term revenue. Recall that an operator’s profit is the difference between its revenue and the one-time upgrade cost. By further considering the symmetric case of T_1 ≤ T_2, we have the following result.

Lemma 2: Consider two operators i, j ∈ \{1, 2\} (with i ≠ j) upgrading at T_i and T_j. Operator i’s long-term profit is
\[ \pi_i(T_i, T_j) = \begin{cases} \pi^{ER}(T_i, T_j), & \text{if } T_i \leq T_j; \\ \pi^{LT}(T_i, T_i), & \text{if } T_i \geq T_j, \end{cases} \]
where \( \pi^{ER}(T_i, T_j) \) and \( \pi^{LT}(T_i, T_i) \) are given in (9) and (10), respectively.

Note that an operator’s profit \( \pi_i(T_i, T_j) \) is continuous in its upgrade time \( T_i \). When operator i’s upgrade time \( T_i \) is less than \( T_j \), it increases its market share at rate \( \alpha\lambda \) during the time period from \( T_i \) to \( T_j \); but when \( T_i > T_j \), operator i loses its market share at rate \( \alpha\lambda \) during the period from \( T_j \) to \( T_i \). This explains why we need two different functions \( \pi^{ER}(T_i, T_j) \) and \( \pi^{LT}(T_j, T_i) \) to completely characterize the long-term profit for each operator.

### 5.2 Duopoly Upgrade Game

Next we consider the non-cooperative game that models the interactions between two operators, where each of them seeks to maximize its long-term profit by choosing the best upgrade time.

**Upgrade Game:** We model the competition between two operators as follows:
- **Players:** Operators 1 and 2.
- **Strategy spaces:** Operator i ∈ \{1, 2\} can choose upgrade time \( T_i \) from the feasible set \( T_i = [0, \infty) \).
- **Payoff functions:** Operator i ∈ \{1, 2\} wants to maximize its profit \( \pi_i(T_i, T_j) \) defined in (8).

13. Note that \( T_i = \infty \) means that operator i never upgrades.
we mentioned at the beginning of this section. After available resources for 4G upgrade at a similar time as decision as time goes on. Also, it should be pointed out operator has complete information about its competitor’s to be planned and prepared. As we consider that each changing decisions frequently, as many upgrade opera-

14. Operators can predict the future market adoption by exploring historical records of the market and some trial of 4G deployment. In the future we will study the incomplete information case, where an operator may learn more information as the time goes and revise its historical records of the market and some trial of 4G deployment. In

6 4G Competitive Market: No Inter-network Switching

In this section, we consider the case of \( \alpha = 0 \), i.e., no user switches operators because of a very high switching cost. This corresponds to the case, for example, where the penalty for terminating a 3G contract to the other operator’s 4G service is very high or where a contract lasts for a very long time. The analysis here helps us better understand the more general case of \( \alpha > 0 \) in Section 7.

Notice that we consider a static game here, where both operators decide when to upgrade at the beginning of time. This is motivated by the fact that operators usually make long-term decisions in practice rather than changing decisions frequently, as many upgrade opera-

\begin{align*}
\pi^{ER}(T_1, T_j) &= \frac{(2 - \gamma)e^{-ST_1} - e^{-\alpha(\lambda(T_j - T_1) - ST_1)}}{2S} (2 - \gamma)\left(\frac{e^{-ST_1} - e^{-\alpha(T_j - T_1) - ST_1}}{\lambda + S}ight) - \frac{(1 - \gamma)e^{-ST_1}}{\lambda + S} \\
&\quad + \frac{1 - \gamma}{2}\left(e^{-ST_1} + e^{-(1+\alpha)\lambda(T_j - T_1) - ST_1} + \frac{e^{-ST_1} - e^{-\gamma\lambda(T_j - T_1) - ST_1}}{(1 + \alpha)\lambda + S} + \frac{e^{-ST_1} - e^{-2\alpha\lambda(T_j - T_1) - ST_1}}{2(2\alpha + S)}\right) \\
&\quad - \frac{1}{2}\left(1 - e^{-\gamma\lambda(T_j - T_1)} + e^{-\alpha\lambda(T_j - T_1)}\right) K e^{-U T_1}. \\
\pi^{LT}(T_j, T_1) &= \frac{e^{-\alpha\lambda(T_j - T_1) - ST_1} - \frac{1}{S} - 1 + \frac{1}{2}\left(e^{-\gamma(T_j - T_1) + e^{-\alpha\lambda(T_j - T_1)}}\right)\left(\frac{1}{\lambda + S} - \frac{1}{2} \frac{1}{\lambda + S}\right)}{2(\alpha + S)} + \gamma \frac{(1 - e^{-ST_1})}{2S} - K e^{-U T_1}. 
\end{align*}

