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Optimal Pricing and Admission Control for Heterogeneous Secondary Users

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***Abstract*—This paper studies how to maximize a spectrum database operator's expected revenue in sharing spectrum to secondary users, through joint pricing and admission control of spectrum resources. A unique feature of our model is the consideration of the stochastic and heterogeneous nature of secondary users' demands. We formulate the problem as a stochastic dynamic programming problem, and present the optimal solutions under both static and dynamic pricing schemes. In the case of static pricing, the prices do not change with time, although the admission control policy can still be time-dependent. In this case, we show that a stationary (time-independent) admission policy is in fact optimal under a wide range of system parameters. In the case of dynamic pricing, we allow both prices and admission control policies to be time-dependent. We show that the optimal dynamic pricing can improve the operator's revenue by more than 30% over the optimal static pricing, when secondary users' demands for spectrum opportunities are highly elastic.**

***Index Terms*—Spectrum Pricing, Heterogeneous Demands.**

I. Introduction

DATABASE-ASSISTED spectrum sharing is a promising approach to improve the utilization of limited spectrum resources [2], [3]. In such an approach, primary licensed users (PUs) report their spectrum usage patterns to a spectrum database, which uses the primary activity records to coordinate the opportunistic spectrum access of secondary unlicensed users (SUs). Several government regulators, such as the FCC in the US and the Ofcom in the UK, strongly advocate such an approach (e.g., for the sharing of TV white space) due to its high reliability compared to sensing. Under such an approach, the database can effectively coordinate SUs' accesses, by mitigating these SUs' mutual usage conflicts and controlling the potential conflicts with PUs. Though researchers have made significant research progress in addressing various technical issues of spectrum database (e.g., database system management and spectrum allocation [4]–[6]), very few studies looked at the economic issue of spectrum database (e.g., [4]–[6]). Without a proper economic mechanism, the database operator may not have enough incentives to coordinate the spectrum sharing process. This motivates us to explore the revenue maximization problem for a spectrum database, in particular, the admission of SUs and pricing of idle spectrum resources.

There are two key challenges when considering such a revenue maximization problem for the database operator. First, SUs' demands can be *heterogeneous* in terms of spectrum occupancy. For example, a large file (e.g., video) downloading takes minutes or even hours to finish (hence we call heavy-traffic), while sending a short text message or accessing location-based services can be completed in seconds (hence we call light-traffic). Second, SUs' demands are often *randomly* generated over time. The heterogeneity and randomness make it difficult for the operator to accurately predict future demands and make proper allocation decisions. Yet the two issues have not been fully considered in the previous studies (e.g., [4]–[6]).

To address the above two challenges, we propose a joint spectrum pricing and admission control scheme for the database operator to maximize its expected revenue. The optimization is over the time period during which the spectrum channel is available for SUs to opportunistically access due to the lack of PU activities. The period is divided into several time slots, and the database operator needs to determine the optimal prices for different types of SUs (e.g., in heavy- and light-traffic types) in each time slot. These prices can be fixed (static pricing) or vary over time (dynamic pricing), and will affect how SUs request to access the spectrum. However, pricing alone may not be enough to mitigate the conflicts between multiple SUs who want to access to the limited spectrum at the same time. The operator also needs to determine the optimal admission control policy to control the total demand. The pricing and admission decisions need to be jointly optimized to achieve the maximum performance.

There are several recent results focusing on the spectrum pricing issues of a spectrum database (e.g., [4]–[6]). These studies focused primarily on the static pricing with complete information in the spectrum database system, without considering the heterogeneous and stochastic SU demands as in our work. Under static pricing, the pricing decisions do not change over time (e.g., [4]–[11]). In particular, Duan *et al.* in [9] considered the static pricing of spectrum under supply uncertainty, and extended to the competitive duopoly pricing in [10]. The literature on general dynamic pricing focuses on dynamic pricing decisions of selling a given stock of items by a deadline (e.g., [12], [13]), and in particular, pricing decisions of airline seats and hotel rooms booking (e.g., [14], [15]).

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The literature on dynamic pricing of wireless resources only emerged recently (e.g., [16]–[18]). Song *et al.* in [16] studied the network revenue maximization problem by using dynamic pricing for multiple flows in a wireless multi-hop network. Ha *et al.* in [17] proposed time-dependent pricing to decrease customers' congestion cost. Ma *et al.* in [18] proposed time and location based pricing for mobile data traffic.

However, in our work spectrum has its unique features to be priced and used. Unlike a traditional product, the unused spectrum resource cannot be stored and is immediately wasted. Moreover, SUs' demands are often heterogeneous and random over time. The characterizations of both the time-sensitivity of spectrum and the SUs' demand heterogeneity and uncertainty make our model and analysis different from prior studies. We tackle these issues by jointly considering the admission control of SUs and the dynamic pricing of spectrum in a dynamic setting over time, and characterize the conditions under which the often complicated optimal pricing and admission decisions degenerate to the stationary pricing and admission schemes.

To our best knowledge, this is the first work that jointly prices and allocates the spectrum resource in a *dynamic* setting to serve heterogeneous and stochastic SU demands. We formulate the operator's revenue maximization problem as a stochastic dynamic programming problem, which is in general challenging to solve. Our main results and key contributions are summarized as follows.

- *Optimal static pricing and dynamic admission policies*. We first constrain ourselves to the simple and widely used approach of static pricing, meanwhile allowing dynamic time-dependent admissions. We show that the complex optimal dynamic admission policy often degenerates to a threshold-based stationary (time-independent) policy under a wide range of system parameters.
- *Optimal dynamic pricing and dynamic admission policies*. We further allow the prices to dynamically change over time based on different SUs' stochastic demands. Although the optimal prices and admission decisions are coupled, we are able to compute the optimal policy through a proper price-and-admission decomposition in each time slot. Similarly, we show that the optimal admission policy often degenerates to a stationary admission policy under a wide range of system parameters. By comparing the optimal pricing and admission policies under both static and dynamic pricing schemes, we show that the dynamic pricing scheme can significantly improve the database operator's revenue (by more than 30%) when SU's demands are highly elastic.

The rest of the paper is organized as follows. We introduce the model and problem in Section II. In Section III, we formulate and solve the optimal static pricing and dynamic admission control problem. In Section IV, we further consider the joint optimization of dynamic pricing and dynamic admission control. In Section V, we extend our model and results to the more general cases of SU types. We show the simulation results in Section VI and conclude the paper in Section VII.

II. System Model and Problem Formulation

We consider a database operator who records PUs' activities and knows a channel that will not used by PUs during a set $\mathcal{N} = \{1, \cdots, N\}$ of consecutive time slots, similar as in [2], [3]. The database operator wants to maximize its revenue through selling the temporary spectrum opportunities to the SUs. The duration of this whole time period depends on the type of PU traffic, and is known in advance as the PUs need to register all traffic with the database (e.g., [2], [3]).

SUs randomly arrive and request channel access at the beginning of each time slot. To gain clear insights into the admission policies of SUs, we first assume that there are two types of SUs depending on the length of the channel access time. A *light-traffic* SU only needs to use the channel for one time slot, and a *heavy-traffic* SU needs to occupy two consecutive time slots. In Section V, we will extend our analysis to the case where a heavy-traffic SU occupies more than two consecutive time slots, and we will show that our main results do not change. Furthermore, we will further consider the general case where (i) there are an arbitrary number of SU types, and (ii) each type of SU may access a channel for an arbitrary number of time slots.

If an SU is admitted in $n \in \mathcal{N}$, the database operator will charge the SU either $r_l(n)$ or $r_h(n)$, depending on whether it is a light- or heavy-traffic SU. SUs are price-sensitive, and their demand probabilities of requesting the spectrum after arriving are non-increasing in the prices.[1] Since we consider a single channel case, the database operator can admit at most one SU in any time slot. Once an SU's service request is rejected by the database, it will leave the system without waiting. This corresponds to the case where SUs have delay-intolerant applications such as VoIP and video conferencing.

Fig. 1 summarizes the database's operations in our model. At the beginning of each time slot $n \in \mathcal{N}$, the database operator first announces prices $r_l(n)$ and $r_h(n)$ for the light- and heavy-traffic SU types, respectively. Then SUs observe the price update and randomly arrive with the probabilities affected by the prices.[2] Finally, the database operator admits at most one SU to the channel (if the channel is available) and rejects the other SUs (if any). After the three phases, the admitted SU will transmit data over the channel during the rest of the slot.

To maximize the expected revenue, the database operator wants to jointly optimize spectrum prices and admissions over all N time slots. In this optimization problem, the database operator's decision of admitting a heavy-traffic SU will prevent admitting a light/heavy-traffic SU (if available) in the next time slot, hence the operation decisions over time are coupled. We will model the problem as a stochastic dynamic programming problem, and propose the optimal admission policies under

[1] We in fact consider two different arrival processes. The first process describes how the SUs arrive at the system, which can be any process such that there is at least one arrival at the beginning of each time slot. The second process characterizes how the arrived SUs request spectrum access from the database. Such a process depends on the prices set by the operator, as a higher price will reduce the demand from SUs.

[2] In addition to prices, the wireless channel condition will also influence each SU's request for the spectrum. Due to page limit, we put the detailed channel modeling, analysis, and simulation results into Appendix A of [23].

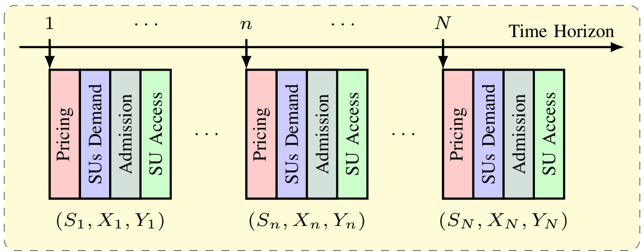

Fig. 1. Database operation in N time slots. At the beginning of each time slot, the database operator announces prices for incoming SUs. After observing the realized demands, the database operator then makes the admission decision and inform the selected SU to access. The notation (S_n, X_n, Y_n) denotes the resultant channel occupancy and two SU types' demand realizations (will be explained in Subsection III-A).

static pricing in Section III and under dynamic pricing in Section IV, respectively. In both sections, we allow dynamic admission decisions over time. Notice that static pricing is a special case of dynamic pricing, and is widely used in industry due to its simplicity and low complexity. Hence, we are interested in exploring the benefits that the flexible dynamic pricing may bring beyond the simplified static pricing. Then we can provide insights into which pricing and admission scheme the operator should choose and under what conditions.

