

# Cooperative Spectrum Sharing: A Contract-based Approach

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**Abstract**—Providing economic incentives to all parties involved is essential for the success of dynamic spectrum access. Cooperative spectrum sharing is one effective way to achieve this, where secondary users (SUs) relay traffics for primary users (PUs) in exchange for dedicated spectrum access time for SUs' own communications. In this paper, we study the cooperative spectrum sharing under *incomplete information*, where SUs' wireless characteristics are private information and not known by a PU. We model the PU-SU interaction as a labor market using contract theory. In contract theory, the employer generally does not completely know employees' private information before the employment and needs to offers employees a contract under incomplete information. In our problem, the PU and SUs are respectively the employer and employees, and the contract consists of a set of items representing combinations of spectrum accessing time (i.e., reward) and relaying power (i.e., contribution). We study the optimal contract design for both weakly and strongly incomplete information scenarios. In the weakly incomplete information scenario, we show that the PU will optimally hire the most efficient SUs and the PU achieves the same maximum utility as in the complete information benchmark. In the strongly incomplete information scenario, however, the PU may conservatively hire less efficient SUs as well. We further propose a Decompose-and-Compare (DC) approximate algorithm that achieves a close-to-optimal contract. We further show that the PU's average utility loss due to the suboptimal DC algorithm and the strongly incomplete information are relatively small (less than 2% and 1.3%, respectively, in our numerical results with two SU types).

**Index Terms**—Dynamic spectrum access, spectrum trading, cooperative spectrum sharing, contract theory, game theory.



## 1 INTRODUCTION

WITH the explosive development of wireless services and networks, spectrum is becoming more congested and scarce. Dynamic spectrum access is a promising approach to increase spectrum efficiency and alleviate spectrum scarcity, as it enables unlicensed secondary users (SUs) to dynamically access the spectrum licensed to primary users (PUs) [2], [3]. The successful implementation of dynamic spectrum access requires many innovations in technology, economics, and policy. In particular, it is important to design the mechanism such that PUs have the incentive to open their licensed spectrum for sharing, and SUs have the incentive to utilize the new spectrum opportunities after considering potential costs.

Market-driven spectrum trading is a promising paradigm to address the incentive issue in dynamic spectrum access [4]. In a market mechanism, PUs temporarily *sell* the spectrum to SUs in exchange of either a monetary reward or a performance improvement. A particular interesting market mechanism is *cooperative*

*spectrum sharing*, where SUs relay traffics for PUs in order to get their own share of spectrum. An illustrative example of cooperative spectrum sharing is shown in Fig. 1. The SUs' transmitters ( $ST_1 \sim ST_3$ ) act as cooperative relays for the PU in Phase I and Phase II (Decoding and Forwarding), and transmit their own data in Phase III.

There has been extensive research on the cooperative relay networks, often within the scope of physical layer performance optimization [17]–[25]. Researchers have only recently started to study the incentive issue in cooperative spectrum sharing mechanisms [13]–[16]. Prior results all assumed *complete network information*, where PUs know SUs' types including information such as channel conditions, resource constraints, and costs of transmission. This assumption is often too strong for practical implementations. In this paper, we study the cooperative spectrum sharing between a single PU and multiple competitive SUs under *incomplete information*, where SUs' types are private information<sup>1</sup>.

We focus on the PU's utility maximization under incomplete information: *How should the PU employ the SUs as relays to maximize its expected performance improvement without knowing the SUs' exact types?* Precisely, which SUs the PU should employ, what relaying efforts the PU want to ask from the employed SUs, and what rewards the PU would like to give to the employed SUs?

To tackle this problem, we propose a contract-based cooperative spectrum sharing mechanism. Contract the-

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1. Consider a cellular network with multiple base stations and multiple licensed mobile phone users. Each link between one base station and one licensed user defines one PU.

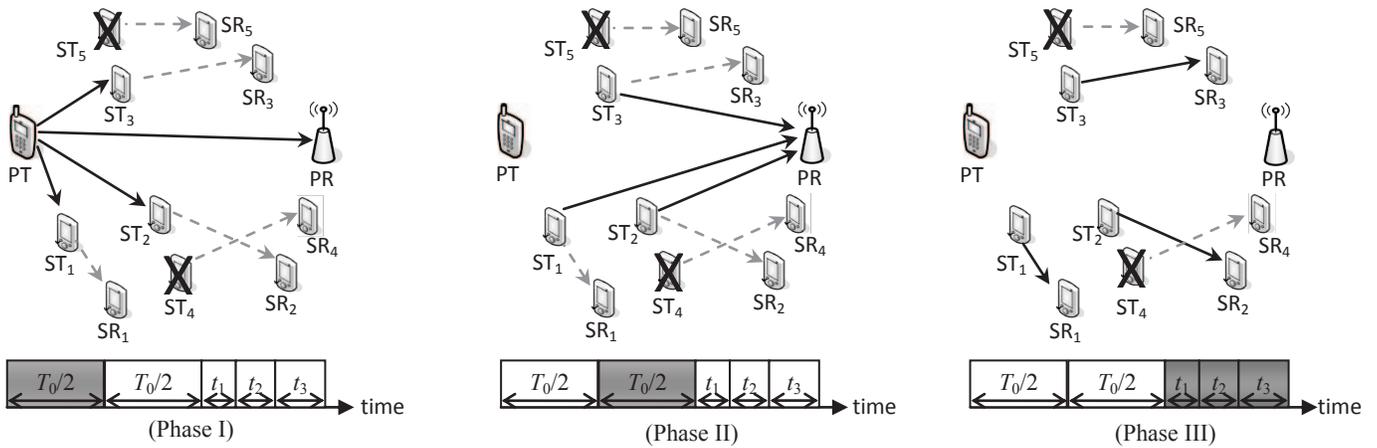


Fig. 1. A single time slot in cooperative spectrum sharing includes three phases. Phase I involves transmission from primary user transmitter PT to primary receiver PR and transmitters of the involved SUs (ST<sub>1</sub>, ST<sub>2</sub>, and ST<sub>3</sub>). Phase II involves the transmission of the involved SUs’ transmitters to the primary receiver PR. Phase III involves the dedicated transmissions of the involved SUs.

ory is effective in designing incentive compatible mechanisms in a monopoly market under incomplete information [30], where the employer needs to sign a contract with employees before fully knowing their private information. The key idea is to offer the right contract items so that all agents have the incentive to truthfully reveal their private information. In our problem, we can imagine the network as a labor market. The PU is the employer and offers a *contract* to the SUs. The contact consists of a set of contract items, which are combinations of spectrum access time (i.e., reward) and relay power (i.e., contribution). The SUs are employees, and each SU selects the best contract item according to its type. We want to design an optimal contract that maximizes the PU’s utility (average data rate) under the incomplete information of SUs’ types.

The main contributions of this paper are as follows:

- *New modeling and solution technique:* As far as we know, this is the first contract paper tackling cooperative spectrum sharing between one PU and *multiple* SUs under incomplete information.
- *Multiple information scenarios* (Sections 3 and 4): We first consider the complete information scenario as a benchmark. Then we consider the optimal contract design in two incomplete information scenarios (where the PU does not know the precise type of every SU): *weakly incomplete* information, where the PU knows the percentage of every type, and *strongly incomplete* information, where the PU knows only the probability distribution of every type.
- *Feasibility of contracts* (Section 5): Under incomplete information, a contract is *feasible* if and only if it is incentive compatible (IC) and individually rational (IR) for each SU. We characterize the necessary and sufficient conditions for a contract to be IC and IR systematically.
- *Optimality of contracts* (Sections 6 and 7): Under

TABLE 1  
 Feasibility conditions and optimality of different information scenarios

Network Information	Feasibility	Optimality	Section No.
Complete (benchmark)	IR	Optimal	4
Weakly Incomplete	IC & IR	Optimal	5, 6
Strongly Incomplete	IC & IR	Close-to-Optimal	5, 7

weakly incomplete information, we derive the optimal contract, which achieves the same maximum PU utility as in the complete information benchmark. Under strongly incomplete information, we propose a Decompose-and-Compare algorithm that achieves a close-to-optimal contract.

- *Performance analysis* (Section 8): Under strongly incomplete information, the PU’s utility loss is caused by two factors: the suboptimal contract and the incomplete information. We quantify the PU’s average utility loss due to the suboptimal algorithm (by comparing it with exhaustive searching) and incomplete information (by comparing it with complete information) through numerical examples. Both losses turn out to be relatively small, e.g., less than 2% and 1.3% for a two-type case, respectively.

The key results and corresponding section numbers in this paper are summarized in Table 1. We also review the related literature in Section 2 and conclude in Section 9.

## 2 RELATED WORK

### 2.1 Spectrum Trading for Dynamic Spectrum Sharing

Recent years have witnessed a growing body of literatures on the market-driven spectrum trading [4], which can be classified into two types: money-exchange and

resource-exchange. In the former type, SUs pay PUs in the form of (virtual) money for the usage of spectrum (e.g., [5]–[12]); in the latter type, SUs provide communication resources (e.g., the power in our model) in exchange for the usage of spectrum (e.g., [13]–[16]).

