Fairness and Efficiency Tradeoffs for User Cooperation in Distributed Wireless Networks

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***Abstract*—We propose a general framework to analyze incentives for user cooperation, and characterize the tradeoff between fairness and efficiency for cooperative networks. More specifically, we define the incentive region as a set of action profiles that provides cooperation benefits to all users and focus on the optimization of efficiency and fairness within this region. We introduce a linear resource allocation (LRA) scheme and show that most existing fairness measures can be converted to LRA with different linear coefficient vectors. We then propose the concept of strong price of fairness (SPoF) to study the network efficiency of the strong equilibrium. We show that both the SPoF and fairness measures are connected to the linear coefficient vector of LRA, which makes it possible to study the fairness and efficiency relationship. We then use the random access (RA) system as an example to show how to use the proposed framework to study a specific wireless network.**

I. Introduction

User cooperation such as resource sharing and information exchanging can improve the performance of distributed wireless networks. There are several common questions for user cooperation-enabled wireless networks: a) How to incentivize users to cooperate in the wireless network? b) How to fairly allocate the network resources to the cooperative users? c) How to optimize the network efficiency? Each of these three questions has been individually investigated in the literature. For example, Niu *et al.* in [1] proposed strategies to stimulate user cooperation in a multicast network. Xiao *et al.* in [2] studied the optimal power allocation and relay selection algorithms to maximize the performance of multi-hop relaying networks. Lan *et al.* in [3] have introduced a unique family of fairness measures and its corresponding axioms. However, the current studies of these three questions are largely disconnected. Recently Joe-Wong *et al.* in [4] jointly studied the resource allocation efficiency and the fairness measure, and showed that a fairness and efficiency tradeoff can only be observed under certain situations. Bertsimas *et al.* in [5] introduced the concept of price of fairness for a centralized system, and derived upper bounds for two popular fairness criteria: proportional fairness and max-min fairness. However, the results proposed in [4] and [5] focused on the case where there is a central network controller, and hence may not be applicable to wireless networks where users may need to make distributed decisions and hence will only choose to cooperate with each other if there are incentives.

What motivates us in this research is the lack of a general framework describing the relationship between the incentive mechanism, network efficiency, and fairness for user cooperation in wireless networks. Our paper is the first step in building such a framework. More specifically, in Section II, we characterize the conditions under which users have incentives to cooperate, and define the *incentive region* as the set of all action profiles[1] that could stimulate this cooperation. We then focus on the network optimization to achieve the best tradeoff between efficiency and fairness within this region. To characterize different fairness measures of resource sharing, we introduce a *linear resource allocation (LRA)* scheme where the payoff division among users satisfies a linear relationship. We show that in LRA, it is possible to use a single linear coefficient vector to characterize different resource allocation schemes. We also show that most of the fairness measures proposed in the literature can be regarded as special cases of LRA by proper choices of the linear coefficient vector. We then propose the concept of strong price of fairness (SPoF) to study the network efficiency under different fairness criteria. We show that both the SPoF and fairness measures are related to the linear coefficient vector of LRA, which makes it possible to explicitly characterize the fairness and efficiency tradeoffs.

As an example, in Section III we apply the proposed framework to investigate random access (RA) network [6]. More specifically, we introduce a general model for RA and investigate the incentive mechanisms and the optimization of the fairness and efficiency relationship for this model. Our model allows the payoff function of each user to be any

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[1]Each user will choose one action, and an action profile characterizes the actions of all users.

performance or efficiency measures, e.g., channel capacity or probability of successful decoding. We derive conditions for the existence of the incentive region, and then use the LRA and SPoF to characterize its fairness and efficiency, respectively. We present the tradeoff curve and show that, because of user heterogeneity, we can not always increase the system efficiency by sacrificing the share (fairness) of an arbitrary user. In fact, we can only achieve the proper efficiency and fairness tradeoff by changing the share of "proper" users.

