Economic Analysis of 4G Network Upgrade

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***Abstract*—As the successor to the 3G standard, 4G provides much higher data rates to address cellular users' ever-increasing demands for high-speed multimedia communications. This paper analyzes the cellular operators' timing of network upgrades and models that users can switch operators and services. Being the first to upgrade 3G to 4G service, an operator increases his market share but takes more risk or upgrade cost because 4G technology matures over time. This paper first studies a 4G monopoly market with one dominant operator and some small operators, where the monopolist decides his upgrade time by trading off increased market share and upgrade cost. The paper also considers a 4G competition market and develops a game theoretic model for studying operators' interactions. The analysis shows that operators select different upgrade times to avoid severe competition. One operator takes the lead to upgrade, using the benefit of a larger market share to compensate for the larger cost of an early upgrade. This result matches well with many industry observations of *asymmetric* 4G upgrades. The paper further shows that the availability of 4G upgrade may decrease both operators' profits due to increased competition. Perhaps surprisingly, the profits can increase with the upgrade cost.**

I. Introduction

The third generation (3G) of cellular wireless networks was launched during the last decade. It has provided users with high-quality voice channels and moderate data rates (up to 2 Mbps). However, 3G service cannot seamlessly integrate the existing wireless technologies (*e.g.*, GSM, wireless LAN, and Bluetooth) [1], and cannot satisfy users' fast growing needs for high data rates. Thus, most major cellular operators worldwide plan to deploy the fourth-generation (4G) networks to provide much higher data rates (up to hundreds of megabits per second) and integrate heterogeneous wireless technologies. The 4G technology is expected to support new services such as high-quality video chat and video conferencing [3].

One may expect competitive operators in the same cellular market to upgrade to a 4G service at about the same time. However, many industry examples show that symmetric 4G upgrades do not happen in practice, even when multiple operators have obtained the necessary spectrum and technology patents for upgrade ([3], [4]). In South Korea, for example, Korean Telecom took the lead to deploy the world's first 4G network using WiMAX technology in 2006, whereas SK Telecom started to upgrade using more mature LTE technology in 2011. In US, Sprint deployed the first 4G WiMAX network in late 2008, Verizon waited until the end of 2010 to deploy his 4G LTE network, and AT&T planed to deploy his 4G LTE network at the end of 2011 [3]. In China, China Mobile and China Unicom are the two dominant cellular operators, and China Mobile has decided to first deploy 4G LTE network during 2012-2013 [4]. Thus, the key question we want to answer in this paper is the following: *How do the cellular operators decide the timing to upgrade to 4G networks?*

In this paper, we analyze the timing of operators' 4G upgrades in different models, including both a 4G *monopoly* market and a 4G *competition* market. Operators need to pay the cost of 4G upgrade, which decreases over time as 4G technology matures. There are two key factors that affect the operators' upgrade decisions: namely, *4G upgrade cost* and *user switching cost*. An existing 3G user can switch to the 4G service of the same operator or of a different operator, depending on how large the switching cost is. In a monopoly market where only a dominant operator can choose to upgrade to 4G, this operator can use the 4G service to capture a larger market share from small operators. In a competition market where multiple operators can choose to upgrade, we analyze the operators' interactions as a non-cooperative game. We study how the users' inter-network switching cost affects the operators' upgrade decisions, and our findings are consistent with the asymmetric upgrades observed in the industry.

Our key results and contributions are as follows.

- *A revenue-sharing model between operators:* Most existing works only study a single network's revenue by exploring the network effect (*e.g.*, [7]), and the results may not apply in a competitive market. In Section III, we study two interconnected networks, where their operators share the revenue of the inter-network traffic.
- *Monopolist's optimal timing of 4G upgrade:* By upgrading early, the 4G monopolist in Section IV obtains a large market share and a large revenue because of 4G's Quality of Service (QoS) improvement, but it cannot benefit from the cost depreciation over time. When the upgrade cost is relatively low, he upgrades at the earliest available time; otherwise he postpones his upgrade.
- *Competitive operators' asymmetric upgrades:* In Sections V and VI, users can switch operators. By upgrading early, an operator captures a large market share and

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the 4G's QoS improvement which can compensate for the large upgrade cost. The other operator, however, postpones his upgrade to avoid severe competition and benefits from cost reduction. *The availability of 4G upgrade may decrease both operators' profits because of the increased competition, and paradoxically, their profits may increase with the upgrade cost.*

II. Related Work

A. Network Effect and Network Value

In telecommunications, the network effect is the added value that a user derives from the presence of other users [13]. In a network with N users, each user perceives a value that increases with N. A reasonable model was suggested by Briscoe *et al.* [7], where each user perceives a value of order $\log(N)$. In that model, a user ranks the other users in decreasing order of importance and assigns a value $1/k$ to the k-th user in that order, for a total value $1+1/2+\cdots+1/(N-1) \approx \log(N)$. The resulting total network value is $N\log(N)$, which is appropriate for cellular networks shown by quantitative studies [7].

B. Network Upgrade

Recently, there has been a growing interest in studying the economics of network upgrades [8]–[10]. Musacchio *et al.* [8] studied the upgrade timing game between two interconnected Internet Service Providers (ISPs), where one ISP's architecture upgrade also benefits the other because of the network effect. This free-riding effect may make the second operator postpone his upgrade or even never decide to upgrade. Jiang *et al.* [9] studied a network security game, where one user's investment (upgrade) can reduce the propagation of computer viruses to all users. In our problem, however, one operator may benefit from the other's upgrade only when he also upgrades, letting his 4G users communicate with existing 4G users in other networks. Moreover, our model characterizes the dynamics of users switching between operators and/or services. These dynamics imply that an operator can obtain a larger market share by upgrading earlier, and this weakens the free-riding effect. Sen *et al.* [10] studied the users' adoption and diffusion of a new network technology in the presence of an incumbent technology. Our work is different from that study in that we are not focusing on technology competition to attract users, but on the operators' competition in upgrade timing to obtain greater profits. Moreover, the switching cost is not considered in [10], whereas it is an important parameter of our model.