III. Optimal Static Pricing and Dynamic Admission

We first consider the case of static pricing, where prices do not change over time. It will serve as a benchmark and help us quantify the performance gain by using dynamic pricing in Section IV. With static pricing, the database only needs to optimize and announce prices once at the beginning of time slot 1, and keeps the prices fixed for the rest $N-1$ time slots, i.e., $r_l(n) = r_l$ and $r_h(n) = r_h$ for each time slot $n \in \mathcal{N}$.

We will formulate the revenue maximization problem with static pricing and dynamic admission as a stochastic dynamic programming problem. In Subsections III-A to III-C, we will formulate and solve the optimal admission control problem through backward induction, given any *fixed* prices. In Subsection III-D, we will optimize the static prices, considering the admission policies developed in Subsections III-A to III-C.

A. Admission Control Formulation under Fixed Prices

Given fixed prices r_l and r_h, we now optimize the channel admission decision in each time slot. Such optimization not only considers the channel availability and SU demands in the current time slot, but also considers SU demands in future time slots. We will formulate it as a stochastic dynamic programming problem.

We first define the system state as follows.

Definition 1 (System State): The system state in time slot n is (S_n, X_n, Y_n). Here, S_n denotes the number of *remaining occupied* time slots at the beginning of time slot n. Since $S_n \in \{0, 1\}$, S_n also indicates the binary channel state, where $S_n = 0$ denotes that the channel is available for admission in time slot n, and $S_n = 1$ otherwise. The parameter $X_n = 1$ denotes that at least one light-traffic SU arrives at the beginning of the time slot (and is willing to pay for price r_l), and $X_n = 0$ otherwise. The parameter Y_n is defined similarly as X_n but for the heavy-traffic SUs. We define the SU demand probabilities in time slot n as $p_l = \Pr\{X_n = 1\}$ and $p_h = \Pr\{Y_n = 1\}$, respectively. As prices are unchanged over time, p_l and p_h are the same for all time slots.

The system state changes over time, depending on the channel admission decisions and SUs' demand realizations over time. The feasible set of admission actions in each time slot depends on the current system state. Formally, we define the state-dependent feasible admission action set as follows.

Definition 2 (Admission Action Set): The set of feasible admission actions in time slot n is a state-dependent set $\mathcal{A}_n(S_n, X_n, Y_n)$. When $S_n = 1$, i.e., the current time slot is not available for new admission as we are still serving the heavy-traffic SU from the last time slot, we have $\mathcal{A}_n(1, X_n, Y_n) = \{0\}$ for all possible (X_n, Y_n). When $S_n = 0$, the admission action set depends on which type of SUs' demands in the current time slot. If no SUs request in time slot n (i.e., $(X_n, Y_n) = (0, 0)$), the set of actions is still $\mathcal{A}_n = \{0\}$. If both light- and heavy-traffic SUs demand, i.e., $(X_n, Y_n) = (1, 1)$, then we can either serve no SU, a light-traffic SU, or a heavy-traffic SU, and thus the set of actions is $\mathcal{A}_n = \{0, 1, 2\}$. To summarize,

$$\mathcal{A}_n(0, X_n, Y_n) = \begin{cases} \{0\}, & \text{if } (X_n, Y_n) = (0, 0), \\ \{0, 1\}, & \text{if } (X_n, Y_n) = (1, 0), \\ \{0, 2\}, & \text{if } (X_n, Y_n) = (0, 1), \\ \{0, 1, 2\}, & \text{if } (X_n, Y_n) = (1, 1). \end{cases} \tag{1}$$

We further define the specific admission decision in time slot n as $a_n \in \mathcal{A}_n(S_n, X_n, Y_n)$.

Now we are ready to introduce the state dynamics. When $S_n = 1$, we will not admit any SU, hence in the next time slot $S_{n+1} = S_n - 1 = 0$, as the remaining occupied time slots decreases by one. When $S_n = 0$, the channel availability of the next time slot only depends on the action a_n. If we admit the light-traffic SU with $a_n = 1$, then the channel is available in the next time slot (as the remaining occupied time slot is 0), i.e., $S_{n+1} = a_n - 1 = 0$. If we admit the heavy-traffic SU with $a_n = 2$, it will occupy two time slots (time slots n and $n+1$). This means that at the beginning of time slot $n+1$, we will have the number of remaining occupied time slot to be 1, i.e., $S_{n+1} = a_n - 1 = 1$. At the beginning of time slot $n+2$, there is no SU occupying the channel, hence $S_{n+2} = S_{n+1} - 1 = 0$ and time slot $n+2$ is available for admission. To summarize, we derive the following state dynamics.

Lemma 1 (State Dynamics): The dynamics of the system state component S_n for each time slot $n \in \mathcal{N}$ satisfies the following equation:

$$S_{n+1} = (S_n + a_n(1 - S_n) - 1)^+, \tag{2}$$

where $(x)^+ := \max(0, x)$, and $S_n \in \{0, 1\}$ for each $n \in \mathcal{N}$.

Lemma 1 captures the change of remaining occupied time slots. The system state components (X_n, Y_n) are the realizations of SU demands in the current time slot, and do not depend on the action a_n in previous time slots. The key notations we introduced so far are listed in Table I.

TABLE I
KEY NOTATIONS

Symbols	Physical Meaning
$\mathcal{N} = \{1, \cdots, N\}$	Set of time slots
(S_n, X_n, Y_n)	System state in time slot n
$a_n(S_n, X_n, Y_n)$ and a_n	Admission action in time slot n
$\mathcal{A}_n(S_n, X_n, Y_n)$	Set of feasible admission actions in time slot n
$r(a_n)$	Immediate revenue by the admission action a_n
$R_n(S_n, X_n, Y_n, a_n)$	Total revenue from time slot n to N
$\mathbb{E}[R_n^*(S_n, X_n, Y_n)]$ and $\bar{R}_n^*(S_n)$	Optimal expected future revenue from n to N
$\pi^*(S_n, X_n, Y_n)$	Optimal admission strategy in time slot n
$\boldsymbol{\pi}^* = \{\pi^*(S_n, X_n, Y_n), n \in \mathcal{N}\}$	Optimal admission policy for all time slots
$R_n^*(r_l, r_h)$	Total revenue from n to N as a function of prices
$r_l(n), r_h(n)$	Price for light/heavy-traffic SUs in time slot n
$\bar{R}_n(r_l(n), r_h(n))$	Expected future revenue from time slot n to N
$p_l(r_l(n)), p_h(r_h(n))$	Probabilities of having at least one light- and heavy-traffic SU requesting spectrum in n
k_l, k_h	Demand elasticity of light/heavy-traffic SUs
$\mathcal{I} = \{1, \cdots, I\}$	Set of SUs' types in the multiple types case
$X_n^{(i)}, \forall i \in \mathcal{I}$	Demand of type-i SUs in time slot n

We are now ready to introduce the revenue maximization problem. We define a policy $\boldsymbol{\pi} = \{a_n(S_n, X_n, Y_n), \forall n \in \mathcal{N}\}$ as the set of decision rules for all possible states and time slots, and we let $\boldsymbol{\Pi} = \{\mathcal{A}_n(S_n, X_n, Y_n), \forall n \in \mathcal{N}\}$ be the feasible set of $\boldsymbol{\pi}$. Given *all possible* system state vectors $\boldsymbol{S} = \{S_n, \forall n \in \mathcal{N}\}$, $\boldsymbol{X} = \{X_n, \forall n \in \mathcal{N}\}$, and $\boldsymbol{Y} = \{Y_n, \forall n \in \mathcal{N}\}$, the database operator aims to find an optimal policy $\boldsymbol{\pi}^*$ (from the set of all admission policies $\boldsymbol{\Pi}$) that maximizes the expected total revenue from time slot 1 to N. Formally, we define Problem **P1** as follows.

P1: *Revenue Maximization by Dynamic Admission*

$$\text{maximize} \quad \mathbb{E}^{\boldsymbol{\pi}}_{\boldsymbol{X},\boldsymbol{Y}}[R(\boldsymbol{S}, \boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\pi})] \tag{3}$$

$$\text{subject to} \quad a_n(S_n, X_n, Y_n) \in \mathcal{A}_n(S_n, X_n, Y_n), \quad \forall n \in \mathcal{N}, \tag{4}$$

$$S_{n+1} = (S_n + a_n(1 - S_n) - 1)^+, \quad \forall n \in \mathcal{N} \setminus \{N\}, \tag{5}$$

$$\text{variables} \quad \boldsymbol{\pi} = \{a_n(S_n, X_n, Y_n), \forall n \in \mathcal{N}\}, \tag{6}$$

where the expectation in the objective function is taken over SUs' random requests $(\boldsymbol{X}, \boldsymbol{Y})$.

We proceed to analyze Problem **P1** by using backward induction [19]. After SUs' demands X_n and Y_n are realized in time slot n, the operator makes the admission action a_n to maximize the total revenue by considering future SU demands. We define the total revenue from time slot n to N as $R_n(S_n, X_n, Y_n, a_n)$. The total revenue computed in time slot n has two parts: i) the *immediate revenue* $r(a_n)$ for the current admission action a_n, where $r(a_n) = 0$, r_l, or r_h if $a_n = 0$, 1, or 2, respectively; and ii) the *expected future revenue* from time slot $n+1$ to N, i.e., $\mathbb{E}[R_{n+1}(S_{n+1}, X_{n+1}, Y_{n+1})]$, where the expectation is taken over the SUs' possible demands in the next time slot $n+1$, i.e., (X_{n+1}, Y_{n+1}). Then the optimization problem of time slot n in the backward induction process is

$$R_n^*(S_n, X_n, Y_n) = \max_{a_n \in \mathcal{A}_n} R_n(S_n, X_n, Y_n, a_n), \tag{7}$$

where the revenue's dynamic recursion is

$$R_n(S_n, X_n, Y_n, a_n) = r(a_n) + \mathbb{E}[R_{n+1}^*(S_{n+1}, X_{n+1}, Y_{n+1})]. \tag{8}$$

As a boundary condition in the last time slot N, we have $R_N^*(S_N, X_N, Y_N, a_N) = r(a_N)$, as there is no future spectrum opportunity and revenue collection after time slot N.