II. A General Framework for Analyzing Incentive, Efficiency, and Fairness

In this paper, we establish the relationship between the incentive, fairness, and efficiency for wireless networks by introducing and characterizing the following new concepts.

A. Incentive Region

Autonomous users are generally assumed to be selfish and will only consider resource sharing when there is an incentive to do so. In a wireless network, a sharing incentive exists when each of users can improve his payoff (e.g., network performance) through participating the cooperation. We can model this as a coalitional game [7] which consists of a set $\mathcal{K}$ of *players* (users), the set $\mathcal{A}$ of *action profiles*, and the *payoff* ϖ_i for each player i. A *coalition* is a nonempty subset of the set of all players. A *payoff division function* is a function that specifies how the total payoff of each coalition is divided among its members. A (Nash) *equilibrium* is an action profile such that no player can do better by unilaterally changing its strategy. A *strong (Nash) equilibrium* [8] is a equilibrium that is resilient to deviations by every coalition. Different from a Nash equilibrium, in a strong equilibrium, no coalition can cooperatively deviate and benefit every member of the coalition. Following the same line as [5], we define the *efficiency* to be the total payoff of all players.

First, let us study the conditions under which all users have incentives to form a coalition. Let us define a *(cooperative) incentive region* as follows.

Definition 1. *The (cooperative) incentive region $\mathcal{R} \subseteq \mathcal{A}$ is a sub-set of the set of all action profiles that could incentivize the coalition formation among players.*

In a communication system based on user cooperation, the incentive region will contain all the action profiles that could provide higher payoff (e.g., capacity and throughput) for each user/device in the network due to cooperation [1]. In a computer system, the incentive region may contain the action profiles that could provide a higher payoff (e.g., computation speed, CPU cycles and memory usage) than equal sharing (equally dividing the payoff among all the jobs/users/tasks) [3].

Coalitional game theory uses the concept of *core* to denote the set of action profiles of the coalition upon which no sub-coalition can improve. The incentive region is different from the core in the sense that the incentive region is decided by the specific user applications and may not always exist even when the core are non-empty. Therefore, it is important to derive the conditions under which the incentive region exists. For a non-empty incentive region, we can also define the *incentive global optimal solution* to be the solution within the incentive region that maximizes the efficiency of the system.

B. Linear Resource Allocation

Fairness is another key consideration in resource allocation, with or without cooperation. To capture this, let us define the linear resource allocation (LRA) scheme where the payoff division among players satisfies a linear relationship. This allows us to denote the payoff division of LRA to be

$$\varpi_1 = \varpi_2/\alpha_2 = \ldots = \varpi_K/\alpha_K \tag{1}$$

for a K-player system, where α_i is a constant and ϖ_i is the payoff allocated to player i. Let us now show LRA is general enough to cover most of the fairness measures in the literature.

In LRA, the linear coefficient vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_K]$ (with $\alpha_1 = 1$) decides the relative resource division among players. It is observed in the literature that network optimization based on different fairness criteria may result in different network resource allocations (what we call the fairness maximum points). For example, if we substitute (1) into Jain's fairness index [9], we have

$$f^{Jain}(\boldsymbol{\alpha}) = \frac{\left(\sum_{i=1}^{K} \varpi_i\right)^2}{\sum_{i=1}^{K} \varpi_i^2} = \frac{\left(1 + \sum_{i=2}^{K} \alpha_i\right)^2}{1 + \sum_{i=2}^{K} \alpha_i^2}, \tag{2}$$

and hence the fairness maximum point using the Jain's fairness index measure is given by $\boldsymbol{\alpha}^{Jain} = \arg\max_{\boldsymbol{\alpha}} f^{Jain}(\boldsymbol{\alpha})$.

Similarly, we can denote the max-ratio, min-ratio, entropy and the unified fairness measure [3] using different $\boldsymbol{\alpha}$'s.

