DORA: Dynamic Optimal Random Access for Vehicle-to-Roadside Communications

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Abstract—In this paper, we study random access in a drive-thru scenario, where roadside access points (APs) are installed on a highway to provide temporary Internet access for vehicles. We consider vehicle-to-roadside (V2R) communications for a vehicle that aims to upload a file when it is within the APs' coverage ranges, where both the channel contention level and transmission data rate vary over time. The vehicle will pay a fixed amount each time it tries to access the APs, and will incur a penalty if it cannot finish the file uploading when leaving the APs. First, we consider the problem of finding the optimal transmission policy with a single AP and random vehicular traffic arrivals. We formulate it as a finite-horizon sequential decision problem, solve it using dynamic programming (DP), and design a general dynamic optimal random access (DORA) algorithm. We derive the conditions under which the optimal transmission policy has a threshold structure, and propose a monotone DORA algorithm with a lower computational complexity for this special case. Next, we consider the problem of finding the optimal transmission policy with multiple APs and deterministic vehicular traffic arrivals thanks to perfect traffic estimation. We again obtain the optimal transmission policy using DP and propose a joint DORA algorithm. Simulation results based on a realistic vehicular traffic model show that our proposed algorithms achieve the minimal total cost and the highest upload ratio as compared with some other heuristic schemes. In particular, we show that the joint DORA scheme achieves an upload ratio 130% and 207% better than the heuristic schemes at low and high traffic densities, respectively.

Index Terms—Random access, medium access control, vehicular ad hoc networks, dynamic programming, Markov decision processes, threshold policy.

I. INTRODUCTION

VEHICULAR ad hoc networks (VANETs) enable autonomous data exchanges among vehicles and roadside access points (APs), and are essential to various intelligent transportation system (ITS) applications. For example, safety applications (such as cooperative forward collision warning, lane change warning, and left turn assistant [1], [2]) have been

proposed to improve the safety of the passengers by informing the vehicles of potential dangers ahead of time. Non-safety applications (such as traffic management, instant messaging, and media content delivery) have been designed to avoid traffic congestion and improve the experience of driving. Clearly, the quality of service (QoS) requirements of various applications are different.

VANETs support various ITS applications through different types of communication mechanisms, including vehicle-to-roadside (V2R) and vehicle-to-vehicle (V2V) communications [3]. V2R communications involve data transmissions between vehicular nodes and roadside APs. V2V communications only involve data exchanges among vehicular nodes. For both types, we can further classify the communications as either single-hop or multi-hop. In this paper, we focus on analyzing *V2R single-hop uplink transmissions* from vehicles to APs. Due to the limited communication opportunities between vehicles and APs, efficient resource allocation (either centralized or distributed) is crucial for the successful deployment of V2R ITS applications.

In the *centralized* setting, the AP schedules the transmissions from different vehicles based on some predefined criteria. Hadaller *et al.* in [4] proposed a scheduling protocol that grants channel access to a vehicle that achieves the maximum transmission rate. Analytical and simulation results showed significant overall system throughput improvement over a benchmark scheme. Zhang *et al.* in [5] considered the case where roadside APs only store the data uploaded by the vehicles. Scheduling priority is determined by two factors: data size and deadline. A request with either a smaller data size or an earlier deadline will be served first. Alcaraz *et al.* in [6] considered both uplink and downlink scheduling of non-real-time traffic for non-safety applications. The scheduling problem was formulated as a constrained linear quadratic regulator design problem that aims to reduce the residual queue backlog for each user. However, because centralized resource allocation is not scalable due to its computational complexity, we focus on distributed resource allocation scheme in this paper.

In the *distributed* setting, the vehicles contend for the channel for transmission based on the applications' QoS requirements. Shrestha *et al.* in [7] considered the scenario where the data packets are first distributed from the roadside units (RSUs) to the onboard units (OBUs). The OBUs then bargain with each other for the missing data packets, and exchange them using BitTorrent protocol. Jarupan *et al.* in [8] proposed a cross-layer protocol for V2R multi-hop communication. The medium access control (MAC) module collects information of

local data traffic, and the routing module finds a path with the minimum delay. Niyato *et al.* in [9] proposed a hierarchical optimization framework for downlink data streaming in V2R communications. The optimal pricing and bandwidth reservation of a service provider is obtained using game theory, and the optimal download policy of an OBU is obtained using constrained Markov decision processes. Tan *et al.* in [10] analyzed the performance of a downlink resource allocation scheme in a V2R communication system with one AP on a road. The distribution of the number of bytes downloaded per drive-thru was derived using Markov reward processes. Roman *et al.* in [11] proposed a cross-layer protocol in the physical and MAC layers that addresses the issues of channel fading, synchronization, and channel contention. Performance analysis was presented for the channel contention scheme, and a testbed was used to evaluate the proposed protocol.

In this paper, we aim to design a *uplink* random access algorithm that is *distributed* in nature, so that it is compatible with the IEEE 802.11p standard that is developed to facilitate the provision of wireless access in vehicular environment [12], [13]. Different from most previous works on *heuristic* distributed uplink V2R communication algorithm design, we aim at designing an *optimal* uplink resource allocation scheme in VANETs analytically in this paper.

In this work, we consider the drive-thru scenario [14], where vehicles pass by several APs located along a highway and obtain Internet access for only a limited amount of time. We assume that a vehicle wants to upload a file when it is within the coverage ranges of the APs, and needs to pay for the attempts to access the channel. As both the channel contention level and achievable data rate vary over time, the vehicle needs to decide when to transmit by taking into account the required payment, the application's QoS requirement, and the level of contention in current and future time slots. Because of the dynamic nature of the problem, we formulate it as a finite-horizon sequential decision problem and solve it using the dynamic programming (DP).

The main contributions of our work are as follows:

- *Optimal Access Policy Design*: In the case of a single AP with random vehicular traffic, we propose a general dynamic optimal random access (DORA) algorithm to compute the optimal access policy. We further extend the results to the case of multiple consecutive APs and propose a joint DORA (JDORA) algorithm to compute the optimal policy.
- *Low Complexity Algorithm*: We consider a special yet practically important case of a single AP with constant data rate. We show that the optimal policy in this case has a threshold structure, which motivates us to propose a low complexity and efficient monotone DORA algorithm.
- *Superior Performance*: Extensive simulation results show that our proposed algorithms achieve the minimal total cost and the highest upload ratio as compared with three other heuristic schemes. In the multi-AP scenario, the performance improvements in upload ratio of the JDORA scheme are 130% and 207% at low and high traffic densities, respectively.

The rest of the paper is organized as follows. We describe our system model in Section II and formulate the DP problem
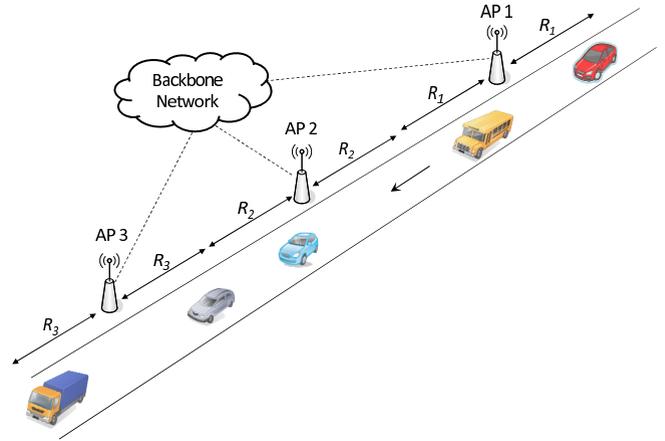

Fig. 1. Drive-thru vehicle-to-roadside (V2R) communications with multiple APs.

in Section III. The general and monotone DORA algorithms for single AP are proposed in Section IV-A, and the JDORA algorithm for multiple APs is discussed in Section IV-B. Simulation results are given in Section V, and the paper is concluded in Section VI.

II. SYSTEM MODEL

We consider a drive-thru scenario on a highway as shown in Fig. 1, where multiple APs are installed and connected to a backbone network to provide Internet services to vehicles within their coverage ranges. We focus on a vehicle that wants to upload a *single* file of size S when it moves through a segment of this highway with a set of APs $\mathcal{J} = \{1, \ldots, J\}$, where the vehicles pass through the ith AP before the jth AP for $i < j$ with $i, j \in \mathcal{J}$. We assume that the jth AP has a transmission radius R_j. We also assume that the vehicle is connected to at most one AP at a time. If the coverage areas of the APs are overlapping, then proper handover between the APs will be performed [15]. For the ease of exposition, we assume that the APs are set up in a way that any position in this segment of highway is covered by an AP. Our work can easily be extended to consider the settings where the coverage areas of adjacent APs are isolated from each other.