The maximum expected revenue from time slot n to N is denoted by $\mathbb{E}_{X_n,Y_n}[R_n^*(S_n, X_n, Y_n)]$, which is a part of the revenue and will be utilized for admission decision-making in previous time slots. Since the expectation $\mathbb{E}_{X_n,Y_n}[R_n^*(S_n, X_n, Y_n)]$ is taken over all possible SU demand combinations (X_n, Y_n), we rewrite it as $\bar{R}_n^*(S_n), \forall n \in \mathcal{N}$ for simplicity. We derive the expected total revenue $R_n(S_n, X_n, Y_n, a_n)$ by adding the immediate revenue as a result of action a_n and the corresponding expected future revenue $\bar{R}_{n+1}^*(S_{n+1})$ (if $a_n = 0$ or 1, i.e., no admission or admitting a light-traffic SU) or $\bar{R}_{n+2}^*(S_{n+2})$ (if $a_n = 2$, i.e., admitting a heavy-traffic SU), considering all possible SU demands (X_n, Y_n) in time slot n:

$$\begin{aligned} R_n(S_n, X_n, Y_n, a_n) = {} & (1 - p_l)(1 - p_h)[0 + \bar{R}_{n+1}^*(S_{n+1})] \\ & + p_l(1 - p_h)[r_l + \bar{R}_{n+1}^*(S_{n+1})] \\ & + (1 - p_l)p_h[(0 + \bar{R}_{n+1}^*(S_{n+1})) \cdot \mathbf{1}_{\{a_n=0\}} \\ & + (r_h + \bar{R}_{n+2}^*(S_{n+2})) \cdot \mathbf{1}_{\{a_n=2\}}] \\ & + p_l p_h[(r_l + \bar{R}_{n+1}^*(S_{n+1})) \cdot \mathbf{1}_{\{a_n=1\}} \\ & + (r_h + \bar{R}_{n+2}^*(S_{n+2})) \cdot \mathbf{1}_{\{a_n=2\}}], \end{aligned} \tag{9}$$

which can be computed according to (7) and (8) recursively and backwardly from time slot N to n. Later, we will calculate $R_n(S_n, X_n, Y_n, a_n)$ by setting the specific values of a_n in the *last two terms* of (9) according to the different admission strategies for time slot n.

Next, we will solve Problem **P1** using (7)-(9).

B. Optimal Dynamic Admission Control

By using backward induction [19], we start with the final time slot N and derive the optimal decisions slot by slot back. In time slot n, the admission decision is made by comparing the corresponding total revenues $R_n(S_n, X_n, Y_n, a_n)$ for different admission a_n in time slot n.

Based on the above discussions, we propose the *optimal* dynamic admission control policy in Algorithm 1. More specifically, this control policy $\boldsymbol{\pi}^*$ is developed by solving Problem **P1** using standard backward induction mentioned earlier. In the following Cases I-III, we formally compare the immediate revenue plus the expected future revenue to make admission decisions (i.e., $r_h + \bar{R}_{n+2}^*(S_{n+2})$, $r_l + \bar{R}_{n+1}^*(S_{n+1})$, and $0 + \bar{R}_{n+1}^*(S_{n+1})$):

- In Case I (lines 5-6) of Algorithm 1, it is more beneficial for the operator to admit a heavy-traffic SU (if it exists) than a light-traffic SU.
- In Case II (lines 7-8) of Algorithm 1, it is more beneficial for the operator to admit a light-traffic SU (if it exists) than a heavy-traffic SU.
- In Case III (lines 9-10) of Algorithm 1, it is more beneficial for the operator to only admit a light-traffic SU (if it exists).

Algorithm 1: Optimal Admission Control Policy

Set $n = N, \bar{R}^*_{N+1} = 0$
The optimal admission for N is $a^*_N = X_N$ and $\bar{R}^*_N = p_l r_l$
for $n = N-1, \cdots, 2, 1$ **do**
  Calculate $\bar{R}^*_{n+1}(S_{n+1})$ using (9).
  if $r_h + \bar{R}^*_{n+2}(S_{n+2}) \geq r_l + \bar{R}^*_{n+1}(S_{n+1})$ **then**
    if $Y_n = 1$, then $a_n = 2$; if $Y_n = 0$, $X_n = 1$, then $a_n = 1$; otherwise $a_n = 0$.
  else if $\bar{R}^*_{n+1}(S_{n+1}) < r_h + \bar{R}^*_{n+2}(S_{n+2}) < r_l + \bar{R}^*_{n+1}(S_{n+1})$ **then**
    if $X_n = 1$, then $a_n = 1$; if $X_n = 0, Y_n = 1$, then $a_n = 2$; otherwise $a_n = 0$.
  else
    if $X_n = 1$, then $a_n = 1$; otherwise $a_n = 0$.
  end if
end for
return the optimal admission policy $\boldsymbol{\pi}^*$

By the principle of optimality [19], $\boldsymbol{\pi}^* = \{\pi^*(S_n, X_n, Y_n), n \in \mathcal{N}\}$ is optimal, as shown in the following proposition.

Proposition 1: Algorithm 1 solves Problem **P1** and computes the optimal admission policy $\boldsymbol{\pi}^*$.

The proof of Proposition 1 is given in Appendix B of [23]. Note that the optimal policy $\boldsymbol{\pi}^*$ is a contingency plan, which contains the optimal admission policy in each time slot $n \in \mathcal{N}$ for any system state. After deriving the optimal policy, we can implement the policy forwardly from time slots 1 to N, after observing SUs' demand realizations.

C. Stationary Admission Policies

The optimal admission control solution in Algorithm 1 does not have a closed-form characterization and the system still needs to check a huge-size table created from the algorithm after knowing the realizations of SU random demands. This motivates us to focus on a class of low complexity stationary admission policies, where the admission rules do not change over time (while the actual admission decisions might change over time). We will characterize the conditions under which these stationary admission policies are optimal.

Recall that there are three possible admission strategies in each time slot, depending on the values of $r_h + \bar{R}^*_{n+2}(S_{n+2})$, $r_l + \bar{R}^*_{n+1}(S_{n+1})$, and $0 + \bar{R}^*_{n+1}(S_{n+1})$. For a particular time slot n, for example, if $r_h + \bar{R}^*_{n+2}(S_{n+2}) > r_l + \bar{R}^*_{n+1}(S_{n+1}) > 0 + \bar{R}^*_{n+1}(S_{n+1})$, we prefer to serve the heavy-traffic SU type rather than the light-traffic one or not serving anyone (i.e., the admission priority follows $\Lambda(2) > \Lambda(1) > \Lambda(0)$). Here, we define the function $\Lambda(a_n)$ to capture the priority order of the admission action $a_n \in \{0, 1, 2\}$. Due to the fact $r_l + \bar{R}^*_{n+1}(S_{n+1}) > 0 + \bar{R}^*_{n+1}(S_{n+1})$ and serving a light-traffic SU is better than serving no one, there are a total of three reasonable admission priority orders, i.e., $\Lambda(2) > \Lambda(1) > \Lambda(0)$, $\Lambda(1) > \Lambda(2) > \Lambda(0)$, and $\Lambda(1) > \Lambda(0) > \Lambda(2)$. We discuss them one by one next.

Table II shows the three stationary policies that we will discuss. Recall that when $S_n = 1$ (i.e., channel is still occupied in the current time slot), we have $a^*_n = 0$ (not admitting any SU) for any values of X_n and Y_n. Table II only focuses on the case of $S = 0$. The three rows/sub-tables, namely, Tab.II–HP: a^{HP*}_n, Tab.II–LP: a^{LP*}_n, and Tab.II–LD: a^{LD*}_n, represent the Heavy-Priority (i.e., $\Lambda(2) > \Lambda(1) > \Lambda(0)$), Light-Priority (i.e., $\Lambda(1) > \Lambda(2) > \Lambda(0)$), and Light-Dominant (i.e., $\Lambda(1) > \Lambda(0) > \Lambda(2)$) admission policies, respectively. For each policy, we will derive the conditions of the static prices r_l and r_h, under which the policy achieves the optimality of Problem **P1**.

TABLE II
THREE STATIONARY ADMISSION POLICIES

Admission Policies	System states (S_n, X_n, Y_n)			
	(0, 0, 0)	(0, 0, 1)	(0, 1, 0)	(0, 1, 1)
Tab.II–HP: a^{HP*}_n	0	2	1	2
Tab.II–LP: a^{LP*}_n	0	2	1	1
Tab.II–LD: a^{LD*}_n	0	0	1	1

We first analyze the Heavy-Priority admission policy (in Tab.II–HP: $a^{HP*}_n, \forall n \in \mathcal{N}$). Under this policy, we will serve a heavy-traffic SU ($a_n = 2$) whenever possible ($Y_n = 1$), and only serve a light-traffic SU ($a_n = 1$) when there is only a light-traffic SU ($X_n = 1$ and $Y_n = 0$).[3] Such a stationary policy is optimal if the following two conditions hold for each and every time slot $n \in \{1, \cdots, N-1\}$,

$$r_h + \bar{R}^*_{n+2}(0) \geq 0 + \bar{R}^*_{n+1}(0), \tag{10}$$

$$r_h + \bar{R}^*_{n+2}(0) \geq r_l + \bar{R}^*_{n+1}(0). \tag{11}$$

Inequality (10) shows that serving a heavy-traffic SU who occupies two consecutive time slots leads to a higher expected total revenue than serving no SU in the current time slot. Inequality (11) shows that serving a heavy-traffic SU leads to a higher expected total revenue than serving a light-traffic SU in the current time slot. Since (11) ensures (10), we only need to consider (11).