III. System Model

A. Value of Cellular Networks

In this paper, we adopt the $N\log(N)$ Law, where the network value with N users is proportional to $N\log(N)$. The operator of a cellular network prefers a large network value; this is because the revenue he obtains by charging users can be proportional to the network value. Notice that the value of a 4G network is larger than a 3G network even when two networks have the same number of users. This is because the communication between two 4G users is more efficient and more frequent than between two 3G users. Because the average data rate in the 4G service is 5-10 times faster than the 3G (both downlink and uplink), a 4G network can support many new applications. We denote the efficiency ratio between 3G and 4G services as $\gamma \in (0,1)$. That is, by serving all his users via QoS-guaranteed 4G rather than 3G services, an operator obtains a larger (normalized) revenue $N\log(N)$ instead of $\gamma N\log(N)$.[1] Note that this result holds for a single operator's network that is not connected to other networks.

Next we discuss the revenues of *multiple* operators whose networks (*e.g.*, two 3G networks) are interconnected. For the purpose of illustration, we consider two networks that contain N_1 and N_2 users, respectively. The whole market covers $N = N_1 + N_2$ users. We assume that two operators' 3G (and later 4G) services are equally good to users, and the efficiency ratio γ is the same for both operators. The traffic between two users can be intra-network (when both users belong to the same operator) or inter-network (when two users belong to different operators), and the revenue calculations in the two cases are different. We assume that the user who originates the communication session (irrespective of whether the same network or to the other network) pays for the communication. This is motivated by the industry observations in EU and many Asian countries.[2] Before analyzing each operator's revenue, we first introduce two practical concepts in cellular market: "termination rate" and "user ignorance".

When two users of the same operator 1 communicate with each other, the calling user only pays operator 1. But when an operator 1's user calls an operator 2's user, operator 2 charges a *termination rate* for the incoming call [11].[3] We denote the two operators' revenue-sharing portion per inter-network call as η, where the value of $\eta \in (0,1)$ depends on the agreement between the two operators or on governments' regulation on termination rate.

User ignorance is a unique problem in the wireless cellular network, where users are often not able to identify which specific network they are calling. Mobile number portability further exacerbates this problem [14]. Thus a typical user's evaluation of two interconnected 3G networks does not depend on which network he belongs to, and equals $\gamma\log(N)$ where $N = N_1 + N_2$. We assume a call from any user terminates at a user in network $i \in \{1,2\}$ with a probability of N_i/N as in [14]. The operators' revenues when they are both providing 3G services are given in Lemma 1.

Lemma 1: When operators 1 and 2 provide 3G services, their revenues are $\gamma N_1\log(N)$ and $\gamma N_2\log(N)$, respectively.

The proof of Lemma 1 is given in our online technical report

[1] We assume that an operator's operational cost (proportional to network value) has been deducted already, and thus the revenue in this paper represents a normalized one.

[2] Our model can also be extended to the case where both involved users in a communication session pay for their communication. This is what happening in US cellular market.

[3] In the US, termination rate follows "Bill and Keep" and is low. Then operator 1 can keep most of the calling user's payment. In EU, however, termination rate follows "Calling Party Pays" and is much higher. Then most of the calling user's payment to operator 1 is used to compensate for the termination rate charged by operator 2 [11].

[20]. Both operators' revenues are linear in their numbers of users (or market share), and are independent of the sharing portion η of the inter-network revenue. Intuitively, the inter-network traffic between two networks is bidirectional: when a user originates a call from network 1 to another user in network 2, his inter-network traffic generates a fraction η of corresponding revenue to operator 1; when the other user calls back from network 2 to network 1, he generates a fraction $1-\eta$ of the same amount of revenue to operator 1. Thus an operator's total revenue is independent of η. Later, in Section IV, we show that such independence on η also applies when the two operators both provide 4G services or provide mixed 3G and 4G services.

B. User Churn during Upgrade from 3G to 4G Services

When 4G service becomes available in the market (offered by one or both networks), the existing 3G users have an incentive to switch to the new service to experience a better QoS. Such user churn does not happen simultaneously for all users, this is because different users have different sensitivities to quality improvements and switching costs [12]. We use two parameters λ and α to model the user churn within and between operators:

- *Intra-network user churn:* If an operator provides 4G in addition to his existing 3G service, his 3G users need to buy new mobile phones to use the 4G service. The users also spend time to learn how to use the 4G service on their new phones. We use λ to denote the users' switching rate to the 4G service within the same network.
- *Inter-network user churn:* If a 3G user wants to switch to another network's 4G service, he either waits till his current 3G contract expires, or pays for the penalty of immediate contract termination. This means that inter-network user churn incurs an additional cost on top of the mobile device update, and thus the switching rate will be smaller than the intra-network user churn. We use $\alpha\lambda$ to denote the users' inter-network switching rate to 4G service, where $\alpha \in (0,1)$ reflects the transaction cost of switching operators.

We illustrate the process of user churn through a continuous time model. The starting time $t=0$ denotes the time when the spectrum resource and the 4G technology are available for at least one operator (see Section IV for monopoly market and Section VI for competition market). We also assume that the portion of users switching to the 4G service follows the exponential distribution (at rate λ for intra-network churn and $\alpha\lambda$ for inter-network churn).

As an example, assume that operator 1 introduces a 4G service at time $t=T_1$ while operator 2 decides not to upgrade. The numbers of operator 1's 4G users and 3G users at any time $t \geq 0$ are $N_1^{4G}(t)$ and $N_1^{3G}(t)$, respectively. The number of operator 2's 3G users at time $t \geq 0$ is $N_2^{3G}(t)$. As time t increases (from T_1), 3G users in both networks start to churn to 4G service, and $\forall t \geq 0$

$$N_1^{3G}(t) = N_1 e^{-\lambda \cdot \max(t-T_1,0)}, N_2^{3G}(t) = N_2 e^{-\alpha\lambda \cdot \max(t-T_1,0)}, \quad (1)$$

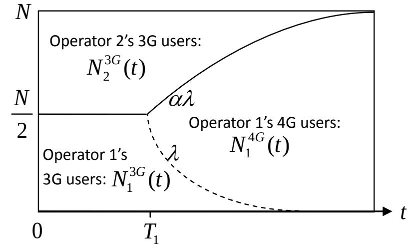

Fig. 1. The numbers of users in the operators' different services as functions of time t. Here, operator 1 upgrades at T_1 and operator 2 does not upgrade.

and operator 1's 4G service gains an increasing market share,

$$N_1^{4G}(t) = N - N_1 e^{-\lambda \cdot \max(t-T_1,0)} - N_2 e^{-\alpha\lambda \cdot \max(t-T_1,0)}. \quad (2)$$

We illustrate (1) and (2) in Fig. 1. We can see that operator 1's early upgrade attracts users from his competitor and increases his market share. Notice that (2) increases with α, thus operator 1 captures a large market share when α is large (*i.e.*, the switching cost is low).