A. Traffic Model

Let λ denote the average number of vehicles passing by a fixed AP per unit time. We assume that the number of vehicles moving into this segment of the highway follows a Poisson process [16] with a mean arrival rate λ. Let ρ denote the vehicle density representing the number of vehicles per unit distance along the road segment, and ν be the speed of the vehicles. From [17], we have

$$\lambda = \rho \nu. \tag{1}$$

The relation between the vehicle density ρ and speed ν is given by the following equation [17]:

$$\nu = \nu_f (1 - \rho/\rho_{max}), \tag{2}$$

where ν_f is the free-flow speed when the vehicle is moving on the road without any other vehicles, and ρ_{max} is the vehicle density during traffic jam.
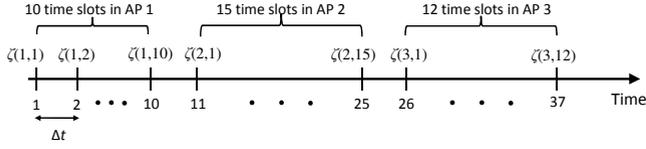
Fig. 2. An example of the time line representation for the events happened with three APs (i.e., $\mathcal{J} = \{1, 2, 3\}$). Here, we assume that $T_1 = 10$, $T_2 = 15$, and $T_3 = 12$. With respect to the time line, we have $\mathcal{T}_1 = \{1, \ldots, 10\}$, $\mathcal{T}_2 = \{11, \ldots, 25\}$, and $\mathcal{T}_3 = \{26, \ldots, 37\}$. It is clear from the figure that $\zeta(j, \tau) = \sum_{i=0}^{j-1} T_i + \tau$, $\forall \tau \in \{1, \ldots, T_j\}$, where $T_0 = 0$.

As we are studying the traffic flow in *steady state*, all the vehicles within the coverage range are assumed to move with the same speed ν in (2). Let $\lfloor \cdot \rfloor$ denote the floor function. The maximum number of vehicles that can be accommodated within the coverage range of the jth AP is given by

$$N_{max,j} = \lfloor 2R_j \rho_{max} \rfloor, \quad \forall j \in \mathcal{J}. \tag{3}$$

B. Channel Model

Wireless signal propagations suffer from path loss, shadowing, and fading. Since the distance between the vehicle and the AP varies in the drive-thru scenario, we focus on the dominant effect of channel attenuation due to path loss. The data rate at time slot t is given by

$$w_t = W \log_2 \left(1 + \frac{P}{N_0 W d_t^\gamma} \right), \tag{4}$$

where W is the channel bandwidth, P is the transmit power of the vehicle, d_t is the distance between the vehicle and the closest AP at time slot t, and γ is the path loss exponent. We assume that the additive white Gaussian noise has a zero mean and a power spectral density $N_0/2$. In addition, we also consider a special case with fixed data rate in Section IV-A1.

C. Distributed Medium Access Control (MAC)

We consider a slotted MAC protocol, where time is divided into equal time slots of length Δt. We assume that there is perfect synchronization between the APs and the vehicles with the use of global positioning system (GPS) [18]. The total number of time slots that the vehicle stays within the coverage range of the jth AP is $T_j = \left\lfloor \frac{2R_j}{\nu \Delta t} \right\rfloor$. We use the notation $\zeta(j, \tau)$ to denote the τth time slot when the vehicle is in the coverage area of the jth AP, i.e.,

$$\zeta(j, \tau) = \sum_{i=0}^{j-1} T_i + \tau, \quad \forall \tau \in \{1, \ldots, T_j\}, \tag{5}$$

where $T_0 = 0$. The set of time slots in the jth AP with respect to this time line representation is $\mathcal{T}_j = \{\zeta(j, 1), \ldots, \zeta(j, T_j)\}$. An example of the time line representation is given in Fig. 2.

When the vehicle first *enters the coverage range* of the jth AP, it declares the type of its application to the AP. In return, the jth AP informs the vehicle the channel contention in the coverage range (λ and $p_t^{succ}, \forall t \in \mathcal{T}_j$), data rate in all the time slots in the jth coverage range (i.e., $w_t, \forall t \in \mathcal{T}_j$), the price q_j, and the estimated number of vehicle departures from
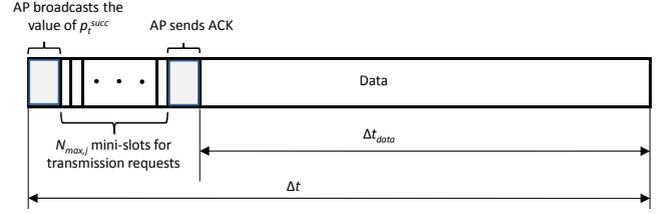

Fig. 3. The structure of a time slot of the jth AP.

the coverage range in all the time slots in the jth coverage range (i.e., $l_t, \forall t \in \mathcal{T}_j$). We further elaborate these system parameters as follows:

- p_t^{succ} represents the probability that the vehicle can successfully obtain access in time slot $t \in \mathcal{T}_j$ after contending with all the vehicles in the jth coverage range. p_t^{succ} is estimated by the AP based on the level of system contention and it varies over time. Since p_t^{succ} is related to the number of vehicles n_t currently in the jth coverage range at time slot t, we define $p_t^{succ} = g_j(n_t)$, where g_j is a strictly decreasing function. An AP knows the value of n_t, since vehicles need to establish and terminate their connections when they enter and leave the coverage range, respectively.

- $q_j \geq 0$ denotes the amount a vehicle needs to pay the AP for each time slot that it sends a transmission request in the jth coverage range, even it fails to access the channel. The value of q_j does not change over time.

- l_t represents the number of vehicle departing at time slot $t \in \mathcal{T}_j$ from the jth coverage range. Since all the vehicles move with constant speed ν in the traffic model, we assume that ($l_t, \forall t \in \mathcal{T}_j$) are accurately known by the jth AP, and are sent to the vehicle when it enters the coverage range.

In each time slot $t \in \mathcal{T}_j$ in the jth coverage range, the jth AP first broadcasts the value of p_t^{succ} to all the vehicles in its coverage range. If a vehicle decides to transmit within this time slot, it sends a request to the jth AP at its scheduled mini-slot, where $N_{max,j}$ mini-slots are reserved for transmission requests. The transmissions of requests are thus collision-free. After collecting the requests from all vehicles in its coverage range, the jth AP assigns the time slot to one of these vehicles. The vehicle, which receives the acknowledgement (ACK), can transmit the data packets in the remaining time Δt_{data} of this time slot, where $\Delta t_{data} < \Delta t$. The structure of a time slot is shown in Fig. 3.

Meanwhile, regardless of which vehicle is granted the time slot, each vehicle which requested to transmit in the time slot needs to pay q_j to the jth AP. Without such pricing, each vehicle would send a request in every time slot, which unnecessarily increases the contention level and prevents efficient allocation of time slots to the most needed application.

The vehicle aims to achieve a good *tradeoff* between the total uploaded file size and the total payment to the APs according to the QoS requirement of the application. For example, a higher priority may be placed on the total uploaded file size for safety applications, but on the total payment for non-safety applications. The problem is further complicated by the time-varying data rate w_t and channel contention level.

Therefore, it is a challenge for the vehicle to decide when to request for data transmission.

III. PROBLEM FORMULATION

In this section, we formulate the optimal transmission problem of *a single vehicle* as a *finite-horizon sequential decision problem* [19]. The *decision epochs* of the vehicle are

$$t \in \mathbb{T} = \bigcup_{j \in \mathcal{J}} \mathcal{T}_j = \bigcup_{j \in \mathcal{J}} \{\zeta(j,1), \ldots, \zeta(j,T_j)\}, \quad (6)$$

where $\mathbb{T}$ is the set of all the time slots in the total of J coverage ranges.

The system *state* of a vehicle is defined as (s, p^{succ}), where the state element $s \in \mathcal{S} = [0, S]$ represents the remaining size (in bits) of the single file to be uploaded. If we denote the number of vehicles in the coverage range of the jth AP as $n \in \mathcal{N}_j = \{1, \ldots, N_{max,j}\}$, then $p^{succ} \in \mathcal{P}_j = \{g_j(n) : n \in \mathcal{N}_j\}$.

At any state (s, p^{succ}), the vehicle has two possible *actions*:

$$a \in \mathcal{A} = \{0, 1\}, \quad (7)$$

where action $a = 1$ implies that the vehicle decides to request to transmit, and action $a = 0$ otherwise.

The *cost* at state (s, p^{succ}) with action $a \in \mathcal{A}$ at time slot $t \in \mathcal{T}_j$ in the jth coverage range is

$$c_t(s, p^{succ}, a) = a q_j, \quad \forall t \in \mathcal{T}_j. \quad (8)$$

After the vehicle has left the Jth coverage range at time $\zeta(J, T_J + 1)$, we define a *self-incurred penalty* of the vehicle for not being able to complete the file uploading as

$$\hat{c}_{\zeta(J,T_J+1)}(s, p^{succ}) = h(s), \quad (9)$$

where $h(s) \geq 0$ is a nondecreasing function of s with $h(0) = 0$. The function depends on the QoS requirement of the application. To sum up, each vehicle is incurred with two costs: the transmission cost in each time slot in (8) and the penalty after leaving the Jth coverage range in (9).