Similarly, we can derive the condition under which the Light-Priority admission policy (in Tab.II–LP: a^{LP*}_n) is optimal, i.e., $0 + \bar{R}^*_{n+1}(0) < r_h + \bar{R}^*_{n+2}(0) < r_l + \bar{R}^*_{n+1}(0)$ for all $n \in \{1, \cdots, N-1\}$. Under this policy, we will admit a light-traffic SU whenever possible ($X_n = 1$), and admit a heavy-traffic SU otherwise ($X_n = 0$ and $Y_n = 1$). Finally, we can derive the condition under which the Light-Dominant admission policy (in Tab.II–LD: a^{LD*}_n) is optimal, i.e., $r_h + \bar{R}^*_{n+2}(0) \leq 0 + \bar{R}^*_{n+1}(0)$ for all $n \in \{1, \cdots, N-1\}$. Under this policy, we will choose to admit a light-traffic SU ($a_n = 1$) whenever possible ($X_n = 1$), and will never admit any heavy-traffic SU, as it leaves no room to accept a light-traffic SU in the next time slot.

To summarize, we have the following theorem. Recall that r_h/r_l denotes the ratio between the prices charged to the heavy-traffic and the light-traffic SUs, and p_l and p_h are the demand probabilities defined in Subsection III-A.

Theorem 1: A stationary admission policy becomes the optimal policy to solve Problem **P1** if one of the following condition is true:

- The Heavy-Priority admission policy a^{HP*}_n in Tab.II–HP for all $n \in \mathcal{N}$ is optimal if $r_h/r_l \geq 2p_l + (1-p_l)/(1-p_h)$.
- The Light-Priority admission policy a^{LP*}_n in Tab.II–LD for all $n \in \mathcal{N}$ is optimal if $p_l \leq r_h/r_l \leq 1 + p_l$.

[3] Note that the discussion is only meaningful for time slot 1 to $N-1$, as in the last time slot N we will admit a light-traffic SU whenever possible.

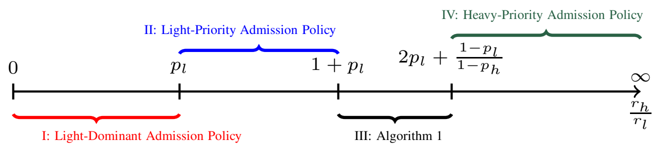

Fig. 2. Optimal stationary admission policies for all price ratio r_h/r_l values (regimes I, II, and IV).

- The Light-Dominant admission policy a_n^{LD*} in Tab.II–LP for all $n \in \mathcal{N}$ is optimal if $r_h/r_l < p_l$.

The proof of Theorem 1 is given in Appendix C of [23]. The theorem shows that each of the three stationary policies is optimal within a particular range of the price ratio r_h/r_l. Fig. 2 illustrates the results of Theorem 1 graphically. In this figure, we divide the feasible range of the price ratio r_h/r_l into four regimes, among which in three regimes (I, II, and IV) the stationary policies are optimal. We are able to further characterize the closed-form optimal total revenues for these three regimes, and the details can be found in Appendix C [23]. It is clear that a larger value of r_h/r_l gives a higher preference to the admission of a heavy-traffic SU. In regime III, we have to use Algorithm 1 to compute the optimal admission policy.

After analyzing the optimal admission control decisions from time slot N to 1 in backward induction, we now optimize the initial pricing decision at the beginning of time slot 1.

D. Optimal Static Pricing

Under static pricing, the database operator optimizes and announces the prices r_h and r_l in time slot 1, and do not change these prices for the remaining $N-1$ time slots. As explained in Section II, we consider the general case where prices will affect SU demands during each time slot. As a concrete example, we consider the widely used linear demand function in economics [20], where the probability of an SU of type $i \in \{l, h\}$ requesting the spectrum resource in a time slot is $p_i(r_i) = 1 - k_i r_i$, where $0 \le r_i \le r_i^{\max} = 1/k_i$.[4] The parameters k_l and k_h characterize the demand elasticity of the light-traffic and the heavy-traffic SUs, respectively, and larger values of k_l and k_h reflect higher price sensitivities.[5]

By using the three stationary admission policies in Theorem 1, we are able to derive three closed-form objective $R_n^*, n \in \mathcal{N}$ as a function of prices r_l and r_h. Next we optimize the prices that maximize the total revenue R_1^* in Problem **P1**.

Proposition 2: Consider the case $r_h/r_l \ge 2p_l + (1-p_l)/(1-p_h)$, in which the heavy-priority admission policy is optimal as shown in Theorem 1. The optimal static pricing (r_l^*, r_h^*) is the optimal solution to the following problem

$$\begin{aligned}
&\text{maximize} && R_1^*(r_l, r_h), && (12)\\
&\text{subject to} && r_h/r_l \ge 2p_l + (1-p_l)/(1-p_h), && (13)\\
& && 0 \le r_l \le r_l^{\max}, 0 \le r_h \le r_h^{\max}, && (14)\\
&\text{variables} && r_l, r_h, && (15)
\end{aligned}$$

where

$$\begin{aligned}
R_1^*(r_l, r_h) = {} & N\left(\frac{p_l r_l + (1-p_l)p_h r_h}{1-(p_l p_h - p_h)}\right)\\
& + \left(\frac{(p_l p_h - p_h)(r_h - p_l r_l)}{1-(p_l p_h - p_h)}\right)\frac{(p_l p_h - p_h)(1-(p_l p_h - p_h)^N)}{1-(p_l p_h - p_h)}. \quad (16)
\end{aligned}$$

The proof of Proposition 2 is given in Appendix D of [23]. The same conclusion holds for the other two cases shown in Theorem 1, and the details are provided in [23]. The function $R_1^*(r_l, r_h)$ turns out to be non-convex in general, and the optimal prices cannot be solved in closed form. However, notice that the key benefit of static pricing is that it does not need to be recomputed and updated frequently over time, thus we can compute the optimal static prices *offline* once and the high computational complexity is not a major practical issue.

IV. Optimal Dynamic Pricing and Dynamic Admission

In Section III, we have considered the static pricing and dynamic admission control problem. Now we consider the case of dynamic pricing, where the prices vary over time. In the following, we will formulate the dynamic pricing and dynamic admission control problem, aiming at deriving the optimal dynamic pricing and admission policies.

A. Dynamic Pricing-and-Admission Problem Formulation

Now we further study the general case of dynamic pricing, where the database operator has the flexibility of changing prices over time. The database operator's goal is to compute the optimal prices $\boldsymbol{r}_l^* = \{r_l^*(n), n \in \mathcal{N}\}$ and $\boldsymbol{r}_h^* = \{r_h^*(n), n \in \mathcal{N}\}$, and the optimal admission policy $\boldsymbol{\pi}^* = \{a_n^*(S_n, X_n, Y_n), n \in \mathcal{N}\}$ for all time slots and system states to maximize its expected revenue, i.e.,

P2: *Joint Dynamic Pricing and Dynamic Admission*

$$\begin{aligned}
&\text{maximize} && \mathbb{E}_{\boldsymbol{X},\boldsymbol{Y}}^{\boldsymbol{\pi}}[R(\boldsymbol{S}, \boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{\pi}, \boldsymbol{r}_l, \boldsymbol{r}_h)] && (17)\\
&\text{subject to} && a_n(S_n, X_n, Y_n) \in \mathcal{A}_n(S_n, X_n, Y_n),\\
& && \qquad \forall n \in \mathcal{N}, && (18)\\
& && S_{n+1} = (S_n + a_n(1-S_n) - 1)^+,\\
& && \qquad \forall n \in \mathcal{N} \setminus \{N\}, && (19)\\
& && 0 \le r_l(n) \le r_l^{\max}, \forall n \in \mathcal{N}, && (20)\\
& && 0 \le r_h(n) \le r_h^{\max}, \forall n \in \mathcal{N}, && (21)\\
&\text{variables} && \{\boldsymbol{\pi}, \boldsymbol{r}_l, \boldsymbol{r}_h\}. && (22)
\end{aligned}$$

We can again use backward induction to solve Problem **P2** in each time slot. Different from Section III, we need to jointly

[4] Changing to common nonlinear functions are unlikely to change the key results. This is because the optimal static pricing can be solved in Proposition 2, even for nonlinear functions, we can still search the optimal static pricing.

[5] In practice, the price elasticity parameters can be estimated according to the market survey or historical data about demand responses (e.g., [21]). By doing independent repeated trials, the operator can estimate the parameters.

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TABLE III
THREE ADMISSION STRATEGIES IN TIME SLOT n

Admission Strategies in time slot n	Conditions
Heavy-Priority Strategy (HP): $a_n = (2 - X_n)Y_n + X_n$	$r_h(n) + \bar{R}^*_{n+2} \geq r_l(n) + \bar{R}^*_{n+1}$
Light-Priority Strategy (LP): $a_n = X_n \cdot \mathbf{1}_{\{Y_n=0\}} + (2 - X_n) \cdot \mathbf{1}_{\{Y_n=1\}}$	$r_h(n) + \bar{R}^*_{n+2} > 0 + \bar{R}^*_{n+1}$ $r_h(n) + \bar{R}^*_{n+2} < r_l(n) + \bar{R}^*_{n+1}$
Light-Dominant Strategy (LD): $a_n = X_n$	$r_h(n) + \bar{R}^*_{n+2} \leq 0 + \bar{R}^*_{n+1}$

determine the prices and the admission decisions from time slot N to 1. The subproblem in each time slot $n \in \mathcal{N}$ is

P3: *Pricing-and-Admission Subproblem in time slot* n

$$\text{maximize} \quad R_n\big(r_l(n), r_h(n), a_n(S_n, X_n, Y_n)\big) \tag{23}$$
$$\text{subject to} \quad a_n(S_n, X_n, Y_n) \in \mathcal{A}_n(S_n, X_n, Y_n), \tag{24}$$
$$0 \leq r_l(n) \leq r_l^{\max}, \tag{25}$$
$$0 \leq r_h(n) \leq r_h^{\max}, \tag{26}$$
$$\text{variables} \quad \{a_n(S_n, X_n, Y_n), r_l(n), r_h(n)\}. \tag{27}$$

The expression of R_n can be similarly derived by using the derivation procedure of (9), except that the demand probabilities p_l and p_h are functions the prices $r_l(n)$ and $r_h(n)$, i.e., $p_l(r_l(n))$ and $p_h(r_h(n))$, respectively. The key challenge of solving Problem **P3** is the coupling between the pricing and admission decisions in each time slot. Next we will propose a decomposition approach for the two decisions that helps us solve Problem **P3** in each time slot n.