Let us denote the cost threshold in the 4G competitive market under \( \alpha = 0 \) as

\[ K_{th1, \alpha=0}^{comp} = \frac{3(1 - \gamma)\lambda^2}{4U(\lambda + S)(2\lambda + S)}. \]  

Theorem 2: With symmetric operators and \( \alpha = 0 \), there exists a unique symmetric 4G upgrade equilibrium with the following characteristics:

- **Low cost regime** (upgrade cost \( K \leq K_{th1, \alpha=0}^{comp} \)): Both operators upgrade at \( T_1 = T_2 = 0 \).
- **High cost regime** (upgrade \( K > K_{th1, \alpha=0}^{comp} \)): Both operators upgrade at

\[ T_1^* = T_2^* = \frac{1}{U - S} \log \left( \frac{K_{th1, \alpha=0}}{K_{th1, \alpha=0}} \right) > 0, \]

which increases with \( K \).

The proof of Theorem 2 is given in Appendix A. Intuitively, in the low cost regime, both operators are willing to upgrade at \( t = 0 \) to maximize the revenue from providing high-quality 4G services. In the high cost regime, on the other hand, both operators postpone their upgrades until the upgrade cost is small enough. If only one operator upgrades, its total market share does not change (since \( \alpha = 0 \) and only its own 3G users switch to the 4G service. Thus the revenue from the 4G service does not compensate for the high upgrade cost. This makes the operator reluctant to upgrade until the 4G technology is mature. As one operator cannot attract the other operator’s users, the two operators independently decide their upgrade timings.

It should be noted that the duopoly case here is not a special case of the monopoly case of \( \alpha = 0 \) in Section 4. Unlike Section 4, both operators here can upgrade to 4G services, and the positive network effect of technology upgrade exists between two operators even when \( \alpha = 0 \). More precisely, when one operator upgrades to 4G, though its market share (total number of 3G and 4G users) remains the same, its new 4G users will enjoy the addition of intercommunication benefits with the 4G users of the other operator (once that operator has chosen to upgrade to 4G). Such positive network effect is mutual between the operators.

To mathematically demonstrate the differences between the two sections, we compare the cost threshold \( K_{th1, \alpha=0}^{comp} \) in (11) and \( K_{th1, \alpha=0}^{mono} \) in (6) for initial upgrade as follows. For a fair comparison between the two sections, we assume that the monopolist in Section 4 covers half of the market (\( N_1 = N/2 \)). In this section, we have assumed
that each competitive operator covers half of the market. By substituting $N_1 = N/2$ and $\alpha = 0$ into (6), we have

$$K^{\text{mono}}_{th}|_{\alpha=0,N_1=N/2} = \frac{(1 - \gamma)\lambda^2}{2U(\lambda + S)(2\lambda + S)},$$

which is smaller than $K^{\text{comp}}_{th1,\alpha=0}$ in (11). This shows that each of the two competitive operators here has a higher incentive to upgrade immediately (even without increasing its market share). Intuitively, a competitive operator can benefit from the 4G upgrade by the other operator due to the positive network effect, and this urges both operators to upgrade earlier. Unlike this section, in Section 4 the monopolist’s 4G users have no 4G users in the other networks to communicate with, which reduces the monopolist’s incentive to upgrade.