In addition, LRA can help us to to study fair resource allocation in a heterogeneous system where different players may have different priorities in using the resource. As will be shown later in this paper, if all players have equal priority in sharing the resource, sacrificing fairness by decreasing the resource share of the higher performance players may not increase the system efficiency. In other words, in this system, deciding which player's resource share should be lowered is more important than deciding how much fairness should be sacrificed/tolerated. In a priority system, some players may have a higher priority in deciding the resource allocation over the other players. In such a system, the high priority players will generally not allow any sacrifice on their share of resource.

C. Strong Price of Fairness

To characterize the system efficiency of each fairness measure, we introduce the concept of strong price of fairness. The SPoF captures the efficiency of the network when all the players are willing to join the coalition under a given fairness criterion. The formal definition of SPoF is given as follows, *Definition 2. The SPoF is defined as the ratio of the worst strong equilibrium of a coalitional game with a given fairness constraint $\mathcal{F}$ to the global optimal outcome without any fairness constraints, i.e.,*

$$\text{SPoF} = \frac{U\left(\boldsymbol{a} \in \mathcal{R}^S|\mathcal{F}\right)}{U\left(\boldsymbol{a} \in \mathcal{R}\right)}, \quad (3)$$

where $\mathcal{R}^S$ is the set of strong equilibria. $\mathcal{F}$ is the fairness criterion agreed among players. $U\left(\boldsymbol{a} \in \mathcal{R}^S|\mathcal{F}\right)$ and $U\left(\boldsymbol{a} \in \mathcal{R}\right)$ are the system efficiency with and without fairness criterion $\mathcal{F}$, respectively. More precisely, we have

$$\begin{aligned} U\left(\boldsymbol{a} \in \mathcal{R}\right) &= \max_{\boldsymbol{a}\in\mathcal{R}} \sum_{i\in\mathcal{K}} \varpi_i(\boldsymbol{a}), \\ U\left(\boldsymbol{a} \in \mathcal{R}^S|\mathcal{F}\right) &= \max_{\substack{\boldsymbol{a}\in\mathcal{R}^S \\ \varpi_i = g_i(\boldsymbol{a}|\mathcal{F}),\, \forall i\in\mathcal{K}}} \sum_{i\in\mathcal{K}} \varpi_i(\boldsymbol{a}), \end{aligned}$$

and $g_i\left(\boldsymbol{a}|\mathcal{F}\right)$ is the payoff division function of player i with fairness criterion $\mathcal{F}$.

SPoF is always no larger than 1. Different from the PoF defined in [5], the SPoF only exists when users have the incentive to form the coalition. In this paper, we focus on characterizing the relationship between the fairness and efficiency among users. Since a fairness criteria is only meaningful when users have incentives to cooperate. For the rest of this paper, we will only discuss the case where a strong equilibrium exists.

To summarize, in this paper, we use the linear coefficient vector $\boldsymbol{\alpha}$ in LRA to evaluate the fairness among the players, and the SPoF to measure the worst case efficiency of the strong equilibrium. More specifically, the denominator of the SPoF is affected by the incentive region and the numerator of the SPoF is a function of the fairness criterion agreed by all the cooperative players (hence is decided by $\boldsymbol{\alpha}$). This makes it possible for us to study the relationship among incentive, efficiency, and fairness.

III. Efficiency and Fairness Tradeoff in a RA System

In this section, we use the random access (RA) system as an example to illustrate the framework we just described. RA has been widely adapted in computer storage system and wireless networks [6].

A. System Model

Let us consider a 2-user RA system where two source destination pairs, S_1 to D_1 and S_2 to D_2, correspond to two users. We denote the set of users as $\mathcal{K} = \{1, 2\}$, and they share the same resource block (e.g., time slot and frequency band) with probabilities p_1 and p_2, respectively. Each user i tries to maximize its payoff ϖ_i. The network efficiency is measured by the total payoff of both users. We use $-i$ to denote the user other than user i.