B. Decomposition of Pricing and Admission in Each Time Slot

First we want to clarify the difference between an *admission strategy* and an *admission policy*. An admission strategy specifies the admission actions for a particular time slot n, while an admission policy applies to all time slots in $\mathcal{N}$ (e.g., those in Table II). Here we will focus on the admission strategy, as we only study Problem **P3** for a particular time slot n.

Next we consider all possible admission control strategies for a time slot n, as shown in Table III. In this table, HP stands for heavy-priority strategy, LP stands for light-priority strategy, and LD stands for light-dominant strategy. Each strategy is accompanied by a condition of the total revenue from time slot n to N. The strategy is optimal for time slot n if the corresponding condition holds.

Let us take the heavy-priority strategy (HP) as an example to explain our decomposition approach. In this strategy, we will serve a heavy-traffic SU ($a_n = 2$) whenever possible ($Y_n = 1$), and only serve a light-traffic SU ($a_n = 1$) if there is no heavy-traffic SU ($X_n = 1$ and $Y_n = 0$). Summarizing these cases together, the decision under the heavy-priority strategy can be written as $a_n = (2 - X_n)Y_n + X_n$. The corresponding condition for the heavy-priority strategy in Table III shows that the total revenue of admitting a heavy-traffic SU is no less than that of admitting a light-traffic SU. The conditions for the other two admission strategies (LP and LD) can be derived similarly.

Using the result in Table III, we can solve Problem **P3** in the following two steps:

- *Price optimization under a chosen admission strategy:* Assume that one of the three admission strategies in Table III will be used in time slot n, we optimize prices $r_l(n)$ and $r_h(n)$ to maximize the expected total revenue.
- *Admission strategy optimization:* Compare the maximized expected total revenues (from slot n to N) under the three admission strategies with the optimized prices, and pick the best admission strategy and pricing combination that leads to the largest revenue.

Notice that the above decomposition method is for each time slot $n \in \mathcal{N}$. The above decomposition procedure guarantees that we obtain the optimal solution of the joint problem **P3** for the following reason. First, the three possible admission strategies in each time slot are exhaustive and mutually exclusive, in the sense that the optimal pricing decision in time slot n guarantees that there is only one strategy that is optimal to adopt in this time slot, depending on the conditions in Table III. Second, if one-out-of-the-three admission strategies is optimal to adopt in time slot n, there must exist an associated optimal pricing accordingly that maximizes the total revenue. We thus conclude that the two-step decomposition procedure is guaranteed to solve Problem **P3** optimally.

Next, we will derive the closed-form optimal pricing under each of the three admission strategies, respectively. We will conduct admission strategy optimization in Subsection IV-C.

1) Optimal Pricing under Heavy-Priority Strategy: Given HP strategy chosen in time slot n, we derive the *expected total revenue* $R_n^{HP}\big(r_l^{HP}(n), r_h^{HP}(n)\big)$ by setting $a_n = 2$ and $a_n = 2$ in the last two terms of (9), respectively, where the probabilities $p_l(r_l^{HP}(n)$ and $p_h(r_h^{HP}(n))$ can also be modeled as the linear function in Subsection III-D. Notice that the database operator may not always use HP in future time slots.

In order to optimize the prices, the database operator needs to solve the following problem.

P4: *Optimal Pricing for time slot* n *under HP*

$$\text{maximize} \quad R_n^{HP}\big(r_l^{HP}(n), r_h^{HP}(n)\big) \tag{28}$$
$$\text{subject to} \quad r_h^{HP}(n) + \bar{R}^*_{n+2} \geq r_l^{HP}(n) + \bar{R}^*_{n+1}, \tag{29}$$
$$0 \leq r_l^{HP}(n) \leq r_l^{\max}, \tag{30}$$
$$0 \leq r_h^{HP}(n) \leq r_h^{\max}, \tag{31}$$
$$\text{variables} \quad r_l^{HP}(n), r_h^{HP}(n). \tag{32}$$

Constraint (29) guarantees that the heavy-priority admission strategy is optimal in time slot n, where $\bar{R}^*_{n+2}$ and $\bar{R}^*_{n+1}$ are determined by the optimal solutions to Problem **P3** in time slots $n+2$ and $n+1$. Since the optimization problem **P4** is a continuous function over a compact feasible set, the maximum is guaranteed to be attainable. It is easy to show that Problem **P4** is not a convex optimization problem due to the three-order polynomial objective function. Thus, a solution satisfying KKT conditions may be either a local optimum or a global optimum. Hence, we need to find all solutions satisfying KKT conditions, and then compare these solutions to find the global optimum.

We will first examine the feasible region of Problem **P4** based on any possible prices $r_l^{HP}(n)$ and $r_h^{HP}(n)$. It turns out that the feasible region is a polyhedron in a two-dimensional

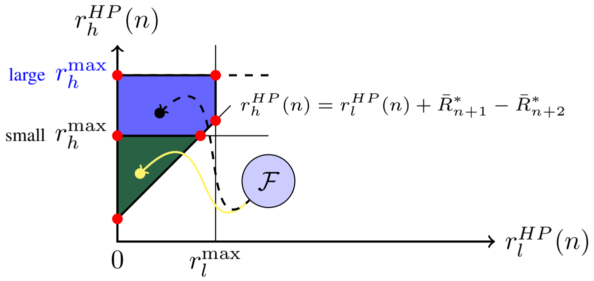

Fig. 3. The feasible region $\mathcal{F}$ of Problem **P4**. There are two possible shapes according to the values of $r_h^{\max}$. The interior points, the red dots, and the line segments between them are all possible solutions.

plane. Fig. 3 shows the feasible region. According to the value of $r_h^{\max}$, the feasible region has two possible cases. The optimal solution can only be either the interior points inside the feasible region or the extreme points on the boundary. As such, we only need to check whether all the possible extreme points and the interior points satisfying KKT conditions are local optima. We skip the details (which can be found in [23]) due to space limit, and summarize the optimal pricing results in the following proposition.

Proposition 3: The optimal pricing in time slot n under the HP strategy is summarized in Table IV, which depends on the values of $\bar{R}^*_{n+1} - \bar{R}^*_{n+2}$ and k_h/k_l. The closed-form optimal pricing solutions in Table IV are given as follows, respectively,

$$I_0^{HP}: \begin{cases} r_l^{HP}(n) = \frac{1}{2k_l} \\ r_h^{HP}(n) = \frac{\left(\frac{1}{4k_l}+\frac{1}{k_h}+\bar{R}^*_{n+1}-\bar{R}^*_{n+2}\right)}{2} \end{cases},$$

$$E_2^{HP}: \begin{cases} r_l^{HP}(n) = \frac{1}{k_h} + \bar{R}^*_{n+2} - \bar{R}^*_{n+1} \\ r_h^{HP}(n) = \frac{1}{k_h} \end{cases}, \text{ and}$$

$$E_1^{HP}: \begin{cases} r_l^{HP}(n) = \frac{-(\bar{R}^*_{n+1}-\bar{R}^*_{n+2})+\sqrt{(\bar{R}^*_{n+1}-\bar{R}^*_{n+2})^2+\frac{3}{k_lk_h}}}{3} \\ r_h^{HP}(n) = \frac{2(\bar{R}^*_{n+1}-\bar{R}^*_{n+2})+\sqrt{(\bar{R}^*_{n+1}-\bar{R}^*_{n+2})^2+\frac{3}{k_lk_h}}}{3} \end{cases}.$$

The proof of Proposition 3 is given in Appendix E of [23]. In Table IV, I_0^{HP}, E_1^{HP}, and E_2^{HP} represent the unique optimal solution in different cases (i.e., one interior point solution and two extreme point solutions). "N/A" represents the cases where the combinations of conditions are infeasible. For example, when $4/3 \le k_h/k_l < 3$, it follows that $\bar{R}^*_{n+1} - \bar{R}^*_{n+2} > (4k_l - 3k_h)/(4k_hk_l)$; when $k_h/k_h \ge 3$, we have $\bar{R}^*_{n+1} - \bar{R}^*_{n+2} \ge (2 - \sqrt{1 + k_h/k_l})/k_h$. Hence, the corresponding cell is labeled as "N/A".

Tables IV shows the optimal dynamic pricing in each time slot n under the HP strategy. Given the demand elasticities k_l and k_h, the solution will be uniquely given by one of the three cases of $\bar{R}^*_{n+1} - \bar{R}^*_{n+2}$ regimes. In Subsection IV-C, we will propose an algorithm to compute $\bar{R}^*_{n+1} - \bar{R}^*_{n+2}$ iteratively for all time slots.