The state transition probability $p_t\big((s', p^{succ'}) \,|\, (s, p^{succ}), a\big)$ is the probability that the system will go into state $(s', p^{succ'})$ if action a is taken at state (s, p^{succ}) at time slot $t \in \mathbb{T}$. Since the transition from p^{succ} to $p^{succ'}$ is independent of the value of s but depends on time t, we have

$$p_t\big((s', p^{succ'}) \,|\, (s, p^{succ}), a\big) = p_t\big(s' \,|\, (s, p^{succ}), a\big) \, p_t\big(p^{succ'} \,|\, p^{succ}\big). \quad (10)$$

With action $a = 1$, we have

$$p_t\big(s' \,|\, (s, p^{succ}), 1\big) = \begin{cases} p^{succ}, & \text{if } s' = [s - w_t \Delta t_{data}]^+, \\ 1 - p^{succ}, & \text{if } s' = s, \\ 0, & \text{otherwise}, \end{cases} \quad (11)$$

where $[x]^+ = \max\{0, x\}$. The first and second cases correspond to the scenarios of successful and unsuccessful packet transmissions, respectively. With action $a = 0$, we have

$$p_t\big(s' \,|\, (s, p^{succ}), 0\big) = \begin{cases} 1, & \text{if } s' = s, \\ 0, & \text{otherwise}, \end{cases} \quad (12)$$

where the remaining size of the file to upload does not change. The derivation of $p_t\big(p^{succ'} \,|\, p^{succ}\big)$ will be discussed in detail in Section IV.

Let $\delta_t : \mathcal{S} \times \mathcal{P}_j \to \mathcal{A}$ be the *decision rule* that specifies the transmission decision of the vehicle at state (s, p^{succ}) at time slot $t \in \mathcal{T}_j$ in the jth coverage range. We define a *policy* as a set of decision rules covering all the states as $\boldsymbol{\pi} = (\delta_t(s, p^{succ}), \forall s \in \mathcal{S}, p^{succ} \in \mathcal{P}_j, t \in \mathcal{T}_j, \forall j \in \mathcal{J})$. We denote $(s_t^{\boldsymbol{\pi}}, p_t^{succ, \boldsymbol{\pi}})$ as the state at time slot t if policy $\boldsymbol{\pi}$ is used, and we let Π be the feasible set of $\boldsymbol{\pi}$. The vehicle aims to find an optimal policy that minimizes the total expected cost, which can be formulated as the following optimization problem

$$\min_{\boldsymbol{\pi} \in \Pi} E_{\boldsymbol{\pi}, (S, p_1^{succ})} \Bigg[\sum_{j=1}^{J} \Bigg[\sum_{\tau=1}^{T_j} c_{\zeta(j,\tau)}\Big(s_{\zeta(j,\tau)}^{\boldsymbol{\pi}}, p_{\zeta(j,\tau)}^{succ, \boldsymbol{\pi}}, \delta_{\zeta(j,\tau)}(s_{\zeta(j,\tau)}^{\boldsymbol{\pi}}, p_{\zeta(j,\tau)}^{succ, \boldsymbol{\pi}})\Big) \Bigg] + \hat{c}_{\zeta(J,T_J+1)}\big(s_{\zeta(J,T_J+1)}^{\boldsymbol{\pi}}, p_{\zeta(J,T_J+1)}^{succ, \boldsymbol{\pi}}\big) \Bigg], \quad (13)$$

where $E_{\boldsymbol{\pi}, (S, p_1^{succ})}$ denotes the expectation with respect to the probability distribution by policy $\boldsymbol{\pi}$ with an initial state (S, p_1^{succ}) at time slot $t = \zeta(1,1) = 1$. In the following section, we will study two scenarios: single AP with random vehicular traffic and multiple APs with traffic pattern estimation.

IV. FINITE-HORIZON DYNAMIC PROGRAMMING

In this section, we describe how to obtain the optimal transmission policies in both the single-AP and multiple-AP scenarios using *finite-horizon dynamic programming*. We first study the single-AP scenario with random vehicular traffic arrival in Section IV-A. In particular, we consider a special case that the optimal policy has a *threshold structure* in Section IV-A1. When the traffic pattern can be estimated accurately, we consider a joint AP optimization in Section IV-B.

A. Single AP Optimization with Random Vehicular Traffic

Since we are considering one AP (i.e., $\mathcal{J} = \{1\}$) in this subsection, we drop the subscript j for simplicity. Although the exact traffic pattern (i.e., the exact number of vehicles in the coverage range of the AP in each time slot) is not known, the vehicles arrive according to a Poisson process with parameter λ. Meanwhile, the parameters $l_t \,(\forall t \in \mathcal{T})$, ρ_{max}, Δt, R, and the function $g(\cdot)$ are available. The transition probability of p^{succ} is given by

$$p_t\big(p^{succ'} \,|\, p^{succ}\big) = p_t\big(g(n') \,|\, g(n)\big) = p_t\big(n' \,|\, n\big)$$
$$= \begin{cases} \frac{(\lambda \Delta t)^{n'-n+l_{t+1}}}{(n'-n+l_{t+1})! \, \phi_t(n)}, & \text{if } n - l_{t+1} \leq n' \leq N_{max}, \\ 0, & \text{otherwise}, \end{cases} \quad (14)$$

where $\phi_t(n) = \sum_{y=0}^{N_{max}-n+l_{t+1}} \frac{(\lambda \Delta t)^y}{y!}$ is a normalization factor. Because $p^{succ} = g(n)$ is a strictly decreasing function of n, there is a one-to-one mapping between p^{succ} and n as shown in the first two equalities in (14). The expression after the third equalities describes the probability with $n' - n + l_{t+1}$

arrivals due to the Poisson process and l_{t+1} deterministic departures at time $t + 1$. n' is lower-bounded by $n - l_{t+1} \geq 0$ when there is no vehicle arrival, and is upper-bounded by N_{max}.

In this subsection, since we consider $\mathcal{J} = \{1\}$, we can simplify problem (13) as

$$\min_{\boldsymbol{\pi} \in \Pi} E_{\boldsymbol{\pi},(S,p_1^{succ})} \left[\sum_{t=1}^{T} c_t \left(s_t^{\boldsymbol{\pi}}, p_t^{succ,\boldsymbol{\pi}}, \delta_t(s_t^{\boldsymbol{\pi}}, p_t^{succ,\boldsymbol{\pi}}) \right) \right. \tag{15}$$
$$\left. + \hat{c}_{T+1}(s_{T+1}^{\boldsymbol{\pi}}, p_{T+1}^{succ,\boldsymbol{\pi}}) \right].$$

Let $v_t(s, p^{succ})$ be the minimal expected total cost that the vehicle has to pay from time t to time $T + 1$ when it is in the coverage range, given that the system is in state (s, p^{succ}) immediately before the decision at time slot $t \in \mathcal{T}$. The *optimality equation* [19, pp. 83] relating the minimal expected total cost at different states for $t \in \mathcal{T}$ is

$$v_t(s, p^{succ}) = \min_{a \in \mathcal{A}} \{\psi_t(s, p^{succ}, a)\}, \tag{16}$$

where

$$\psi_t(s, p^{succ}, a) = c_t(s, p^{succ}, a) +$$
$$\sum_{s' \in \mathcal{S}} \sum_{p^{succ'} \in \mathcal{P}} p_t \left((s', p^{succ'}) \mid (s, p^{succ}), a \right) v_{t+1}(s', p^{succ'}) \tag{17}$$

$$= aq + \sum_{p^{succ'} \in \mathcal{P}} p_t \left(p^{succ'} \mid p^{succ} \right) \left[a p^{succ} v_{t+1} \left([s - w_t \Delta t_{data}]^+, p^{succ'} \right) \right.$$
$$\left. + \left(1 - a p^{succ}\right) v_{t+1}(s, p^{succ'}) \right]. \tag{18}$$

The first and second terms on the right hand side of (17) are the *immediate cost* and the *expected future cost* in the remaining time slots in the coverage range for choosing action a, respectively. Equation (18) follows directly by evaluating (17) using (10) - (12). For time $t = T + 1$, we have the boundary condition that

$$v_{T+1}(s, p^{succ}) = \hat{c}_{T+1}(s, p^{succ}) = h(s). \tag{19}$$

Lemma 1: The value of $\psi_t(s, p^{succ}, a)$, $\forall t \in \mathcal{T}$, can be obtained as

$$\psi_t(s, p^{succ}, a) = aq + \sum_{m=0}^{N_{max}-n+l_{t+1}} \frac{(\lambda \Delta t)^m}{m! \phi_t(n)}$$
$$\times \left[a p^{succ} v_{t+1} \left([s - w_t \Delta t_{data}]^+, g(n + m - l_{t+1}) \right) \right. \tag{20}$$
$$\left. + \left(1 - a p^{succ}\right) v_{t+1} \left(s, g(n + m - l_{t+1}) \right) \right],$$

where $n = g^{-1}(p^{succ})$ is the number of vehicles in the coverage range of the AP.