We assume that each user i knows neither the signal structure (e.g., symbol/packet intervals, transmit power), nor the transmission schedule of the other, but user i can access a common resource block with each other with a probability $p_i \in (0, 1]$. The payoff of user i is given by

$$\varpi_i\left(\boldsymbol{p}, \boldsymbol{\Pi}_i\right) = p_i p_{-i} \varpi_i^{\mathcal{K}} + p_i\left(1 - p_{-i}\right) \varpi_i^{\{i\}}, \quad (4)$$

where $\boldsymbol{p} = [p_1, p_2]$, $\boldsymbol{\Pi}_i = \left[\varpi_i^{\mathcal{K}}, \varpi_i^{\{i\}}\right]$, and $\varpi_i^{\mathcal{L}}$ is user i's payoff when only users in set $\mathcal{L}$ transmit signals with probability 1. Moreover, the performance for each source-to-destination pair depends not only on its own transmit probability but also on the transmit probability of the other. Note that if both users transmit with probability one during the entire transmission process, i.e., $p_1 = p_2 = 1$, we have $\varpi_i = \varpi_i^{\mathcal{K}}$ for $i = 1, 2$.

In this paper, we consider a general model and the payoff function ϖ_i can be any performance measure, e.g., capacity, probability of successful decoding or throughput. For instance, if user i's payoff is the average channel capacity of the ith source destination pair, we have $\varpi_i^{\mathcal{L}} = \mathbb{E}_{h_{-i}} \log\left(1 + \frac{h_{ii} w_i}{1 + h_{-ii} w_{-i}}\right)$, where h_{ij} is the ratio of the instantaneous channel gain between S_i and D_j to the additive noise of D_j for $i, j \in \mathcal{K}$ and w_i is the transmit power of S_i. If $\varpi_i^{\mathcal{L}}$ is the probability of successful decoding at D_i when users in set $\mathcal{L}$ are transmitting signals at the same time, i.e., $\varpi_i^{\mathcal{L}} = \Pr\{D_i \text{ decodes successfully} \mid \text{all users in } \mathcal{L} \text{ transmit}\}$, then our model corresponds to an RA-based ALOHA system model. More specifically, if $\varpi_i^{\mathcal{L}} = 0$ when $|\mathcal{L}| = 2$, our model becomes an RA model for the collision channel. One the other hand, if $\varpi_i^{\mathcal{L}} \neq 0$ when $|\mathcal{L}| = 2$, our model becomes the RA model for the multi-packet reception-based system.

B. Incentive Region

In a wireless network, users can either compete for the limited resources or cooperate with each other if there exists an incentive to do so. In an RA system, users can cooperate by decreasing their transmission probabilities on the common resource block. A selfish user can obtain a higher performance by refusing to reduce its transmit probability when the other user does. Hence, to make sure that both users are willing to simultaneously reduce their transmit probabilities, a negotiating process between users is necessary. This makes it natural to model the interaction among users in a RA system as a coalitional game[2], in which the set $\mathcal{K}$ of *players* are the source destination pairs, the *action* of each player i is its transmit probability p_i, and each player i tries to maximize its *payoff* ϖ_i according to fairness criterion $\mathcal{F}$ agreed between cooperative users, i.e., $\mathcal{F} = \{LRA\}$ if both users agree to divide the payoff using LRA.

[2]As observed in [10], it is generally inefficient to allow all the users in wireless networks to cooperate with each other, especially in large networks. In other words, it is more natural to model a K-user RA as a coalition formation game. However, in this paper, we focus on the relationship between the fairness and efficiency in one coalition. If the users refuse to join the coalition and choose to act selfishly, a non-cooperative game model applies and should be studied by using the concept of price of anarchy [11].