2) Optimal Pricing under Light-Priority Strategy: Given LP strategy chosen in time slot n, we derive the expected total revenue $R_n^{LP}\big(r_l^{LP}(n), r_h^{LP}(n)\big)$ by setting $a_n = 2$ and

TABLE IV
OPTIMAL PRICING UNDER HEAVY-PRIORITY STRATEGY

	$\bar{R}^*_{n+1} - \bar{R}^*_{n+2}$		
	$\le \frac{4k_l-3k_h}{4k_hk_l}$	$\left(\frac{4k_l-3k_h}{4k_hk_l}, \frac{2-\sqrt{1+\frac{k_h}{k_l}}}{k_h}\right)$	$\ge \frac{2-\sqrt{1+\frac{k_h}{k_l}}}{k_h}$
$\frac{k_h}{k_l} < \frac{4}{3}$	I_0^{HP}	E_1^{HP}	E_2^{HP}
$\frac{4}{3} \le \frac{k_h}{k_l} < 3$	N/A	E_1^{HP}	E_2^{HP}
$\frac{k_h}{k_l} \ge 3$	N/A	N/A	E_2^{HP}

$a_n = 1$ in the last two terms of (9), respectively. The database operator needs to solve the following problem.

P5: *Optimal Pricing for time slot n under LP*

$$\begin{aligned} \text{maximize} \quad & R_n^{LP}\big(r_l^{LP}(n), r_h^{LP}(n)\big) & (33)\\ \text{subject to} \quad & r_h^{LP}(n) + \bar{R}^*_{n+2} \le r_l^{LP}(n) + \bar{R}^*_{n+1}, & (34)\\ & r_h^{LP}(n) + \bar{R}^*_{n+2} \ge \bar{R}^*_{n+1}, & (35)\\ & 0 \le r_l^{LP}(n) \le r_l^{\max}, & (36)\\ & 0 \le r_h^{LP}(n) \le r_h^{\max}, & (37)\\ \text{variables} \quad & r_l^{LP}(n), r_h^{LP}(n). & (38) \end{aligned}$$

Constraints (34) and (35) guarantee that the light-priority strategy is optimal in time slot n.

The analysis for Problem **P5** is similar to that for Problem **P4**, due to the similar structures of the two problems. We thus have Proposition 4 as follows.

Proposition 4: The optimal solution to Problem **P5** can also be summarized in a table as in Table IV, only with different conditions in the rows and the columns and expressions of I_0^{LP}, E_1^{LP} and E_2^{LP}.

Due to space limitation, the proof of Proposition 4 and the detailed solutions can be found in Appendix F of [23].

3) Optimal Pricing under Light-Dominant Strategy: Given LD strategy chosen in time slot n, we derive the expected total revenue $R_n^{LD}\big(r_l^{LD}(n), r_h^{LD}(n)\big)$ by setting $a_n = 0$ and $a_n = 1$ in the last two terms of (9), respectively. The database operator needs to solve the following problem.

P6: *Optimal Pricing for time slot n under LD*

$$\begin{aligned} \text{maximize} \quad & R_n^{LD}\big(r_l^{LD}(n), r_h^{LD}(n)\big) & (39)\\ \text{subject to} \quad & r_h^{LD}(n) + \bar{R}^*_{n+2} \le 0 + \bar{R}^*_{n+1}, & (40)\\ & 0 \le r_l^{LD}(n) \le r_l^{\max}, & (41)\\ & 0 \le r_h^{LD}(n) \le r_h^{\max}, & (42)\\ \text{variables} \quad & r_l^{LD}(n), r_h^{LD}(n). & (43) \end{aligned}$$

Unlike the HP and the LP cases, we can derive the optimal prices under LD in closed-form.

Proposition 5: The optimal prices in time slot n under the LD strategy are given by the interior point solution I_0^{LD} :

$$r_l^{LD}(n) = \frac{1}{2k_l}, r_h^{LD}(n) = \min(\bar{R}^*_{n+1} - \bar{R}^*_{n+2}, r_h^{\max}). \quad (44)$$

The proof of Proposition 5 is given in Appendix G of [23]. We have analyzed the price optimization under any chosen admission strategy. Next, we will compare the expected total revenues R_n^{HP*}, R_n^{LP*}, and R_n^{LD*} to pick the optimal pricing-admission strategy.

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Algorithm 2: Optimal Dynamic Pricing and Admission Policy

1: Set $n = N+1$, $\bar{R}^*_{N+1} = 0$
2: Set $r^*_l(N), r^*_h(N)$ by (44) and $\bar{R}^*_N$ by $\bar{R}^{LD}_N(r^*_l(N), r^*_h(N))$.
3: **for** $n = N-1, \cdots, 2, 1$ **do**
4:     Derive $r^{HP*}_l(n), r^{HP*}_h(n)$, R^{HP*}_n by Table IV.
5:     Derive $r^{LP*}_l(n), r^{LP*}_h(n)$, R^{LP*}_n by Prop. 4.
6:     Derive $r^{LD*}_l(n), r^{LD*}_h(n)$ and R^{LD*}_n by (44).
7:     $\bar{R}^*_n \leftarrow \max\{R^{HP*}_n, R^{LP*}_n, R^{LD*}_n\}$ and $r^*_l(n), r^*_h(n) \leftarrow \arg\max\{R^{HP*}_n, R^{LP*}_n, R^{LD*}_n\}$.
8:     **if** $r^*_l(n), r^*_h(n) = r^{HP*}_l(n), r^{HP*}_h(n)$ **then**
9:         The heavy-priority strategy is optimal.
10:     **else if** $r^*_l(n), r^*_h(n) = r^{LP*}_l(n), r^{LP*}_h(n)$ **then**
11:         The light-priority strategy is optimal.
12:     **else**
13:         The light-dominant strategy is optimal.
14:     **end if**
15: **end for**
16: **return** Pricing-Admission policy $\boldsymbol{r}^*$ and $\boldsymbol{\pi}^*$.

C. Optimal Dynamic Pricing and Admission Policies

After deriving the optimal prices under each admission strategy, we can now compare the corresponding revenues and choose the best admission strategy for time slot n. We need to do this for each of the N time slots. We show this process in Algorithm 2, which involves the previous solutions (Table IV, Proposition 4, and Equation (44)). More specifically, the algorithm iteratively computes the prices and revenues under the three admission strategies, respectively, and then selects the optimal prices and the corresponding admission strategy which lead to the largest revenue (lines 3 to 15). The complexity of Algorithm 2 is low and in the order of the total time slots $\mathcal{O}(N)$, as it only needs to check the tables and Equation (44) we derived. We summarize the optimality result as follows.

Theorem 2: The dynamic prices $\boldsymbol{r}^* = \{\boldsymbol{r}^*(n), \forall n \in \mathcal{N}\}$ and the dynamic admission policy $\boldsymbol{\pi}^* = \{a^*_n(S_n, X_n, Y_n), \forall n \in \mathcal{N}\}$ derived in Algorithm 2 are the unique optimal solution to Problem **P2**.

The proof of Theorem 2 is given in Appendix H of [23]. Note that the optimal prices and admission policy form a *contingency* plan that contains information about the optimal prices and admission decisions at all the possible system states (S_n, X_n, Y_n) in any time slots $n \in \mathcal{N}$. To implement the optimal policy from time slot 1 to N, the database operator needs to decide the actual admission actions according to the realizations of random demands and the transition of system states. More specifically, at the beginning of each time slot n, the operator first announces prices $\boldsymbol{r}^*(n)$ according to $\boldsymbol{r}^*$ and checks the actual demands (X_n, Y_n). Then, the admission decisions are determined by checking the optimal policy $\boldsymbol{\pi}^*$ and the state component S_n is updated accordingly.

V. Extensions

The analysis of the simplified case in Sections II to IV paves the way for the analysis of the general case of multiple types of SUs. Next, we will first consider the case of arbitrary spectrum occupancies of two SU types, and then the general case of more than two SU types.

A. Extension to Arbitrary Spectrum Occupancies of Two SU Types

In Sections II to IV, we have assumed that a heavy-traffic SU occupies 2 consecutive time slots. Now we proceed to consider the general case where a heavy-traffic SU occupies M consecutive time slots. The channel occupancy of a light-traffic SU is still normalized to a unit time slot. Naturally, we have $2 \le M \le N$. Following similar notations as in Section II, in order to characterize the spectrum occupancy information over time, we define S_n as the number of *remaining occupied* time slots *before* making the admission action a_n in time slot n, where $S_n \in \{0, 1, \cdots, M-1\}$. At the beginning of time slot n, we first check the SU occupancy of the current time slot, i.e.,

$$S_n = \begin{cases} 1, \cdots, M-1, & \text{if time slot } n \text{ is occupied,} \\ 0, & \text{if time slot } n \text{ is idle.} \end{cases} \tag{45}$$

For example, if $M = 3$ and we start admitting a heavy-traffic SU in time slot n, then $S_{n+1} = 2, S_{n+2} = 1$, and $S_{n+2} = 0$. If we define the possible admission action as $a_n = 0$ (admitting no SU), $a_n = 1$ (admitting a light-traffic SU), and $a_n = M$ (admitting a heavy-traffic SU), then the dynamics of the system state in (2) still holds here, i.e., $S_{n+1} = (S_n + a_n(1-S_n)-1)^+, \forall n \in \{1, \cdots, N-1\}$, and we define the whole system state in time slot n as (S_n, X_n, Y_n) similarly as in Section III. The problem formulation turns out to be the same as Problem **P1**. As a result, the optimal admission policy can also be computed similarly as Algorithm 1.

1) Stationary Admission Policy under Static Pricing: When we analyze the static pricing for this general case, a new challenge is to understand that under which combination of system parameters the stationary admission policies are optimal, which is different from those in Subsection III-C. Next we take the "Heavy-Priority Admission Policy" as an example, and derive the condition of the parameters p_l, p_h, r_l, and r_h, under which the stationary admission policy is optimal under static pricing.