Proof: The result follows directly by evaluating (18) using (14). ∎

Intuitively, the minimal expected cost $v_t(s, p^{succ})$ should be smaller when the remaining file size s to be uploaded is smaller. It is confirmed by the following lemma:

Lemma 2: $v_t(s, p^{succ})$ is a nondecreasing function in s, $\forall p^{succ} \in \mathcal{P}, t \in \mathcal{T}$.

The proof of Lemma 2 is given in Appendix A.

Algorithm 1 *General DORA Algorithm for single AP optimization (i.e., problem (15)).*

1: Planning Phase:
2: Input the traffic parameters: ν, λ, ρ_{max}, l_t $(\forall t \in \mathcal{T})$;
3: Input the system parameters: $h(\cdot)$, S, R, w_t $(\forall t \in \mathcal{T})$, q, Δt, Δt_{data}, σ, $g(\cdot)$;
4: Set the boundary condition $v_{T+1}(s, p^{succ})$, $\forall s \in \mathcal{S}, \forall p^{succ} \in \mathcal{P}$ using (19);
5: $t := T$;
6: **while** $t \geq 1$
7:     **for** $p^{succ} \in \mathcal{P}$
8:         $s := 0$;
9:         **while** $s \leq S$
10:             Calculate $\psi_t(s, p^{succ}, a)$, $\forall a \in \mathcal{A} = \{0, 1\}$ using (20);
11:             $\delta_t^*(s, p^{succ}) := \arg\min_{a \in \mathcal{A}} \{\psi_t(s, p^{succ}, a)\}$;
12:             $v_t(s, p^{succ}) := \psi_t \left(s, p^{succ}, \delta_t^*(s, p^{succ}) \right)$;
13:             $s := s + \sigma$;
14:         **end while**
15:     **end for**
16:     $t := t - 1$;
17: **end while**
18: Output the optimal policy $\boldsymbol{\pi}^*$ for use in the transmission phase;
19: Transmission Phase:
20: $t := 1$ and $s := S$;
21: **while** $t \leq T$
22:     Receive the information of p^{succ} from the AP;
23:     Set action $a := \delta_t^*(s, p^{succ})$ based on the policy $\boldsymbol{\pi}^*$;
24:     **If** action $a = 1$
25:         Send a request to the AP;
26:         **If** ACK is received from the AP
27:             Transmit packets with total size $w_t \Delta t_{data}$;
28:             $s := [s - w_t \Delta t_{data}]^+$;
29:         **end if**
30:     **end if**
31:     $t := t + 1$;
32: **end while**

Using the optimality equation and *backward induction* [19, pp. 92], we propose the general dynamic optimal random access (DORA) algorithm in Algorithm 1 to obtain the *optimal policy* $\boldsymbol{\pi}^* = (\delta_t^*(s, p^{succ}), \forall s \in \mathcal{S}, p^{succ} \in \mathcal{P}, t \in \mathcal{T})$, where

$$\delta_t^*(s, p^{succ}) = \arg\min_{a \in \mathcal{A}} \{\psi_t(s, p^{succ}, a)\}. \tag{21}$$

Theorem 1: The policy $\boldsymbol{\pi}^*$ obtained from Algorithm 1 is the optimal solution of problem (15).

Proof: Using the principle of optimality [20, pp. 18], we can show that $\boldsymbol{\pi}^*$ is the optimal solution of problem (15). ∎

The proposed DORA algorithm consists of two phases: Planning phase and transmission phase. The planning phase starts when the vehicle enters the coverage range. The vehicle then obtains information from the AP and computes the optimal policy $\boldsymbol{\pi}^*$ offline using dynamic programming. In fact, $\boldsymbol{\pi}^*$ is a *contingency plan* that contains information about the optimal decisions at *all* possible states (s, p^{succ}) in the coverage range. In the transmission phase, the transmission decision in each time slot is made according to the optimal policy $\boldsymbol{\pi}^*$, and s is updated depending on whether the time slot is granted to the vehicle for transmission or not. We let $\sigma > 0$ be the granularity of the discrete state element s in the algorithm.

1) Special Case: Convex Penalty Function and Fixed Data Rate: In this subsection, we further investigate a special yet

practically important case with convex penalty function and non-adaptive data rate [10]. Specifically, if the self-incurred penalty function $h(s)$ is *convex* and the data rate w_t is fixed within the coverage range (i.e., $w_t = w, \forall t \in \mathcal{T}$), we can show in Appendix B that $\psi_t(s, p^{succ}, a)$ is *subadditive* [19, pp. 103] on $\mathcal{S} \times \mathcal{A}, \forall t \in \mathcal{T}$, which is defined as follows.

Definition 1: Given p^{succ}, the function $\psi_t(s, p^{succ}, a)$ is subadditive on $\mathcal{S} \times \mathcal{A}$ if for $\hat{s}, \check{s} \in \mathcal{S}$ and $\hat{a}, \check{a} \in \mathcal{A}$, where $\hat{s} \geq \check{s}$ and $\hat{a} \geq \check{a}$, we have

$$\psi_t(\hat{s}, p^{succ}, \hat{a}) + \psi_t(\check{s}, p^{succ}, \check{a}) \leq \psi_t(\hat{s}, p^{succ}, \check{a}) + \psi_t(\check{s}, p^{succ}, \hat{a}). \tag{22}$$

Furthermore, with $\delta_t^*(s, p^{succ})$ as defined in (21), we can establish the threshold structure of the optimal policy [19, pp. 104, 115], [21].

Theorem 2: If $h(s)$ is a convex and nondecreasing function in s, and the data rate w_t is fixed such that $w_t = w, \forall t \in \mathcal{T}$, then we have a threshold optimal policy $\boldsymbol{\pi}^* = (\delta_t^*(s, p^{succ}), \forall s \in \mathcal{S}, p^{succ} \in \mathcal{P}, t \in \mathcal{T})$ in s as follows:

$$\delta_t^*(s, p^{succ}) = \begin{cases} 1, & \text{if } s > s_t^*(p^{succ}), \\ 0, & \text{otherwise,} \end{cases} \tag{23}$$

where $s_t^*(p^{succ})$ is the threshold that depends on both p^{succ} and t.

The proof of Theorem 2 is given in Appendix C. By modifying Algorithm 1, we are ready to propose the *monotone* DORA algorithm with a lower computational complexity in Algorithm 2 using monotone backward induction [19, pp. 111]. Let $\tilde{\mathcal{A}} \subseteq \mathcal{A}$ be the set of actions that we need to consider in the minimization in line 11 in Algorithm 2. When $\delta_t^*(s, p^{succ}) = 1$ and $flag = 0$ are satisfied (line 13), which means that the threshold $s_t^*(p^{succ})$ is reached, set $\tilde{\mathcal{A}}$ is reduced from $\{0, 1\}$ to $\{1\}$ and the threshold $s_t^*(p^{succ})$ is recorded (line 14). Then the minimization in line 11 is readily known, since set $\tilde{\mathcal{A}} = \{1\}$ is a singleton. The computational complexity is thus reduced. Moreover, memory can be saved, because we do not need to store the complete optimal policy $\boldsymbol{\pi}^* = (\delta_t^*(s, p^{succ}), \forall s \in \mathcal{S}, p^{succ} \in \mathcal{P}, t \in \mathcal{T})$. We just need to store the thresholds $(s_t^*(p^{succ}), \forall p^{succ} \in \mathcal{P}, t \in \mathcal{T})$, which completely characterize the optimal policy $\boldsymbol{\pi}^*$ as shown in (23).