There has been extensive research on the money-exchange spectrum trading model, often in forms of pricing (e.g., [5], [6]), auction (e.g., [7]–[10]), and contract (e.g., [11], [12]). *Pricing* is often used when the seller knows precisely the value of the resource being sold. *Auction* becomes a suitable approach when the seller has no knowledge about the value of the resource. *Contract* is more effective in the case where the seller only knows limited information (e.g., distribution) of the buyers’ valuations of the resource. By motivating the buyers truthfully reveal their private valuations, the seller can optimally allocate the resource. In [11], Gao *et al.* proposed a quality-price contract for the spectrum trading in a monopoly spectrum market. In [12], Kalathil *et al.* focused on 1 PU and 1 SU and proposed a contract-based spectrum sharing mechanism to avoid possible manipulating in auction. Yet all these monetary-exchange spectrum trading schemes require a trustworthy and widely acknowledged billing system [33], and such a system is far from industry practice yet.

Money-exchange spectrum trading is most effective when PUs have some temporarily unused spectrum. However, when PUs’ own demands are high or the primary channels’ capacities are low (e.g., due to shadowing and deep fading), there will be hardly any resource left for sale. Also, such a money-exchange trading requires a trustworthy billing system which is still far from practical implementation. In this case, resource-exchange spectrum trading can be a better choice. *Cooperative spectrum sharing* is an effective form of resource-exchange spectrum trading [13]–[16], wherein PUs utilize SUs as cooperative relays. Such cooperation can significantly improve PUs’ data rate and thus can free up spectrum resources for SUs. Existing cooperative spectrum sharing mechanisms are usually based on Stackelberg game formulations with complete information [13]–[16]. All of these works cannot be easily extended to the scenario of incomplete information. Taking [13] as an example. Under incomplete information, an SU can increase its utility by misreporting its private information (e.g., its own channel gain and its cost per unit energy consumption). Thus, it is no longer optimal for the PU to stick to the equilibrium of the proposed Stackelberg game. In this case, some incentive compatible mechanism is necessary for eliciting the SUs’ private information.

In this paper, we consider the cooperative spectrum sharing under *incomplete information*, and propose a contract-based cooperative spectrum sharing mechanism (which is incentive compatible). As far as we know, this is the first work considering the cooperative spectrum sharing under incomplete information. Compared to those works on monetary trading contracts, the resource-exchange contract design is more challenging, as we

TABLE 2  
 A summary of spectrum trading literatures

Network Information	Money-Exchange	Resource-Exchange
Complete	Pricing: [5], [6]	Stackelberg: [13]–[16]
Incomplete	Contract: [11], [12] Auction: [7]–[10]	Contract: This paper

need to consider the PU’s and SUs’ non-transferable utilities (since not everything can be translated to a single currency) as well as other physical constraints. For example, the resource allocation to one SU will affect other SUs directly, and the physical constraints include transmission power constraint, spectrum opportunity limit, and relay number constraint. Thus our contract model is more complex to analyze than the pure money exchange (e.g., in [12]). We summarize the key literatures of spectrum trading in Table 2.

## 2.2 Cooperative Relay

The performance of cooperative spectrum sharing mechanism greatly depends on the underlying cooperative relay protocol. There has been extensive research on the cooperative relay protocols, often within the scope of physical layer performance optimization [17]–[25]. The results in [17]–[19] have shown that the collaboration among mobile users may significantly increase the system performance. In [20], Hua *et al.* studied the optimization of power (rate) subject to the rate (power) constraint. In [21], Gazor *et al.* studied the problem of relay and protocol selection using three criteria: rate, energy, and resource. In [22], Hou *et al.* proposed a polynomial time algorithm solving the optimal relay node assignment problem. In [23], Rong proposed the optimal structure of source and relay matrices for multi-hop MIMO relay systems with QoS constraints. In [24], Sung *et al.* proposed a new amplify-and-forward (AF) relay systems using superimposed pilot signals. In [25], Viberg *et al.* proposed a new technique for self-interference suppression in full-duplex MIMO relays.

In this work, we consider the *distributed space-time-coded protocol* [26] as an example of the underlying cooperative scheme. Our contract design can be used with other choices of relay protocols as well.

## 3 SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cognitive radio network with a primary licensed user (PU) and multiple secondary unlicensed users (SUs) as shown in Fig. 1.<sup>2</sup> Each user is a dedicated

2. The results of the single PU case in this paper can be extended to the multiple PU case. Motivated by the fact that the number of SUs is generally much larger than that of PUs, we can simply divide the whole network into multiple sub-networks. Each sub-network still consists of one PU and a set of nearby SUs and our approach still applies. Note that such division will not cause significant loss of system performance.

transmitter-receiver pair. The PU has the exclusive usage right of the licensed spectrum band, but suffers from the poor channel condition between its transmitter PT and receiver PR. We represent  $M$  SUs by distinct transmitter-receiver pairs  $\{ST_k - SR_k\}_{k=1}^M$ . Each SU wants to have dedicated time to access the licensed band and transmit its own data, and it cannot transmit without the PU's permission. The PU can employ a subset of or all SUs to relay its traffic; the *involved* SUs will obtain dedicated transmission time for their own data. The interaction between the PU and the SUs involves three phases as in Fig. 1: Phases I and II for the cooperative communications with a total fixed length of time  $T_0$ , and Phase III for the SUs' own transmissions. More specifically,

- Phase I: In the first half of the cooperative communication period ( $T_0/2$ ), the primary transmitter PT broadcasts its data to the primary receiver PR and the involved SUs' transmitters (e.g.,  $ST_1$ ,  $ST_2$ , and  $ST_3$  in Fig. 1). Note SU 4 and SU 5 are not involved in this example.
- Phase II: In the remaining half of the cooperative communication period ( $T_0/2$ ), the involved SUs' transmitters (STs) decode the data received in Phase I, and forward to the primary receiver PR simultaneously using the space-time codes assigned by PU from its random codebook. Through proper choice of space-time codes, SUs' simultaneous relay signals do not interference with each other at the primary receiver PR [26], [28].
- Phase III: PU rewards each involved SU with a dedicated time allocation for that SU's own data (e.g.,  $\{t_k\}_{k=1}^3$  for three involved SUs). SUs access the spectrum using TDMA and do not interfere with each other.<sup>3</sup>

We assume each involved SU can successfully decode PU's data in the first phase of cooperative communications. Thus we focus on the relay links between the secondary transmitters STs and primary receiver PR, which are the performance bottleneck of cooperative communications.<sup>4</sup> It should be noted that the PU may adopt a more efficient relay scheme where it can also transmit in Phase II, and we will show that our main results also extend to that scenario in Appendix G.

The PU and SUs have conflicting objectives in the above interactions. The PU wants the SUs to relay its traffic with high power levels, which will increase the PU's data rate but reduce the SUs' battery levels. An SU  $k$  wants to obtain a large dedicated transmission time  $t_k$ , which will increase the SU's own performance

3. Our results also apply to the case where the SUs share the spectrum using FDMA, where a more efficient relay will be rewarded with a larger bandwidth.

4. This assumption can be relaxed if the PU performs an initial screening over all SUs as follows. The PU first broadcasts a pilot signal to all SUs. Only those SUs replying correctly can choose to accept the contract and involve in the cooperative communications later. Notice that the involved SUs are often the ones whose transmitters are close to PT, and thus may be not close to PR. This explains why a PT - ST channel is often better than the corresponding ST - PR channel.

TABLE 3  
Key Notations

$p_k$	The received power at PR from SU $k$ (relay);
$t_k$	The dedicated access time for SU $k$ ;
$\Phi$	The power-time contract, $\Phi = \{(p_k, t_k), \forall k \in \{1, \dots, K\}\}$ ;
$R^{dir}$	The PU's direct transmission rate;
$R_{PU}^{tot}$	The PU's total achievable rate with SUs' cooperation;
$U_{PU}$	The PU's utility (average transmission rate);
$p_k^{rl}$	The relay power of SU $k$ for PU's traffic;
$p_k^{tr}$	The transmission power of SU $k$ for its own traffic;
$C_k$	The SU $k$ 's cost for one unit power consumption;
$R_k$	The SU $k$ 's own transmission rate;
$\Pi_k$	The SU $k$ 's payoff, $\Pi_k = t_k R_k - (t_k p_k^{tr} + 0.5 p_k^{rl}) C_k$ ;
$\tilde{\Pi}_k$	The SU $k$ 's normalized payoff, also denoted by $\pi_k$ ;
$\theta_k$	The type of SU $k$ , $\theta_k := \frac{2h_{ST_k, PR}(R_k - C_k p_k^{tr})}{C_k}$ .

but reduce the PU's utility (time average data rate). In Sections 3.1 and 3.2, we will explain in details how the PU and SUs evaluate the trade-off between relay powers and time allocations. In Section 3.3, we propose a contract-based framework which brings the PU and SUs together and resolves the conflicts.

We list key notations used in this paper in Table 3. The meaning of each notation will be explained later.

### 3.1 Primary User Modeling

In this subsection, we discuss how PU evaluates relay powers and time allocations.