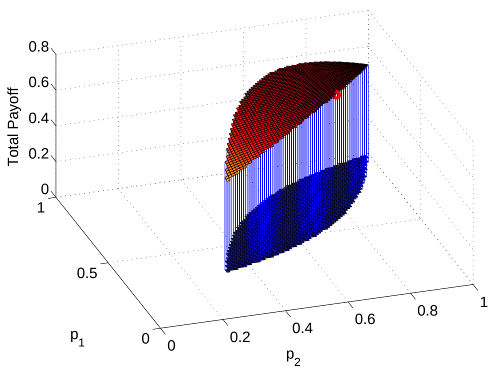


Fig. 1. Total payoff and the incentive region of a two-user RA system. The red dot represents the incentive global optimal solution.

It is easy to understand that allowing both users to transmit with probability 1 may not always maximize users' total payoff [6]. We hence need first to derive the condition for which there exists at least one probability pair (p_1, p_2) for $0 < p_1, p_2 < 1$ such that both users can improve their payoffs as compared to the benchmark of transmitting with probability 1. Let us define the incentive region for RA as follows.

Definition 3. *The incentive region $\mathcal{R}^{RA}$ is the set of transmit probability vectors where any $\boldsymbol{p} \in \mathcal{R}^{RA}$ satisfies the following two properties: 1) $0 < p_i < 1$, 2) $\varpi_i(\boldsymbol{p}) > \varpi_i^{\mathcal{K}}$ for each $i \in \mathcal{K}$, where $\varpi_i^{\mathcal{K}}$ is the user i's payoff when both users in $\mathcal{K}$ transmit with probability 1.*

We have the following result.

Theorem 1. *For a two-user RA system, there exists at least one pair of transmit probabilities $(p_1,\ p_2)$ with $0 < p_1 < 1$ and $0 < p_2 < 1$ in incentive region $\mathcal{R}^{RA}$ if and only if*

$$0 < \frac{\varpi_1^{\mathcal{K}}}{\varpi_1^{\{1\}}} + \frac{\varpi_2^{\mathcal{K}}}{\varpi_2^{\{2\}}} < 1. \tag{5}$$

The above theorem can be proved by substituting (4) into Definition 3. We omit the detailed proof due to space limitations.

Theorem 1 provides a basic condition for both source-to-destination pairs to improve their payoffs by decreasing their transmit probabilities. In other words, as long as (5) is satisfied, we can always find at least one pair of (p_1, p_2) for $0 < p_1, p_2 < 1$ that can improve the payoffs of both source-to-destination pairs.

In Figure 1, we present the incentive region and total payoff of a two-user RA system that satisfies (5). It is observed that if one user i increases its transmit probability to 1, then the transmit probability of the other user $-i$ will also approach 1, i.e., if $p_i \to 1$, then $p_{-i} \to 1$.

Note that if we only consider the non-cooperative behaviors between users in the RA system, then we can easily prove that *the only equilibrium of the non-cooperative RA game is* $(p_1 = 1, p_2 = 1)$. This shows that the non-cooperative equilibrium is always inefficient if (5) is satisfied, as users can achieve higher payoffs through cooperation.

C. Linear Resource Allocation

Let us now consider the LRA introduced in Section II-B. We can rewrite the total network payoff maximization problem with the fairness constraint in (1) as follows,

$$\begin{aligned} &\max_{\boldsymbol{p} \in \mathcal{R}^{RA}} \ U\left(\boldsymbol{p} | \mathcal{F} = \{LRA\}\right) \\ &\text{s.t.} \quad \varpi_1 = \varpi_2 / \alpha_2. \end{aligned} \tag{6}$$

The above problem can be easily solved by using standard optimization methods (substitute the subjective function into the objective function, then take the first order derivative and set it to zero). Solving the above problem, we can obtain the solution $\boldsymbol{p}^{LRA} = [p_1^{LRA}, p_2^{LRA}]$ where p_i^{LRA} for $i = 1, 2$ is a function of $\boldsymbol{\alpha}$. It can be easily observed that $\boldsymbol{p}^{LRA}$ is the only strong equilibrium if (5) is satisfied. We omit the detailed equations due to space limitations.