B. Joint AP Optimization with Deterministic Vehicular Traffic

In the previous subsection, we consider the optimization problem in a single AP. In this subsection, we extend the result to the case of *multiple* APs, where we assume that the traffic pattern (i.e., the exact number of vehicles in the coverage ranges of the APs in each time slot) can be estimated accurately. The traffic pattern can be estimated in various ways, such as by installing a traffic monitor at a place before the first AP to observe the actual traffic pattern when the vehicles pass by (e.g., using computer vision [22] and pattern recognition [23]). If the traffic flow reaches the steady state (as discussed in Section II-A), the estimation of the number of vehicles n_t at time $t \in \mathbb{T}$ can be reasonably accurate. As a result, the values of $p_t^{succ} = g_j(n_t), \forall t \in \mathbb{T}$ can be obtained accurately. As an example, we consider that the traffic model is as described in Section II-A, and all the APs have the same transmission radii. After the traffic monitor has estimated the values of $p_\tau^{succ}, \forall \tau \in \mathcal{T}_1$ for the first coverage range, it

Algorithm 2 *Monotone DORA Algorithm for single AP optimization (i.e., problem (15)) for the special case with convex penalty function $h(s)$ and fixed data rate w_t.*

1: Planning Phase:
2: Input the traffic parameters: $\nu, \lambda, \rho_{max}, l_t (\forall t \in \mathcal{T})$;
3: Input the system parameters: $h(\cdot), S, R, w_t (\forall t \in \mathcal{T}), q, \Delta t,$ $\Delta t_{data}, \sigma, g(\cdot)$;
4: Set the boundary condition $v_{T+1}(s, p^{succ}), \forall s \in \mathcal{S}, \forall p^{succ} \in \mathcal{P}$ using (19);
5: $t := T$;
6: **while** $t \geq 1$
7:     **for** $p^{succ} \in \mathcal{P}$
8:         Set $s := 0, flag := 0$, and $\tilde{\mathcal{A}} := \{0, 1\}$;
9:         **while** $s \leq S$
10:           Calculate $\psi_t(s, p^{succ}, a), \forall a \in \tilde{\mathcal{A}}$ using (20);
11:           $\delta_t^*(s, p^{succ}) := \arg\min_{a \in \tilde{\mathcal{A}}}\{\psi_t(s, p^{succ}, a)\}$;
12:           $v_t(s, p^{succ}) := \psi_t(s, p^{succ}, \delta_t^*(s, p^{succ}))$;
13:           **if** $\delta_t^*(s, p^{succ}) = 1$ and $flag = 0$
14:             Set $\tilde{\mathcal{A}} := \{1\}, s_t^*(p^{succ}) = s$, and $flag = 1$;
15:           **end if**
16:           $s := s + \sigma$;
17:         **end while**
18:     **end for**
19:     $t := t - 1$;
20: **end while**
21: Output the thresholds $(s_t^*(p^{succ}), \forall p^{succ} \in \mathcal{P}, t \in \mathcal{T})$ for use in the transmission phase;
22: Transmission Phase:
23: $t := 1$ and $s := S$;
24: **while** $t \leq T$
25:     Receive the information of p^{succ} from the AP;
26:     **If** $s > s_t^*(p^{succ})$
27:         Send a request to the AP;
28:         **If** ACK is received from the AP
29:           Transmit packets with total size $w_t \Delta t_{data}$;
30:           $s := [s - w_t \Delta t_{data}]^+$;
31:         **end if**
32:     **end if**
33:     $t := t + 1$;
34: **end while**

---

can set $p_{\zeta(j, \tau)}^{succ} := p_\tau^{succ}$ for the remaining coverage ranges $j \in \mathcal{J} \backslash \{1\}$.

The optimality equations relating the minimal expected total cost at different time $t \in \mathbb{T}$ for problem (13) are similar to that described in Section IV-A, but are simplified because we assume that $p_t^{succ}, \forall t \in \mathbb{T}$ are known. At time $t \in \mathcal{T}_j$, we have

$$v_t(s, p_t^{succ}) = \min_{a \in \mathcal{A}}\{\psi_t(s, p_t^{succ}, a)\}, \tag{24}$$

where

$$\psi_t(s, p_t^{succ}, a)$$
$$= c_t(s, p_t^{succ}, a) + \sum_{s' \in \mathcal{S}} p_t\big((s', p_{t+1}^{succ}) | (s, p_t^{succ}), a\big) v_{t+1}(s', p_{t+1}^{succ})$$
$$= aq_j + ap_t^{succ} v_{t+1}\big([s - w_t \Delta t_{data}]^+, p_{t+1}^{succ}\big)$$
$$+ \big(1 - ap_t^{succ}\big) v_{t+1}(s, p_{t+1}^{succ}). \tag{25}$$

The second line in (25) is obtained by using (11) and (12). After the vehicle has left the Jth coverage range at $t = \zeta(J, T_J + 1)$, the boundary condition is

$$v_{\zeta(J, T_J+1)}(s, p_{\zeta(J, T_J+1)}^{succ}) = \hat{c}_{\zeta(J, T_J+1)}(s, p_{\zeta(J, T_J+1)}^{succ}) = h(s). \tag{26}$$

Algorithm 3 *JDORA Algorithm for joint AP optimization (i.e., problem (13)).*

1: Planning Phase:
2: Input the traffic parameters: ν, p_t^{succ} ($\forall t \in \mathbb{T}$);
3: Input the system parameters: $h(\cdot)$, S, R_j ($\forall j \in \mathcal{J}$), w_t ($\forall t \in \mathbb{T}$), $q_j (\forall j \in \mathcal{J})$, Δt, Δt_{data}, σ;
4: Set the boundary condition $v_{\zeta(J,T_J+1)}(s, p_{\zeta(J,T_J+1)}^{succ})$, $\forall s \in \mathcal{S}$ using (26);
5: $t := \zeta(J, T_J)$;
6: **while** $t \geq 1$
7:     $s := 0$;
8:     **while** $s \leq S$
9:       Calculate $\psi_t(s, p_t^{succ}, a)$, $\forall a \in \mathcal{A} = \{0, 1\}$ using (25);
10:       $\delta_t^*(s, p_t^{succ}) := \arg\min_{a \in \mathcal{A}}\{\psi_t(s, p_t^{succ}, a)\}$;
11:       $v_t(s, p_t^{succ}) := \psi_t(s, p_t^{succ}, \delta_t^*(s, p_t^{succ}))$;
12:       $s := s + \sigma$;
13:     **end while**
14:     $t := t - 1$;
15: **end while**
16: Output the optimal policy $\boldsymbol{\pi}^*$ for use in the transmission phase;
17: Transmission Phase:
18: $t := 1$ and $s := S$;
19: **while** $t \leq \zeta(J, T_J)$
20:     Set action $a := \delta_t^*(s, p_t^{succ})$ based on the policy $\boldsymbol{\pi}^*$;
21:     **If** action $a = 1$
22:       Send a request to the AP;
23:       **If** ACK is received from the AP
24:         Transmit packets with total size $w_t \Delta t_{data}$;
25:         $s := [s - w_t \Delta t_{data}]^+$;
26:       **end if**
27:     **end if**
28:     $t := t + 1$;
29: **end while**

The JDORA algorithm for joint AP optimization is given in Algorithm 3. In Algorithm 3, the vehicle first needs to obtain the values of p_t^{succ}, $\forall t \in \mathbb{T}$, from the traffic monitor. In the planning phase, for each $s \in \mathcal{S}$ and $t \in \mathbb{T}$, the optimal decision rule $\delta_t^*(s, p_t^{succ})$ is the action that minimizes the expected total cost (line 10), where the expected total cost $\psi_t(s, p_t^{succ}, a)$ for all possible actions is calculated (line 9) based on v_{t+1} obtained (line 11) in the previous iteration $t + 1$. After the process is repeated for all $t \in \mathbb{T}$ (line 6) and $s \in \mathcal{S}$ (line 8), we obtain the optimal policy $\boldsymbol{\pi}^*$. In the transmission phase, the transmission decision in each time slot is made according to the optimal policy $\boldsymbol{\pi}^*$, and it follows the MAC protocol described in Section II-C.

Theorem 3: The policy $\boldsymbol{\pi}^* = (\delta_t^*(s, p_t^{succ}), \forall s \in \mathcal{S}, t \in \mathbb{T})$ obtained from Algorithm 3 is the optimal solution of problem (13) when p_t^{succ}, $\forall t \in \mathbb{T}$ are accurately known.

Proof: The result follows directly from the principle of optimality [20, pp. 18]. ∎

V. PERFORMANCE EVALUATIONS

In this section, we first compare Algorithms 1 and 3 with three heuristic schemes using the traffic model described in Section II-A in both the single-AP and multiple-AP scenarios. In particular, we study the performance of Algorithm 3 under imperfect estimations of the p_t^{succ} in the multiple-AP scenario. We then study the threshold policies obtained by Algorithm 2.

TABLE I
SIMULATION PARAMETERS

Parameters	Values
Number of APs J	1, 5
AP's transmission radius R	100 m
Free-flow speed ν_f	110 km/hr
Vehicle jam density ρ_{max}	100 veh/km
Duration of a time slot Δt	0.02 sec
Duration for data transmission in a time slot Δt_{data}	0.018 sec
Channel bandwidth W	20 MHz
Transmit signal-to-noise ratio $\frac{P}{N_0 W}$	60 dB
Path loss exponent γ	3
Payment per time slot q	1
Contention window $cw \in [cw_{min}, cw_{max}]$	[1, 8]
MCBC parameter K (used in [11])	3
MCBC parameter $[p_1 p_2, p_3]$ (used in [11])	[0.12, 0.77, 0.86]
MCBC parameter F (used in [11])	15

The three heuristic schemes that we consider are as follows. The first heuristic scheme is a *greedy* algorithm, in which each vehicle sends transmission requests at all the time slots if its file upload is not complete. That is, the greedy algorithm aims to maximize the total uploaded file size. The second heuristic scheme is the *exponential backoff* algorithm that is similar to the one used in the IEEE 802.11. We have slightly modified it for the system that we consider as follows. Each vehicle has a counter, which randomly and uniformly chooses an initial integer value cnt from the interval $[0, cw)$, where cw is the contention window size. The value of cnt is decreased by one after each time slot. When $cnt = 0$, the vehicle will send a request. If the vehicle has sent a request in a time slot, the size of $cw \in [cw_{min}, cw_{max}]$ will change according to the response from the AP: If an ACK is received from the AP, cw is set to cw_{min}. Otherwise, cw is doubled until it reaches cw_{max}. For the DORA, JDORA, greedy, and exponential schemes, we assume that the APs allow the vehicles to share the channel with an equal probability. Therefore, $p_t^{succ} = 1/n_t$. The third heuristic scheme is the MAC protocol in the multi-carrier burst contention (MCBC) scheme [11]. Similar to the greedy scheme, a vehicle will send a request if it has data to send in each time slot. However, the vehicles need to undergo K rounds of contention in each time slot. First, in round r, a vehicle survives the contention with probability p_r. Each of these vehicles will choose a random integer in $\{1, \ldots, F\}$. Vehicles that have chosen the largest number can proceed to round $r + 1$. The transmission is successful if there is only one vehicle left in round K. Otherwise, packet collision will occur.