### 7 4G Competition Market: Practical Inter-network Switching Rate

In this section, we consider the case of $\alpha > 0$, i.e., 3G users may switch to the 4G service of a different operator. The equilibrium analysis in this general case depends on the relationship between $U$ (upgrade cost discount rate) and $S$ (money depreciation rate). We assume that $U$ is much larger than $S$, i.e., $U > S + \alpha \lambda$. This represents the practical case where the advance of technology is the dominant factor in determining $U$, and not many 3G users choose to switch operators when the 4G service is just deployed (i.e., small $\alpha$) [30]. For example, Sprint deployed the first 4G network in US by using WiMAX technology in 2008 when LTE technology was not mature yet. Only two years later, in 2010, LTE could already offer a much lower cost per bit than WiMAX [5]. From 2012, LTE is expected to be the leading technology choice for 4G networks. This example motivates that $U$ is much larger than $S$, and we will study whether the operators’ symmetric 4G upgrades happen in this scenario.

Recall that by upgrading at $T_1$ and $T_2$, the operators receive the profits given in (8). In our game theoretic model, one operator’s best response function is its upgrade time that achieves the largest long-term profit, as a function of a fixed upgrade time of the other operator [33]. A fixed point of the two operators’ best response functions is the Nash equilibrium, and in general there can be more than one such fixed point.

We can show that the operators’ best response functions ($T^{\text{best}}_1(2)$ and $T^{\text{best}}_2(1)$) depend on the upgrade cost $K$, and in particular, they depend on two cost thresholds ($K^{\text{comp}}_{th1} < K^{\text{comp}}_{th2}$) that lead to three cost regimes: low, medium, and high.

When the upgrade cost $K$ is less than the first threshold $K^{\text{comp}}_{th1}$ (i.e., low cost regime), both operators will upgrade at $t = 0$ to maximize the revenue from the 4G service. By solving

$$\frac{\partial \pi^{LT}(0, T_i)}{\partial T_i}|_{T_i=0} = 0, \forall i \in \{1, 2\},$$

we have

$$K^{\text{comp}}_{th1} = \left(1 - \gamma\right)\frac{(1 - \alpha)\lambda}{\lambda + S} + \frac{\alpha \lambda}{S} + \frac{1 - \gamma}{2}(S + 3\alpha \lambda + 2S)$$

$$\cdot \left(\frac{1}{\lambda + S} - \frac{1}{2\lambda + S}\right) \frac{2U}{(2\lambda + S)}.$$ (13)

When the upgrade cost $K$ is larger than $K^{\text{comp}}_{th1}$ (i.e., medium or high cost regimes), at least one operator postpones its upgrade until the upgrade cost decreases sufficiently. In particular, when $K$ is larger than the second threshold $K^{\text{comp}}_{th2}$ (i.e., high cost regime), both operators postpone their upgrades. When operator $i \in \{1, 2\}$ upgrades at $t = 0$, operator $j \neq i$ postpones its upgrade to $T^{\text{best}}_j(0)$, which is the unique solution to

$$\frac{\partial \pi^{LT}(0, T_j)}{\partial T_j}|_{T_j=T^{\text{best}}_j(0)} = 0.$$ (14)

The threshold $K^{\text{comp}}_{th2}$ can be obtained by solving

$$\frac{\partial \pi^{ER}(T_i, T^{\text{best}}_j)}{\partial T_i}|_{T_i=0} = 0.$$ (15)

Next we illustrate numerically how the two operators’ best response functions ($T^{\text{best}}_1(2)$ and $T^{\text{best}}_2(1)$) change with the upgrade cost $K$.

Figure 9 shows that each operator’s best response function is discontinuous in the medium cost regime, and the two best response functions with the same value of $K$ intersect at two points: $0 = T^{\text{best}}_1 < T^{\text{best}}_2$ and (symmetrically) $0 = T^{\text{best}}_2 < T^{\text{best}}_1$. To illustrate this situation, consider operator 2’s best response $T^{\text{best}}_2(T_1)$ in the case $K = 0.062$. If operator 1 upgrades early such that $T_1$ is less than 0.05, operator 2 does not upgrade at the same time to avoid a severe competition. If operator 1 upgrades later such that $T_1$ is larger than 0.05, operator 2 chooses to upgrade earlier than operator 1 to increase its market share. Thus $T^{\text{best}}_2(T_1)$ is discontinuous at $T_1 = 0.05$.

Figure 10 shows that each operator’s best response function is discontinuous in the high cost regime, and the two functions (with the same value of $K$) intersect at two points, equilibria $0 < T^{\text{best}}_1 < T^{\text{best}}_2$ and (symmetrically)
0 < T^+_2 < T^*_2. Unlike Fig. 9, the high cost here prevents any operator from choosing the upgrade time $t = 0$.