We first compute the PU's achievable data rate during cooperative communications (i.e., Phases I and II in Fig. 1). Let us denote the set of involved SUs as  $\mathcal{N}$  (e.g.,  $\mathcal{N} = \{1, 2, 3\}$  in Fig. 1). The received power (at the primary receiver PR) from SU  $k$  is  $p_k$ , and the time allocation to SU  $k$  is  $t_k$ . *Without loss of generality, we normalize  $T_0$  to be 1 in the rest of the paper. Then  $t_k$  can be viewed as the percentage of the  $T_0$ .*

- In Phase I, PT broadcasts its data, and PR achieves a data rate (per unit time) of

$$R^{dir} = \log(1 + \text{SNR}_{PT, PR}), \quad (1)$$

which remains as a constant throughout the analysis. Here  $\text{SNR}_{a,b}$  denotes the Signal-to-interference ratio of the signal transmitted from node  $a$  to  $b$ .

- In Phase II, each involved SU decodes PT's data (received in Phase I) and forwards to PR.

The PU's total achievable rate during Phases I and II is given by [26]:

$$\begin{aligned} R_{PU}^{tot} &= \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \sum_{k \in \mathcal{N}} \text{SNR}_{ST_k, PR} \right) \\ &= \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{N}} p_k}{n_0} \right), \end{aligned} \quad (2)$$

where  $n_0$  is the noise power, and  $1/2$  denotes equal partition of the cooperative communication time into Phase I and Phase II. We can think (2) as the sum of transmission rates of two "parallel" channels, one from

the PT to PR and the other from the set of involved SUs' transmitters to PR.

Based on the above discussion, we next compute the PU's average data rate during entire time period (i.e., Phases I, II, and III). The cooperative communications only utilizes  $1/(1 + \sum_{k \in \mathcal{N}} t_k)$  fraction of the entire time period. The PU's objective is to maximize its **utility** (i.e., average transmission rate during the entire time period) as follows

$$\begin{aligned} U_{\text{PU}} &= \frac{1}{1 + \sum_{k \in \mathcal{N}} t_k} R_{\text{PU}}^{\text{tot}} \\ &= \frac{1}{1 + \sum_{k \in \mathcal{N}} t_k} \left( \frac{R^{\text{dir}}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{N}} p_k}{n_0} \right) \right), \end{aligned} \quad (3)$$

which is decreasing in the total time allocations to SUs (i.e.,  $\sum_{k \in \mathcal{N}} t_k$ ), and is increasing in the total received power from SUs (i.e.,  $\sum_{k \in \mathcal{N}} p_k$ ).

We want to emphasize that the utility calculation in (3) assumes that the PU involves at least one SU in the cooperative communication. The PU can also choose to have direct transmissions only in both Phases I and II and does not interact with the SUs (and thus there is no Phase III). The total rate in this direct transmission only approach is  $R^{\text{dir}}$ . This means that the PU will only choose to use cooperative communications if the utility in (3) is larger than  $R^{\text{dir}}$ . In the rest of the analysis, we assume that  $R^{\text{dir}}$  is small such that the PU wants to use cooperative communications. In Section 8, we will further explain what will happen when this is not true.

### 3.2 Secondary User Modeling

Next we discuss how SUs evaluate relay powers  $\{p_k\}_{k \in \mathcal{N}}$  and time allocations  $\{t_k\}_{k \in \mathcal{N}}$ . We want to emphasize that the relay power is measured at the PU's receiver, not at the SUs' transmitters. We consider a general model where SUs are heterogeneous in three aspects:

- SUs have different relay channel gains between their transmitters and the PU's receiver ( $h_{\text{ST}_k, \text{PR}}$ ). If SU  $k$  wants to reach a received power  $p_k$  at the PU's receiver, it needs to transmit with a power  $p_k^{r,l} = p_k / h_{\text{ST}_k, \text{PR}}$ .
- SUs achieve different (fixed) rates to their own receivers (i.e., data rate  $R_k$  over link  $\text{ST}_k - \text{SR}_k$ ) with different (fixed) transmission power (i.e.,  $p_k^{tr}$ ).
- SUs have different cost  $C_k$  per unit transmission power.

Note that the parameters  $C_k$ ,  $R_k$ ,  $p_k^{r,l}$ , and  $h_{\text{ST}_k, \text{PR}}$  are SU  $k$ 's private information and are only known to itself.

We define an SU  $k$ 's payoff as  $\Pi_k$ , which is the difference between its own transmitted rate (during time allocation  $t_k$  in Phase III) and its cost of power consumption (during Phase II and its allocated time in Phase III). That is,

$$\begin{aligned} \Pi_k &= t_k \cdot R_k - (t_k \cdot p_k^{tr} + 0.5 \cdot p_k^{r,l}) \cdot C_k \\ &= t_k \cdot R_k - \left( t_k \cdot p_k^{tr} + 0.5 \cdot \frac{p_k}{h_{\text{ST}_k, \text{PR}}} \right) \cdot C_k \end{aligned} \quad (4)$$

Without loss of generality, we assume that every SU is willing to transmit if it does not need to relay the PU's traffic, i.e.,  $R_k - p_k^{tr} \cdot C_k \geq 0$  for all  $k \in \mathcal{N}$ . Notice that (4) is increasing in time allocation  $t_k$ , but is decreasing in relay power  $p_k$  (received at PR).

We can further simplify (4) by multiplying both sides by  $2h_{\text{ST}_k, \text{PR}}/C_k$ , which leads to the normalized payoff

$$\tilde{\Pi}_k := \Pi_k \frac{2h_{\text{ST}_k, \text{PR}}}{C_k} = \frac{2h_{\text{ST}_k, \text{PR}}(R_k - C_k p_k^{tr})}{C_k} t_k - p_k. \quad (5)$$

Such normalization does not affect SUs' choice among different relay powers and time allocations. Thus it will not affect the contract design introduced later.

To facilitate later discussions, we define an SU  $k$ 's **type** as

$$\theta_k := \frac{2h_{\text{ST}_k, \text{PR}}(R_k - C_k p_k^{tr})}{C_k} > 0, \quad (6)$$

which captures all private information of this SU.<sup>5</sup> A larger  $\theta_k$  means that the SU's own transmission is more efficient (a larger  $R_k$  or a smaller  $p_k^{tr}$ ), or it has a better channel condition over relay link  $\text{ST}_k - \text{PR}$  (a larger channel gain  $h_{\text{ST}_k, \text{PR}}$ ), or it has a more efficient battery technology (a smaller  $C_k$ ). With (6), we can simplify SU's **normalized payoff** in (5) as

$$\pi_k(p_k, t_k) := \tilde{\Pi}_k = \theta_k t_k - p_k. \quad (7)$$

which is decreasing in PU's received power  $p_k$  and increasing in SU  $k$ 's achieved time  $t_k$ .

Since each SU is selfish, a type- $\theta_k$  SU wants to choose relay power and time allocation to maximize its normalized payoff in (7). Notice that an SU always has the option of not helping the PU and thus receiving zero time allocation and zero payoff (i.e.,  $t_k = p_k = 0$ ). Thus the PU needs to provide proper incentives to attract the SUs to help.

### 3.3 Contract Formulation

After introducing PU's utility (3) and SUs' normalized payoffs (7), we are ready to introduce the contract mechanism that resolves the conflicting objectives between the PU and SUs.

Contract theory studies how economic decision-makers construct contractual arrangements, generally in the presence of private information [30]. In our case, the SUs' types are private information. The PU does not know each SU's type, and needs to design a contract to attract the SUs to participate in cooperative communications to maximize PU's utility.

To better understand the contract design in this paper, we can imagine the PU as the employer and SUs as employees in a labor market. The employer optimizes the contract, which specifies the relationship between the employee's performance (i.e., received relay powers at PU's receiver) and reward (i.e., time allocations to SUs).

5. From a practical implementation point of view, we need to discretize the users' types (which are related to users' channel conditions, data rates, and energy consumption costs) to a finite number of levels (similar to [34], [35]).

If we denote  $\mathcal{P}$  as the set of all possible relay powers and  $\mathcal{T}$  as the set of all possible time allocations, then the contract specifies a  $t \in \mathcal{T}$  for every  $p \in \mathcal{P}$ . Each distinct power-time association becomes a contract item. Once a contract is given, each SU will choose the contract item that maximizes its payoff in (7). The PU wants to optimize the contract items to maximize its utility in (3).

We consider  $K$  SU types denoted by set  $\Theta = \{\theta_1, \theta_2, \dots, \theta_K\}$ .<sup>6</sup> Without loss of generality, we assume that  $\theta_1 < \theta_2 < \dots < \theta_K$ . The total number of type- $\theta_k$  SUs is  $N_k$ . According to the revelation principle [29], it is enough to consider the class of contract that enables the SUs to truthfully reveal their types. Because of this, it is enough to design a contract that consists  $K$  contract items, one for each type. We denote contract item designed for type- $\theta_k$  as  $(p_k, t_k)$ . The contract can be written as  $\Phi = \{(p_k, t_k), \forall k \in \mathcal{K}\}$  where  $\mathcal{K} = \{1, 2, \dots, K\}$ .<sup>7</sup>

We consider optimal contract design for three information scenarios.