For the evaluations of all the schemes, we use the convex self-incurred penalty function

$$h(s) = bs^2, \quad \forall s \in \mathcal{S}, \qquad (27)$$

where $b \geq 0$ is a constant. The three heuristic schemes are evaluated using a similar transmission phase as in Algorithms 1 and 3, but with $\boldsymbol{\pi}^*$ in Algorithms 1 and 3 replaced by the corresponding policies. The simulation parameters are listed in Table I.

We first study the impact of penalty parameter b on the total uploaded file size for $S = 100$ Mbits and $\rho = 20$ veh/km in one AP. As shown in Fig. 4, by increasing b, a larger penalty
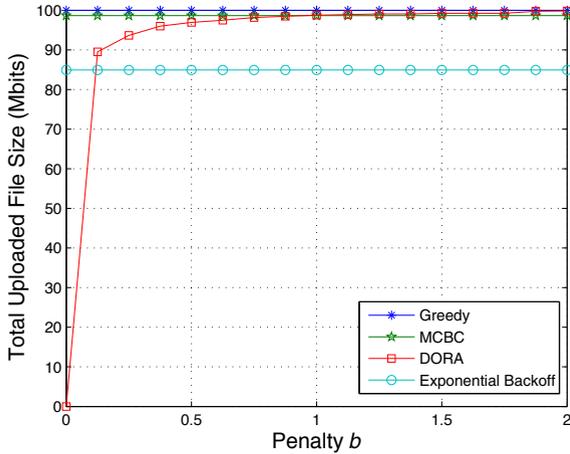
Fig. 4. Total uploaded file size against the penalty parameter b for $S = 100$ Mbits and $\rho = 20$ veh/km with a single AP. As b increases, a larger file size is uploaded for the DORA scheme.
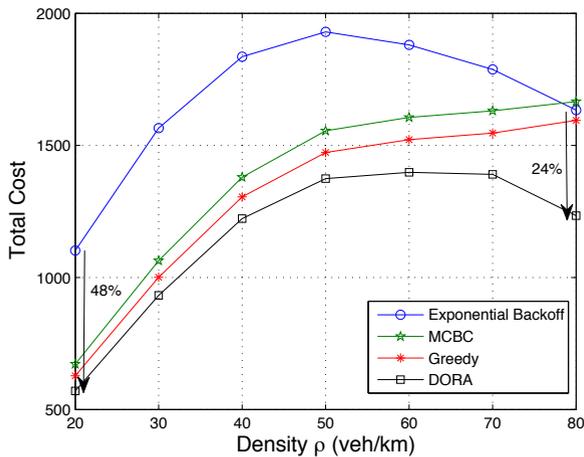

Fig. 6. Upload ratio (i.e., total uploaded file size / total payment to the APs) versus traffic density ρ for file size $S = 200$ Mbits with a single AP. The DORA scheme achieves the highest upload ratio.

Fig. 5. Total cost versus traffic density ρ for file size $S = 200$ Mbits with a single AP. The DORA scheme has the minimal total cost.

Fig. 7. Total cost versus traffic density ρ for file size $S = 500$ Mbits with five APs. The JDORA scheme with perfect estimation of p_t^{succ} has the minimal total cost. Moreover, a higher total cost is required when the precision of the estimation reduces (i.e., when the variance of the estimation θ increases).

is incurred on the size of the file not yet uploaded by using Algorithm 1. As a result, a larger file size is uploaded to reduce the penalty. Depending on the QoS requirements of different applications, different values of b should be chosen that tradeoff the total uploaded file size and total payment to the AP by a different degree. Taking safety application as an example, it may be more important to maximize the uploaded file size than to reduce the total payment to the APs, so a large value of b should be be chosen. Also, since the transmission policies of the greedy, MCBC, and exponential backoff schemes do not consider the self-incurred penalty in (27), their total uploaded file size are independent of b.

Next, we plot the total cost against the traffic density ρ for $S = 200$ Mbits with $b = 0.1$ for the case of one AP in Fig. 5. It is clear that the DORA scheme in Algorithm 1 achieves the minimal total cost as stated in Theorem 1, with 48% and 24% cost reduction as compared with the exponential backoff scheme at low and high ρ, respectively. To measure the cost effectiveness of the file uploading for the four schemes, we propose a metric called the *upload ratio*, which is defined
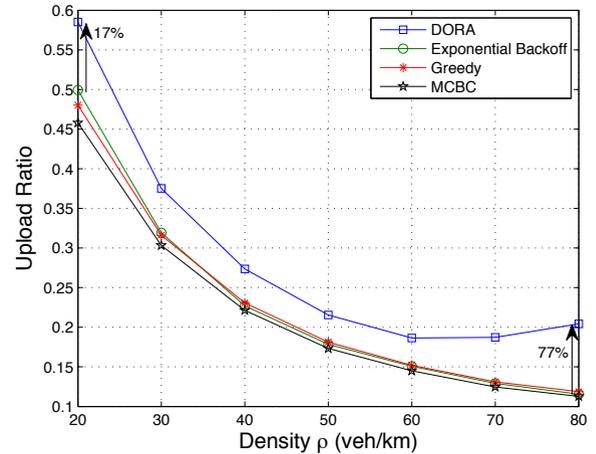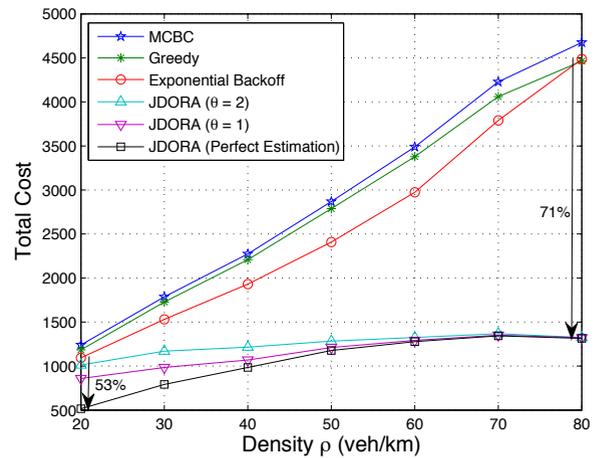
as the total uploaded file size divided by the total payment to the APs. In other words, it represents the size of the file uploaded per unit payment. As shown in Fig. 6, since the DORA algorithm takes into account the varying channel contention level and data rate in determining the transmission policy, it is cost effective and achieves the highest upload ratio. In particular, the performance gains in upload ratio over the exponential backoff scheme are 17% and 77% at low and high ρ, respectively.

Furthermore, we consider the case with five APs, where we assume that all of them have the same transmission radii R and price q. For the JDORA scheme in Algorithm 3, we consider that the estimated number of vehicles $\tilde{n}_t$ at time $t \in \mathbb{T}$ is obtained by rounding off a normally distributed random variable with a mean n_t and a variance θ to the nearest nonnegative integer. Thus, the lower the variance θ, the higher is the precision of the estimation. The value of p_t^{succ} is obtained by setting $p_t^{succ} = g_j(\tilde{n}_t)$, $\forall t \in \mathcal{T}_j, j \in \mathcal{J}$. We plot the total cost and upload ratio in Figs. 7 and 8 for $S = 500$ Mbits with
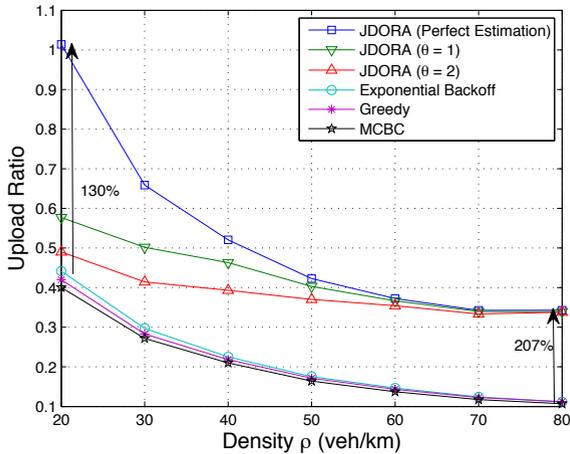
Fig. 8. Upload ratio versus traffic density ρ for file size $S = 500$ Mbits with five APs. The JDORA scheme with perfect estimation of p_t^{succ} achieves the highest upload ratio as compared with three other heuristic schemes. Moreover, a lower upload ratio is achieved when the precision of the estimation reduces (i.e., when the variance of the estimation θ increases).