Figure 11 summarizes how operators' upgrade equilibrium changes as cost $K$ increases: starting with $T^*_i = T^*_j = 0$ in low cost regime, then $0 < T^*_i < T^*_j$ with increasing $T^*_j$ in medium cost regime, and finally $0 < T^*_i < T^*_j$ with increasing $T^*_i$ and $T^*_2$ in high cost regime.

In the following theorem, we prove that the operators do not choose symmetric upgrades as long as the cost is not low.

**Theorem 3:** The two operators' 4G upgrade equilibria satisfy the following properties:

- **Low cost regime ($K \leq K_{th1}^{comp}$):** Both operators upgrade at $T^*_i = T^*_j = 0$.
- **Medium cost regime ($K_{th1}^{comp} < K \leq K_{th2}^{comp}$):** Operators do not upgrade at the same time, and only one operator may upgrade at $t = 0$. The possible equilibrium can only be $0 < T^*_1 < T^*_2$ and (symmetrically) $0 < T^*_2 < T^*_1$.
- **High cost regime ($K > K_{th2}^{comp}$):** Operators do not upgrade at the same time, and none of them upgrade at $t = 0$. The possible equilibria can only be $0 < T^*_1 < T^*_2$ and (symmetrically) $0 < T^*_2 < T^*_1$.

**Proof:** We divide our proof into three parts. First, we prove the equilibrium in the low cost regime. Then we prove the asymmetric upgrade structure in the other two regimes. Finally, we prove that both operators postpone their upgrades in the high cost regime.

**Proof of equilibrium in the low cost regime:** We want to prove that $T^*_1 = T^*_2 = 0$ is the unique equilibrium under $K \leq K_{th1}^{comp}$.

- We first show $T^*_1 = T^*_2 = 0$ is an equilibrium by proving that no operator has an incentive to deviate. Given $T^*_1 = 0$, operator 2's profit is $\pi^{LT}(0, T_2)$ when upgrading at $T_2$. We can show that $\pi^{LT}(0, T_2)$ has a unique maximum $T^{best}_2(0) = 0$. Thus operator 2 will not deviate from $T^*_2 = 0$. Similarly, we can prove that given $T^*_2$, operator 1 will not deviate from $T^*_1 = 0$.

- We then prove the uniqueness of the equilibrium by contradiction. Without the loss of generality, we suppose there is another equilibrium with $0 < T^*_1 \leq T^*_2$. Note that $T^*_j$ cannot equal zero, otherwise operator 2 will also upgrade at $t = 0$ and we reach the same equilibrium. Next we show that $0 < T^*_1 < T^*_2$ cannot be an equilibrium by proving operator 2 will choose its upgrade at the same time. This can be proved by showing operator 2's profit $\pi^{LT}(T_1, T_2)$ with any $T_1 < T^*_2$ decreases with $T_2$. Thus the possible other equilibrium can only be $T^*_1 = T^*_2 > 0$. However, this is not possible since operator 1's profit $\pi^ER(T_1, T_2)$ with any $T_1 = T^*_2$ decreases with $T_1$, i.e.,

$$\frac{\partial \pi^ER(T_1, T_2)}{\partial T_1} |_{T_1 = T^*_2} < 0.$$
Thus, finally, we prove that no operator upgrades at $T^*$ which means that (16) and (17) cannot be satisfied at the same time. However, we can show that an equilibrium, where we have proved that only asymmetric upgrades exist at any possible equilibrium. We need to explore whether any operator has an incentive to deviate.

- Given $T_1^* = 0$, operator 2’s profit is $\pi^{LT}(0, T_2)$. It will not deviate only when $T_2^*$ already maximizes $\pi^{LT}(T_1, T_2)$ (i.e., $T_2^* = T^{best}_2(0)$).
- Given $T_2^* = T^{best}_2(0)$, operator 1’s profit is $\pi^{ER}(T_1, T^{best}_2(0))$ by upgrading at $T_1 \leq T^{best}_2(0)$. We find that operator 1 has an incentive to deviate from $T_1^* = 0$. Indeed, since $K > K_{th2}$,

$$\left. \frac{\partial \pi^{ER}(T_1, T_2^{best}(0))}{\partial T_1} \right|_{T_1 = 0} > 0. \tag{19}$$

Thus $0 = T_1^* < T_2^*$ cannot be an equilibrium, and both operators postpone their upgrades.