- *Complete information in Section 4:* This is a benchmark case, where the PU knows each SU's type. We will compute the maximum utility the PU can achieve in this case, which serves as an upper-bound of the PU's achievable utility under any information scenario.
- *Weak incomplete information in Section 6:* The PU does not know each SU's type, but has knowledge of the set of types and the number of each type SUs in the market (i.e.,  $N_k$  for type- $\theta_k$  SUs). We will show that the optimal contract in this case achieves the same maximum PU's utility as in the complete information benchmark.
- *Strong incomplete information in Section 7:* The PU only knows the total number of SUs ( $N$ ) and the distribution of each type, but does not know the number of each type ( $N_k$ ). The PU needs to design a contract to maximize its *expected* utility.

Once a contract is given, the interactions between the PU and SUs follow 4 steps.

- 1) The PU broadcasts the contract  $\Phi = \{(p_k, t_k), \forall k \in \mathcal{K}\}$  to all SUs.
- 2) After receiving the contract, each SU chooses one contract item that maximizes its payoff and informs the PU its choice.
- 3) After receiving all SUs confirmations, the PU informs the involved SUs (i.e., those choosing positive contract items) the space-time codes to use in Phase II and the transmission schedule in Phase

6. Although  $\theta_k$  is a continuous random variable depending on the realisation of the channel coefficient, here we assume there are  $K$  distinct types for simplicity, where any two distinct types can be arbitrary close to each other. A large enough  $K$  can well approximate the continuous case with only a slightly decreased system performance.

7. Note that an SU can always choose not to work for the PU, which implies an implicit contract item  $(p, t) = (0, 0)$  in the contract (often not counted in the total number of contract items).

III.<sup>8</sup> Note that the length for transmission time for each involved SU is specified by the contract item and the PU can no longer change.

- 4) The communications start by following three phases in Fig. 1.

It is worth noting that our contract design focuses on the issue of hidden (private) information, that is, it aims at screening the private type of SUs so as to maximize the PU's utility under incomplete information. We do not consider other issues regarding contract design (such as hidden action or moral hazard), since the contracts for eliciting private information and for hedging moral hazard are very different and are usually studied independently. Once a contract has been agreed by the PU and one SU, we assume that neither the PU nor the SU will deviate from the contract. This is reasonable in our model, because the action of each SU can be observed by the PU through measuring the received relay power from the SU, and the action of the PU can also be observed by the SU through measuring the duration of the awarded access time. In this case, it is not difficult to solve the moral hazard or hidden action problem by introducing simple penalty mechanisms as potential threat. For example, the PU can impose a large penalty on the SU (e.g., not cooperating with the SU in the future), once observing the SU's deviation from the agreement, and vice versa.

## 4 OPTIMAL CONTRACT DESIGN WITH COMPLETE INFORMATION

In the complete information scenario, the PU knows precisely the type of each SU. We will use the maximum PU's utility achieved in this case as a benchmark to evaluate the performance of the proposed contracts under incomplete information in Sections 6 and 7. Without loss of generality, we assume that  $N_k \geq 1$  for all  $k \in \mathcal{K}$ .

As the PU knows each SU's type, it can monitor and make sure that each SU accepts only the contract item designed for its type. The PU needs to ensure that the SUs have non-negative payoffs so that they are willing to accept the contract. In other words, the contract needs to satisfy the following individual rationality constraint.

*Definition 1:* [IR: Individual Rationality] A contract satisfies the IR constraint if for each type- $\theta_k$  SU, it receives a non-negative payoff by accepting the contract item for  $\theta_k$ , i.e.,

$$\theta_k t_k - p_k \geq 0, \forall k \in \mathcal{K}. \quad (8)$$

8. The exchange of such information (e.g., space-time codes and other details) between the PU and SUs will introduce additional communication overhead. However, as long as the cooperation lasts for a sufficiently long time (e.g., including many rounds of Phases I-III), the overhead is relatively small. The overhead may become a major issue if SUs are mobile and their wireless characteristics (e.g., relay channel gains and their own channel gains) change frequently. In that case, only a few rounds of Phases I-III last before the user types change, and the PU needs to tradeoff the cooperation benefit and the communication overhead. Detailed analysis and results are provided in Appendix G.

We say a contract is *optimal* if it yields the maximum utility for the PU under the current information scenario. Different information scenarios may lead to different optimal contracts.

In the complete information scenario, an optimal contract maximizes the PU's utility as follows

$$\max_{\{(p_k, t_k) \geq \mathbf{0}, \forall k \in \mathcal{K}\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{K}} N_k p_k}{n_0} \right)}{1 + \sum_{k \in \mathcal{K}} N_k t_k}, \quad (9)$$

subject to IR Constraints in Eq. (8).

In this paper, the vector operations are component-wise (e.g.,  $(p_k, t_k) \geq \mathbf{0}$  means that  $p_k \geq 0$  and  $t_k \geq 0$ ) unless specified otherwise. Then we have the following result.

*Lemma 1:* In an optimal contract with complete information, each SU achieves zero payoff by accepting the corresponding contract item, i.e.,  $t_k \theta_k = p_k, \forall k \in \mathcal{K}$ .

*Proof.* We prove by contradiction. Suppose that there exists an optimal contract item  $(p_k, t_k)$  with  $\theta_k t_k - p_k > 0$ . Since PU's utility in (9) is increasing in  $p_k$  and decreasing in  $t_k$ , the PU can increase its utility by decreasing  $t_k$  until  $\theta_k t_k - p_k = 0$ . This contradicts with the assumption that  $(p_k, t_k)$  with  $\theta_k t_k - p_k > 0$  belongs to an optimal contract, and thus completes the proof. ■

Using Lemma 1, we can replace  $p_k$  by  $\theta_k t_k$  for each  $k \in \mathcal{K}$  and simplify the PU's utility maximization problem in (9) as

$$\max_{\{t_k \geq 0, \forall k \in \mathcal{K}\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{K}} N_k \theta_k t_k}{n_0} \right)}{1 + \sum_{k \in \mathcal{K}} N_k t_k}. \quad (10)$$

*Theorem 1:* In an optimal contract with complete information, only the contract item for the highest type is positive and all other contract items are zero, i.e.,  $(p_K, t_K) > \mathbf{0}$  and  $(p_k, t_k) = \mathbf{0}, \forall k < K$ .<sup>9</sup>

*Proof.* By contradiction, suppose there exists an optimal contract item with  $t_k > 0$  and  $k < K$  (for type- $\theta_k$  SUs). The total time allocation is  $T' = \sum_{k \in \mathcal{K}} N_k t_k$ . Then the PU's utility is

$$U_{PU}^1 = \frac{1}{1 + T'} \left( \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{K}} \theta_k N_k t_k}{n_0} \right) \right). \quad (11)$$

Next we show that given a fixed total time allocation  $T'$ , allocating positive time only to the highest type SUs (i.e.,  $N_K t_K = T'$ ) achieves a larger utility for the PU as follows

$$U_{PU}^2 = \frac{1}{1 + T'} \left( \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\theta_K N_K t_K}{n_0} \right) \right). \quad (12)$$

This is because  $\theta_K N_K t_K = \theta_K T'$  in (12) and  $\sum_{k \in \mathcal{K}} \theta_k N_k t_k < \theta_K T'$  in (11), thus (12) is larger than (11). This contradicts with the optimality of the contract, and thus completes the proof. ■

Intuitively, the highest type SUs can offer the most help to the PU given total time allocation in Phase III.

9. As  $N_K$  and  $t_K$  always appear together in (10), the PU's optimal utility does not depend on the specific value of  $N_K$ , and the complexity in calculating the optimal time allocations does not depend on  $N_K$  either. It is also worth noting that when SUs have maximum transmission power constraints, it may not be optimal for the PU to involve the highest type SUs only. For details, please see Appendix A.

Using Theorem 1, the optimization problem in (10) can be simplified as

$$\max_{t_K \geq 0} \frac{1}{1 + N_K t_K} \left( \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\theta_K N_K t_K}{n_0} \right) \right). \quad (13)$$

Since  $N_K$  and  $t_K$  always appear as a product in (13), we can redefine the optimization variable as  $\tilde{t}_K = N_K t_K$  and rewrite (13) as

$$\max_{\tilde{t}_K \geq 0} \frac{1}{1 + \tilde{t}_K} \left( \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\theta_K \tilde{t}_K}{n_0} \right) \right), \quad (14)$$

This means the PU's optimal utility does not depend on  $N_K$ . When  $N_K$  changes, the optimal time allocation per user  $t_K^*$  changes inversely proportional to  $N_K$ .

At this point, we have successfully simplified the PU's optimization problem from involving  $2K$  variables  $\{(p_k, t_k), \forall k \in \mathcal{K}\}$  in (9) to a single variable  $\tilde{t}_K$  in (14).