$b = 0.01$, respectively. In Fig. 7, we can see that the JDORA scheme with perfect estimation (i.e., $\theta = 0$) of p_t^{succ}, $\forall t \in \mathbb{T}$ achieves the minimal total cost as stated in Theorem 3, where it achieves 53% and 71% cost reduction as compared with the exponential backoff scheme at low and high traffic density ρ, respectively. In Fig. 8, we can see that the JDORA scheme with perfect estimation achieves the highest upload ratio. In particular, it achieves an upload ratio 130% and 207% better than the exponential backoff scheme at low and high traffic density ρ, respectively. As shown in Figs. 7 and 8, the total cost is increased and the upload ratio is reduced, respectively, when the estimation precision decreases. However, this result based on equal share of bandwidth that $p_t^{succ} = 1/\tilde{n}_t$, $\forall t \in \mathbb{T}$ is less sensitive to the estimation error when the traffic density ρ is high. It suggests that the JDORA algorithm is suitable especially for VANETs with high traffic densities.

Finally, we study the threshold policy in a single AP obtained by Algorithm 2 when the penalty function $h(s)$ is convex and data rate w_t is fixed. We consider that $S = 100$ Mbits, $\nu = 100$ km/hr, $w_t = 54$ Mbps, $\forall t \in \mathcal{T}$, and $h(s)$ is defined as in (27). From Theorem 2, we know that the optimal policy has a threshold structure. In Fig. 9, we plot the thresholds $s_t^*(p^{succ})$ of the optimal policy against the decision epoch t for different values of p^{succ}. With the use of the convex penalty function, we can see that the threshold increases with t. In Fig. 9(a), for $b = 0.1$, we can observe that the threshold increases when p^{succ} decreases. It is because a small penalty parameter is chosen, which places a higher priority on the total payment than on the uploaded file size. When p^{succ} is small, the chance of successful transmission is low, so the vehicle chooses a higher threshold and transmits less aggressively to reduce the amount of payment. In Fig. 9(b), we choose a larger penalty parameter $b = 10$ such that a higher priority is placed on the uploaded file size than on the total payment. We observe that the threshold decreases when p^{succ} decreases. It is because when p^{succ} is small, the vehicle needs to transmit more aggressively (i.e., with a lower
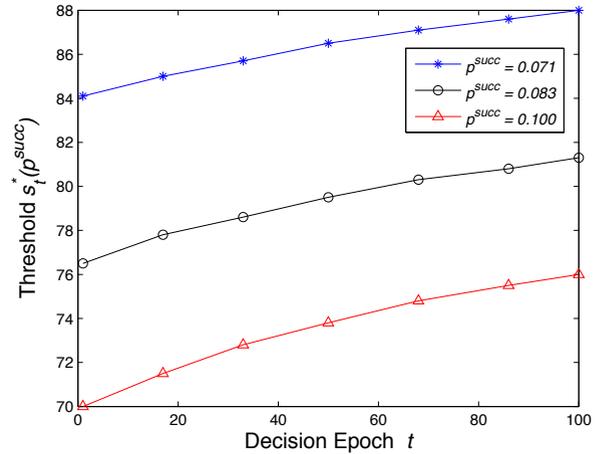
(a) $b = 0.1$
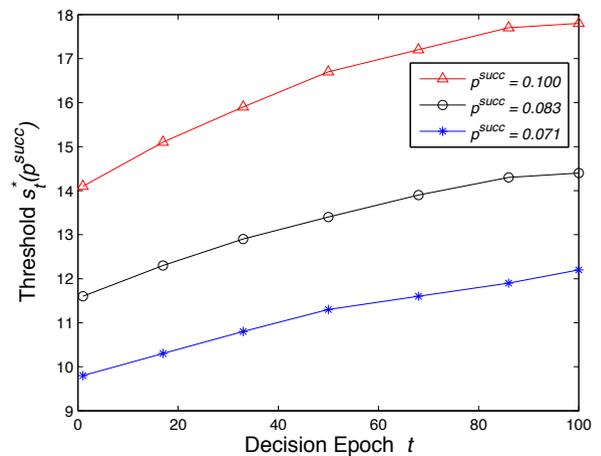
(b) $b = 10$

Fig. 9. The thresholds $s_t^*(p^{succ})$ of the optimal policy against the decision epoch t for different penalty parameters b.

threshold) to prevent a large penalty. Moreover, we can see that the thresholds presented in Fig. 9(b) is lower than that in Fig. 9(a) due to the higher incentive to transmit when the penalty is large.

VI. CONCLUSION

In this paper, we studied the V2R uplink transmission from a vehicle to the APs in a dynamic drive-thru scenario, where both the channel contention level and data rate vary over time. Depending on the applications' QoS requirements, the vehicle can achieve different levels of tradeoff between the total uploaded file size and the total payment to the APs by tuning the self-incurred penalty. For a single AP with random vehicular traffic, we proposed a DORA algorithm based on DP to obtain the optimal transmission policy for the vehicle in a coverage range. We prove that if the self-incurred penalty function $h(s)$ is convex and the data rate w_t is non-adaptive and fixed, then the optimal transmission policy has a threshold structure. A monotone DORA algorithm with a lower computational complexity was proposed for this special case. Next, for multiple APs with known vehicular patterns, we considered the transmission policy in multiple

coverage ranges jointly and proposed an optimal JDORA algorithm. Simulation results showed that our schemes achieve the minimal total cost and the highest upload ratio as compared with three other heuristic schemes. An interesting topic for future work is to consider joint AP optimization without traffic pattern estimation.

APPENDIX

A. Proof of Lemma 2

We prove it by induction. From (19), since $v_{T+1}(s, p^{succ}) = h(s)$, $v_{T+1}(s, p^{succ})$ is a nondecreasing function in s, $\forall p^{succ} \in \mathcal{P}$. Assume that $v_{t+1}(s, p^{succ})$ is a nondecreasing function in s, $\forall p^{succ} \in \mathcal{P}$. Since $p_t(p^{succ'} \mid p^{succ}) \geq 0$ and $0 \leq ap^{succ} \leq 1$, it can be inferred from (18) that $\psi_t(s, p^{succ}, a)$ is a nondecreasing function in s, $\forall p^{succ} \in \mathcal{P}$. Thus, $v_t(s, p^{succ})$ in (16) is a nondecreasing function in s, $\forall p^{succ} \in \mathcal{P}$. ∎

B. Subadditivity of $\psi_t(s, p^{succ}, a)$

Because $w_t = w, \forall t \in \mathcal{T}$, we let $\omega = w\Delta t_{data}$. Since $\delta_t^*(s, p^{succ})$ is defined as in (21), we can establish the threshold policy if we can prove that $\psi_t(s, p^{succ}, a)$ is subadditive on $\mathcal{S} \times \mathcal{A}$, $\forall t \in \mathcal{T}$ [19, pp. 104, 115]. The following results from Lemma 3 and 4 establish the subadditivity of $\psi_t(s, p^{succ}, a)$. First, Lemma 3 shows that $v_t(s, p^{succ})$ has a nondecreasing difference in s if $h(s)$ is a convex and nondecreasing function.

Lemma 3: If $h(s)$ is a convex and nondecreasing function in s, then

$$v_t(s, p^{succ}) - v_t([s-\omega]^+, p^{succ}) \geq v_t([s-\sigma]^+, p^{succ})$$
$$-v_t([s-\sigma-\omega]^+, p^{succ}), \forall s \in \mathcal{S}, p^{succ} \in \mathcal{P}, t \in \mathcal{T} \cup \{T+1\}. \quad (28)$$

Proof: We prove it by induction. Since $h(s)$ is a nondecreasing convex function, we have

$$h(s) - h([s-\omega]^+) \geq h([s-\sigma]^+) - h([s-\sigma-\omega]^+), \quad \forall s \in \mathcal{S}. \quad (29)$$

Let $s \in \mathcal{S}$, $p^{succ} \in \mathcal{P}$ be given. For $t = T+1$, we have

$$v_{T+1}(s, p^{succ}) - v_{T+1}([s-\omega]^+, p^{succ}) = h(s) - h([s-\omega]^+)$$
$$\geq h([s-\sigma]^+) - h([s-\sigma-\omega]^+)$$
$$= v_{T+1}([s-\sigma]^+, p^{succ}) - v_{T+1}([s-\sigma-\omega]^+, p^{succ}), \quad (30)$$

where the equalities are due to (19) and the inequality is due to (29).