C. Operators' Revenues and Upgrade Costs

Because of the time discount, an operator values the current revenue more than the same amount of revenue in the future. We denote the discount rate over time as S, and the discount factor is thus e^{-St} at time t according to [16].

We approximate one operator's 4G upgrade cost as a one-time investment. This is a practical approximation, as an operator's initial investment of wireless spectrum and infrastructure can be much higher than the maintenance costs in the future. For example, spectrum is a very scarce resource that is allocated (auctioned) infrequently by government agencies. Thus an operator cannot obtain additional spectrum frequently after his 4G upgrade. To ensure a good initial 4G coverage, an operator also needs to update many base stations to cover at least a whole city all at once. Otherwise, 4G users would be unhappy with the service, and this would damage the operator's reputation. That is why Sprint and Verizon covered many markets in their initial launch of their 4G services [18].

More specifically, we denote the 4G upgrade cost at $t=0$ as K, which discounts over time at a rate U. Thus if an operator upgrades at time t, he needs to pay an upgrade cost Ke^{-Ut} according to [16]. We should point out that the upgrade cost decreases faster than the normal discount rate (*i.e.*, $U > S$). This happens because the upgrade cost decreases because of both technology improvement and time discount. Very often the advance of technology is the dominant factor in determining U, and this is discussed further in Section VI.

Based on these discussions on revenue and upgrade cost, we define an operator's profit as the difference between his revenue in the long run and the one-time upgrade cost. Without loss of generality, we will normalize an operator's revenue rate (at any time t), total revenue, and upgrade cost by $N\log(N)$, where N is the total number of users in the market.[4]

[4]Our model and analytical results later can be extended to the case where N increases over time. As new users prefer 4G service to 3G service, and can easily switch at rate λ, operators will have more incentives to upgrade earlier.

IV. 4G Monopoly Market

We first look at the case where only operator 1 can choose to upgrade from 3G to 4G, while the other operators (one or more) always offer the 3G service because of the lack of financial resources or the necessary technology. This can be a reasonable model, for example, for countries such as Mexico and some Latin American ones, where America Movil is the dominant cellular operator in the 3G market. As the world's fourth-largest cellular network operator, America Movil has the advantage over other small local operators in winning additional spectrum via auctions and obtaining LTE patents, and he is expected to be the 4G monopolist in that area [15].

The key question in this section is how operator 1 should choose his upgrade time T_1 from the 3G service to the 4G service. $T_1 = 0$ means that operator 1 upgrades at the earliest time that the spectrum and technology are available, and $T_1 > 0$ means that operator 1 chooses to upgrade later to take advantage of the reduction in the upgrade cost. Because of user churn from the 3G to the 4G service, the operators' market shares and revenue rates change after time T_1. For that reason, we analyze periods $t \leq T_1$ and $t > T_1$ separately.

- *Before 4G upgrade ($t \leq T_1$):* Operator 1's and other operators' market shares do not change over time. Operator 1's revenue rate at time t is
$$\pi_1^{3G-3G}(t) = \gamma \frac{N_1}{N},$$
which is independent of time t. His revenue during this time period is
$$\pi_{1,t\leq T_1}^{3G-3G} = \int_0^{T_1} \pi_1^{3G-3G}(t) e^{-St} dt = \frac{\gamma N_1}{SN}(1 - e^{-ST_1}). \tag{3}$$
- *After 4G upgrade ($t > T_1$):* Operator 1's market share increases over time, and the other operators' total market share (denoted by $N_2^{3G}(t)/N$) decreases over time. We denote operator 1's numbers of 3G users and 4G users as $N_1^{3G}(t)$ and $N_1^{4G}(t)$, respectively, and we have $N_1^{3G}(t) + N_1^{4G}(t) + N_2^{3G}(t) = N$. This implies that
$N_2^{3G}(t) = (N - N_1) e^{-\alpha\lambda(t-T_1)}$, $N_1^{3G}(t) = N_1 e^{-\lambda(t-T_1)}$, and
$$N_1^{4G}(t) = N - N_1 e^{-\lambda(t-T_1)} - (N - N_1) e^{-\alpha\lambda(t-T_1)}.$$
Note that a 3G user's communication with a 3G or a 4G user is still based on the 3G standard, and only the communication between two 4G users can achieve a high 4G standard QoS. Operator 1's revenue rate is
$$\pi_1^{4G-3G}(t) = \frac{\gamma N_1^{3G}(t)}{N} + \frac{N_1^{4G}(t)}{N} \cdot \left(\frac{N_1^{4G}(t) + \gamma N_1^{3G}(t)}{N} + \frac{\gamma N_2^{3G}(t)}{N} \right),$$
which is independent of the revenue sharing ratio η between the calling party and receiving party. Operator 1's revenue during this time period is then
$$\pi_{1,t>T_1}^{4G-3G} = \int_{T_1}^{\infty} \pi_1^{4G-3G}(t) e^{-St} dt, \tag{4}$$
where $t \to \infty$ is an approximation of the long-term 4G service provision (*e.g.*, one decade) before the emergence of the next generation standard. This approximation is reasonable since the revenue in the distant future becomes less important because of discount.

Figure 1 illustrates how the numbers of users of operators' different services change over time. Before operator 1's upgrade (*e.g.*, $t \leq T_1$ in Fig. 1), the number of total users in each network does not change;[5] after operator 1's upgrade, operator 1's and the other operators' 3G users switch to the new 4G service at rates λ and $\alpha\lambda$, respectively.