Let us denote the objective of the problem (14) as  $U_{PU}(\tilde{t}_K)$ , which is a function of  $\tilde{t}_K$  but is not concave. Any local optimal solution (denoted as  $\hat{t}_K$ ) to the problem (14) satisfies

$$\frac{dU_{PU}(\tilde{t}_K)}{d\tilde{t}_K} \Big|_{\tilde{t}_K = \hat{t}_K} = \frac{\frac{\theta_K(1 + \hat{t}_K)}{n_0 + \theta_K \hat{t}_K} - (R^{dir} + \log(1 + \frac{\theta_K \hat{t}_K}{n_0}))}{(1 + \hat{t}_K)^2} = 0. \quad (15)$$

We can further compute the second order derivative as

$$\begin{aligned} \frac{d^2 U_{PU}(\tilde{t}_K)}{d\tilde{t}_K^2} \Big|_{\tilde{t}_K = \hat{t}_K} &= \frac{1}{(1 + \hat{t}_K)^4} \left( -\frac{\theta_K^2 (1 + \hat{t}_K)^3}{(n_0 + \theta_K \hat{t}_K)^2} \right. \\ &\quad \left. + 2(1 + \hat{t}_K) \left( \frac{\theta_K (1 + \hat{t}_K)}{n_0 + \theta_K \hat{t}_K} - \left( R^{dir} + \log \left( 1 + \frac{\theta_K \hat{t}_K}{n_0} \right) \right) \right) \right). \end{aligned} \quad (16)$$

By substituting (15) into (16), we can show that  $d^2 U_{PU}(\tilde{t}_K)/d\tilde{t}_K^2$  is always negative at  $\hat{t}_K$ . This means that the local optimal solution (without considering the nonnegative constraint) to the problem (14) is unique and is actually the global optimal solution  $t_K^*$ . Thus we conclude that  $t_K^* = \max(\hat{t}_K, 0)$ , where  $\hat{t}_K$  is the unique solution to (15) and can be computed numerically.

## 5 FEASIBLE CONTRACTS UNDER INCOMPLETE INFORMATION

In this section, we study the necessary and sufficient conditions for a feasible contract under incomplete information. This will help us derive optimal contracts in Sections 6 and 7.

A feasible contract includes  $K$  power-time items such that any type- $\theta_k$  SU prefers the contract item for its type, i.e.,  $(p_k, t_k)$ , to any other contract item. Under incomplete information, a feasible contract must satisfy both the individual rationality (IR) constraint in Definition 1 (introduced in Section 4) and the incentive compatibility (IC) constraint defined next.

*Definition 2:* [IC: Incentive Compatibility] A contract satisfies the IC constraint if for each type- $\theta_k$  SU, it prefers to choose the contract item for  $\theta_k$ , i.e.,

$$\theta_k t_k - p_k \geq \theta_k t_j - p_j, \forall k, j \in \mathcal{K}. \quad (17)$$

In summary, the PU's optimization problem is

$$\begin{aligned} & \max_{\{(p_k, t_k), \forall k \in \mathcal{K}\}} U_{\text{PU}}(\{(p_k, t_k), k \in \mathcal{K}\}), \quad (18) \\ & \text{subject to } \theta_k t_k - p_k \geq \theta_k t_j - p_j, \quad \forall k, j \in \mathcal{K}, \\ & \quad \theta_k t_k - p_k \geq 0, \quad \forall k \in \mathcal{K} \\ & \quad t_k \geq 0, p_k \geq 0, \quad \forall k \in \mathcal{K}. \end{aligned}$$

The first two constraints correspond to IC and IR constraints, respectively. It is worth noting that with IC and IR constraints, each SU will truthfully reveal its private type. Specifically, the IR constraint ensures that a type  $\theta_k$  SU will get a nonnegative payoff by choosing the  $k$ -th contract item. The IC constraint ensures that a type  $\theta_k$  SU will get the maximum payoff by choosing the  $k$ -th contract item. In other words, the IC constraint will incentivise SUs to truthfully reveal their types to the PU.<sup>10</sup>

### 5.1 Sufficient and Necessary Conditions for Feasibility

Next we provide several necessary and sufficient conditions for the contract feasibility. Let  $\Phi = \{(p_k, t_k), \forall k\}$  denote a feasible contract.

*Proposition 1:* [Necessary Condition 1] For any  $i, j \in \mathcal{K}$ ,  $p_i > p_j$  if and only if  $t_i > t_j$ .

The proof of Proposition 1 is given in Appendix B. This proposition shows that an SU contributing more in terms of received power at the PU receiver should receive more time allocation, and vice versa. From Proposition 1, we have the following corollary, saying that the same relay powers must have the same time allocations, and vice versa.

*Corollary 1:* For any  $i, j \in \mathcal{K}$ ,  $p_i = p_j$  if and only if  $t_i = t_j$ .

The following Proposition 2 shows the second necessary condition for contract feasibility.

*Proposition 2:* [Necessary Condition 2] For any  $i, j \in \mathcal{K}$ , if  $\theta_i > \theta_j$ , then  $t_i \geq t_j$ .

The proof of Proposition 2 is given in Appendix C. This proposition shows that a higher type SU should be allocated more transmission time. Combined with Proposition 1, we know that a higher type of SU should also contribute more in terms of PU's received power.

From Propositions 1 and 2, we conclude that for a feasible contract, all power-time combination (contract items) satisfy

$$0 \leq p_1 \leq p_2 \leq \dots \leq p_K, \quad 0 \leq t_1 \leq t_2 \leq \dots \leq t_K, \quad (19)$$

with  $p_k = p_{k+1}$  if and only if  $t_k = t_{k+1}$ .

The previous propositions help us obtain Theorem 2 as follows.

*Theorem 2:* [Sufficient and Necessary Conditions for Contract Feasibility]: For a contract  $\Phi = \{(p_k, t_k), \forall k\}$ , it is feasible if and only if all the following three conditions hold:

10. In fact, this contract belongs to the class of screening contract, and the main purpose of a screening contract is to elicit the private information of agents, so as to avoid the false reports of agents.

- Contd.a:  $0 \leq p_1 \leq p_2 \leq \dots \leq p_K$  and  $0 \leq t_1 \leq t_2 \leq \dots \leq t_K$ ;
- Contd.b:  $\theta_1 t_1 - p_1 \geq 0$ ;
- Contd.c: For any  $k = 2, 3, \dots, K$ ,  

$$p_{k-1} + \theta_{k-1}(t_k - t_{k-1}) \leq p_k \leq p_{k-1} + \theta_k(t_k - t_{k-1}). \quad (20)$$

The proof of Theorem 2 is given in Appendix D. The conditions in Theorem 2 are essential to the optimal contract design under weakly and strongly incomplete information in Sections 6 and 7.

## 6 OPTIMAL CONTRACT DESIGN UNDER WEAKLY INCOMPLETE INFORMATION

In this section, we will look at the weakly incomplete scenario where the PU does not know each SU's type, but knows the set of SUs' type (i.e.,  $\Theta$ ) and the number of each type (i.e.,  $N_k$  for any  $k \in \mathcal{K}$ ). Without loss of generality, we assume that  $N_k \geq 1$  for all  $k \in \mathcal{K}$ .<sup>11</sup> Different from the complete information case, here the PU cannot determine the optimal contract item by Lemma 1 for an SU since the PU does not know the SU's type. The major difference is that we need to consider IC constraint here.

A conceptually straightforward approach to derive the optimal contract is to solve (18) directly. Going through this route, however, is very challenging as (18) is a non-convex and involves complicated constraints. Here we adopt a sequential optimization approach instead: we first derive the best relay powers  $\{p_k^*(\{t_k, \forall k\}), \forall k\}$  given fixed feasible time allocations  $\{t_k, \forall k\}$ , then derive the best time allocations  $\{t_k^*, \forall k\}$  for the optimal contract, and finally show that there is no gap between the solution  $\{(p_k^*, t_k^*), \forall k\}$  obtained from this sequential approach and the one obtained by directly solving (18).

*Proposition 3:* Let  $\Phi = \{(p_k, t_k), \forall k\}$  be a feasible contract with fixed time allocations  $\{t_k, \forall k : 0 \leq t_1 \leq \dots \leq t_K\}$ . The optimal unique relay powers satisfy

$$\begin{aligned} p_1^*(\{t_k, \forall k\}) &= \theta_1 t_1, \\ p_k^*(\{t_k, \forall k\}) &= \theta_1 t_1 + \sum_{i=2}^k \theta_i (t_i - t_{i-1}), \quad \forall k = 2, \dots, K. \end{aligned} \quad (21)$$

The proof of this proposition is given in Appendix E. Using Proposition 3, we can simplify the PU's optimization problem in (18) as

$$\max_{\{t_k, \forall k \in \mathcal{K}\}} \frac{\frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{k \in \mathcal{K}} N_k (\theta_1 t_1 + \sum_{i=1}^k \theta_i (t_i - t_{i-1}))}{n_0} \right)}{1 + \sum_{k \in \mathcal{K}} N_k t_k}, \quad (22)$$

subject to  $0 \leq t_1 \leq \dots \leq t_K$ .

We can further simplify (22) using Theorem 3 below.

*Theorem 3:* In an optimal contract with weakly incomplete information, only the contract item for the highest SU type is positive and all other contract items are zero, i.e.,  $(p_K, t_K) > \mathbf{0}$  and  $(p_k, t_k) = \mathbf{0}, \forall k < K$ .