To understand the intuition behind the asymmetric structure, we summarize the advantages of earlier and later upgrades with $\alpha > 0$ as follows:

- Earlier upgrade gives an operator the advantage to attract more users (from the other operator), and enables the operator to collect a higher revenue from the 4G service.
- Later upgrade allows an operator to incur a reduced upgrade cost when it upgrades.

In order to fully enjoy the two advantages of earlier or later upgrades, operators will avoid symmetric upgrade.

Fig. 12. The equilibrium profits ($\pi_1^*, \pi_2^*$) change as cost $K$ increases under a large $\gamma = 0.5$. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, and $\lambda = 1$ (hence $U > S + \alpha \lambda$).

If one operator upgrades much earlier to capture a larger market share that can compensate for a large upgrade cost, the second operator will not upgrade at the same time to avoid severe competition in market share; instead, the second operator will wait until its loss of users and revenue is compensated by the reduction of upgrade cost (with $U > S + \alpha \lambda$).

Next we study how operators’ equilibrium profits change with cost $K$ and the economic efficiency ratio $\gamma$ between 3G and 4G services in the three cost regimes. Note that an operator may or may not be able to charge a significantly higher price from a 4G user though 4G does improve a lot over 3G in QoS. Figures 12 and 13 show operators’ equilibrium profits under large and small $\gamma$ values, respectively.

We first study the large $\gamma$ scenario in Fig. 12 which means that an operator only charges a slightly higher price in 4G service than 3G. Without loss of generality, we focus on the case where operator 1 upgrades no later than operator 2 (i.e., $T_1^* \leq T_2^*$).

- In the low cost regime, by upgrading at $T_1^* = T_2^* = 0$, the two operators’ profits are the same and decrease with cost $K$.
- In the medium cost regime, Fig. 12 shows that operator 1 receives a larger profit than operator 2 by upgrading at $T_1^* = 0$. Perhaps surprisingly, its profit increases with $K$, whereas operator 2’s profit decreases with $K$. Intuitively, the increase of $K$ encourages operator 2 to further postpone its upgrade and lose more users to operator 1. The change of operator 1’s profit trades off the increase of its market share and upgrade cost. As operator 1’s market share increases, its growing 4G users communicate more with its 3G users via the efficient 3G service under large $\gamma$. Operator 1’s 3G revenue increases because of a more efficient intra-network traffic, which helps compensate for the upgrade cost.

- In the high cost regime, Fig. 12 shows that both operators have to postpone their upgrades and, surprisingly, both operators’ profits increase with $K$. As $K$ increases, operator 1 further postpones its upgrade and operator 2’s market share decreases more slowly. Thus operators’ competition in the market share is postponed, and under large $\gamma$ operator 2 can obtain more 3G revenue before operator 1’s upgrade. Operator 1, on the other hand, also benefits from its postponement to decrease its upgrade cost. Since operator 2 also postpones its upgrade, operator 1 can still capture a large market share even though it upgrades later. As $K \rightarrow \infty$, no operator upgrades and operators’ profits approach the symmetric 3G profits. Under large $\gamma$, the 4G service is not much better than 3G and the availability of 4G upgrade only intensifies operators’ competition. Compared to traditional 3G scenario, both operators’ profits decrease when the upgrade cost is high. In other words, both operators will be better off if 4G technology is not available in this case.
Operator 2’s equilibrium profit \( \pi_2 \) can be derived as:

\[
\pi_2 = \int_0^T \frac{4G-3G(t)e^{-St}}{S + \alpha_2 \lambda} dt + \int_0^\infty \frac{4G-4G(t)e^{-St}}{S} dt - Ke^{-UT_2} - \frac{\gamma}{\lambda} e^{-(\alpha_2 \lambda + S)T_2} \lambda + S
\]

which is not concave. However, we notice that \( \pi_2(0, T_2) \) is first monotonically increasing and then monotonically decreasing in \( T_2 \), we can show that \( \pi_2(0, T_2) \) is has a unique extremum on \( T_2 \), which maximizes operator 2’s profit. If cost \( K \) is small enough such that

\[
d\pi_2(0, T_2)/dT_2 |_{T_2=0} \leq 0,
\]

then operator 2’s best response is \( T_2(0) = 0 \). Otherwise, operator 2 will postpone its upgrade (i.e., \( T_2(0) > 0 \)).