By considering (3), (4), and the decreasing cost Ke^{-UT_1}, operator 1's long-term profit when choosing an upgrade time T_1 is
$$\begin{aligned}\pi_1(T_1) &= \pi_{1,t\leq T_1}^{3G-3G} + \pi_{1,t>T_1}^{4G-3G} - Ke^{-UT_1} \\ &= e^{-ST_1} \left(\frac{1}{S} + (1-\gamma) \frac{\left(\frac{N_1}{N}\right)^2}{2\lambda + S} + (1-\gamma) \frac{\left(\frac{N-N_1}{N}\right)^2}{2\alpha\lambda + S} \right) \\ &- e^{-ST_1} \left(2(1-\gamma) \frac{\frac{N_1}{N}}{\lambda + S} + (2-\gamma) \frac{\frac{N-N_1}{N}}{\alpha\lambda + S} \right) - Ke^{-UT_1} \\ &+ 2e^{-ST_1}(1-\gamma) \frac{\frac{N_1(N-N_1)}{N^2}}{(1+\alpha)\lambda + S} + \frac{N_1 \gamma}{NS}(1 - e^{-ST_1}). \end{aligned} \tag{5}$$
We can show that $\pi_1(T_1)$ in (5) is strictly concave in T_1, thus we can compute the optimal upgrade time T_1^* by solving the first-order condition. The optimal upgrade time depends on the following upgrade cost threshold in the monopoly 4G market,
$$\begin{aligned} K_{th}^{mono} =& (1-\gamma) \frac{\frac{\left(\frac{N_1}{N}\right)^2}{2\lambda+S} + \frac{\left(\frac{N-N_1}{N}\right)^2}{2\alpha\lambda+S} - \frac{2\frac{N_1}{N}}{\lambda+S} + \frac{2\frac{N_1(N-N_1)}{N^2}}{(1+\alpha)\lambda+S}}{U/S} \\ &+ \frac{1 - \gamma\frac{N_1}{N} - (2-\gamma)\frac{N-N_1}{N}\frac{S}{\alpha\lambda+S}}{U}. \end{aligned} \tag{6}$$

Theorem 1: Operator 1's optimal upgrade time in a 4G monopoly market is:

- *Low cost regime* (upgrade cost $K \leq K_{th}^{mono}$): operator 1 upgrades at $T_1^* = 0$.
- *High cost regime* ($K > K_{th}^{mono}$): operator 1 upgrades at
$$T_1^* = \frac{1}{U-S} \log \left(\frac{K}{K_{th}^{mono}} \right) > 0. \tag{7}$$

Intuitively, an early upgrade gives operator 1 a larger market share and enables him to get a higher revenue via the more efficient 4G service. Such advantage is especially obvious in the low cost regime where the upgrade cost K is small.

Next we focus on the high cost regime, and explore how the network parameters affect operator 1's upgrade time.

Observation 1: Operator 1's optimal upgrade time T_1^* increases with the upgrade cost K, and decreases with α (*i.e.*, increases with the users' inter-network switching cost).

The proofs of Observation 1 and the following observations are given in our online technical report [20].

Observation 2 (Figure 2): When $K_{th}^{mono} < K < \frac{U}{S} K_{th}^{mono}$, T_1^* first increases and then decreases in U. When $K \geq \frac{U}{S} K_{th}^{mono}$, T_1^* monotonically decreases in U.

Figure 2 shows T_1^* as a function of U and K. When K is large or U is large, operator 1 wants to postpone his upgrade until the upgrade cost decreases significantly. A larger U (thus

[5] It is possible for users to switch between networks in the same 3G service class, but such minor switching does not change the average market shares.

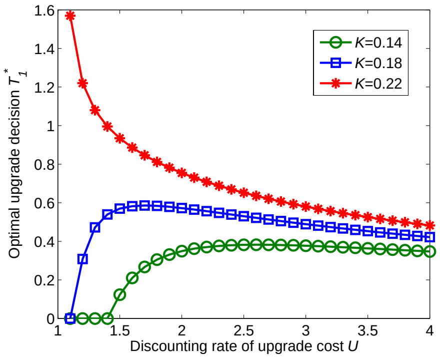

Fig. 2. Operator 1's optimal upgrade time T_1^* as a function of cost K and the discount rate U of cost. Other parameters are $N = 1000$, $N_1 = 500$, $S = 1$, $\lambda = 1$, and $\alpha = 0.5$.

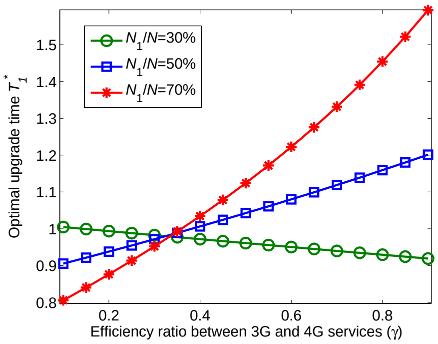

Fig. 3. Operator 1's optimal upgrade time T_1^* as a function of the efficiency ratio between 3G and 4G services (γ) and his original market share $\frac{N_1}{N}$. Other parameters are $N = 1000$, $K = 2000$, $S = 1$, $\lambda = 1$, and $U = 2$.

a faster cost-decreasing rate) strengthens his willingness to postpone. When K is small and U is small, the upgrade cost is small and does not decrease fast enough. In this case, operator 1 chooses to upgrade early if the revenue increase can compensate for the small upgrade cost.

Observation 3 (Figure 3): When operator 1's original market share $\frac{N_1}{N}$ is large, T_1^* increases in the efficiency ratio γ (between 3G and 4G). When $\frac{N_1}{N}$ is small, T_1^* decreases in γ.

Figure 3 shows T_1^* as a function of γ and $\frac{N_1}{N}$. When $\frac{N_1}{N}$ is large, operator 1 cannot attract many users from other operators and has smaller incentives to upgrade. As γ increases (and thus the QoS gap between 3G and 4G shrinks), he is less interested in 4G service and thus postpones his upgrade. When $\frac{N_1}{N}$ is small, operator 1 has limited market share and can attract many more users by upgrading early. As γ increases, operator 1 obtains a higher revenue between his existing 3G users and the increasing number of 4G users. Operator 2, however, does not benefit much from this, as he loses his market share due to operator 1's 4G services. Thus T_1^* decreases in γ.