11. Otherwise we can remove type  $\theta_k$  from  $\Theta$  and solve a problem with  $K - 1$  types.

Theorem 3 can be proved in a similar way as Theorem 1 and we skip the proof here due to page limit. Following a similar analysis as in Appendix A, we can show that it may not be optimal for the PU to involve the highest type SUs if we impose additional constraints, e.g., SUs' transmission power constraints. However, even without such constraints, we will later show that the PU under strongly incomplete information needs to involve multiple types of SUs to maximize the expected utility.

Using Theorem 3, we can simplify the optimization problem in (22) further as

$$\max_{t_K \geq 0} \frac{1}{1 + N_K t_K} \left( \frac{R_k^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\theta_K N_K t_K}{n_0} \right) \right). \quad (23)$$

Notice that (23) under weakly incomplete information is the same as (13) under complete information. We thus conclude that our sequential optimization approach (first over  $\{p_k, \forall k\}$  and then over  $\{t_k, \forall k\}$ ) results in no loss in optimality, since it achieves the same maximum utility as in the complete information scenario.

To solve problem (23), we can use an efficient one-dimensional exhaustive search algorithm to find the global optimal solution  $t_K^*$ . We will provide numerical results in Section 8.

## 7 OPTIMAL CONTRACT DESIGN UNDER STRONGLY INCOMPLETE INFORMATION

In this section, we study the strongly incomplete information scenario, where the PU does not even know the number of each type. The PU only knows the total number of SUs  $N$  and the probability  $q_k$  of any SU belonging to type- $\theta_k$ . Obviously  $q_k \in [0, 1]$  and  $\sum_{k \in \mathcal{K}} q_k = 1$ .

Similar to the weakly incomplete information scenario, here the PU needs to consider the IC constraint since it cannot determine the optimal contract item by Lemma 1 for an SU as well. The difference from Section 6 is that here the PU does not know which type is the highest type among all SUs in the current system and how many SUs are in the highest type, and thus the simple approach of only providing a positive contract item for type  $\theta_K$  as in Theorem 3 may not be optimal. If the PU does that and it turns out that  $N_K = 0$  in the current system, then there will be no SUs participating in the cooperative communications.

The proper target for the PU is design a contract to maximize the *expected* utility subject to the IC and IR constraints. As the PU knows the total number of SUs  $N$ , then the probability density function of the number of SUs  $\{N_k, \forall k\}$  is

$$\begin{aligned} & Q_{(n_1, \dots, n_{K-1})} \\ & := \Pr(N_1 = n_1, \dots, N_{K-1} = n_{K-1}, N_K = N - \sum_{i=1}^{K-1} N_i) \\ & = \frac{N!}{n_1! \dots n_{K-1}! (N - \sum_{i=1}^{K-1} n_i)!} q_1^{n_1} \dots q_{K-1}^{n_{K-1}} q_K^{N - \sum_{i=1}^{K-1} n_i}. \end{aligned} \quad (26)$$

The PU's optimization problem can be written in (24) subject to the IC and IR constraints.

Similar to Section 6, here we adopt a sequential optimization approach: we first derive the optimal relay powers  $\{p_k^*(\{t_k, \forall k\}), \forall k\}$  with fixed feasible time allocations  $\{t_k, \forall k\}$ , then derive the optimal time allocations  $\{t_k^*, \forall k\}$  for the optimal contract. The difference is that optimality is no longer guaranteed here as explained later.

*Proposition 4:* Let  $\Phi = \{(p_k, t_k), \forall k\}$  be a feasible contract with fixed time allocations  $\{t_k, \forall k : 0 \leq t_1 \leq \dots \leq t_K\}$ , then the unique optimal relay powers satisfy

$$\begin{aligned} p_1^*(\{t_k, \forall k\}) &= \theta_1 t_1, \\ p_k^*(\{t_k, \forall k\}) &= \theta_1 t_1 + \sum_{i=2}^k \theta_i (t_i - t_{i-1}), \forall k = 2, \dots, K. \end{aligned} \quad (27)$$

Notice that Proposition 4 under strongly incomplete information is actually the same as Proposition 3 under weakly incomplete information. Proposition 4 can be proved in a similar manner as Proposition 3, using the fact that PU's expected utility is increasing in  $\{p_k, \forall k\}$ . The proof is omitted here due to space limit. Based on Proposition 4, we can simplify the PU's optimization problem in (24) as (25).

Note that (25) is a non-convex optimization problem and all  $K$  variables are coupled in the objective function. Furthermore, the number of terms in the objective increases exponentially with the number of types  $K$ . Thus it is hard to solve efficiently.

Next, we propose a low computation complexity approximate algorithm, Decompose-and-Compare algorithm, to compute a close-to-optimal solution to (25) efficiently. In this algorithm, we will compare  $K$  simple candidate contracts, and pick the one that yields the largest utility for the PU. There are two key ideas behind this heuristic algorithm.

- *One positive contract item per contract:* In each of the  $K$  individually optimized candidate contracts, there is only one positive contract item (for one or more types). For the contract item in the  $k$ th candidate contract, this positive contract item is offered to SUs with types equal to or larger than type- $\theta_k$ . In other words, optimization of each candidate contract only involves a scalar optimization, instead of  $K$  variables as in (25).
- *A tradeoff between efficiency and uncertainty:* Among  $K$  optimized candidate contracts, we will pick the best one that achieves the best tradeoff between efficiency and uncertainty (i.e., maximizing the PU's expected utility). Under strongly incomplete information, it is not clear which type is the highest among all SUs existing in the current system. If a candidate contract offers the same positive contract item for types equal to or larger than  $\theta_k$ , then all SUs in these types will choose to accept that contract item. The corresponding SU's payoff is increasing in

$$\max_{\{(p_k, t_k), \forall k\}} \sum_{n_1=0}^N \sum_{n_2=0}^{N-n_1} \dots \sum_{n_{K-1}=0}^{N-\sum_{i=1}^{K-2} n_i} Q_{(n_1, \dots, n_{K-1})} \left( \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{i=1}^{K-1} n_i p_i + (N - \sum_{i=1}^{K-1} n_i) p_K}{n_0} \right) \right) \quad (24)$$

subject to, IC and IR constraints

$$\max_{\{t_k, \forall k\}} \sum_{n_1=0}^N \sum_{n_2=0}^{N-n_1} \dots \sum_{n_{K-1}=0}^{N-\sum_{i=1}^{K-2} n_i} Q_{(n_1, \dots, n_{K-1})} \left( \frac{R^{dir}}{2} + \frac{1}{2} \log \left( 1 + \frac{\sum_{i=1}^{K-1} n_i p_i^* (\{t_k, \forall k\}) + (N - \sum_{i=1}^{K-1} n_i) p_K^* (\{t_k, \forall k\})}{n_0} \right) \right) \quad (25)$$

subject to,  $0 \leq t_1 \leq \dots \leq t_K$ .

type, i.e., a type  $\theta_k$  SU receives zero payoff and a type- $\theta_K$  SU receives the maximum positive payoff. Thus choosing a candidate contract with a threshold  $\theta_k$  too low will give too much payoffs to the SUs (and thus reduce the PU's expected utility), but choosing a candidate contract with a threshold too high might lead to the undesirable case that no SUs are willing to participate. This requires us to examine all possibilities (i.e.,  $K$  candidate contracts) and pick the one with the best performance.

The Decompose-and-Compare algorithm works as follows.

- 1) *Decomposition*: Construct  $K$  candidate contracts, where for the  $k$ th contract,
  - Offers the same contract item  $t_k > 0$  to SUs with a type equal to or larger than type  $\theta_k$  (called *critical type*), and zero for the SUs below the critical type. That is,  $t_1 = t_2 = \dots = t_{k-1} = 0$  and  $t_k = t_{k+1} = \dots = t_K$ .
  - Computes the optimal  $t_k^*$  that maximizes PU's expected utility in (25) under the constraints of  $t_1 = t_2 = \dots = t_{k-1} = 0$  and  $t_k = t_{k+1} = \dots = t_K$ . The corresponding relay power is  $p_k^* = \theta_k t_k^*$  as specified in the contract.
- 2) *Comparison*: Choose the best contract out of the  $K$  candidates to maximize the PU's expected utility.

Intuitively, we can view each candidate contract (corresponding to a critical type  $\theta_k$ ) in the above algorithm as a simple *take-it-or-leave contract* (with time allocation  $t_k$  and relay power  $p_k = \theta_k t_k$ ). Obviously, SUs with types higher than or equal to  $\theta_k$  will accept the contract, while SUs with types lower than  $\theta_k$  will refuse the contract. The decompose-and-compare algorithm computes all  $K$  optimal take-it-or-leave contracts, and chooses the best one that maximizes the expected revenue.