Thus we can derive the first cost threshold for operator 2 as

\[
\hat{K}_{th2}'(N_2, \alpha_2) = \frac{U}{N} \left( (1 - \gamma) \frac{(1 - \alpha_2) \lambda}{\lambda + S} - \frac{1}{2\lambda + S} \right) \pi_2(0, T_2)
\]

which depends on \( N_2 \) where \( N_1 = N - N_2 \). Similarly, we have \( \hat{K}_{th1}'(N_1) \) for operator 1. We can further check

\[
\frac{\partial \hat{K}_{th2}'(N_2, \alpha_2)}{\partial N_2} = \frac{1 - \gamma}{N} \left( \frac{1 - \alpha_2}{\lambda + S} - \frac{1}{2\lambda + S} \right) > 0,
\]

where all three terms in the multiplication are positive. Thus \( \hat{K}_{th2}'(N_2, \alpha_2) \) is increasing in \( N_2 \). As a linear function of \( \alpha_2 \), we can check that \( \hat{K}_{th2}'(N_2, \alpha_2) \) is linearly increasing in \( \alpha_2 \) as long as \( S \) is small. Similarly, we can develop the cost threshold \( \hat{K}_{th1}'(N_1, \alpha_1) \) for operator 1. Both operators will upgrade immediately only when the upgrade cost \( K \) is less than both cost thresholds, or equivalently, the minimum of the two thresholds. Then we have the following result.

**Theorem 4:** An operator having a larger subscriber number (than its competitor) or having a smaller power of keeping its subscribers has more incentive to upgrade immediately. This characterizes its fear to lose many of its 3G users to its competitor. Two operators will only reach the symmetric equilibrium \( T_1^* = T_2^* = 0 \) when

\[
K \leq \hat{K}_{th1}' := \min(\hat{K}_{th1}'(N_1, \alpha_1), \hat{K}_{th2}'(N_2, \alpha_2))
\]

The fact that \( \hat{K}_{th1}' < K_{th1}' \) (with \( K_{th1}' \) in (13)) under \( N_1 \neq N_2 \) shows that the competition between operators become less severe if they have asymmetric market power. When \( K \) is larger than \( \hat{K}_{th1}' \), we can similarly show that operators will only reach an asymmetric equilibrium as in Theorem 3. This means that two operators with heterogeneous parameters are more likely to reach an asymmetric equilibrium in 4G upgrades.

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**8 Extension to the Heterogeneous Operator Scenario**

Here we present some preliminary results by considering operators’ network heterogeneity, and show its impact on our previous results. More specifically, we assume that the numbers of 3G users in the two networks are different (\( N_1 \neq N_2 \) with \( N_1 + N_2 = N \)), and users have asymmetric switching costs between the two operators to 4G service. We denote the switching cost factor from operator 1’s 3G service to operator 2’s 4G service as \( \alpha_1 \), and the switching rate is thus \( \alpha_1 \lambda \). Similarly, the switching cost factor from operator 2’s 3G service to operator 1’s 4G service is \( \alpha_2 \), which may be different from \( \alpha_1 \). We still focus on the most practical setting with \( U > \max(\alpha_1, \alpha_2) \lambda + S \) as in Section 7.

We first fix operator 1’s upgrade decision at \( T_1 = 0 \) and study operator 2’s best response. Similar to Section 5.1, we can derive operator 2’s real time revenue before and after \( T_1 \) as

\[
\pi_{2,t\leq T_2}^{4G-3G}(t) = \frac{N_2}{N} e^{-\alpha_2 \lambda t},
\]

and

\[
\pi_{2,t>T_2}^{4G-3G}(t) = \frac{N_2}{N} e^{-\alpha_2 \lambda t} \left( e^{-\lambda(t-T_2)} + (1 - e^{-\lambda(t-T_2)}) \right)
\]

\[
\cdot \left( 1 - (1-\gamma)\frac{N_1}{N} e^{-\lambda t} - (1-\gamma)\frac{N_2}{N} e^{(1-\alpha_2)\lambda T_2 - \lambda t} \right)
\]
9 CONCLUSIONS AND FUTURE WORK