Proposition 6: The optimal policy for solving the revenue maximization Problem **P1** degenerates to the heavy-priority stationary admission policy when price ratio between the heavy-traffic SU and the light-traffic SU is larger than a threshold $\theta^{HP}_{th}(p_l, p_h)$, i.e.,

$$r_h / r_l > \theta^{HP}_{th}(p_l, p_h), \tag{46}$$

where the threshold ratio $\theta^{HP}_{th}(p_l, p_h)$ can be determined by solving the following:

$$r_h + \bar{R}^*_{n+M} = r_l + \bar{R}^*_{n+1}, \forall n \in \{1, 2, \cdots, N-M+1\}. \tag{47}$$

The proof of Proposition 6 is given in Appendix I of [23]. We give the proof sketch as follows. First, we derive the expected revenue $\bar{R}^*_n$ as a function of r_l, r_h, p_l, p_h, given the heavy-priority stationary admission policy. Second, we determine r_h/r_l in terms of p_l, p_h, and n, by plugging $\bar{R}^*_{n+1}$ and $\bar{R}^*_{n+M}$ into the condition (47), i.e., $r_h/r_l = f(p_l, p_h, n)$. Third, we denote $f(p_l, p_h, n)$ as $\theta^{HP}_{th}(p_l, p_h, n)$, and derive the final threshold $\theta^{HP}_{th}(p_l, p_h)$ by optimizing $\theta^{HP}_{th}(p_l, p_h, n)$ over $n \in \{1, 2, \cdots, N-M+1\}$. It thus follows that the heavy-priority stationary admission policy is optimal to solve the operator's revenue maximization problem if (46) holds. Proposition 6 shows that our analysis in Section III also applies to the general case. We can also derive the threshold condition for the light-priority admission policy by considering $\bar{R}^*_{n+1} \leq r_h + \bar{R}^*_{n+M} \leq r_l + \bar{R}^*_{n+1}$, and the light-dominant admission policy by considering $r_h + \bar{R}^*_{n+M} < \bar{R}^*_{n+1}$ similarly. The related analysis are similar to Theorem 1. We skip the detailed analysis due to space constraints.

2) Dynamic Pricing and Performance Evaluation: The analysis under dynamic pricing is also similar to that in Section IV, where we decompose the problem into three subproblems in each time slot. We show the main result in the following proposition, by focusing on the heavy-priority strategy for the illustration purpose.

Proposition 7: Given an arbitrary value of spectrum occupancy M, the optimal dynamic pricing under the heavy-priority strategy is the same as that in Proposition 3 and Table IV, once we replace $\bar{R}^*_{n+1} - \bar{R}^*_{n+2}$ by $\bar{R}^*_{n+1} - \bar{R}^*_{n+M}$.

The proof of the proposition is given in Appendix J of [23]. Proposition 7 shows that previous analysis for dynamic pricing can be directly extended to the arbitrary occupancy case.

B. Extension to Multiple Types of SUs

In this subsection, we further extend the analysis in Sections III to IV and Subsection V-A to the case with a total of I types of SUs seeking for spectrum access, including one type of light-traffic SUs and $I-1$ types of heavy-traffic SUs who occupy $2, 3, \cdots, I$ consecutive time slots, respectively.

1) Problem Formulation: We use $\mathcal{I} = \{1, 2, \cdots, I\}$ to denote the set of SU types. To analyze the stationary admission policy, we need to compare a total of $I+1$ admission choices (including no admission) as in the analysis in Section III and Subsection V-A. The difference is that there are two revenue constraints for each policy in Section III and Subsection V (e.g., (10) and (11)), while there are $I+1$ revenue constraints here. We continue the procedure and derive the associated thresholds, then determine the stationary admission policy by comparing the price relations with those thresholds.

More specifically, we define the prices charged to all types of SUs as $\mathcal{R} = \{r_i, \forall i \in \mathcal{I}\}$, where r_i is the price charged to a type-i SU for using the spectrum resource. Let the demand probabilities of all types of SUs be $\mathcal{P} = \{p_i, \forall i \in \mathcal{I}\}$, and the realizations of all types of SUs' demands in time slot n be $X_n^{(i)}, \forall i \in \mathcal{I}, n \in \mathcal{N}$. Given $r_i \in \mathcal{R}$ and $p_i \in \mathcal{P}$, the expected total revenue in time slot n, i.e., $R_n(S_n, X_n^{(1)}, \cdots, X_n^{(I)}, a_n)$, is the summation of the immediate revenue (as a result of the immediate action a_n) and the expected future revenue $\bar{R}^*_{n+1}(S_{n+1})$ (if $a_n = 0$ with no admission) or $\bar{R}^*_{n+i}(S_{n+i})$ (if $a_n = i$, admitting a type-i SU), considering all possible SU demands $(X_n^{(1)}, \cdots, X_n^{(I)})$ in time slot n. The detailed expression is similar to (9), and can be found in [23].

At the beginning of time slot n, we determine the optimal admission decision by comparing the total revenue of admitting a particular type of SU, which involves both the immediate revenue r_i and the maximum expected future revenue $\bar{R}^*_{n+i}(S_{n+i})$. Given SUs' demands in time slot n, if the optimal decision is no admission ($a_n = 0$) due to a more profitable type of SU in the next time slot, the total revenue in time slot n is $0 + \bar{R}^*_{n+1}(S_{n+1})$. To summarize, the optimal decision in time slot n is

$$\begin{aligned} a_n^* = \arg\max_{a_n \in \mathcal{I} \cup \{0\}} & \{0 + \bar{R}^*_{n+1}(S_{n+1}), \\ & (r_i + \bar{R}^*_{n+i}(S_{n+i})) \cdot \mathbf{1}_{\{X_n^{(i)}=1\}}, \forall i \in \mathcal{I}\}. \end{aligned} \quad (48)$$

The above argument reveals a backward induction algorithm of determining the optimal admission decision in each time slot, which is similar to Algorithm 1. We are interested in the optimality of the stationary admission policies as discussed in Subsection III-C.

2) Stationary Admission Policies under Static Pricing: We first consider a type-i and a type-j SU ($i > j > 1$) who seek to occupy arbitrarily consecutive time slots i and j, respectively. In this case, the priority of admitting a particular type of SUs depends on the values of $r_i + \bar{R}^*_{n+i}$, $r_j + \bar{R}^*_{n+j}$, and $0 + \bar{R}^*_{n+1}$. For a particular time slot n, for example, if $r_i + \bar{R}^*_{n+i} > r_j + \bar{R}^*_{n+j} > 0 + \bar{R}^*_{n+1}$, we prefer to serve the type-i SU type rather than the type-j SU (i.e., the admission priority follows $\Lambda(i) > \Lambda(j) > \Lambda(0)$). By specifying the values of a_n according to this admission priority in $R_n(S_n, X_n^{(1)}, \cdots, X_n^{(I)}, a_n)$, we determine the differences $\bar{R}^*_{n+j} - \bar{R}^*_{n+i}$ and $\bar{R}^*_{n+1} - \bar{R}^*_{n+j}$ similarly as Theorem 1 and Proposition 6. The threshold that guarantees the condition $r_i + \bar{R}^*_{n+i} > r_j + \bar{R}^*_{n+j} > 0 + \bar{R}^*_{n+1}$ can be derived by solving this condition. Further, by optimizing the derived threshold over all time slots $n \in \mathcal{N}$, we derive the final threshold that guarantees the optimality of the admission priority $\Lambda(i) > \Lambda(j) > \Lambda(0)$ for all time slots. Hence, this admission priority becomes one of the stationary admission policies. Similarly, the thresholds for the other five admission priorities can be determined by solving the corresponding revenue conditions.

The above discussions can be generalized to the case of multiple types of SUs as follows. For a particular time slot n, for example, if the revenue conditions satisfy $r_I + \bar{R}^*_{n+I} > r_{I-1} + \bar{R}^*_{n+I-1} > \cdots > r_1 + \bar{R}^*_{n+1} > 0 + \bar{R}^*_{n+1}$, the admission priority follows $\Lambda(I) > \Lambda(I-1) > \cdots > \Lambda(1) > \Lambda(0)$. By specifying the values of a_n according to this admission priority in $R_n(S_n, X_n^{(1)}, \cdots, X_n^{(I)}, a_n)$, we determine the difference $\bar{R}^*_{n+j} - \bar{R}^*_{n+I}, \forall j \in \{1, \cdots, I-1\}$ similarly as Theorem 1 and Proposition 6, respectively. We then proceed to derive the thresholds such that the revenue conditions hold for all time slots. These thresholds guarantee that the admission priority $\Lambda(I) > \Lambda(I-1) > \cdots > \Lambda(1) > \Lambda(0)$ is optimal for all time slots, and hence it becomes a stationary admission policy.

Proposition 8: Given the set $\mathcal{I}$ of I types of SUs, there are $(I+1)!$ admission priorities. For each admission priority, there exist thresholds of the price ratios such that the optimal admission priority for a time slot is optimal for all time slots (corresponding to an optimal stationary admission policy).

The proof of Proposition 8 is given in Appendix K of [23]. Proposition 8 shows that the threshold-based stationary policy still holds in the general scenario, and there exist $(I+1)!$

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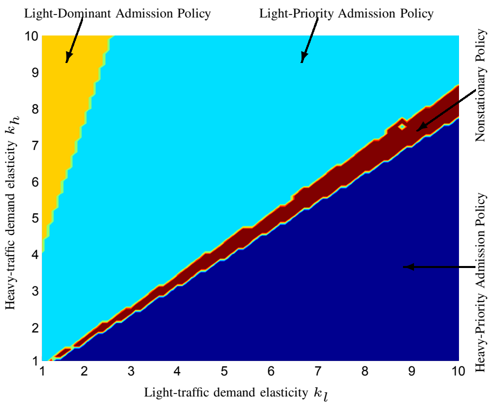

Fig. 4. Optimal choices of admission policies for different values of elasticity parameters k_l and k_h. The yellow (Light-Dominant Admission Policy), cyan (Light-Priority Admission Policy), and blue (Heavy-Priority Admission Policy) regimes represent three stationary admission policies, i.e., I, II, and IV regimes in Fig. 2, respectively. The brown (Nonstationary Policy) regime requires "Algorithm 1" to compute the optimal policy.