V. 4G Competition Market: Duopoly Model and Game Formulation

In this section, we focus on the competition between multiple operators who can choose to upgrade to 4G services. To make the analysis tractable and to derive clear engineering insights, we focus on the case of two operators (duopoly) in this paper. This analysis serves as the first step in understanding the more general oligopoly case. This duopoly model is reasonable in a country like China, where China Mobile and China Unicom are the two dominant cellular operators in the 3G market. A similar situation exists in several other Asian and European countries as well.

The focus of this and the following section is to understand why in so many existing industry examples (*e.g.*, [3]–[5]) operators choose to upgrade to 4G services at different times even though they have the resources to upgrade simultaneously. In particular, we examine whether such asymmetric upgrades emerge even when the two operators are similar (*e.g.*, having similar market shares before upgrades (*e.g.*, Verizon has 106.3 million users and AT&T has 98.6 million users in the US). In our online technical report [20], we also examine the case where networks are heterogeneous in nature, and we show that asymmetric operators (e.g., with different market shares) have more incentives to upgrade at different times. Thus in the following analysis we can consider two operators that have the same market shares before the 4G upgrades ($N_1 = N_2$), the same upgrade cost K, and the same cost discount rate U. We will first derive the operators' profits under any upgrade decisions, and then analyze the duopoly game where each operator chooses the best upgrade time to maximize his profit.

A. Operators' Long-term Profits

Let us denote two operators' upgrade times as T_1 and T_2, respectively. Because the two operators are symmetric, without loss of generality, we assume in the following example (before Lemma 2) that operator 1 upgrades no later than operator 2 (*i.e.*, $T_1 \leq T_2$). To calculate the operators' profits, we first need to understand how users churn from 3G to 4G services, and how this affects the operators' revenue rates over time. Figure 4 shows that user churn is different in three phases, depending on how many operators have upgraded.

- *Phase I ($0 \leq t \leq T_1$):* No operator has upgraded and both operators' market shares do not change.
- *Phase II ($T_1 < t \leq T_2$):* Operator 1 has upgraded to 4G service but operator 2 has not. The 3G users of two operators switch to operator 1's 4G service at different rates. The numbers of users in the operators' different services are
$$N_1^{3G}(t) = \frac{N}{2}e^{-\lambda(t-T_1)}, N_2^{3G}(t) = \frac{N}{2}e^{-\alpha\lambda(t-T_1)},$$
and
$$N_1^{4G}(t) = N - \frac{N}{2}e^{-\lambda(t-T_1)} - \frac{N}{2}e^{-\alpha\lambda(t-T_1)}.$$
- *Phase III ($t > T_2$):* Both operators have upgraded, and 3G users only switch to the 4G service of their current operator. The numbers of users in operators' different services are
$$N_1^{3G}(t) = \frac{N}{2}e^{-\lambda(t-T_1)},$$
$$N_1^{4G}(t) = N - \frac{N}{2}e^{-\lambda(t-T_1)} - \frac{N}{2}e^{-\alpha\lambda(T_2-T_1)},$$
$$N_2^{3G}(t) = \frac{N}{2}e^{\alpha\lambda(T_2-T_1)-\lambda(t-T_2)},$$

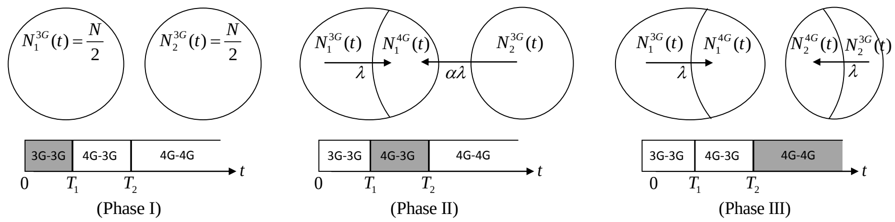

Fig. 4. Users' switches over the operators' services: Phase I with 3G services only, Phase II with operator 1's 4G service, and Phase III with both operators' 4G services.

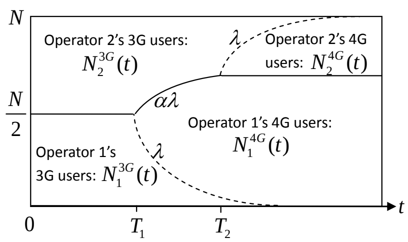

Fig. 5. The numbers of users in the operators' different services as functions of time t. Here, operator 1 chooses his upgrade time at T_1 and operator 2 chooses T_2 with $T_1 \leq T_2$.

and

$$N_2^{4G}(t) = \frac{N}{2} e^{\alpha\lambda(T_2 - T_1)} \left(1 - e^{-\lambda(t-T_2)}\right).$$

Figure 5 summarizes how users churn in the three phases. Similar to Section IV, we can derive operators' revenue rates based on users' churn over time. By integrating each operator's revenue rate over all three phases, we obtain that operator's long-term revenue. Recall that an operator's profit is the difference between his revenue and the one-time upgrade cost. By further considering the symmetric case of $T_1 \leq T_2$, we have the following result.

Lemma 2: Consider two operators $i, j \in \{1, 2\}$ (with $i \neq j$) upgrading at T_i and T_j. Operator i's long-term profit is

$$\pi_i(T_i, T_j) = \begin{cases} \pi^{ER}(T_i, T_j), & \text{if } T_i \leq T_j; \\ \pi^{LT}(T_j, T_i), & \text{if } T_i \geq T_j, \end{cases} \quad (8)$$

where $\pi^{ER}(T_i, T_j)$ and $\pi^{LT}(T_j, T_i)$ are given in (9) and (10), respectively.

Note that an operator's profit $\pi_i(T_i, T_j)$ is continuous in his upgrade time T_i. When operator i's upgrade time T_i is less than T_j, he increases his market share at rate $\alpha\lambda$ during the time period from T_i to T_j; but when $T_i > T_j$, operator i loses his market share at rate $\alpha\lambda$ during the period from T_j to T_i. This explains why we need two different functions $\pi^{ER}(T_i, T_j)$ and $\pi^{LT}(T_j, T_i)$ to completely characterize the long-term profit for each operator.