Now we analyze the computational complexity of the above decompose-and-compare algorithm. Without such an algorithm, we can simply use exhaustive search to optimally solve the non-convex optimization Problem (25). With such an algorithm, we can still use exhaustive search to solve each of  $K$  subproblems to derive a candidate contract. To make a fair comparison between the complexities with and without this algorithm, we adopt the basic exhaustive search for complexity measurement. Let us denote the possible range of  $t_k$  as  $[0, W]$ . We approximate the continuity of this range through a

proper discretization, i.e., representing all possibilities of any  $t_k$  by  $T$  equally spaced values (with the first and last values equal to 0 and  $W$ , respectively). Then, with the optimal contract design, we need to search over all possible time allocation ranges for  $K$  types jointly (i.e.,  $\{t_k, \forall k \in \mathcal{K}\}$ ) to directly solve Problem (25), and the computation complexity is  $\mathcal{O}(T^K)$ . As we use the decompose-and-compare algorithm, we decompose the complex Problem (25) into  $K$  simple sub-problems, each associated with one critical type. The computation complexity for each sub-problem is  $\mathcal{O}(T)$ , and thus the overall computation complexity is  $\mathcal{O}(K \cdot T)$ .

Unlike complete or weakly incomplete information, here the computational complexity increases with the number of SU types. This is because that the PU may want to involve more than one type of SUs in the contract to mitigate the uncertainty and avoid risk of having no relay in a particular network realisation. However, the complexity of this approach will increase with the number of types.

In Section 8, we show by numerical results that the proposed Decompose-and-Compare algorithm achieves a performance very close to the optimal solution to (25) in most cases.

## 8 NUMERICAL RESULTS

### 8.1 Complete and Weak Incomplete Information Scenarios

As shown in Sections 4 and 6, the optimal contract is the same for complete and weakly incomplete information scenarios. By examining the PU's optimization in (23) that applies to both scenarios, we have the following observations.

*Observation 1*: The PU's optimal utility is increasing in the highest SU type  $\theta_K$  and the PU's direct transmission rate  $R^{dir}$ .

Figure 2 shows that PU's utility under the optimal contract is increasing in  $\theta_K$  and  $R^{dir}$ .<sup>12</sup> We also plot the dotted baseline that represents the rate  $R^{dir}$  achieved by direct transmission only. As  $R^{dir}$  increases, the PU has less incentive to share spectrum with the SUs. When  $R^{dir}$  is very large, the PU chooses not to use SUs at all, which corresponds to *No Relay Region* in Fig. 2 (where the three curves with different  $\theta_K$ 's are below the baseline).

12. Without loss of generality, we normalize  $n_0$  to be 1 in the rest of this paper.

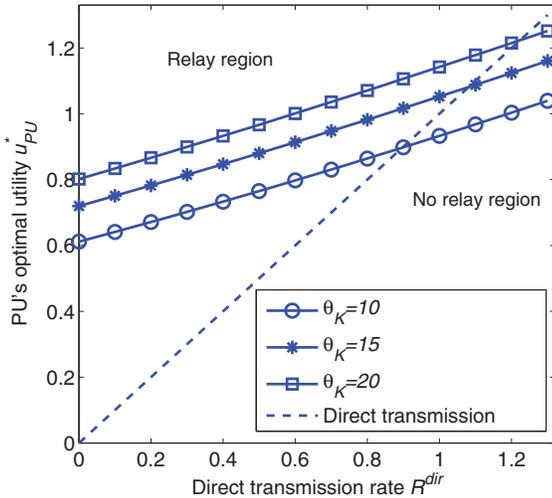


Fig. 2. PU's optimal utility  $U_{PU}^*$  as a function of the highest type  $\theta_K$  and direct transmission rate  $R^{dir}$ . The dotted 45 degree line divides the figure into two regions: Relay Region and No Relay Region. When the maximum utility achievable under the optimal contract (by using the largest type  $\theta_K$ ) falls into the No Relay Region, the PU will choose direct transmission only.

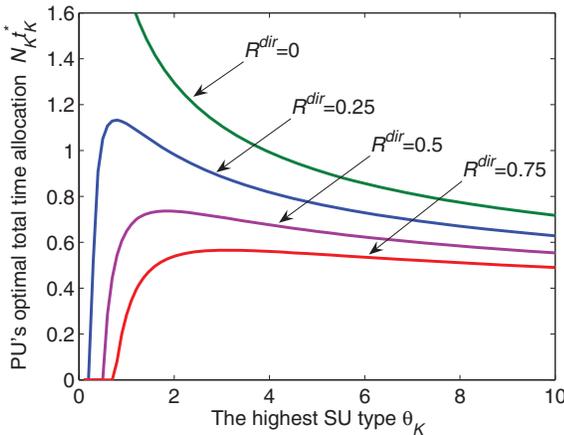


Fig. 3. PU's optimal total time allocation  $N_{Kt_K}^*$  as a function of the PU's direct transmission rate  $R^{dir}$  and the highest type  $\theta_K$ .

For the rest of the numerical results, we will only examine the PU's choice of optimal contract, without restating the need to compare with  $R^{dir}$  and choose direct transmission only if needed.

*Observation 2:* Figure 3 shows that the PU's optimal total time allocation to the highest type- $\theta_K$  SUs,  $N_{Kt_K}^*$ , decreases in PU's direct transmission rate  $R^{dir}$ . When  $R^{dir} = 0$ , the PU's optimal total time allocation is strictly decreasing in  $\theta_K$ ; when  $R^{dir} > 0$ , the total time allocation first increases in  $\theta_K$  and then decreases in  $\theta_K$ .

This can be explained as follows. When  $R^{dir} = 0$ , the PU can only rely on SUs for transmissions and will

always allocate positive transmission time to the highest type- $\theta_K$  SUs. If we look at the PU's utility in (23) with  $R^{dir} = 0$ , the logarithmic term  $\log(1 + \theta N_K t_K^*)$  plays a more important role than the scaling term  $\frac{1}{1 + N_K t_K^*}$  in this case. When  $\theta_K$  is small, the PU needs to allocate a large amount time to the SUs to achieve its desirable rate. When  $\theta_K$  becomes large, the PU can reach a high relay rate by allocating less transmission time to the SUs. This explains why we observe a decrease of  $N_K t_K^*$  in  $\theta_K$ . Appendix F provides a rigorous proof of Observation 2 under  $R^{dir} = 0$ .

When  $R^{dir} > 0$  (the lower three curves in Fig. 3), the PU has less incentive to allocate transmission time to the SUs, especially when the highest SU type- $\theta_K$  is small. As  $\theta_K$  becomes large, PU is willing to allocate more time in exchange of efficient help from SUs. As  $\theta_K$  becomes very large, the PU only needs to allocate a small amount of time to the SUs in order to obtain enough relay help. This explains why the lower three curves in Fig. 3 first increase and then decrease in  $\theta_K$ .

## 8.2 Strong Incomplete Information Scenario

Here we show how PU design the contract to maximize its expected utility. As a performance benchmark, we first compute the optimal solution to the PU's expected utility maximization problem in (25) via an  $K$ -dimensional exhaustive search. We denote the corresponding optimal solution as  $E[U_{PU}]^*$ . Notice that  $E[U_{PU}]^*$  is often smaller than the maximum utility achieved under complete information. The performance gap is due to the strongly incomplete information. Next, we will compare the PU's expected utility achieved by the proposed Decompose-and-Compare algorithm with  $E[U_{PU}]^*$ .

For illustration purposes, we consider only two types of SUs:  $\theta_1 < \theta_2$ . The PU only knows the total number of SUs  $N$  and the probabilities  $q_1$  and  $q_2$  of two types, with  $q_1 + q_2 = 1$ .

In the Decompose-and-Compare algorithm, we first consider two candidate contracts. The first candidate contract optimizes the same positive contract item  $t_1 = t_2 > 0$  for both types. The PU's corresponding maximum expected utility is  $E[U_{PU}]^{1-2}$ . The second candidate contract sets  $t_1 = 0$  and optimizes the positive contract item  $t_2 > 0$ . The PU's corresponding maximum expected utility is  $E[U_{PU}]^2$ . Then we pick the candidate contract that leads to a larger PU's expected utility as the solution of the Decompose-and-Compare algorithm.

To evaluate the performance of the algorithm, we consider two different parameter regimes.

### 8.2.1 Large $q_1^N$

This means that the probability that all SUs belong to the low type- $\theta_1$  is large. This happens when the total number of SUs  $N$  is small and probability  $q_1$  is large. Figure 4 shows the PU's expected utility obtained

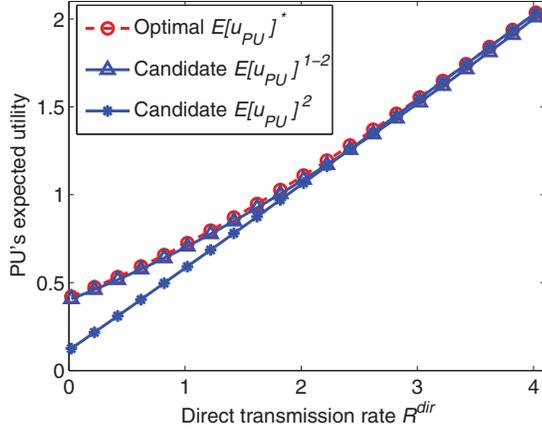


Fig. 4. Comparison between PU's optimal expected utility using optimal exhaustive search method ( $E[U_{PU}]^*$ ) and the two candidate contracts of the Decompose-and-Compare algorithm, as a function of the PU's direct transmission rate  $R^{dir}$ . We set  $q_1 = 0.9$ ,  $N = 2$ ,  $\theta_1 = 4$ , and  $\theta_2 = 10$ .