D. The Strong Price of Fairness

Before calculating the SPoF, we need first to calculate the incentive global optimal solution. As stated in Definition 3, two users agree on a transmission profile (p_1, p_2) with $0 < p_1, p_2 < 1$ as long as each user achieves a payoff higher than the one under $(p_1, p_2) = (1, 1)$. The incentive global optimal solution of the RA system can be obtained by solving the following problem,

$$\max_{\boldsymbol{p} \in \mathcal{R}^{RA}} \ U(\boldsymbol{p}) \tag{7}$$

Let us denote the solution of the above problem as $\boldsymbol{p}^G = [p_1^G, p_2^G]$. We can show that

Proposition 1. *If (5) is satisfied, there exists a unique global optimal transmit probability pair $\left(p_{i^*}^G, p_{-i^*}^G\right)$ for a two-user RA where $i^* = \arg\max_{i=1,2}\{U(p_i^G, p_{-i}^G)\}$.*

Next we explain the intuition behind Proposition 1. If player i fixes its transmission probability p_i, we can write $U(\boldsymbol{p})$ as

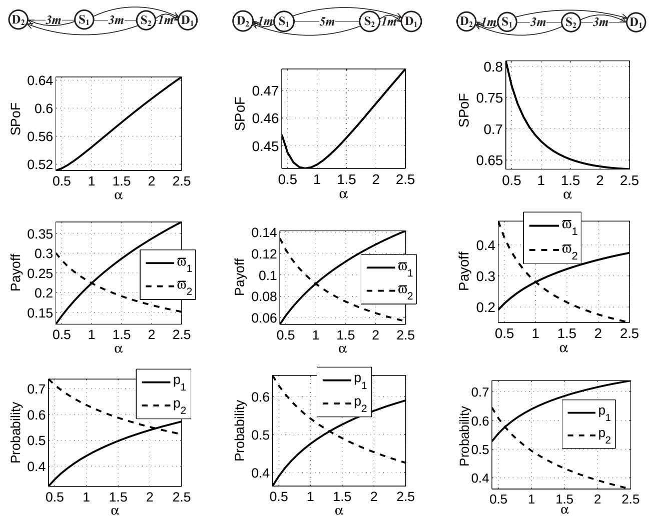
$$\begin{aligned} U(\boldsymbol{p}) \ &= \ p_{-i}\Big(p_i\left(\varpi_i^{\mathcal{K}} + \varpi_{-i}^{\mathcal{K}} - \varpi_i^{\{i\}} - \varpi_{-i}^{\{-i\}}\right) \\ &\quad + \varpi_{-i}^{\{-i\}}\Big) + p_i \varpi_i^{\{i\}} \end{aligned} \tag{8}$$

which is a linear function of p_{-i}. Whether $U(\boldsymbol{p})$ increases or decreases with p_{-i} depends on the value of p_i. Using the conditions of the incentive region, i.e., $\varpi_i > \varpi_i^{\mathcal{K}}$ and $\varpi_{-i} > \varpi_{-i}^{\mathcal{K}}$, we can obtained the upper and lower bounds of p_{-i} as follows,

$$\frac{\varpi_{-i}^{\mathcal{K}}}{\varpi_{-i}^{\{-i\}} - p_i\left(\varpi_{-i}^{\{-i\}} - \varpi_{-i}^{\mathcal{K}}\right)} \le p_{-i} \le \frac{p_i \varpi_i^{\{i\}} - \varpi_i^{\mathcal{K}}}{p_i\left(\varpi_i^{\{i\}} - \varpi_i^{\mathcal{K}}\right)} \tag{9}$$