B. Duopoly Upgrade Game

Next we consider the non-cooperative game theoretical interactions between two operators, where each of them seeks to maximize his long-term profit by choosing the best upgrade time.

Upgrade Game: We model the competition between two operators as follows:

- Players: Operators 1 and 2.
- Strategy spaces: Operator $i \in \{1, 2\}$ can choose upgrade time T_i from the feasible set $\mathcal{T}_i = [0, \infty]$.[6]
- Payoff functions: Operator $i \in \{1, 2\}$ wants to maximize his profit $\pi_i(T_i, T_j)$ defined in (8).

Notice that we consider a static game here, where both operators decide when to upgrade at the beginning of time. This is motivated by the fact that operators usually make long-term decisions in practice rather than changing decisions frequently, as many upgrade operations (e.g., financial budget and technological trials) need to be planned and prepared. As we consider that each operator has complete information about his competitor's and users' parameters, he will not deviate from his initial decision as time goes on. Also, it should be pointed out that in many competition markets operators can obtain available resource for 4G upgrade at a similar time as we mentioned at the beginning of this section. After making decision at $t = 0$, one operator does not change his decision later on. This is reasonable when operators can predict the future 4G market adoption.[7]

In Section VI, we analyze the duopoly upgrade game under different switching costs (*i.e.*, the value of $\alpha \in (0, 1)$). Nash equilibrium is a commonly used solution concept for a static game. At a Nash equilibrium, no player can increase his payoff by deviating unilaterally [19]. We are interested in characterizing the conditions under which an asymmetric upgrade equilibrium emerges between symmetric operators.

VI. 4G Competition Market: Practical Inter-network Switching Rate

In this section, we consider the case of $\alpha > 0$, *i.e.*, 3G users may switch to the 4G service of a different operator. The equilibrium analysis in this general case depends on the relationship between U (upgrade cost discount rate) and S (money depreciation rate). We assume that U is much larger than S, *i.e.*, $U > S + \alpha\lambda$. This represents the practical case where the advance of technology is the dominant factor in determining U, and not many 3G users choose to switch operators when the 4G service is just deployed (*i.e.*, small α) [17]. For example, Sprint deployed the first 4G network in US by using WiMAX technology in 2008 when LTE technology

[6] Note that $T_i = \infty$ means that operator i never upgrades.

[7] Operators can predict the future market adoption by exploring historical records of the market and some trial of 4G deployment. In the future we will study the incomplete information case, where an operator may learn more information as the time goes and revise his upgrade decision (if he has not upgraded yet).

$$
\begin{aligned}
\pi^{ER}(T_i,T_j) =& \frac{\gamma+(2-\gamma)e^{-ST_i}-e^{-\alpha\lambda(T_j-T_i)-ST_j}}{2S} - \frac{(2-\gamma)(e^{-ST_i}-e^{-\alpha\lambda(T_j-T_i)-ST_j})}{2(\alpha\lambda+S)} - (1-\gamma)\frac{e^{-ST_i}}{\lambda+S} \\
&+ \frac{1-\gamma}{2}\left(\frac{e^{-ST_i}+e^{-(1+\alpha)\lambda(T_j-T_i)-ST_j}}{2(2\lambda+S)} + \frac{e^{-ST_i}-e^{-(1+\alpha)\lambda(T_j-T_i)-ST_j}}{(1+\alpha)\lambda+S} + \frac{e^{-ST_i}-e^{-2\alpha\lambda(T_j-T_i)-ST_j}}{2(2\alpha\lambda+S)}\right) \\
&- \frac{1-\gamma}{2(\lambda+S)}e^{-\alpha\lambda(T_j-T_i)-ST_j}\left(1-\frac{e^{-\lambda(T_j-T_i)}+e^{-\alpha\lambda(T_j-T_i)}}{2}\right) - Ke^{-UT_i}. \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
\pi^{LT}(T_j,T_i) =& \frac{e^{-\alpha\lambda(T_i-T_j)-ST_i}}{2}\left(\frac{1}{S}-\frac{1-\gamma}{\lambda+S}-\frac{1-\gamma}{2}\left(e^{-\lambda(T_i-T_j)}+e^{-\alpha\lambda(T_i-T_j)}\right)\left(\frac{1}{\lambda+S}-\frac{1}{2\lambda+S}\right)\right) + \frac{\gamma}{2S}(1-e^{-ST_j}) \\
&+ \frac{\gamma}{2(\alpha\lambda+S)}(e^{-ST_j}-e^{-\alpha\lambda(T_i-T_j)-ST_i}) - Ke^{-UT_i}. \qquad (10)
\end{aligned}
$$

was not mature yet. Only two years later, in 2010, LTE could already offer a much lower cost per bit than WiMAX [3]. From 2012, LTE is expected to be the leading technology choice for 4G networks. This example motivates that U is much larger than S, and we will study whether the operators' symmetric 4G upgrades happen in this scenario.

Recall that by upgrading at T_1 and T_2, the operators receive the profits given in (8). In game theory, one operator's *best response function* is his upgrade time that achieves the largest long-term profit, as a function of a fixed upgrade time of the other operator [19]. A fixed point of the two operators' best response functions is the Nash equilibrium, and in general there can be more than one such fixed point.

We can show that the operators' best responses functions ($T_1^{best}(T_2)$ and $T_2^{best}(T_1)$) depend on the upgrade cost K, and in particular, they depend on two cost thresholds ($K_{th1}^{comp} < K_{th2}^{comp}$) that lead to three cost regimes: low, medium, and high.