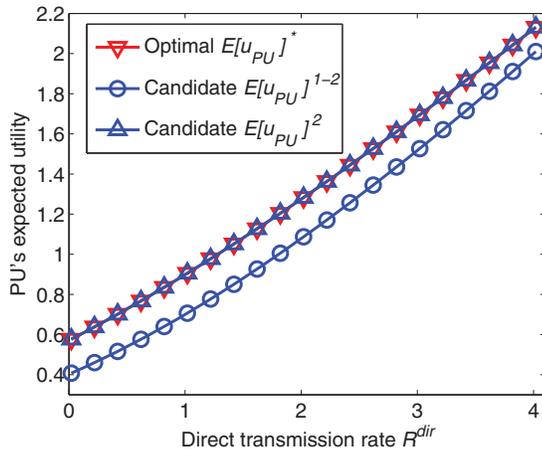


Fig. 5. Comparison between PU's optimal expected utility using optimal exhaustive search method ( $E[U_{PU}]^*$ ) and the two candidate contracts of the Decompose-and-Compare algorithm, as a function of the PU's direct transmission rate  $R^{dir}$ . We set  $q_1 = 0.5$ ,  $N = 5$ ,  $\theta_1 = 4$ , and  $\theta_2 = 10$ .

with the two candidate contracts of the Decompose-and-Compare algorithm ( $E[U_{PU}]^{1-2}$  and  $E[U_{PU}]^2$ ) and the optimal exhaustive search method ( $E[U_{PU}]^*$ ), as functions of PU's direct transmission rate  $R^{dir}$ . We can see that the candidate contract that offers the same positive contract items to both types (i.e.,  $E[U_{PU}]^{1-2}$ ) achieves a close-to-optimal performance with all values of  $R^{dir}$  simulated here. This is because very often the PU needs to rely on the low type- $\theta_1$  SUs to relay its traffic.

We also notice that the candidate contract that offers a positive contract item to the high type SU (i.e.,  $E[U_{PU}]^2$ ) also achieves a close-to-optimal performance (even larg-

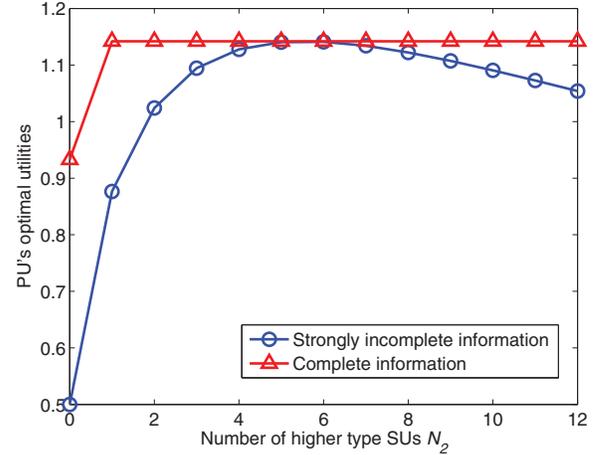


Fig. 6. Comparison between PU's optimal utilities for different user number realizations under different information scenarios. We set  $q_1 = 0.5$ ,  $N = 12$ ,  $R^{dir} = 1$ ,  $\theta_1 = 10$ , and  $\theta_2 = 20$ .

er than  $E[U_{PU}]^{1-2}$ ) when  $R^{dir}$  is large. This is because the PU with a large  $R^{dir}$  relies less on the SUs, and will have more incentive to employ only the high type SUs when they are available.

### 8.2.2 Small $q_1^N$

This means that the probability that at least one SU belongs to the high type- $\theta_2$  (i.e.,  $1 - q_1^N$ ) is large. Figure 5 shows the PU's expected utility obtained from Decompose-and-Compare algorithm and the optimal exhaustive search method as a function of  $R^{dir}$ . We can see that  $E[U_{PU}]^2$  is always better than  $E[U_{PU}]^{1-2}$  and achieves a close-to-optimal performance under all choices of  $R^{dir}$ . This is because very often the PU can find a high type  $\theta_2$  SU to relay its traffic.

*Observation 3:* The performance of the proposed Decompose-and-Compare algorithm achieves a close-to-optimal performance (i.e., less than 2% according to Fig. 4 and Fig. 5) under the strongly incomplete information.<sup>13</sup>

Next we study how the strongly incomplete information reduces PU's utility comparing with the complete information benchmark. First, we note that PU's contract in the strongly incomplete information scenario does not depend on  $N_1$  and  $N_2$ , as the PU targets at optimizing the expected rate and does not know the realization information. However, the actual PU's utility (not the expected value) does depend on  $N_1$  and  $N_2$ . Figure 6 shows the PU's utility under different information scenarios and different user number realizations (i.e., any realization of  $N_1$  and  $N_2$  for a fixed  $N_1 + N_2 = N$ ). Here,  $q_1^N$  is small and the curve with strongly incomplete information corresponds to offering  $t_2 > 0$  and

13. Due to space limit, we only discuss two-type case here. Similar results can be obtained for cases of more than two types.

$t_1 = 0$  (as in Fig. 5). The (very close to) optimal contract under strongly incomplete information can be obtained by using the Decompose-and-Compare algorithm. The optimal contract under complete information changes as  $N_1$  and  $N_2$  change.

*Observation 4:* Figure 6 shows that the PU's optimal utility under strongly incomplete information achieves the maximum value (close to the one under complete information) when the realized SU numbers is close to the expected value (i.e.,  $N_1 = N_2 = 6$  in this example), as the PU determines the contract for SUs in the expected sense.

In Fig. 6, the largest performance gap between the two curves happens when  $N_2 = 0$ . In this case, the optimal contract under complete information satisfies  $t_1 > 0$  and  $t_2 = 0$ , as there are no type- $\theta_2$  users. However, the PU under strongly incomplete information chooses  $t_1 = 0$  and  $t_2 > 0$  to maximize the PU's expected utility. Such mismatch means that the PU under strongly incomplete information has no SUs serving as relays. However, this parameter setting only happens with a very small probability  $q_1^N = 0.5^{12} \approx 2.4 \times 10^{-4}$ .

Another useful comparison is the PU's *average* utility loss due to strongly incomplete information. We first compute the PU's optimal average utility under strongly incomplete information, which is the solution of (25) (via a  $K$ -dimensional exhaustive search rather than the Decompose-and-Compare algorithm). Then we can compute the PU's average utility under complete information by calculating the weighted sum (weighted by the probability of each parameter ( $N_1, N_2$ )) of the 13 values on the upper curve in Fig. 6.<sup>14</sup> In this example, the ratio is 0.9874. This means that the PU's average utility loss due to strongly incomplete information is very small (i.e., less than 1.3%).

## 9 CONCLUSION AND FUTURE WORK

We study the cooperative spectrum sharing between one PU and multiple SUs, where the SUs' types are private information. We model the network as a monopoly market, in which the PU offers the contract and each SU selects the best contract item according to its type. We study the optimal contract design for multiple information scenarios. We first provide the necessary and sufficient conditions for feasible contracts under incomplete information. For the weakly incomplete information scenario, we derive the optimal contract that achieves the same PU's utility as in the complete information benchmark. For the strongly incomplete information scenario, we propose a Decompose-and-Compare algorithm that achieves a close-to-optimal PU's expected utility. Both the PU's average utility loss due to the suboptimal algorithm and the strongly incomplete information are

14. With complete information, the PU's optimal utility in a time slot depends on the number of users in each class and may not be the same all the time (see  $N_2 = 0$  and  $N_2 \geq 1$  in Fig. 6).

small in our numerical example (less than 2% and 1.3% in our numerical results with two SU types).

This work represents an important step towards establishing a general framework of understanding incomplete information in dynamic spectrum sharing. As the next step, we will consider more incomplete information structures: (1) the PU does not know the distribution of SUs' types, and (2) each SU knows other SUs' types but PU does not know (a similar setting with a different application has been studied in [27]).

We also want to understand how to design incentive compatible mechanisms in a market with multiple PUs and multiple SUs. The mechanism proposed in this paper can be viewed as a sub-optimal solution in the multiple PU scenario. More precisely, we can imagine the scenario where the number of SUs is much larger than that of PUs. In this case, we can divide the whole network into multiple sub-networks: each sub-network consists of one PU (as a monopolist) and a set of nearby SUs. Our proposed contract approach can work in each of the sub-network. Of course, the way that we divides the network will affect the performance loss of the sub-optimal algorithm. In an optimal trading scheme between multiple PUs and multiple SUs, a PU needs to consider not only the information asymmetry (between the PU and SUs), but also the asymmetric information and potential competition of other PUs. This will make the optimal contract design much more difficult.

Finally, we plan to study the contact design for continuous SU type distributions where each type  $\theta_k$  can be a continuous random variable depending on the realization of the channel coefficient. Then the contract would become no longer a discrete vector (with each element corresponding to an SU-type), but a continuous function of SU types [36].

## 10 NOTICE FOR APPENDICES OF PROOFS

Most proofs for this paper in Appendices are removed to another separate supplemental file along with this submission, according to TMC new requirement on Supplemental Material. These appendices can also be found in our online technical report [32].

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