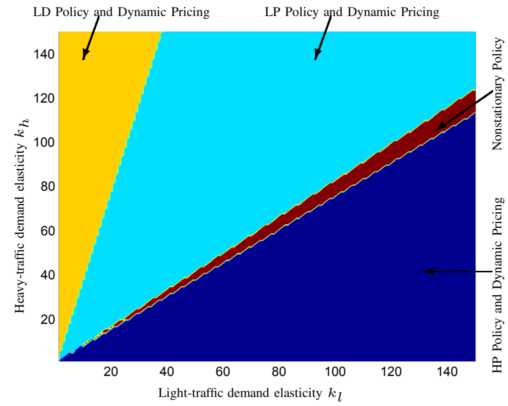

Fig. 5. Optimal admission policies under dynamic pricing over $N = 100$ time slots. The yellow (LD Policy and Dynamic Pricing), cyan (LP Policy and Dynamic Pricing), and blue (HP Policy and Dynamic Pricing) regimes are the three stationary admission policies, i.e., I, II, and IV regimes in Fig. 2, respectively. The brown (Nonstationary Policy) regime requires Algorithm 2 to compute the optimal policy.

thresholds[6] for all types of SUs $\mathcal{I}$, which are completely determined by the values of $\{0 + \bar{R}^*_{n+1}, r_i + \bar{R}^*_{n+i}, \forall i \in \mathcal{I}\}$ in each time slot. Recall that in Subsection III-C, we should have $(2+1)!$ stationary admission policies. However, due to the fact $r_1 + \bar{R}^*_{n+1} > 0 + \bar{R}^*_{n+1}$, finally we have a total of $(2+1)!/2! = 3$ stationary admission policies.

3) Optimal Dynamic Pricing and Dynamic Admission: In the dynamic pricing setting, the joint pricing and admission problem in time slot n can be formulated similarly as Problem **P3** in Section IV, by changing the objective function to $R_n(S_n, X_n^{(1)}, \cdots, X_n^{(I)}, a_n)$. Since there are $I+1$ possible revenues in (48) and we need to determine their value orders, there are $(I+1)!$ admission strategies (admission priorities) as in Proposition 8. We follow the same pricing-admission decomposition procedure to transform the joint problem into $(I+1)!$ subproblems corresponding to the $(I+1)!$ admission strategies in this time slot. As such, we can also derive the optimal pricing for maximizing the revenue in each time slot by solving those subproblems as we did in Section IV, and then choose the admission strategy that leads to the largest revenue as shown in Algorithm 2. The analysis procedure is identical with that in the previous scenario. The only difference is that there are I rather than two constraints (revenue conditions) in each optimization problem when assuming a particular admission strategy, hence it will be more complicated to optimize the prices in each subproblem.

VI. Simulation Results

In this section, we provide the simulation results to illustrate our key insights regarding the performances of the dynamic admission control under both static pricing and dynamic pricing. We first illustrate the stationary admission policies for the dynamic admission control under static pricing and dynamic pricing, respectively. We then compare the revenue improvement of dynamic pricing over static pricing under a wide range of system parameters.

[6]To determine the specific admission strategy (priority) in each time slot, we need to sort the $I+1$ revenues in (48) to the corresponding order. Hence, we have a $I+1$ permutation of $I+1$, which involves $(I+1)!$ admission strategies (priorities).

A. Optimal Static Pricing and Stationary Admission Policy

In Subsection III-D, we derived the optimal static pricing by first assuming that one of the stationary admission policies is optimal. Recall that the three conditions in Theorem 1 are characterized by the price ratio r_h/r_l. Given any demand elasticities k_l and k_h (hence any r_h/r_l relation with respect to p_l and p_h), it is natural to ask whether the optimal static pricing satisfies one of the conditions in Theorem 1, so that it is indeed optimal to choose a stationary admission policy after we optimize the static prices. Fig. 4 illustrates the corresponding result, showing when a stationary admission control policy is optimal under the optimal static prices for particular system parameters k_l and k_h. As we can see, except the small brown (Nonstationary Policy) regime which corresponds to regime III in Fig. 2, the stationary policies are optimal in most cases.

B. Optimal Dynamic Pricing and Stationary Admission Policy

In Subsection IV-C, we have shown that in the general case of dynamic pricing and dynamic admission control, the optimal admission strategies in different time slots may be different. On the other hand, it would be interesting to study under what system parameters the optimal admission decisions of different time slots (under dynamic pricing) will coincide with one of the stationary admission policies defined in Table II.

Recall that in our system model, as long as we adopt the linear demand functions, the system only has two parameters k_l and k_h, and the other parameters (e.g., probabilities p_l and p_h) are determined by k_l and k_h. Fig. 5 illustrates the optimal admission and pricing decisions under dynamic pricing. We can see that the optimal admission strategies in Algorithm 2 degenerate to stationary admission policies in most cases, and it is only optimal to switch between different admission strategies (HP, LD, and LP) in a small regime (the brown regime in Fig. 5).

Observation 1: Under a wide range of system parameters k_l and k_h, the optimal admission decisions developed in Algorithm 2 (with the optimized optimal dynamic prices) degenerate to stationary admission policies over all time slots.

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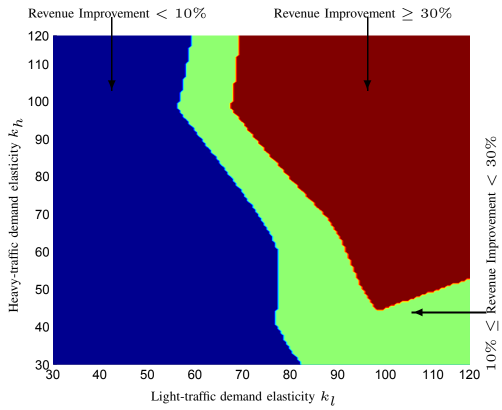

Fig. 6. The revenue improvement of dynamic pricing over static pricing for different k_l and k_h distributions.

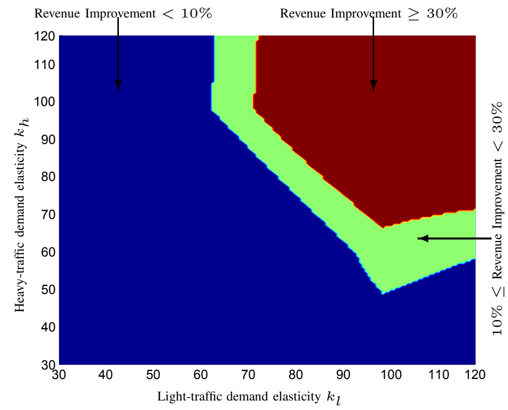

Fig. 7. The revenue improvement of dynamic pricing over static pricing for different k_l and k_h distributions. Here, the spectrum occupancy $M = 3$, while $M = 2$ in Fig. 6.

When the stationary admission policy is optimal, we have the following claims.

- If light-traffic SUs are much more price-sensitive than heavy-traffic SUs (i.e., k_l is significantly larger than k_h), the optimal dynamic pricing degenerates to the heavy-priority admission policy which is stationary over time.
- If heavy-traffic SUs are much more sensitive to prices than light-traffic SUs (k_l is significantly less than k_h), the optimal dynamic pricing degenerates to the light-dominant admission policy which is stationary over time.
- If both light- and heavy-traffic SUs' sensitivities k_l and k_h are comparable, the optimal dynamic pricing degenerates to the light-priority admission policy which is stationary over time.

C. Performance Comparison of Optimal Dynamic Pricing with Optimal Static Pricing

In addition to the optimal pricing and admission policies, it is also important to compare the performance of dynamic pricing with that of static pricing. The key benefit of static pricing is that it does not change over time. Unlike static pricing, the advantage of dynamic pricing is to achieve the maximum operator revenue. However, dynamic pricing has a higher implementational complexity. Next, we compare the optimal revenue of optimal dynamic pricing obtained in Theorem 2 with that of optimal static pricing obtained in Subsection III-D. Fig. 6 shows the revenue improvement of dynamic pricing over static pricing under different demand elasticity values (k_l and k_h). Here, we set the total time slots $N = 100$, so that the time horizon is long enough to approximate the time-average performance.

Observation 2: As shown in Fig. 6, dynamic pricing outperforms static pricing by more than 30% when both types of SUs are sensitive to prices (i.e., both k_l and k_h are high). When both types of SUs are not price-sensitive (i.e., k_l and k_h are low), dynamic pricing only leads to limited revenue improvement (less than 10%) than static pricing, and it is better to adopt static pricing due to its low complexity.

The above comparison is based on the assumption that heavy-traffic SUs request two consecutive time slots. In Section V, we have extended the model to arbitrary spectrum occupancies. Hence, it is also interesting to show the comparison with more spectrum occupancies. Fig. 7 shows the revenue improvement of dynamic pricing over static pricing with three consecutive time slots occupancy of heavy-traffic SUs ($M = 3$). We can see that dynamic pricing significantly outperforms static pricing when SUs' demands are highly elastic, which is similar to Observation 2. Comparing with Fig. 6 with $M = 2$, the difference here is that a larger value of M reduces the benefit of dynamic pricing. For example, when $k_l \in (90, 120)$ and $k_h \in (60, 70)$, the revenue improvement of dynamic pricing over static pricing is more than 30% in Fig. 6, but is only around 10% in Fig. 7. The intuition is that a larger spectrum occupancy reduces the flexibility of dynamic pricing, since more slots will be occupied and cannot be dynamically allocated to new demands. Consider the extreme case $M = N$, then all slots will be occupied when admitting a heavy-traffic SU initially and dynamic pricing degenerates to static pricing. This implies that as the channel occupancy gap between the two SU types increases, it becomes increasingly attractive for the operator to choose the simple static pricing approach in order to achieve a close-to-optimal revenue.

VII. Conclusion

In this paper, we consider a spectrum database operator's revenue maximization problem through joint spectrum pricing and admission control. We incorporate the heterogeneity of SUs' spectrum occupancy and demand uncertainty into the model, and consider both the static and the dynamic pricing schemes. In static pricing, we show that stationary admission policies can achieve optimality in most cases. In dynamic pricing, we compute optimal pricing through a proper pricing-and-admission decomposition in each time slot. Furthermore, we show that dynamic pricing significantly improves revenue over static pricing when SUs are sensitive to prices change. Finally, we show that when the gap of the channel occupation length between two types of SUs increases, the gap between static pricing and dynamic pricing shrinks.

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