When the upgrade cost K is less than the first threshold K_{th1}^{comp} (*i.e.*, low cost regime), both operators will upgrade at $t=0$ to maximize the revenue from the 4G service. By solving

$$\frac{\partial \pi^{LT}(0,T_i)}{\partial T_i}|_{T_i=0} = 0, \forall i \in \{1,2\},$$

we have

$$
\begin{aligned}
K_{th1}^{comp} =& \left((1-\gamma)\frac{(1-\alpha)\lambda}{\lambda+S} + \frac{\alpha\lambda}{S} - \frac{1-\gamma}{2}(\lambda+3\alpha\lambda+2S)\right. \\
&\left. \cdot \left(\frac{1}{\lambda+S}-\frac{1}{2\lambda+S}\right)\right)\frac{1}{2U}. \qquad (11)
\end{aligned}
$$

When the upgrade cost K is larger than K_{th1}^{comp} (*i.e.*, medium or high cost regimes), at least one operator postpones his upgrade until the upgrade cost decreases sufficiently. In particular, when K is larger than the second threshold K_{th2}^{comp} (*i.e.*, high cost regime), both operators postpone their upgrades. When operator $i \in \{1,2\}$ upgrades at $t=0$, operator $j \neq i$ postpones his upgrade to $T_j^{best}(0)$, which is the unique solution to

$$\frac{\partial \pi^{LT}(0,T_j)}{\partial T_j}|_{T_j=T_j^{best}(0)} = 0. \qquad (12)$$

The threshold K_{th2}^{comp} can be obtained by solving

$$\frac{\partial \pi^{ER}(T_i,T_j^{best}(0))}{\partial T_i}|_{T_i=0} = 0. \qquad (13)$$

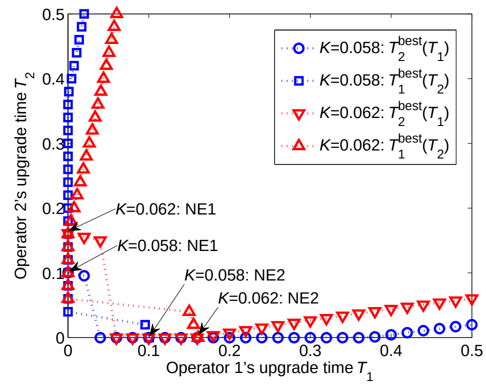

Fig. 6. Two operators' best upgrade responses to each other according to different cost values in the medium cost regime. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, $\lambda = 1$ and $\gamma = 0.5$ such that $U > S + \alpha\lambda$.

Next we illustrate numerically how the two operators' best response functions ($T_1^{best}(T_2)$ and $T_2^{best}(T_1)$) change with the upgrade cost K.

Figure 6 shows that each operator's best response function is discontinuous in the medium cost regime, and the two best response functions with the same value of K intersect at two points: $0 = T_1^* < T_2^*$ and (symmetrically) $0 = T_2^* < T_1^*$. To illustrate this situation, consider operator 2's best response $T_2^{best}(T_1)$ in the case $K = 0.062$. If operator 1 upgrades early such that T_1 is less than 0.05, operator 2 does not upgrade at the same time to avoid a severe competition. If operator 1 upgrades later such that T_1 is larger than 0.05, operator 2 chooses to upgrade earlier than operator 1 to increase his market share. Thus $T_2^{best}(T_1)$ is discontinuous at $T_1 = 0.05$.

Figure 7 shows that each operator's best response function is discontinuous in the high cost regime, and the two functions (with the same value of K) intersect at two points, equilibria $0 < T_1^* < T_2^*$ and (symmetrically) $0 < T_2^* < T_1^*$. Unlike Fig. 6, the high cost here prevents any operator from choosing the upgrade time $t = 0$.

Figure 8 summarizes how operators' upgrade equilibrium changes as cost K increases: starting with $T_i^* = T_j^* = 0$ in low cost regime, then $0 = T_i^* < T_j^*$ with increasing T_j^* in medium cost regime, and finally $0 < T_i^* < T_j^*$ with increasing T_1^* and T_2^* in high cost regime.

In the following theorem, we prove that the operators do not choose symmetric upgrades as long as the cost is not low.

Theorem 2: The two operators' 4G upgrade equilibria sat-

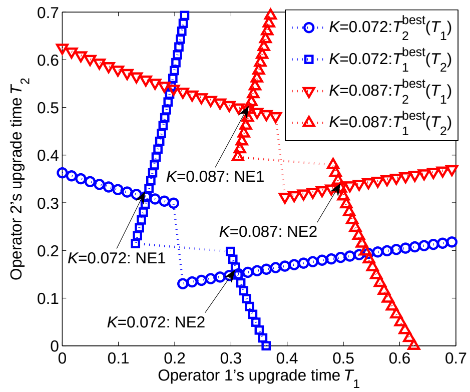

Fig. 7. Two operators' best upgrade responses to each other according to different cost values in the high cost regime. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, $\lambda = 1$ and $\gamma = 0.5$ such that $U > S + \alpha\lambda$.

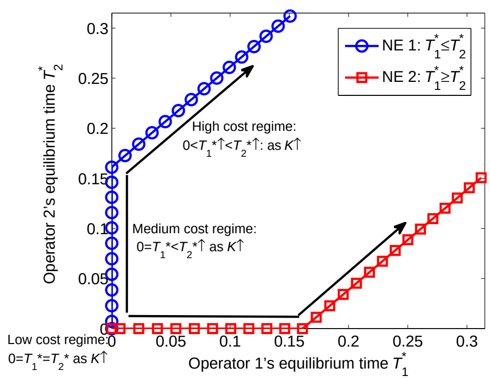

Fig. 8. Two operators' equilibrium (T_1^*,T_2^*) changes as cost K increases. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, $\lambda = 1$, and $\gamma = 0.5$ such that $U > S + \alpha\lambda$.

isfy the following properties:

- *Low cost regime* ($K \le K_{th1}^{comp}$): Both operators upgrade at $T_1^* = T_2^* = 0$.
- *Medium cost regime* ($K_{th1}^{comp} < K \le K_{th2}^{comp}$): Operators do not upgrade at the same time, and only one operator may upgrade at $t = 0$. The possible equilibria can only be $0 \le T_1^* < T_2^*$ and (symmetrically) $0 \le T_2^* < T_1^*$.
- *High cost regime* ($K > K_{th2}^{comp}$): Operators do not upgrade at the same time, and none of them upgrade at $t = 0$. The possible equilibria can only be $0 < T_1^* < T_2^*$ and (symmetrically) $0 < T_2^* < T_1^*$.

The proof of Theorem 2 is given in our online report [20].

To understand the intuition behind the asymmetric structure, we summarize the advantages of earlier and later upgrades with $\alpha > 0$ as follows:

- *Earlier upgrade* gives an operator the advantage to attract more users (from the other operator), and enables the operator to collect a higher revenue from the 4G service.
- *Later upgrade* allows an operator to incur a reduced upgrade cost and to take advantage of the network effect in the 4G market (with more existing 4G users) when he upgrades.