This paper presents the first analytical study of operators’ 4G upgrade decisions. In a 4G monopoly market, we show that the monopolist’s optimal upgrade time trades off an increased market share and the decreasing upgrade cost. In a non-cooperative upgrading game with two operators, we show that operators select different upgrade times to avoid the severe competition. We further show that the availability of 4G upgrade may decrease both operators’ profits due to competition, and their profits may increase with the upgrade cost.

There are several possible ways to extend the results in this paper. First, we could consider an oligopoly market with more than two competitive operators. We expect the asymmetric upgrading equilibrium will still occur. Second, our model can also be extended to include different pricing plans (e.g., student plan, corporate plan, and residential plan), which contribute differently to an operator’s revenue. By following a similar revenue analysis for more than one network as in Section 3, we can derive the total revenue that accounts for the different prices and weights of different plans. Finally, there may be some correlation in users’ usage patterns (e.g., from those belonging to the same family account). According to [21], one Verizon account has 2.72 connections on average, and users belong to the same account communicate with each other more often. We may approximate users of the same account as a super user, and model traffic and revenue collection from many super users.

REFERENCES

APPENDIX A

PROOF OF THEOREM 2

Proof: We establish the properties of the operators’ equilibria in the two cost regimes, respectively.

Proof of equilibrium in the low cost regime: We want to prove that \( T_1^* = T_2^* = 0 \) is the unique equilibrium under \( K \leq K_{th1,\alpha=0}^{comp} \).

- We first show \( T_1^* = T_2^* = 0 \) is an equilibrium by proving that no operator has an incentive to deviate. Given \( T_1^* = 0 \), operator 2’s profit is \( \pi^{LT}(0, T_2^*) \) when upgrading at \( T_2 \). We can show that \( \pi^{LT}(0, T_2^*) \) has a unique maximum \( T_2^{LT}(0) = 0 \). Thus operator 2 will not deviate from \( T_2^* = 0 \). Similarly, we can prove that given \( T_2^* \), operator 1 will not deviate from \( T_1^* = 0 \).

- We then prove the uniqueness of the equilibrium by contradiction. Without the loss of generality, we suppose there is another equilibrium with \( 0 < T_1^* \leq T_2^* \). Note that \( T_1^* \) cannot equal zero, otherwise operator 2 will also upgrade at \( t = 0 \) and we reach the same equilibrium. Next we show that \( 0 < T_1^* < T_2^* \) cannot be an equilibrium by proving operator 2 will choose its upgrade at the same time. This can be proved by showing operator 2’s profit \( \pi^{LT}(T_1, T_2) \) with any \( T_1 < T_2 \) decreases with \( T_2 \). Thus the possible other equilibrium can only be \( T_1^* = T_2^* > 0 \). However, this is not possible since operator 1’s profit \( \pi^{ER}(T_1, T_2) \) with any \( T_1 = T_2 \) decreases with \( T_1 \), i.e.,

\[
\frac{\partial \pi^{ER}(T_1, T_2)}{\partial T_1} \bigg|_{T_1=T_2} \leq 0
\]

\[
< \frac{1 - \gamma}{4} e^{-ST_2} \left[ ST_1^* - \frac{3\lambda^2}{(2\lambda + S)(\lambda + S)} + K_{th1,\alpha=0}^{comp} U e^{-UT_1} \right]
\]

\[
< -e^{-ST_1^*} \left[ ST_1^* - \frac{3(1 - \gamma)\lambda^2}{4(2\lambda + S)(\lambda + S)} + K_{th1,\alpha=0}^{comp} U e^{-ST_1^*} \right] = 0.
\]

Proof of equilibrium in high cost regime: By following a similar proof as in the low cost regime, we can prove that \( T_1^* = T_2^* \) in (12) is the unique equilibrium under \( K > K_{th1,\alpha=0}^{comp} \). The difference is that given \( T_1^* \), we need to consider the possibilities of the other operator \( j \) upgrading earlier or later than \( T_1^* \).

\( \square \)