The solution of (7) can be easily calculated by substituting either the lower bound (if $U(\boldsymbol{p})$ decreases with p_{-i}) or the upper bound (if $U(\boldsymbol{p})$ increases with p_{-i}) of p_{-i} shown in (9). In Figure 1, we mark the incentive global optimal solution of (7) by a red dot. It is shown that the optimal solution is

TABLE I
FAIRNESS AND EFFICIENCY TRADEOFF FOR RA ($\alpha = \frac{1}{\alpha_2}$)



(a) (b) (c)

always at the boundary of the incentive region $\mathcal{R}^{RA}$. In other words, RA allocates all the improvement of the total payoff $\Delta U = U(\boldsymbol{p}) - U(\boldsymbol{p}|\mathcal{F})$ to the user that provides a higher contribution to the efficiency, and the other user achieves the same payoff without RA. The above observation shows that the incentive global optimal solution is also the most *unfair* resource allocation scheme. This motivates us to study the proper tradeoff between fairness and efficiency.

We can write the PoA of a two-user RA with LRA as $\rho^{RA} = \frac{\sum_{i=1}^{2} \varpi_i(p_1^{LRA}, p_2^{LRA})}{\sum_{i=1}^{2} \varpi_i(p_1^{G}, p_2^{G})}$. It can be observed that if (5) is satisfied and $\alpha_i \neq \frac{\varpi_{-i}(\boldsymbol{p}^G)}{\varpi_i(\boldsymbol{p}^G)}$, the SPoF of the RA game is strictly less than 1.

E. A Fundamental Tradeoff between Fairness and Efficiency

If we substitute the unique strong equilibrium $\boldsymbol{p}^{LRA}(\boldsymbol{\alpha})$ derived in Section III-C into the numerator of SPoF and $\boldsymbol{p}^G$ in Section III-D into the denominator of SPoF, we can obtain the exact relationship between the efficiency and the fairness for the RA.

In Table I, we simulate a two-user RA wireless system in a linear network, where each user's payoff is related to its channel capacity as discussed in Section III-A. Let channel gain $h_{ij} = \hat{h}_{ij}/d_{ij}^{\xi}$ for $i, j \in \{S_1, S_2, D_1, D_2\}$, where $\hat{h}_{ij}$ is a Rayleigh distributed random variable, d_{ij} is the distance between i and j which is shown in the first row of Table I, and ξ is the fading exponent. For different α values, we present the fairness and efficiency tradeoff curves (second row), payoff division schemes (third row), and transmit probabilities of the users (fourth row) under three different network settings. Clearly the efficiency-fairness tradeoff heavily depends on the network topology. For example, for the first topology (the first column) in Table I, even transmitting with the same probabilities, user 1 can achieve a higher payoff than user 2 (see the third row in Table I). In this case, if we choose the absolute utility fairness [1] as the fairness criterion (such that $\alpha = 1$ is the fairness maximum point), we can observe from the third row of Table I that improving the network efficiency cannot be achieved by decreasing the payoff of user 1 (through decreasing α), but can only be obtained by sacrificing the payoff of user 2 (through increasing α). This means that decreasing the fairness may not always increase the efficiency. In other words, there always exists a tradeoff between α and SPoF if we can choose the appropriate user to scarifies the performance (such as the user 2 in left column of Table I). To summarize, a fair resource allocation scheme generally results in a lower-than-optimal efficiency for the RA system. In other words, a high efficiency resource allocation scheme is always achieved by sacrificing the benefit of the low performance user.

IV. CONCLUSION

In this paper, we introduce a general framework to study the relationship among the incentive, fairness and efficiency of user cooperation-based wireless networks. We focus on the actions of users within the incentive region and propose the concept of strong price of fairness (SPoF) to study the network efficiency under different fairness measures. We then use a two-user RA system as an example to show how to use our proposed framework to optimize the network fairness and efficiency. We observe that it is important to identify the right users to sacrifice the performance in order to achieve the desirable fairness-efficiency tradeoff. Our future work is to apply our proposed framework to study more general network systems.

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