Assume that for a given $t \in \mathcal{T}$, we have

$$v_{t+1}(s, p^{succ}) - v_{t+1}([s-\omega]^+, p^{succ}) \geq v_{t+1}([s-\sigma]^+, p^{succ})$$
$$-v_{t+1}([s-\sigma-\omega]^+, p^{succ}), \forall s \in \mathcal{S}, p^{succ} \in \mathcal{P}. \quad (31)$$

From (16), let actions a_1, a_2, a_3, and a_4 be defined such that

$$v_t(s, p^{succ}) = \min_{a \in \mathcal{A}} \{\psi_t(s, p^{succ}, a)\} = \psi_t(s, p^{succ}, a_1), \quad (32)$$

$$v_t([s-\omega]^+, p^{succ}) = \min_{a \in \mathcal{A}} \{\psi_t([s-\omega]^+, p^{succ}, a)\}$$
$$= \psi_t([s-\omega]^+, p^{succ}, a_2), \quad (33)$$

$$v_t([s-\sigma]^+, p^{succ}) = \min_{a \in \mathcal{A}} \{\psi_t([s-\sigma]^+, p^{succ}, a)\}$$
$$= \psi_t([s-\sigma]^+, p^{succ}, a_3), \quad \text{and} \quad (34)$$

$$v_t([s-\sigma-\omega]^+, p^{succ}) = \min_{a \in \mathcal{A}} \{\psi_t([s-\sigma-\omega]^+, p^{succ}, a)\}$$
$$= \psi_t([s-\sigma-\omega]^+, p^{succ}, a_4). \quad (35)$$

We thus have

$$v_t(s, p^{succ}) - v_t([s-\omega]^+, p^{succ})$$
$$- v_t([s-\sigma]^+, p^{succ}) + v_t([s-\sigma-\omega]^+, p^{succ})$$
$$= \psi_t(s, p^{succ}, a_1) - \psi_t([s-\omega]^+, p^{succ}, a_2)$$
$$- \psi_t([s-\sigma]^+, p^{succ}, a_3) + \psi_t([s-\sigma-\omega]^+, p^{succ}, a_4)$$
$$= \overbrace{\psi_t(s, p^{succ}, a_1) - \psi_t([s-\sigma]^+, p^{succ}, a_1)}^{A}$$
$$+ \overbrace{\psi_t([s-\sigma]^+, p^{succ}, a_1) - \psi_t([s-\sigma]^+, p^{succ}, a_3)}^{B}$$
$$\overbrace{- \psi_t([s-\omega]^+, p^{succ}, a_2) + \psi_t([s-\omega]^+, p^{succ}, a_4)}^{C}$$
$$- \left(\overbrace{\psi_t([s-\omega]^+, p^{succ}, a_4) - \psi_t([s-\sigma-\omega]^+, p^{succ}, a_4)}^{D}\right)$$
$$= A + B + C - D. \quad (36)$$

We have

$$A = \sum_{p^{succ'} \in \mathcal{P}} p_t(p^{succ'} \mid p^{succ}) \Big[a_1 p^{succ} [v_{t+1}([s-\omega]^+, p^{succ'})$$
$$- v_{t+1}([s-\sigma-\omega]^+, p^{succ'})] + (1 - a_1 p^{succ})$$
$$\times [v_{t+1}(s, p^{succ'}) - v_{t+1}([s-\sigma]^+, p^{succ'})]\Big]$$
$$\geq \sum_{p^{succ'} \in \mathcal{P}} p_t(p^{succ'} \mid p^{succ}) [v_{t+1}([s-\omega]^+, p^{succ'})$$
$$- v_{t+1}([s-\sigma-\omega]^+, p^{succ'})]$$
$$\geq \sum_{p^{succ'} \in \mathcal{P}} p_t(p^{succ'} \mid p^{succ}) \Big[a_4 p^{succ} [v_{t+1}([s-2\omega]^+, p^{succ'})$$
$$- v_{t+1}([s-\sigma-2\omega]^+, p^{succ'})] + (1 - a_4 p^{succ})$$
$$\times [v_{t+1}([s-\omega]^+, p^{succ'}) - v_{t+1}([s-\sigma-\omega]^+, p^{succ'})]\Big]$$
$$= D, \quad (37)$$

where the two equalities are obtained by using (18) and the two inequalities are due to the induction hypothesis in (31). From (34) and (33), we have $B \geq 0$ and $C \geq 0$, respectively. Overall, from (36), we obtain

$$v_t(s, p^{succ}) - v_t([s-\omega]^+, p^{succ})$$
$$-v_t([s-\sigma]^+, p^{succ}) + v_t([s-\sigma-\omega]^+, p^{succ}) \geq 0, \quad (38)$$

which completes the proof. ∎

Lemma 4 shows that $\psi_t(s, p^{succ}, a)$ is subadditive if $v_t(s, p^{succ})$ has a nondecreasing difference in s.

Lemma 4: If $\forall \hat{s}, \check{s} \in \mathcal{S}$, $p^{succ} \in \mathcal{P}$, $t \in \mathcal{T}$ with $\hat{s} \geq \check{s}$, where

$$v_{t+1}(\hat{s}, p^{succ}) - v_{t+1}([\hat{s}-\omega]^+, p^{succ})$$
$$\geq v_{t+1}(\check{s}, p^{succ}) - v_{t+1}([\check{s}-\omega]^+, p^{succ}), \quad (39)$$

then $\psi_t(s, p^{succ}, a)$ is subadditive on $\mathcal{S} \times \mathcal{A}$, $\forall\, t \in \mathcal{T}$.

Proof: Let $\hat{s}, \check{s} \in \mathcal{S}$, $\hat{a}, \check{a} \in \mathcal{A}$, $p^{succ} \in \mathcal{P}$, and $t \in \mathcal{T}$ be given, where $\hat{s} \geq \check{s}$ and $\hat{a} \geq \check{a}$. Then

$$-\psi_t(\hat{s}, p^{succ}, \hat{a}) + \psi_t(\check{s}, p^{succ}, \check{a}) - \psi_t(\hat{s}, p^{succ}, \check{a}) - \psi_t(\check{s}, p^{succ}, \hat{a})$$

$$= -\sum_{p^{succ\prime} \in \mathcal{P}} p_t\big(p^{succ\prime} \,|\, p^{succ}\big) p^{succ}(\hat{a} - \check{a})\Big[v_{t+1}(\hat{s}, p^{succ\prime}) -$$

$$v_{t+1}\big([\hat{s} - \omega]^+, p^{succ\prime}\big) - v_{t+1}(\check{s}, p^{succ\prime}) + v_{t+1}\big([\check{s} - \omega]^+, p^{succ\prime}\big)\Big]$$

$$\leq 0, \tag{40}$$

where the equality is obtained by using (18). The inequality at the end is due to the fact that $p_t\big(p^{succ\prime} \,|\, p^{succ}\big) \geq 0$, $p^{succ} \geq 0$, $\hat{a} \geq \check{a}$, and the given condition in Lemma 4. From Definition 1 and (22), the result follows. $\blacksquare$

C. Proof of Theorem 2

Let $\hat{s}, \check{s} \in \mathcal{S}$, $\omega \geq 0$, $p^{succ} \in \mathcal{P}$, and $t \in \mathcal{T}$ be given. Moreover, let $\check{s} = [\hat{s} - k\sigma]^+$, where $k > 0$. If the condition of the theorem is satisfied, by repetitively applying Lemma 3, we have

$$v_t(\hat{s}, p^{succ}) - v_t\big([\hat{s} - \omega]^+, p^{succ}\big)$$
$$\geq v_t\big([\hat{s} - \sigma]^+, p^{succ}\big) - v_t\big([\hat{s} - \sigma - \omega]^+, p^{succ}\big) \geq \cdots$$
$$\geq v_t\big([\hat{s} - k\sigma]^+, p^{succ}\big) - v_t\big([\hat{s} - k\sigma - \omega]^+, p^{succ}\big)$$
$$= v_t\big(\check{s}, p^{succ}\big) - v_t\big([\check{s} - \omega]^+, p^{succ}\big). \tag{41}$$

From Lemma 4, $\psi_t(s, p^{succ}, a)$ is subadditive on $\mathcal{S} \times \mathcal{A}$, $\forall\, t \in \mathcal{T}$. From [19, pp. 104, 115], $\delta_t^*(s, p^{succ})$ is a monotone nondecreasing function in s. Since $\delta_t^*(s, p^{succ}) \in \mathcal{A} = \{0, 1\}$, $\delta_t^*(s, p^{succ})$ is in the form of (23). $\blacksquare$

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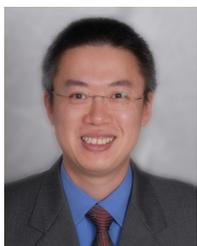
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