In order to fully enjoy the two advantages of earlier or

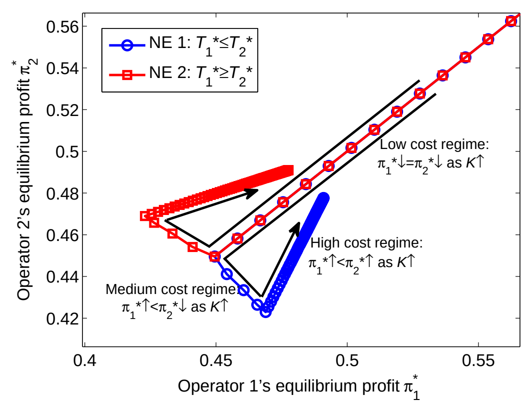

Fig. 9. Two operators' equilibrium profits (π_1^*,π_2^*) change as cost K increases under large $\gamma = 0.5$. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, and $\lambda = 1$ such that $U > S + \alpha\lambda$.

later upgrades, operators will avoid symmetric upgrade. If one operator upgrades much earlier to capture a larger market share that can compensate for a large upgrade cost, the second operator will not upgrade at the same time to avoid severe competition in market share; instead, the second operator will wait until his loss of users and revenue is compensated by the reduction of upgrade cost (with $U > S + \alpha\lambda$).

Next we study how operators' equilibrium profits change with cost K and the economic efficiency ratio γ between 3G and 4G services in the three cost regimes. Note that an operator may or may not be able to charge a significantly higher price from a 4G user though 4G does improve a lot over 3G in QoS. Figures 9 and 10 show operators' equilibrium profits under large and small γ values, respectively.

We first study the large γ scenario in Fig. 9 which means an operator to charge a slightly higher price in 4G service than 3G. Without loss of generality, we focus on the case where operator 1 upgrades no later than operator 2 (*i.e.*, $T_1^* \le T_2^*$).

- In the low cost regime, by upgrading at $T_1^* = T_2^* = 0$, the two operators' profits are the same and decrease with cost K.
- In the medium cost regime, Fig. 9 shows that operator 1 receives a larger profit than operator 2 by upgrading at $T_1^* = 0$. *Perhaps surprisingly, his profit increases with K, whereas operator 2's profit decreases with K.* Intuitively, the increase of K encourages operator 2 to further postpone his upgrade and lose more users to operator 1. The change of operator 1's profit trades off the increases of his market share and upgrade cost. As operator 1's market share increases, his growing 4G users communicate more with his 3G users via the efficient 3G service under large γ. Operator 1's 3G revenue increases because of a more efficient intra-network traffic, which helps compensate for the upgrade cost.
- In the high cost regime, Fig. 9 shows that both operators have to postpone their upgrades and, surprisingly, *both operators' profits increase with K*. As K increases, operator 1 further postpones his upgrade and operator

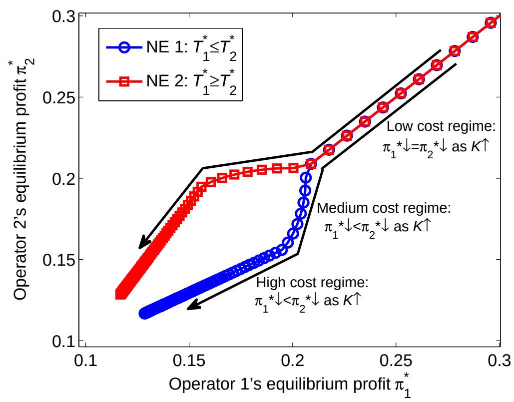

Fig. 10. Two operators' equilibrium profits (π_1^*,π_2^*) change as cost K increases under small $\gamma = 0.1$. Other parameters are $\alpha = 0.5$, $N = 1000$, $U = 2$, $S = 1$, and $\lambda = 1$ such that $U > S + \alpha\lambda$.

2's market share decreases more slowly. Thus operators' competition in the market share is postponed, and under large γ operator 2 can obtain more 3G revenue before operator 1's upgrade. Operator 1, on the other hand, also benefits from his postponement to decrease his upgrade cost. Since operator 2 also postpones his upgrade, operator 1 can still capture a large market share even though he upgrades later. As $K \to \infty$, no operator upgrades and operators' profits approach the symmetric 3G profits. Under large γ, the 4G service is not much better than 3G and the availability of 4G upgrade only intensifies operators' competition. Compared to traditional 3G scenario, both operators' profits decrease when the upgrade cost is high. *In other words, both operators will be better off if 4G technology is not available in this case.*

Figure 10 shows how operators' profits change with K under small γ. The results are more intuitive; this is because the operators' profits decrease with K in all three cost regimes. Under small γ, the availability of the 4G upgrade significantly improves the revenue in each network. A larger K reduces the benefit of upgrades. However, the operators' profits will not be smaller after the 4G upgrade under any value of K.

VII. Conclusions and Future Work

This paper presents the first analytical study of operators' 4G upgrade decisions. We first analyze a 4G monopoly market, where the monopolist's optimal upgrade time trades off an increased market share and the decreasing upgrade cost. We then develop a non-cooperative game model to study the competition between operators. Our results show that operators select different upgrade times to avoid severe competition in market share. We further show that the availability of 4G upgrade may decrease both operators' profits due to their competition, and their profits may increase with the upgrade cost.

There are some possible ways to extend the results in this paper. For example, we could consider an oligopoly market with more than two competitive operators. Intuitively, multiple operators under inter-network switching would still select asymmetric upgrade times to avoid severe competition. Compared to duopoly, the operator who is the last to upgrade loses more market share to the others and but enjoys the smallest upgrade cost and the largest network effect. It is also interesting to study operators' usage of 4G plan announcements before actual upgrades. One operator (who actually decides to upgrades later than what he announces) can prevent some of his users switching to other networks. We can use a signaling game to study operators' announcements. Moreover, we can study operators' mixed strategies in 4G upgrades, where one operator chooses a probability distribution of upgrading at different times. Although theoretically interesting, mixed strategies are hard to implement in practice [19].

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